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# Hybrid IIR/FIR Wavelet Filter Banks for ECG Signal Denoising

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***Abstract*—ElectroCardioGram (ECG) signals are usually corrupted with various types of noise/artifacts such as baseline wander and muscle contraction artifacts which degrade the signal quality and might lead to misdiagnosis of the patient. The wavelet denoising technique is widely studied in the artifact removal literature which employs conventional Finite Impulse Response (FIR) wavelet filter banks for decomposing, thresholding and reconstructing the noisy signal to obtain high fidelity and clean ECG signal. However, the use of high order FIR wavelet filters increases the hardware complexity and cost of the system. This paper presents novel hybrid Infinite Impulse Response (IIR)/FIR Discrete Wavelet Transform (DWT) filter banks that can be employed in ambulatory health monitoring applications for denoising purposes. The proposed systems are evaluated and compared to the conventional FIR based DWT systems in terms of the computational complexity as well as the denoising performance. For this purpose, raw ECG data from MIT-BIH arrhythmia database are contaminated with synthetic noise and denoised with the aforementioned filter banks. The results from 100 Monte Carlo simulations demonstrated that the proposed filter banks provide better denoising performance with fewer arithmetic operations than those reported in the open literature.**

***Keywords*—ECG denoising, Discrete Wavelet Transform, FIR wavelets, IIR wavelets, Hybrid Filter Bank**

## I. Introduction

Biomedical signals are usually contaminated by various noise sources where the noise and signal spectra overlap and the conventional filtering techniques are insufficient to suppress the noise. DWT is a popular tool in the field of non-stationary signal processing that provides simultaneous time and frequency information, and has been used to detect such overlapping noise. In ECG denoising literature a vast amount of research employed FIR filter banks with various wavelet families, most popular ones being the Daubechies (e.g. $db4$), Symmlets (e.g. $sym4$) and Coiflets (e.g. $coif4$) [1]–[3]. On the other hand, the use of IIR wavelet filter banks are less extensive and limited to image processing and compressing applications [4], since the synthesis filter bank requires anti-causal filters for cancelling out the phase distortion caused by the phase non-linearity. The casual implementation of an anti-causal filter is only possible if the data fed to the system is finite and can be time-reversed for signal reconstruction which is a restriction for continuous infinite data. This paper presents the design of novel hybrid IIR/FIR DWT filter banks and their novel application in ECG signal denoising. The proposed hybrid systems are composed of IIR and FIR wavelet filters in the analysis and synthesis filter banks, respectively. The FIR wavelet filters are derived from the analysis filters in order to ensure the near Perfect Reconstruction (PR) of the input data. To the best knowledge of the authors, this is a first in the wavelet literature for ECG denoising and the results demonstrated that the proposed hybrid DWT filter banks achieve better denoising performance with reduced hardware complexity compared to the conventional FIR wavelets. The rest of this document provides details of the IIR wavelet analysis and FIR wavelet synthesis filter bank designs. Section IV presents and compares the computational complexity of the proposed systems in terms of arithmetic operations. Furthermore, Section V introduces the wavelet thresholding technique employed and the noisy test data generated. Comparative analysis on the noise suppression performance of the proposed hybrid and FIR wavelet filter banks for different noise scenarios are presented in Section VI. Finally, Section VII presents the conclusions.

## II. IIR Wavelet Analysis Filter Bank Properties

The analysis part of a two channel PR IIR filter bank can be realized with a halfband lowpass ($H_0(z)$) and a halfband high-pass ($H_1(z)$) filter, which are based on the parallel connection of two real all-pass filters and are power complementary to each other, since they satisfy, $|H_0(z)|^2+|H_1(z)|^2=1$. Thus, a 1-level transform matrix for the analysis filter bank is [5],

$$\boldsymbol{H}(\boldsymbol{z})=\begin{bmatrix}H_0(z)\\H_1(z)\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}A_0(z^2)+z^{-1}A_1(z^2)\\A_0(z^2)-z^{-1}A_1(z^2)\end{bmatrix}, \quad (1)$$

where $A_0(z)$ and $A_1(z)$ are $M^{th}$ order allpass filters with a general transfer function,

$$A(z)=z^{-M}\frac{\sum_{m=0}^{M}\alpha_m z^m}{\sum_{m=0}^{M}\alpha_m z^{-m}} \quad (2)$$

It is well known that, the regularity of wavelets defines the smoothness of the wavelet function and has a crucial effect for noise reduction applications. This is directly related to the wavelet's vanishing moments which is the number of times the wavelet spectrum goes to zero at $\omega=0$ i.e $\left|\Psi(e^{j\omega})\right|_{\omega=0}=0$ where $z=e^{j\omega}$. Hence an additional flatness condition given in (3) is required while designing $H_0(z)$ and $H_1(z)$ [5].

$$\left.\frac{\partial^k\left|H_1\left(e^{j\omega}\right)\right|}{\partial\omega^k}\right|_{\omega=0}=\left.\frac{\partial^k\left|H_0\left(e^{j\omega}\right)\right|}{\partial\omega^k}\right|_{\omega=\pi}=0 \quad (3)$$

for $k = 0, 1, ...K - 1$, where $K$ corresponds to the number of zeros of $H_1(z)$ at $z = 0$ and $H_0(z)$ at $z = -1$ i.e. Nyquist frequency. In this study, IIR wavelet design methodology introduced by Zhang *et. al* [5] is adopted for implementing IIR wavelet filters with 3 and 5 vanishing moments and are referred to as $ilet3$ and $ilet5$, respectively in the rest of this document. Both wavelets filters are designed as maximally flat filters in order to achieve the maximum number of zeros at the Nyquist frequency leading to the maximum possible smoothness of the scaling and wavelet functions. The number of vanishing moments are selected based on the similarity between the wavelet function derived from the designed filters and the ECG signal morphology. Recalling (1), $H_0(z)$ can be re-written as,

$$H_0(z) = \frac{1}{2} A_0(z^2) \left( 1 + z^{-1} U(z^2) \right) \tag{4}$$

where $U(z)$ is an allpass filter with a general transfer function given in (2). For $ilet3$ wavelets $U(z)$ is chosen to be a first order filter with real coefficients $a_1$, and $a_0 = 1$. The frequency response of $H_0(z)$ is calculated by evaluating (4) on the unit circle and the magnitude response is given by,

$$\left| H_0(e^{j\omega}) \right| = \cos \frac{\theta(\omega)}{2} \tag{5}$$

where $\theta(\omega)$ is the phase response of $z^{-1}U(z^2)$. Therefore, for $ilet3$, $\theta(\omega)$ and $\left| H_0(e^{j\omega}) \right|$ are respectively computed as,

$$\theta(\omega) = -2 \tan^{-1} \frac{\sin\left(\frac{3\omega}{2}\right) - a_1 \sin\left(\frac{\omega}{2}\right)}{\cos\left(\frac{3\omega}{2}\right) + a_1 \cos\left(\frac{\omega}{2}\right)} \tag{6}$$

$$\left| H_0(e^{j\omega}) \right| = \frac{\cos\left(\frac{3\omega}{2}\right) + a_1 \cos\left(\frac{\omega}{2}\right)}{\sqrt{1 + 2a_1 \cos(2\omega) + a_1^2}} \tag{7}$$

As mentioned before, the smoothness of the wavelet function is determined by the number of zeros at the Nyquist frequency, which is computed by substituting the numerator of (7) into (3). Then, the filter coefficient $a_1 = 3$ is calculated by solving the linear equations obtained. When the same steps are applied for $ilet5$ in a similar manner by setting $M = 2$ and $K = 5$, two filter coefficients are calculated as $a_1 = 5$ and $a_2 = 10$. Following (4), the poles of $U(z)$ that lies inside the unit circle corresponds to the poles of $A_1(z)$ and the reciprocal poles outside the unit circle corresponds to the zeros of $A_0(z)$. By assigning the poles correctly, one and two stable allpass filters are obtained for $ilet3$ and $ilet5$, respectively. The magnitude responses and pole-zero locations of $H_0(z)$ and $H_1(z)$ for $ilet3$ and $ilet5$ are presented in Fig. 1.

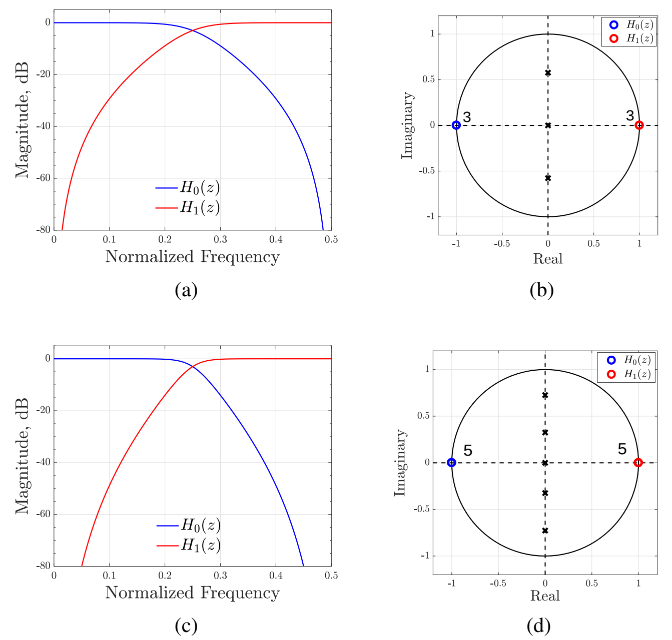



Fig. 1. $ilet3$; (a) Magnitude response and (b)Pole-Zero locations. $ilet5$; (c) Magnitude response and (d) Pole-Zero locations.

## III. FIR Wavelet Synthesis Filter Bank Properties

Orthogonality of the wavelet basis is a significant property for PR filter banks which ensures cancellation of aliasing and maintains a linear phase between the input and the output [5]. Thus, orthogonal wavelet filter banks must satisfy,

$$\begin{aligned} H_0(z) H_0(z^{-1}) + H_0(-z) H_0(-z^{-1}) &= 1, \\ H_1(z) H_1(z^{-1}) + H_1(-z) H_1(-z^{-1}) &= 1, \\ H_0(z) H_1(z^{-1}) + H_0(-z) H_1(-z^{-1}) &= 0. \end{aligned} \tag{8}$$

Terms $H_0(z^{-1})$ and $H_1(z^{-1})$ represent the synthesis filters which are simply the time-reversed (i.e. flipped) versions of the analysis filters for FIR wavelet filter banks. The coefficient flipping eliminates the non-linear phase effects of the analysis filters and guarantees perfect reconstruction. However, in the case of IIR wavelets the coefficients cannot be time-reversed since this will lead to unstable anti-causal filters, thus implementation of the synthesis filters requires more attention. In [6], the authors proposed the use of IIR wavelets for real-time discontinuity detection in an ECG data where the anti-causal filtering problem is solved by using the *block processing* method that requires two Last-In-First-Out (LIFO) registers and two identical allpass filters in parallel branches for each polyphase branch of the synthesis filter bank [6]. However, block processing technique increases both the storage and hardware requirements of the system while the read/write operations increases the system complexity and power dissipation. In order to determine the suitable block size, the impulse response of the IIR filter needs to be truncated which is equivalent to obtaining coefficients of an FIR filter. In this work, the storage and power hungry block processing technique is replaced by the use of the FIR equivalents of the IIR wavelets in the synthesis filter bank. This way, the FIR filter coefficients can be simply flipped hence avoiding the high storage requirements and memory read/write

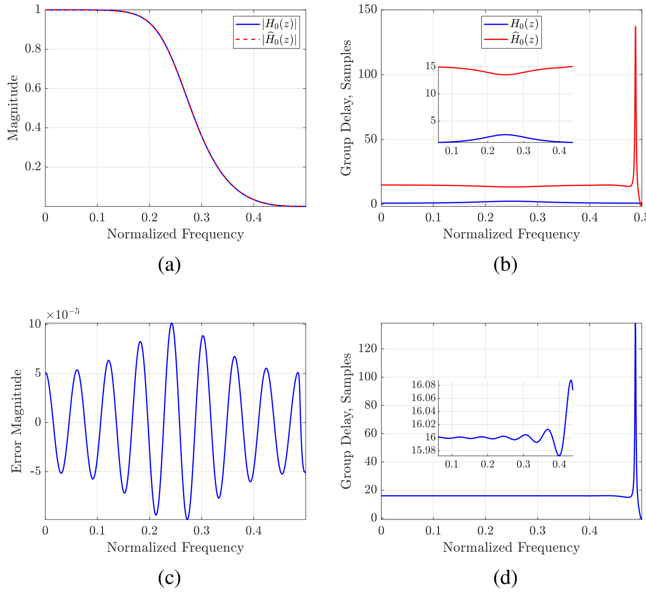



Fig. 2. Hybrid $ilet3$ analysis and synthesis lowpass filter (a) Magnitude response and (b) Group Delay. (c) Magnitude error between analysis and synthesis lowpass filter and (d) Total group delay of the filter bank.

operations required by the LIFO registers while maintaining the near PR property of the filter banks. Therefore, by carefully evaluating the error introduced in the filter magnitude and phase responses as well as the error between the input and reconstructed output, the impulse responses of the $ilet3$ and $ilet5$ filters are truncated to 16 and 24 samples, respectively. Fig. 2 presents the magnitude and group delay responses of the $ilet3$ lowpass analysis ($H_0(z)$) and synthesis filters ($\widehat{H}_0(z)$) along with the magnitude error with maximum error of -80 dB and the total group delay of 16 samples which is approximately constant and represents almost linear phase between the input and the output.

## IV. Computational Complexity

The proposed hybrid systems and the most commonly used FIR wavelet filter banks ($db4$, $db6$, $db8$, $sym4$ and $coif4$) are designed as tree structured 1-level filter banks where the analysis filters are followed by downsamplers and synthesis filters are leaded by upsamplers enabling the polyphase implementation by using the Noble identities [7]. For the hybrid systems, the analysis allpass sections are designed to have the Numerator-Denominator Tapped-Delay Line (ND-TDL) structure with two adders, two registers and one multiplier, whereas the synthesis FIR filters are implemented as time-multiplexed structures which conventionally employs an input and a coefficient memory, a multiplier and an accumulator. The hybrid $ilet3$ and $ilet5$ analysis and synthesis filters require 1 and 8, and 2 and 15 distinct coefficients, respectively. Since the coefficient multiplications are the most hardware and power demanding arithmetic operations, Reconfigurable Multiplier Blocks (ReMBs) are used for replacing the coefficient memory and the multipliers of the synthesis filters [8]. The allpass

TABLE I

Resource Utilization and Power Consumption of One level FIR and Hybrid Filter Bank Architectures

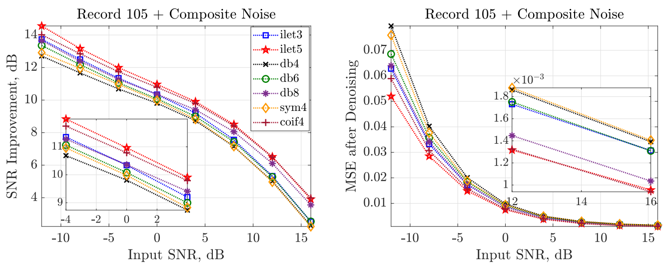
| | $db4$ | $db6$ | $db8$ | $sym4$ | $coif4$ | $ilet3$ | $ilet5$ |
|---|---|---|---|---|---|---|---|
| Adders | 8 | 8 | 8 | 8 | 8 | 12 | 22 |
| Multipliers | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| LUTs | 964 | 1010 | 1009 | 999 | 1024 | 317 | 672 |
| Registers | 684 | 700 | 700 | 688 | 710 | 271 | 501 |
| Power ($\mu$W)[1] | 13.617 | 15.54 | 14.671 | 13.698 | 13.661 | 5.2 | 9.67 |

[1] Power estimated at system clock frequency ($f_{clk}$) = 10 kHz.

coefficient multipliers are also implemented as hard-wired shifts and adds. It is a well known fact that, FIR filters are more sensitive to coefficient quantization which require higher word-lengths compared to the allpass based halfband polyphase IIR filters, hence the IIR allpass and FIR filter coefficients are quantized to 8 and 11 bits, respectively. In addition, the conventional FIR wavelet filter banks are also implemented as polyphase structures employing time-multiplexed FIR filters in each polyphase branch. Therefore, the fixed-point models of the aforementioned filter banks are designed using the System Generator for DSP in the Matlab/Simulink environment and are synthesized on a Kintex-7 (xc7k325tffg900) Field-Programmable-Gate-Array (FPGA) with Vivado v16.2. Performance validations of the system are done by feeding 11-bit ECG dataset from the MIT-BIH Arrhythmia database. The resource utilization as well as the dynamic power consumption for all filter banks are presented in Table I. As expected the number of adders are smaller for the conventional FIR wavelets due to resource sharing and the parallel multipliers, whereas the hybrid filter banks are completely multiplier free. However, as it can be observed from the number of LUTs and registers the FIR filter banks employ more resources due to the increased number of coefficients and the increased coefficient word-length. The conventional filter banks can also be implemented using the method proposed in [8], however this is out of scope of this work and will be presented in further publications. The synthesis results shows that proposed hybrid systems employ approximately 32.5 % and 65 % less resources for $ilet5$ and $ilet3$ wavelets against the selected FIR wavelets, respectively where $ilet3$ and $ilet5$ save 62 % and 30 % power against $db4$ filter bank.

## V. ECG Denoising Performance Evaluation

The aforementioned filter banks are employed for ECG signal denoising where wavelet thresholding/denoising method is used. For this purpose, clean ECG records ('103', '105', '109', and '118') obtained from the MIT-BIH Arrhythmia database are contaminated by synthetically generated additive noise with different noise power in order to obtain noisy ECG signals with Signal-to-Noise Ratio (SNR) ranging from $-12$ to 16 dB. Thus, the noisy ECG signals are decomposed into 7 levels where each of the detail coefficients (i.e. outputs of $H_1(z)$ at each level) are thresholded and the finest level approximation coefficients (i.e. $H_0(z)$ output at level 7) are nullified and the denoised signal is reconstructed. The ECG




(a) (b)

Fig. 3. Average (a) SNR Improvement (dB), and (b) MSE, after wavelet denoising with hybrid $ilet3$, hybrid $ilet5$, $db4$, $db6$, $db8$, $sym4$, and $coif4$.

TABLE II
SNR IMPROVEMENT ($dB$) AND MSE AFTER WAVELET DENOISING: INPUT SNR = $-8\ dB$.

| | $ilet3$ | $ilet5$ | $db4$ | $db6$ | $db8$ | $sym4$ | $coif4$ |
|---|---|---|---|---|---|---|---|
| | **SNR Improvement ($dB$)** | | | | | | |
| '103' | 11.95 | **12.5** | 11.19 | 11.57 | 11.69 | 11.43 | 12.27 |
| '105' | 12.5 | **13.17** | 11.67 | 12.18 | 12.39 | 11.95 | 12.85 |
| '109' | 13 | **13.78** | 12.16 | 12.68 | 12.94 | 12.28 | 13.33 |
| '118' | 12.18 | **12.86** | 11.35 | 11.82 | 12.08 | 11.51 | 12.53 |
| | **Mean Square Error (MSE)** | | | | | | |
| '103' | 0.038 | **0.034** | 0.046 | 0.042 | 0.041 | 0.043 | 0.036 |
| '105' | 0.033 | **0.028** | 0.04 | 0.036 | 0.034 | 0.038 | 0.037 |
| '109' | 0.061 | **0.051** | 0.074 | 0.066 | 0.062 | 0.072 | 0.057 |
| '118' | 0.064 | **0.056** | 0.078 | 0.07 | 0.066 | 0.075 | 0.059 |

signal denoising performance of the wavelet filter banks are evaluated and compared by computing the SNR improvement and Mean Square Error (MSE). The details regarding the generated data, the denoising method and the evaluation metrics can be found in [9].

## VI. Results and Discussions

For each data record at each SNR level, 100 Monte Carlo Simulations are performed and the average SNR improvement and MSE are computed. Results for the noisy record '105' are shown in Fig. 3, whereas Table II presents the SNR improvement and MSE results for different ECG data records with an input SNR of -8 dB. Following Table II, it can be seen that the hybrid $ilet5$ wavelet filter bank provides the highest SNR improvement and the lowest MSE for different ECG records when compared to the rest of the wavelet families. This is due to the better frequency selectivity achieved with the $ilet5$ wavelet despite having lower vanishing moments. Although, $coif4$ wavelet provides close enough denoising performance it utilizes 47% more resources compared to the hybrid $ilet5$ filter bank. In addition, the hybrid $ilet3$ filter bank achieves better denoising performance while using 65 % less resources against $db4$, $db6$, $db8$, and $sym4$ filter banks. For applications where the denoising performance and the power consumption is critical then the hybrid $ilet5$ can be employed instead of the conventional FIR wavelet filter banks. On the other hand, hybrid $ilet3$ filter bank has the lowest computational complexity among the others and provides better denoising performance except from $coif4$ which makes it a favourable choice for power and area limited applications.

## VII. Conclusions

In this paper, novel hybrid IIR/FIR wavelet filter banks and their novel use for ECG signal denoising is presented. For this purpose, two maximally flat and stable IIR analysis wavelet filters, $ilet3$ and $ilet5$, and FIR synthesis filters are designed and implemented. Vivado synthesis results demonstrated that both filter banks are computationally and power efficient against conventional FIR filter banks of most popular mother wavelets in the ECG denoising literature. In addition, wavelet thresholding method is used for denoising synthetically contaminated ECG signals in which the aforementioned hybrid and state-of-the-art FIR wavelet filter banks are employed. The results presented that the hybrid $ilet5$ wavelet filter bank achieves the best ECG denoising performance with the least signal distortion while utilizing 65% less resources with 47% improved power consumption amongst the others. It should be noted that, wavelet thresholding is not always sufficient on its own and required to be used in junction with other methods such as adaptive filtering. This study demonstrates that hybrid IIR/FIR wavelet filter banks can reduce the computational complexity of the DWT while providing better denoising performance. Therefore, they can be included in more sophisticated denoising applications in portable devices.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## References


[1] H. Hong and T. Yonghong, "A novel adaptive wavelet thresholding with identical correlation shrinkage function for ecg noise removal," *Chinese Journal of Electronics*, vol. 27, no. 3, pp. 507–513, 2018.

[2] C.-I. Ieong, M. Li, M.-K. Law, P.-I. Mak, M. I. Vai, and R. P. Martins, "A 0.45 v 147-375 nw ecg compression processor with wavelet shrinkage and adaptive temporal decimation architectures." *IEEE Trans. VLSI Syst.*, vol. 25, no. 4, pp. 1307–1319, 2017.

[3] J. Zhu and X. Li, "Electrocardiograph signal denoising based on sparse decomposition," *Healthcare technology letters*, vol. 4, no. 4, p. 134, 2017.

[4] J. M. Abdul-Jabbar and R. W. Hmad, "Allpass-based design, multiplierless realization and implementation of IIR wavelet filter banks with approximate linear phase," in *Innovation in Information & Communication Technology (ISIICT), 2011 Fourth International Symposium on*. IEEE, 2011, pp. 118–123.

[5] X. Zhang and T. Yoshikawa, "Design of orthonormal IIR wavelet filter banks using allpass filters," *Signal Processing*, vol. 78, no. 1, pp. 91–100, 1999.

[6] S. Damjanovic and L. Milic, "Examples of orthonormal wavelet transform implementad with IIR filter pairs," *Proc. SMMSP 2005, Riga, Latvia*, pp. 19–27, 2005.

[7] Y. Eminaga, A. Coskun, S. A. Moschos, and I. Kale, "Low complexity all-pass based polyphase decimation filters for ECG monitoring," in *11th Conference on Ph.D. Research in Microelectronics and Electronics (PRIME)*. IEEE, 2015, pp. 322–325.

[8] Y. Eminaga, A. Coskun, and I. Kale, "Multiplier Free Implementation of 8-tap Daubechies Wavelet Filters for Biomedical Applications," in *2017 New Generation of CAS (NGCAS)*. IEEE, 2017, pp. 129–132.

[9] ——, "IIR Wavelet Filter Banks for ECG Signal Denoising," in *22nd Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA), 2018*. IEEE, 2018.