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Conversions of Relief: On the perception of depth in drawings

Abstract:
Of the many ways in which depth can be intimated in drawings, perspective has undoubtedly been one of the most frequently examined. But there is also an equally rich history associated with other forms of pictorial representation. Alternatives to perspective became particularly significant in the early twentieth century as artists and architects, intent on throwing off the conventions of their predecessors, looked to new ways of depicting depth. In architecture, this tendency was exemplified by Modernism’s preference for parallel projection – most notably axonometric and oblique. The use of these techniques gave architects the opportunity to convey a new and uniquely modern form of spatial expression. At once shallow and yet expansive, a key feature of these drawings was their ability to support perceptual ambiguity. This paper will consider the philosophy and science of vision, out of which these preoccupations emerged. In this context, the nineteenth-century discovery of stereopsis and the invention of the stereoscope will be used to illustrate the way in which attempts to test the limits of spatial perception led to an opening up of visual experience; and provided a definition of visual experience that could encompass the representational ambiguities later exploited by the early twentieth-century avant-garde.

Key words:
architectural representation; visual space perception; stereopsis; drawing.

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Introduction

That architectural drawings are often constructed almost exclusively from lines seems hardly worthy of comment. But the more complex concerns surrounding the representational and expressive qualities of drawings can sometimes serve to divert attention from the more fundamental but equally remarkable nature of the line itself. In architectural representation, especially today, there are, of course, numerous alternatives. But even in the ethereal space of the computer, lines have an enduring value. Some lines trace edges or contours – defining limits or clinging to surfaces. Others perform a less direct role by providing the projective linkages from one point or surface to another. And sometimes, as is the case with the grid or datum, they are merely a frame of reference – a marker, against which other lines can be measured.[1] Whether as illuminated points on a computer screen or as ink on paper, lines have colour, they have weight and sometimes they even have texture and yet at the same time, notionally speaking, they have no thickness at all – or they are at least, infinitely thin. Little wonder then that one of the primary concerns throughout the history of architectural representation has been how best to employ these abstract linear traces to convey three-dimensional depth. But whilst the abstract nature of lines might seem at first to be their most significant limitation, operating in the inherently ambiguous gap between the flatness of the paper or the canvas and the implied three-dimensionality of representation, this apparent flaw has proved to be one of the most important opportunities offered by drawing.

Of the many ways in which depth can be intimated in a drawing, perspective has (at least since the fifteenth century) been one of the most dominant – and certainly the most talked about. But there is an equally rich history associated with other forms of spatial representation. Rudolf Arnheim and Massimo Scolari, amongst others, have, for example, pointed to the prevalence of oblique and ‘inverted’ forms of perspective in art and architectural drawing throughout history.[2] But even in the explicitly two-dimensional space of orthographic drawings, devices such as occlusion, shading, and the tracing of shadows (sciagraphy) have often been used to bring depth to otherwise flat representations.

Alternatives to perspective became particularly significant in the early twentieth century as artists and architects intent on throwing off the conventions of their predecessors looked to alternative forms of pictorial representation. In architecture, this tendency is exemplified by Modernism’s preoccupation with parallel projection. The use of these techniques provided artists and architects, such as Theo van Doesburg and El Lissitzky, the opportunity to convey a new and uniquely modern form of spatial expression (Figs. 1, 2). An important aspect of their success lay in the perceptual ambiguities that these drawings were able to support.[3] Concentrating predominantly on axonometric and oblique, , these artists were able to create representations devoid of the usual perspectival recession. The drawings are at once both shallow and expansive; three dimensional and yet lacking perspectival depth. As a result, they hover in an elusive and ambiguous space somewhere between the flatness of conventional orthographic drawings and the infinite extensibility offered by parallel projection.

As Hilary Bryon has recently pointed out, the majority of architectural drawings made using parallel projection in the twentieth century were drawn with one particular form of parallel projection, know as oblique.[4] Oblique projection is defined, as the name suggests, by the fact that the projection lines run obliquely to the plane of projection. This means that one plane of the object represented can be aligned with the picture plane and thereby preserve some of the qualities of the two-dimensional drawing. It is possible that one of the reasons for twentieth-century architects’ preference for oblique was that it emphasised the duality that exists in this form of representation between its two-dimensional and three-dimensional aspects: between the plan or the elevation and three-dimensional space. But the ambiguity which characterises these drawings is not restricted to the oblique and there are many examples of isometric and other forms of axonometric projection in which this quality is equally emphasised (Fig. 3). There are also numerous examples in which hybrid or intentionally inconsistent forms of representation are employed.[5]

Central to this use of parallel projection is the challenge that it makes to the assumed necessary equivalence between perspective and human perception. As Lissitzky describes it, the perspectival illusion of three-dimensional space on a plane has been ‘swept away’ by ‘the ultimate illusion of irrational space with infinite extensibility in depth and foreground.’[6] By contrast, in perspective representations (Fig. 4):

The world is put into a cubic box and transformed within the picture plane into something resembling a pyramidal form... This is a facade view of the world, where depth becomes a stage viewed statically... Here the apex of the visual cone has its location either in our eye, ie. in front of the object, or is projected to the horizon, ie. behind the object. The former approach has been taken by the East, the latter by the West. [7]

In pointing to the reversibility of perspective, Lissitzky is also drawing attention to the fact that perspective is reductive. Containing only one viewpoint, the monocular view eliminates the effects of binocular vision and, unlike the celebration of ambiguity found in avant-garde axonometric drawings, ignores the limitations of human perception.[8] Reversible figures and similar pictorial illusions were, however, at the heart of nineteenth and early twentieth research into the nature of visual space perception.

Visual Depth

For many centuries philosophers and scientists have puzzled over the mysteries of visual perception. Even to this day, there are significant questions that remain unanswered. But since the late nineteenth century, psychology (and more recently neuroscience), have provided an unprecedented insight into the mechanisms that underpin spatial perception.

Before the nineteenth-century, the study of space perception had been founded primarily on philosophical investigation, and on the optical relation between the viewer and the external world (as characterised by perspective). Both purely optical and perspectival accounts of vision demonstrate a correspondence between the relative position of objects in space and the patterns of light striking the retina. But these measurable parameters do little to explain how these patterns translate into sensations on the part of the viewer or how spatial depth might be inferred from this information. Even in the seventeenth century, this problem did not go unnoticed and concern both with how perceptions allows us to see objects in a particular position, and how the mind understands the broader spatial framework within which objects sit, were central to philosophical discourse on visual perception. For René Descartes, for example, space was a matter of extension.[9] Consequently, he saw location, distance, size and shape as the primary concerns for vision. Incorporating the Keplerian dioptrics of the retinal image, his theoretical account of perception relates positions contained in the image to the perception of space (Fig. 5)[10]

Similarly, if eye D is turned toward object E, the soul will be able to know the position of this object, inasmuch as [in the brain] the nerves from this eye are differently arranged than if it were turned toward some other object. And [the soul] will be able to know the shape, inasmuch as rays from point I assembling on the nerve termed optic [the retina]at point 2 – and those from point 3 at point 4, and so forth – will trace there a shape corresponding exactly to the shape of E.[11]

But surprisingly, despite Descartes belief in the innate capacity of the 'soul' to derive the extent and shape of the object based on the retinal image, he was not concerned with its picture-like quality. The visual image itself thus has relatively little status in Descartes theoretical account. Despite demonstrating that coherent images are formed on the back of the eye, Descartes was convinced that perception was not dependent on the resemblance between this image and the object:

Now although this picture, in being so transmitted into our head, always retains some resemblance to the objects from which it proceeds, nevertheless, as I have already shown, we must not hold that it is by means of this resemblance that the picture causes us to perceive the objects, as if there were yet other eyes in our brain with which we could apprehend it; but rather that it is the movements of which the picture is composed which, acting immediately on our minds inasmuch as it is united to our body, are so established by nature as to make it have such perceptions…[12]

Descartes theory lacks some of the more detailed knowledge of physiology which would characterise later accounts of vision. But he does, nevertheless, concentrate on the role played by the optic nerves and, as suggested by the extract above, on the visual dynamic of motion parallax. He concludes that knowledge of position and direction with respect to the body does not depend on the image but on locating the points in the brain which correspond to the nerve fibres at the back of the eye. Imaginary lines extending from these points provide the means by which the attention can be directed to specific locations. Using the analogy of a blind person searching out the position of objects in space with two sticks (Fig. 6), Descartes describes the convergence of the eyes as a means by which 'as if by natural geometry' we might know the location of a point. And just as an object located with two hands is not perceived as two objects, so the two eyes working in collaboration see a single object, despite the fact that a separate picture is formed in each eye.

Established 'by nature' in this way and acting therefore directly on the mind, Descartes saw no need for the viewing subject to be aware of the parameters from which the perception was derived. In an equivalent manner, the size and shape of the perceived object would not be based on the absolute shape or size of the object as it appeared in the retinal image, but on judgments made about its position. Pictures, he points out, may 'contain only oval and diamond shapes, yet they cause us to see circles and squares'. In summary, he concludes that it is the mind, not the eye, which is central to vision. But whilst appealing to some form of judgement, for Descartes this is, nevertheless, an immediate and natural capacity, not something that has to be learnt through experience. [13]

In 1709, George Berkeley would go further to conclude that judgments about distance, (or at least proximity), could not be made through vision alone. Like Descartes, Berkeley, concluded that perception of depth relies in part on the direction and convergence of the eyes. He is also equally uncompromising on the question of the role played by the image formed on the retina.[14] We make the mistake, he says, of 'imagining that the pictures of external objects are painted on the bottom of the eye'. These images are understood to be a copy or representation of an original object as if looking with our own eyes at the images formed in the eye of another, and seeing in it a picture of a scene in miniature. The error here, he points out, is in assuming that the image can be compared with the tangible qualities of the object itself, when in fact the likeness can only be with another image. The image in the eye should not therefore, he concludes, be considered as a picture, but rather as itself the original object of vision.[15] For Berkley it follows then, that distance cannot in fact be seen at all, being largely out of the plane of the image, or as he describes it, 'end wise' to the eye. He resorts instead to ambiguous visual cues such as the apparent size of objects and to the sensations associated with binocular convergence. Yet, although both Descartes and Berkeley discount the image in favour of convergence, Berkeley, unlike his predecessor, denies the possibility that depth can be extrapolated from binocular convergence via some innate geometrical sense. Our estimation of distance is, he suggests, informed instead by experience and knowledge of the objects we see. For Berkeley then, the perception of distance is not really a sense at all but rather a judgment based on relevant cues. He takes exception, in particular, to Descartes notion that the angle between the two optic axes provides the means by which the proximity of an object can be determined.[16] Such lines and angles are not, he points out, perceived, nor do they feature in the experience. The angle between the eyes cannot therefore, he concludes, be responsible for introducing the idea of distance:

> *But those lines and angles, by means whereof some men pretend to explain the perception of distance, are themselves not at all perceived, nor are they in truth ever thought of by those unskilful in optics. I appeal to anyone's experience whether upon sight of an object he computes its distance by the bigness of the angle made by the meeting of the two optic axes?*[17]

Descartes, of course, had not in fact suggested that a measurement of this angle was a conscious part of vision; simply that it was the basis for an understanding of the geometric relationship between the object and the observer. But Berkeley's observation regarding the nature of the experience, is not, in any case central to his argument, which hinges instead on the fact that a geometric analysis of distance (whether consciously, or as part of some innate mechanism) is not actually a necessary requirement for the association of distance with convergence. If, he concludes, a sensation derived from the disposition of the eyes can provide information relating to their position, and can therefore be related to the action of directing the eyes towards an object. It should be possible to infer the proximity of the object directly from the sensation, without relying on any innate capacity for calculating distance. The inference in this case is made, not as Descartes might have described it, in terms of the geometry relating the position of the eyes to their distance away from the object, but rather as a correspondence derived solely from experience. The mind, Berkeley suggests, through constant experience, finds that the sensations associated with different dispositions of the eyes can be related to different degrees of distance and that 'there has grown an habitual or customary connexion between those two sorts of ideas.'

By the nineteenth-century experimental devices and methods, borrowed from the physical sciences, had begun to be employed, not simply to account for the external world, but also to investigate the internal mechanisms at work in the sensations that Descartes, Berkeley and others had described – a shift, as Jonathan Crary describes it, from the mechanics of light to the 'physiological makeup' of human sight.[18] Central to this reorientation of human knowledge were studies conducted into the function of binocular vision.

In 1838, Sir Charles Wheatstone, then chair of Experimental Physics at King's College London, delivered the first of two papers to the Royal Society in which he demonstrated an experimental device known as the stereoscope and with it outlined a series of experiments on binocular vision and the unique sense of visual depth which we now know as *stereopsis*.[19] From the outset, however, the stereoscope did more than simply reveal the effects of binocular vision – it provided the opportunity, not only to increase the fidelity of exiting forms of representation, but also to challenge established explanations of spatial perception and to construct new kinds of visual experience.[20]

Key to Wheatstone's analysis is the gradually varying set of conditions experienced in vision from close proximity to the far distance: Noting that each eye must necessarily afford a slightly different perspective, Wheatstone considers the relative disparities that result from viewing objects at different distances. When looking at objects in the far distance, he pointed out, the axes of both eyes will be parallel and no perceptible difference between the views from the left and the right eye will be observed. However, as the eyes converge on objects at closer proximity, the relative disparity between the two views will increase and significantly different 'perspective projections' will be created. [21] Many different factors including perspective, movement parallax and focal accommodation contribute to our perception of visual space, but it is these small and sometimes barely detectable differences between the view from the left and right eye that are responsible for the vivid sense of three-dimensionality that we know as stereoscopic vision.

Having established that stereoscopic vision stems from the difference in viewpoint from the left and right eye, it became obvious to Wheatstone that it should be possible to replicate the visual impression of a solid object simply by simultaneously presenting suitably constructed perspectives to each eye:

> *It being thus established that the mind perceives an object of three dimensions by means of the two dissimilar pictures projected by it on the two retinæ, the following question occurs: What would be the visual effect of simultaneously presenting to each eye, instead of the object itself, its projection on a plane surface as it appears to that eye? To pursue this inquiry it is necessary that means should be contrived to make the two pictures, which must necessarily occupy different places, fall on similar parts of both retinæ.*[22]

Wheatstone's *reflecting* stereoscope (Figs. 7-9), produced the desired result by virtue of a pair of mirrors set directly in front of the viewer. Each of these mirrors, angled at forty-five degrees, reflects a different image positioned on panels at either end of the device.[23] On one side, he placed a perspective drawing constructed as if viewed from the left eye, and on the other, a drawing constructed as if from the right. By controlling the view of these images such that only one image is seen by each eye, the stereoscope therefore approximately reproduces the conditions under which a three-dimensional object is normally viewed. Rudimentary though this apparatus was, by presenting the images in this manner Wheatstone found he was able to artificially stimulate stereopsis and in the process create a pronounced sense of three-dimensionality in the image. As a consequence, he not only provided the key to a new understanding of three-dimensional visual experience but also drew attention to an aspect of vision that had always existed but of which, arguably, the viewing subject had previously been unaware.[24]

Wheatstone's analysis simultaneously places emphasis on the pictorial aspect of vision, and on the inadequacy of the static two-dimensional representation to capture the vivid sensation of relief experienced in viewing a solid object at close proximity, and with both eyes. So although

Wheatstone's concern here was ultimately with the nature of binocular vision, his argument was nonetheless constructed in terms relating to representation and its inability to capture the vivid sense of relief experienced in viewing a solid object.[25] He points out, for example, that the Diorama (which was enjoying considerable popularity at the time)[26] shows how the similar views afforded by each eye of a distant object can be exploited to produce a convincing depiction with a single view. Unlike the view of distant objects, however, when viewed at close proximity an object will present quite different aspects to each eye. As a consequence, it is impossible, he claims, for an artist to make a faithful representation of an object positioned close to the observer.

> *It will now be obvious why it is impossible for the artist to give a faithful representation of any near solid object, that is, to produce a painting which shall not be distinguished in the mind from the object itself. When the painting and the object are seen with both eyes, in the case of the painting two similar pictures are projected on the retinæ, in the case of the solid object the pictures are dissimilar; there is therefore an essential difference between the impressions on the organs of sensation in the two cases, and consequently between the perceptions formed in the mind …*[27]

But significantly for the kinds of instability associated with parallel projection, Wheatstone also describes the way a three-dimensional wire figure outlining a cube can be turned into an ambiguous figure by viewing it with one eye.[28] Under these circumstances a cube may be imagined to be a truncated pyramid or to turn itself inside out. It is the same 'indetermination of judgment' which, he says, can cause a two-dimensional drawing to be perceived by the mind as two different figures.[29] The typical example of such figures is the so-called Necker cube. This drawing was first described by the Swiss crystallographer Louis Necker in 1832 and features a wireframe cube the nearest and furthest faces of which remain impossible to judge with any certainty. This made the Necker cube an important example against which Wheatstone could test his theories on depth perception. It was, however, his scientific rival Sir David Brewster who brought Necker's observation to a wider audience when he published a letter he had received from Necker in the *London and Edinburgh Philosophical Magazine*.[30]

Necker's letter described how when examining engraved illustrations of crystalline forms a 'sudden and involuntary change in the apparent position of a crystal or solid' would occur. The example given is, in fact, an axonometric drawing depicting some form of rectilinear crystalline block (Fig. 10). Studying this figure, Necker noticed that sometimes the solid would seem as if to be arranged in such a way that one corner presented itself as the nearest but then, moments later, his visual interpretation of the drawing would change so that this point seemed the furthest away. Necker explains this in terms of the need to bring a particular point into focus within the area of the retina capable of distinct vision. The point seen most clearly, he suggests, will seem closest. But, as with many other aspects of visual perception, debate would rage between Brewster and Wheatstone on the relative merits of different explanations and the role played by different aspects of vision.[31] But perhaps most significantly for architectural representation, the spatial ambiguities contained in figures such as the Necker cube point the way to more complex forms of visual space in which the inherent contradictions between the two-dimensional representation and the implied three-dimensional form are made explicit. As such, the ambiguity is obviously heightened by the absence of conventional perspective cues. Also important here is the role played by contours and edges. In line drawings, even when some surfaces are evident, the absence of stereoscopic depth facilitates a certain freedom from the limitations of a real three-dimensional form.

On geometric lines

Line drawings are clearly notoriously unreliable but they have nevertheless always been part of the way architects conceptualise buildings. This aspect of architectural drawing is suitably illustrated by Robin Evans in his essay, *Architectural Projection*.[32] Evans cites a particular drawing by Bertrand the Elder (drawn just twenty years before Wheatstone's ground breaking paper) in which the shadows cast by a Tuscan column are carefully plotted (Fig. 11). The drawing employs projective geometry to trace sectional slices through the column, the direction of the light and ultimately, the profiles of the shadows – all in precisely rendered ink lines. Evans notes how the vividly portrayed shape of the shadows in drawings such as this brought out certain characteristics of classical architecture. One might expect that such an exercise might emphasise the solidity of the form but although technically precise, the results are far from stolid. The shadows, as Evans points out, challenge the static qualities of the structural form by superimposing a projection of the column capital onto itself. Despite the 'frozen sharpness of geometric delineation', the insubstantial and transient nature of the shadow becomes integral to the depiction.[33]

The origin of drawings such as these lies in the mathematics of descriptive geometry and nowhere is the abstract nature of the line more evident than in its linear tracery. As a form of parallel projection, this kind of sciagraphy has much in common with oblique projection. And whilst in this case the lines (projected here in the imagined direction of the sun's rays) are terminated by their intersection with another surface, as with oblique, lines extend at an angle from the orthographic view in order to draw out a sense of depth and sculptural form. But, like most of the geometry that had preceded it, descriptive geometry was nevertheless largely employed in the representation of forms and shapes analogous to things found in the real world. In fact it did that better than ever before – an effective and precise description of three-dimensional form and space that made it possible to control and manipulate the relations between solids and surfaces. As Robin Evans describes it, a more abstract and generalised version of architectural drawing – and hence supremely well suited to applications in stonecutting, carpentry and other forms of engineering.[34]

For some, most notably Alberto Perez-Gomez, this signifies a mathematisation of architecture – the triumph of function over intuition, in which design becomes a purely instrumental technological building science.[35] But far from grounding it in practical application, the universality and abstract nature of descriptive geometry would ultimately lead to a questioning of the very relation to reality that this functionality implies. As Evans points out, despite its application to technical problems, descriptive geometry was not concerned to show what things were actually like, but the relation between geometrically defined bodies and surfaces. The solidity of objects vaporises leaving only geometric outlines.[36] Any notion of depth hinges on representational qualities derived from the way the lines are configured, and particularly, the relationships, angles and connections between them. And when the arrangement is inconclusive, the line's inescapable attachment to the surface on which it is drawn leaves it powerless to resolve these inconsistencies. In the service of representation, the subservient line is pulled and pushed around in space, dragged forward or back as required to make local pictorial sense. This quality is particularly in evidence in drawings constructed using axonometric or oblique projections.

Jules de la Gournerie, author of one of the standard texts in descriptive geometry was typical in recommending that shadows should be introduced to counteract this effect.[37] However, in Gournerie's drawings (Figs. 12,13), as with Bertrand the Elder's column capital, the results serve as much to compete with the form as to clarify it. The ambiguity contained in these figures is, it could be said, a direct result of the fact that we are presented with just one view. In this regard, all two-dimensional drawings are essentially ambiguous. An obvious advantage to stereoscopic drawings is therefore that they can overcome such ambiguities. The parallels with geometry did not escape Wheatstone, and in the first of his papers on binocular vision he describes the similarity between descriptive geometry and binocular vision. In this science, he explains, the position of any point on a line is determined by its projection onto two fixed planes. In geometry, these planes are usually at right angles, whereas in vision they are determined by the optic axes and the notional plane occupied by the retinal image.[38] Employed in architecture stereoscopic drawings might then have offered not only vivid depictions but also a key to precisely describing three-dimensional form in a manner akin to descriptive geometry.

Stereoscopic Representation

As we have already seen, the mechanism through which the stereoscope conjures up a convincing illusion of three-dimensional form hinges on the slight differences in view that result from the lateral separation between the left and right eye. And it was not long before the newly invented medium of photography was used to record these viewpoints. Photography's ability to accurately capture the irregular detail, texture and shadows found in a real scene made it ideal for this purpose.[39] And with the introduction of stereoscopic photography, the stereoscope quickly attained a much wider appeal.

Even the physicist Hermann von Helmholtz, author of the most comprehensive account of visual perception in the nineteenth century, could not resist effusive descriptions of its realism:

> *These stereoscopic photographs are so true to nature and so life-like in their portrayals of material things, that after viewing such a picture and recognising in it some object like a house, for instance, we get the impression, when we actually do see this object, that we have already seen it before and are more or less familiar with it. In cases of this kind, the actual view of the thing itself does not add anything new or more accurate to the previous apperception we got from the picture, so far at least as mere form relations are concerned.*[40]

The popularity of stereoscopic photography was clearly derived from the compelling nature of the effect, but its success was also, in part, the result of enhancements to the design of the viewing device itself. Devised by David Brewster in 1849, the *lenticular* stereoscope employed lenses rather than mirrors and was consequently a more compact and portable device (Fig. 14). As a result the new form of stereoscope succeeded in folding the exposed and explicit mechanism of Wheatstone's design into a seemingly more natural extension of vision.[41] Given then, the pervasive nature of the stereoscopic image and its seemingly unsurpassed ability to represent three-dimensional experience, one might reasonably expect that the stereoscope would also have informed the spatial sensibilities of architects and artists. Indeed, David Brewster, in his book *The Stereoscope: Its History, Theory and Construction*, made a point of describing its use for the arts of painting, sculpture, and architecture.[42] In each case, his argument is primarily based on the recording and subsequent study of three-dimensional form – replacing the need for casts or drawings from life. But, although there are countless stereoscopic photographs of buildings, there is no particular evidence to suggest that this form of photography was used creatively by architects in this period. Indeed, representational techniques and conventions seem to change little during the years in which the stereoscope became commonplace, and what few developments there are can more easily be attributed to photography as a whole than to the particular qualities of the stereoscopic image.

It is important, however, to remember that in the 1830s when Wheatstone was working on early versions of his stereoscope, photography was still in its infancy and although the stereoscope was designed to explore the mechanisms and limits of natural perception, it was artificially constructed perspective line drawings that he employed for this task. Wheatstone himself recognised the potential practical application of such drawings in architecture and other disciplines:

> *Stereoscopic drawings afford a means of illustrating works with figures of three dimensions, instead of with mere plane representations. Works on crystallography, solid geometry, spherical trigonometry, <u>architecture</u>, machinery, &c., might be thus rendered more instructive, from the perfect counterpart of the solid figure seen from a single point of view being represented, instead of merely one of its projections.*[43]

There are certainly examples of drawings made for the stereoscope which depict architectural subjects. A stereoscopic drawing of an architectural gateway is, for example, included amongst the drawings made by Wheatstone (Fig. 15). This particular drawing is far from convincing, but although better examples can be found, the stereoscopic drawing, at least in terms of its application to architecture, remains problematic.[44] Significantly, drawings such as these appear in texts about stereoscopy, not in an architectural context, and in many cases may well have been traced or copied from stereoscopic photographs. The difficulty associated with producing architectural drawings in sufficient detail, and small enough to fit in the popular forms of stereoscope, would no doubt have made this approach impractical for most architects. Also, despite their stereoscopic effect, drawings of this kind would undoubtedly have appeared insubstantial in comparison with stereoscopic photographs and as a consequence would have been no match for the elaborate architectural perspectives produced at this time.

Wheatstone's drawings are, of course, also perspective projections, precisely calculated to simulate two slightly different viewing positions (See Fig. 9), but importantly they lack the detail and texture found in material objects. His drawings include projections of cubes and similar geometric figures; but seldom attempt to create solid forms. Instead the illusion is one of drawings that retain their linear quality but yet seem as if to extend out of the paper. As a result the lines now break free and leap out to occupy three dimensional space. They are still lines, but they are no longer drawn **on** anything. There was obviously a certain level of expediency in these simply constructed line drawings but the lack of materiality was also an intentional quality. Wheatstone was, after all, seeking to isolate the effects of stereopsis and eliminating other visual cues such as occlusion, texture and shading was therefore a necessary part of the process. [45] It is significant then that despite being about the tangibility of visual space the objects depicted in Wheatstone's drawings are, in fact, the idealised and abstract forms of geometry. Indeed sometimes, they don't even recognise the need to occlude the obscured parts of the object. They are therefore, in effect, simply the 'wire-frame' outlines of an entirely immaterial form. Importantly the limitations of two-dimensional representation, which Wheatstone sought to overcome, are most pronounced in this kind of skeletal drawing.

Conversions of Relief

Two-dimensional figures such as the Necker cube sustain their ambiguity by virtue of the fact that all the lines which make up the drawing share the same visual depth. Ambiguity in two dimensions relies, in other words, on lack of information about the third. In theory, when binocular vision intervenes, the actual three-dimensional structure of the model becomes abundantly clear. Hermann von Helmholtz, describing the stereoscopic effect, notes how it is most conspicuous in pictures that show simply outlines and where there are no other cues such as shading to promote an illusion of three-dimensions. And although, as he admits, the vividness is greatest in stereo photographs, even the most complicated drawings, 'scarcely intelligible' without a stereoscope can in this way, he claims, be made perfectly clear.[46] But whilst an object constructed in three-dimensions as a stereoscopic image will mostly preclude spatial ambiguity, in contrast to, say, a physical model, stereoscopic drawings combine a compelling spatial impression with all the illusory qualities of the line; and with this, the suggestion that some kind of ambiguity might nevertheless be attained.

Indeed, stereoscopic photographs and drawings have an inherent ambiguity of their own. Stereopsis undoubtedly gives these images an enhanced sense of depth but, at the same time, the fixed viewpoint and close focus of the surface ensures that the stereoscopic image remains bound to its pictorial composition. In this sense, the stereoscopic image, as a three dimensional picture, succeeds in attaining a status somewhere between a solid form and a two dimensional image. Held to the spot by the viewpoint of the image, the experience is simultaneously three-dimensional and pictorial.[47] The stereoscope's capacity to unravel the usual alliance between perspective, focal accommodation and binocular depth therefore ensures that Wheatstone's experiments do more than simply reinforce the solidity of otherwise shallow or flattened representations; they also unlocked the fixity of visual depth. And unlike Jules de la Gournerie, Wheatstone embraces the potential freedom from the limitations of conventional forms of spatial experience, prefiguring Modernism's preoccupation with indeterminate representational depth.

In the second of Wheatstone's papers on binocular vision he describes an instrument that will facilitate the necessary variability. A reworking of his original reflecting stereoscope, this new device allowed for the adjustment of both the convergence between the eyes and the focal distance at which the images were placed (Fig. 16). These parameters, combined with the intrinsic disparity contained in the drawings themselves, allowed Wheatstone to affect changes in apparent depth, and in the process transform depth into a malleable property. [48]

Under normal circumstances when an object moves towards the observer, the attendant increase in apparent size of the image would be accompanied both by adjustments in the convergence and focal accommodation of the eyes; and by an increasing disparity between the images. Wheatstone's second stereoscope facilitated the disassociation of these measures, recombining them in 'unusual manners, so that they may be associated under circumstances that never naturally occur'.[49] So, for example, as the images are swung back to reduce convergence, the depicted object will, (by virtue of being assumed more distant), appear to increase in relative magnitude – despite the fact that the retinal image remains unchanged. This fractured relationship between convergence, accommodation and the scale of the image is an intrinsic feature of the illusion constructed in the stereoscope and may go some way to explain the peculiar, artificially heightened, sense of space that is produced in stereoscopic images.

Wheatstone's dependence on mirrors also meant that he was quick to realise that stereopsis is easily reversible. In pursuit of what he describes as ‘conveying false perceptions to the mind’, Wheatstone observed that if a pair of stereoscopic images is reversed, or if each image were mirrored left to right, a visual space is constructed in which depth becomes inverted. In this experience, to which Wheatstone gives the name, ‘pseudoscopic’ space, volumes that are concave can be made to appear convex and those that are convex, made to appear concave. To affect this inversion, Wheatstone devised an instrument called the pseudoscope, which employs glass prisms to reverse the image seen by each eye (Fig. 17). Under these conditions, more distant objects will appear nearer and smaller, and nearer objects will appear more distant and larger. Equally, (provided the effects of other cues do not impinge on the experience), when an object is seen against a background, the foreground and background may become reversed. The pseudoscope, he suggests, provides a glance ‘into another visible world, in which external objects and our internal perceptions have no longer their habitual relation...’ It is also, importantly, a highly unstable illusion, stereopsis vying with other cues for precedence in the experience. And when, as Wheatstone reports, other external factors **do** interfere with the illusion to provide competing ideas, the two possible readings will alternate – first seeming to adopt one configuration and then another. [50]

Central to the peculiarity of this image, is not only the pseudoscopic nature of some of its components but also the effects of binocular rivalry. That is, when what one sees with the right eye contradicts what one sees with the left. And like pseudoscopic space, binocular rivalry was also explored by early researchers. The experiments of, amongst others, Hermann von Helmholtz, and Peter Panum reveal a condition which leads to instability in the image and to some peculiar superimpositions, as a result of which, a dynamic patchwork of competing elements vie for prominence in the visual field.[51] The resultant effect is an ambiguous mix of different views which refuse to combine – sometimes alternating between different readings and sometimes fragmenting and intermingling. Panum’s drawings show lines configured as if to represent a geometric figure that can be combined in several different ways as different parts of the figure are brought together by binocular vision (Fig. 18). Such experiments can be seen as part of a wider opening up of vision that was to facilitate a new definition of visual space in the twentieth century.

Reversibility and the avant-garde

The possibilities presented by retinal disparity for new forms of visual space were further explored in 1903 by scientist and philosopher Henri Poincaré. In a series of observations about the nature, content and optics of vision, Poincaré seeks to reveal a distinction between the conventional understanding of space (as defined by three-dimensional Euclidean geometry), and the sensory clues upon which spatial experience is based.[52] In his analysis of geometry, space and vision, Poincaré points to the difference between the ‘pure visual space’ of the retinal image and the ‘complete visual space’ of three-dimensional perception. But in making this comparison, Poincaré also does more than simply draw attention to the conventionalised nature of geometry. He points also to the likely potential for exploiting the disparities that he exposes to create the conditions under which other, more complex kinds of space might be invoked. Of particular interest here is his observation that, unlike geometrical space, the space of visual experience is not *isotropic*, that is, it is not the same in all directions. Sight, Poincaré concedes, does enable us to appreciate distance and consequently to perceive three dimensional space. But while an appreciation of distances in all directions is obviously essential to forming a fully three dimensional impression, the means by which we determine distances are not, he suggests, equal in all directions. Distances left, right, up and down can be derived, Poincaré assumes, from the visual sensations produced by a two dimensional image formed on the retina. Our measure of proximity on the other hand appears to be different in kind. ‘Everyone knows’, he says (in terms reminiscent of Descartes or Berkeley) ‘that this perception of the third dimension reduces to a sense of the effort of accommodation which must be made, and to a sense of the convergence of the two eyes..’.

The sensations associated with convergence and focal accommodation Poincaré observes must ordinarily work in constant relation, and as such be indistinguishable in the experience. At the very least, these two mechanisms appear to operate in such a highly coordinated fashion as to require a conscious effort for us to be aware that they exist as separate faculties.[53] This fact, as a defining feature of our visual experience, exemplifies for Poincaré the necessary relationship between the sensations we receive and the external conditions from which they are derived. And yet, intriguingly, as a consequence of this very consistency, Poincaré is able to progress from this observation, to speculate, that by altering those conditions so as to allow the two sensations to vary independently, we might fundamentally change the form of the space we perceive. Poincaré imagines a world experienced as if through a refracting medium capable of altering the parameters by which we make judgements about proximity and space.[54]

> *And so in this there is also a fact of external experiment. Nothing prevents us from assuming that a being with a mind like ours, with the same sense-organs as ourselves, may be placed in a world in which light would only reach him after being passed through a refracting media of complicated form. The two indications which enable us to appreciate distances would cease to be connected by a constant relation.* [55]

Some years earlier, Helmholtz had also conducted experiments in which lenses and prisms were used to alter the nature of the image reaching the eye, and these experiments may well be the inspiration for Poincaré's hypothetical scenario (Fig. 19). Significantly, however, Helmholtz's experiments had demonstrated that it was possible to quickly learn to compensate for such illusions, lending weight to his contention that no formal similarity was required between the sensations experienced and the external conditions that they signify, (assuming that a consistent relation exists between the external cause and the sensory effect).[56] Poincaré's example, however, notionally extends these experiments to include an additional discontinuity between the parameters involved: a relation potentially consistent, not with regular space, but with a space in which focal distance and binocular disparity might vary independently. What it represents, of course, is a kind of inversion, in which the pattern of sensations in some way informs the nature of the space. But frustratingly, no detail is provided in Poincaré's description, as to the exact nature of the refracting media, or of the effects it produces — although we might infer that this space would exhibit an effect something like the varying scale and proximity described by Wheatstone as the various parameters of vision were adjusted or reversed in the stereoscope.

But while Wheatstone's speculations were largely intended for a scientific audience, Poincaré's writing and popular lectures were influential well beyond the scientific community and were regularly discussed in artistic circles.[57] In this way these, and similar ideas about vision, continue to resonate through theories of space and representation in the early part of the twentieth-century. Albert Gleizes and Jean Metzinger's 1912 account of Cubism, for example, cites the effects of convergence and accommodation as part of their critique of perspective:

> *As for visual space, we know that it results from the harmony of the sensations of convergence and accommodation of the eye.*
>
> *For the picture, a flat surface, the accommodation is negative. Therefore the convergence which perspective teaches us to simulate cannot evoke the idea of depth. Moreover, we know that the most serious infractions of the rules of perspective will by no means compromise the spatiality of a painting. Do not the Chinese painters evoke space, despite their strong partiality for divergence?* [58]

Perhaps it was this critique of perspective that Lissitzky had in mind when he wrote about the reversibility of the visual cone. Inherent in their case for a dismantling of conventional perspective is certainly a suggestion, no doubt derived from Poincaré, that the relation between the various components of vision might be considered independently variable in the representational space of painting. And although they seem to confuse the convergence of lines in perspective with the convergence of the eyes in vision there is undoubtedly a relation intimated here which links the convergence of eyes with the increasing divergence of perspective at close proximity – a quality which might well be most apparent in the familiar still-life subjects of cubist painting. But in their promotion of divergent lines is also a suggestion of a space that extends forward of the painting and which is constructed, like the variable illusions created by the stereoscope, in the mind of the observer:

Certain forms must remain implicit, so that the mind of the spectator is the chosen place of their concrete birth. [59]

No longer concerned with eliminating ambiguity but rather with celebrating the versatility of human perception, twentieth-century artists and architects sought new ways of appealing to the cognitive mechanisms underlying vision – a process that Hermann von Helmholtz describes as 'unconscious inference' [60] Lissitzky, it seems, was especially interested in the ambiguities that had also fascinated nineteenth-century scientists. In the infinite extensibility of parallel projection he found a technique that released the drawing from the fixed point of view that perspective had necessarily entailed (Fig. 20). And in the drawings that he made for his various exhibition rooms in the 1920s, parallel projection provided the means to destabilise the space of the drawing both in terms of orientation and reversibility (Fig. 21). The agenda for drawing had clearly changed and attempts made by exponents of nineteenth-century engineering drawing such as Jules de la Gournerie to eliminate ambiguity had been replaced by a need to exploit its extraordinary qualities.

The impetus for this shift clearly came from a desire on the part of artists and architects such as El Lissitzky and Theo van Doesburg to revolutionise the nature of drawing. But it was the work of scientists like Wheatstone that, by questioning the limits and fixity of human perception, paved the way for a new attitude towards the representation of depth. From Necker's early fascination with ambiguous figures to Wheatstone and Panum's interests in pseudoscopic inversions and binocular rivalry, it was not simply that the old conventions of perspective were no longer adequate to convey the newly recognised complexity of human perception. It was also that, through the alternating and unstable visual space of instruments such as the pseudoscope, scientists had opened a door onto a new visible world – a world in which conventional spatial relations could be overturned and replaced with a dynamic visual field of competing elements. What early twentieth century artists shared with their scientific predecessors was both an interest in the mechanisms that underpin pictorial representation, and a determination to harness its potential to create new kinds of spatial experience. Stereoscopic drawing may not have impacted directly on architectural representation in the nineteenth century but contained in Wheatstone's stereoscopic experiments there are nevertheless clues to a very different kind of legacy – one that is located in the psychological dimension of visual experience and in the shifting ambiguities of solid and void that would be exploited by early twentieth-century avant-garde artists and architects.

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[1] Robin Evans, 'In front of lines that leave nothing behind', *AA Files*, 6 (May 1984). Republished in K. Michael Hays, *Architectural Theory Since 1968* (Cambridge Mass., MIT Press, 2000), p.485.
[2] Rudolf Arnheim, 'Inverted Perspective in Art: Display and Expression', *Leonardo*, vol.5, no.2 (Spring 1972), pp.125-135. Also, Massimo Scolari, *Oblique Drawing: A history of anti-perspective*, (Cambridge Mass., MIT Press, 2012).
[3] Yve-Alain Bois, 'Metamorphosis of Axonometry', *Daidalos*, no.1 (1981), p.56.
[4] On the distinction between oblique and axonometric projections, see Hilary Bryon, 'Revolutions in space: parallel projections in the early modern era', *Architectural Research Quarterly*, vol.12, nos.3/4 (2008), pp.337-346. As Bryon quite correctly points out, the majority of twentieth-century architectural drawings employing parallel projection were drawn using the oblique, and these two conventions have distinct spatial qualities (see page 345 of Bryon's article).
[5] See Richard Difford, 'Proun: an exercise in the illusion of four-dimensional space', James Madge and Andrew Peckham (eds.), *Narrating Architecture: a retrospective anthology* (London: Routledge, 2006), pp.73-104.
[6] El Lissitzky, 'K. und Pangeometrie', *Europa Almanach* (1925). Translated as 'A. and Pangeometry' in Eric Dluhosch, *Russia: An Architecture for World Revolution* (Cambridge Mass., MIT Press, 1984), p.145.
[7] *Ibid*., p.143.
[8] Yve-Alain Bois, 'From -∞ to +∞ : axonometry, or Lissitzky's mathematical paradigm', in *El Lissitzky, 1890-1941, Architect, Painter, Photographer, Typographer* (Eindhoven, Municipal van Abbemuseum, 1990), pp.30-31.
[9] Cornelis van de Ven, *Space in architecture : the evolution of a new idea in the theory and history of the modern movements* (Assen, Van Gorcum, 1987), p.30.
[10] Nicholas Wade, *A Natural History of Vision* (Cambridge, Mass., MIT Press, 1998), p.316. In 1604 Johannes

Kepler successfully described the manner in which images are formed in the eye. His analysis was therefore focussed primarily on the optics of the eye. See Nicholas Wade, *A Natural History of Vision, op. cit.*, pp.9-11.

[11] René Descartes, *Traité de l'Homme* (1664). *Treatise of Man*, Trans. T. S. Hall (Cambridge, Mass., Harvard University Press, 1972), pp.59-62.

[12] René Descartes, *La Dioptrique*, (1637). *Discourse on Method, Optics, Geometry and Meteorology,* Trans. Paul J. Olscamp (Indianapolis, Bobbs-Merrill Co., 1965), p.101.

[13] *Ibid.*, pp. 104-108.

[14] George Berkeley, 'An Essay Towards A New Theory of Vision', (1709). George Berkeley, *Philosophical Works, Including the Works on Vision*, ed. Michael R. Ayers based on the 4th Edition, (1732) (London, J. M. Dent & Sons, 1980), p.11.

[15] *Ibid.*, pp.42-43.

[16] *Ibid.*, p.9.

[17] *Ibid.*, p.10.

[18] Jonathan Crary, *Techniques of the Observer: On Vision and Modernity in the Nineteenth Century* (Cambridge, Mass., MIT Press, 1990), p.70.

[19] Stereopsis is defined as 'The ability to perceive depth and relief by stereoscopic vision', *The New Shorter Oxford English Dictionary* (Oxford, Clarendon Press, 1993). On Wheatstone's paper see, 'Obituary of Charles Wheatstone', (probably by C. Brooks), *Proceedings of the Royal Society* (1875-76), cited in Nicholas Wade, *Brewster and Wheatstone on Vision* (London, Academic Press, 1983), pp. 13-14. See also, 'Prof. Wheatstone: On binocular vision; and on the stereoscope, an instrument for illustrating its phenomena ...' *Report of the Eighth Meeting of the British Association for the Advancement of Science*, held in Newcastle in August 1838, VII, (London, John Murray, 1839), pp.16-17.

[20] Martin Jay, *Downcast Eyes: The Denigration of Vision in Twentieth-Century French Thought* (Berkeley, University of California Press, 1993), pp.151-152. See also, J. W. Baird, 'The Influence of Accommodation and Convergence Upon the Perception of Depth', *American Journal of Psychology* 14 (1903), pp.158-169 and, Nicholas Wade, *A Natural History of Vision*, *op. cit.*, pp.396-397.

[21] Charles Wheatstone, 'Contributions to the physiology of vision – Part the first', Republished in Nicholas Wade, Brewster *and Wheatstone on Vision*, *op. cit*., p.65.

[22] *Ibid.*, p.67.

[23] *Ibid.*, pp.70-71.

[24] Nicholas Wade, *A Natural History of Vision*, *op. cit*., pp.300-301.

[25] Jonathan Crary concludes that Wheatstone's aim was to simulate the presence of a physical object, not to discover another method for presenting drawings or paintings. This is, of course, true but by discussing his experiment in the context of other kinds of representation, Wheatstone sees the effects of the stereoscope as an extension of existing representational practices. His use of the term 'perspective projection' likewise suggests the artificial and constructed nature of the component images. Jonathan Crary, *Techniques of the Observer*, *op. cit.*, p.122.

[26] Charles Wheatstone, 'Contributions to the physiology of vision – Part the first.', *op. cit.*, p.65. See also, Richard Difford 'Infinite horizons: Le Corbusier, the Pavillon de l'Esprit Nouveau dioramas and the science of visual distance', *Journal of Architecture*, 14/3 (June 2009). Louis Daguerre's London Diorama opened in 1823 and was open to the public until 1851. See Stephan Oettermann, *The panorama : history of a mass medium* (New York, Zone, 1997), pp.80-83.

[27] Charles Wheatstone, 'Contributions to the physiology of vision – Part the first', *op. cit*., p.66.

[28] *Ibid*., p.82.

[29] *Ibid*., p.80.

[30] Louis Necker, 'Observations on some remarkable optical phenomena seen in Switzerland; and on an optical phenomenon which occurs on viewing a figure of a crystal or geometrical solid.', *London and Edinburgh Philosophical Magazine and Journal of Science* (November 1832), Vol.1, No.5, pp.329-337.

[31] See for example David Brewster, 'On the law of visible position in single and binocular vision, and on the representation of solid figures by the union of dissimilar plane pictures on the retina', *Transactions of the Royal Society of Edinburgh* (1844). Republished in Wade, Brewster *and Wheatstone on Vision, op. cit.*, p.113.

[32] Robin Evans, 'Architectural Projection' in Eve Blau and Edward Kaufman (eds.), *Architecture and its image: four centuries of representation: works from the collection of the Canadian Centre for Architecture* (Montreal, Canadian Centre for Architecture, 1989), pp.18-35.

[33] *Ibid.*, pp.27-30.

[34] *Ibid.*, p.28.

[35] Alberto Perez-Gomez, *Architecture and the Crisis of Modern Science* (Cambridge Mass., MIT Press,1983), pp.272-295.

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[36] Robin Evans, 'Architectural Projection', *op. cit.*, pp.28-9.
[37] Jules de la Gournerie, *Traité de Géométrie Descriptive* (1860-64), pp.127-8. Cited in Yve-Alain Bois, 'Metamorphosis of Axonometry', op. cit., p.56.
[38] Charles Wheatstone, 'Contributions to the physiology of vision – Part the first', *op. cit.*, p.74
[39] Removed from the scientific constraints of Wheatstone's experiments the potential for using photography to record the kind of solidity encountered in the real world was quickly recognised and as early as 1841 Wheatstone had acquired photographs for use in the stereoscope. See, Charles Wheatstone, 'Contributions to the physiology of vision – Part the second. On some remarkable, and hitherto unobserved phenomena of binocular vision', *Philosophical Transactions of the Royal Society,* Vol.142 (1852), pp.371-394. Republished in Nicholas Wade, *Brewster and Wheatstone on Vision*, *op. cit*., p.156. According to Henry Collen, Wheatstone had also previously acquired daguerreotypes from Richard Beard. Henry Collen, 'Earliest Stereoscopic portraits', *Journal of the Photographic Society* , Vol.1, No.16 (April, 1854). For an account of the early developments in stereo photography, see Nicholas Wade, *Brewster and Wheatstone on Vision* , *op. cit*., pp.35-39.
[40] Hermann von Helmholtz, *Handbuch der Physiologischen Optik* (1856-1866). Trans., James Southall (ed.), *Treatise on Physiological Optics* (Wisconsin, Optical Society of America, 1924-25), Vol. 3. p.303.
[41] Jonathan Crary, *Techniques of the observer: on vision and modernity in the nineteenth century*, *op. cit.*, p.129.
[42] Sir David Brewster, *The Stereoscope: Its History, Theory, and Construction* (London, John Murray, 1856) , pp.166-188.
[43] Charles Wheatstone, 'Contributions to the physiology of vision – Part the second', *op. cit.*, p.156. My emphasis.
[44] See for example, Peter Ludvig Panum, *Physiologische Untersuchungen Über das Sehen mit Zwei Augen*. (Kiel, Schwerssche Buchhandlung, 1858), fig 57. Louis Figuier, 'Le Stéréoscope', *Les Merveilles de la Science*, (Paris: Furne Jouvet,1869), fig.118, p.196.
[45] Charles Wheatstone, 'Contributions to the physiology of vision – Part the first', *figs. 10-20*. Nicholas Wade, *Brewster and Wheatstone on Vision*, *op. cit*., pp.72-74.
[46] Hermann von Helmholtz, *Physiological Optics*, vol.3, *op. cit.*, p.302.
[47] For a more detailed discussion of the pictorial qualities of stereoscopic images see, Richard Difford, 'In Defence of Pictorial Space', in Andrew Higgott and Timothy Wray (eds.), *Camera Constructs* (Farnham, Ashgate, 2012), pp.295-312.
[48] Charles Wheatstone, 'Contributions to the physiology of vision – Part the second', *op. cit.*, pp.150-154.
[49] *Ibid.*, p.150.
[50] *Ibid*, pp.162-167.
[51] Peter Ludvig Panum, *Physiologische Untersuchungen Über das Sehen mit Zwei Augen*, *op. cit.*
[52] Henri Poincaré, *La Science et L'hypothèse* (Paris, 1903). Trans. Henri Poincaré, *Science and Hypothesis* (London, Walter Scott Publishing Co., 1905), pp. 52-55.
[53] Helmholtz notes that directing the eyes towards a particular object is not necessarily a conscious act of 'innervation' in a particular nerve or muscle but rather that the intention is to produce an observable effect. Hermann von Helmholtz, 'The Facts of Perception', in Russell Kahl (ed.), *Selected Writings of Hermann Von Helmholtz* (Middletown, Wesleyan University Press, 1971), p.396.
[54] Henri Poincaré, *Science and Hypothesis*, *op. cit.*, pp. 52-55.
[55] *Ibid.*, pp54-55.
[56] Hermann von Helmholtz, 'The Facts of Perception', *op. cit.*, p.397.
[57] See Linda Dalrymple Henderson, *The Fourth Dimension and Non-Euclidean Geometry in Modern Art* (Princeton, Princeton University Press, 1983).
[58] Albert Gleizes and Jean Metzinger, 'Cubism' (1912). Translated in, Robert L. Herbert (ed.), *Modern Artists on Art* (New York, Dover, 2000), p.7.
[59] Gleizes and Metzinger, 'Cubism', *op cit*., p.7-8.
[60] Hermann von Helmholtz, *Physiological Optics*, *op cit*. See Nicholas Wade, *Brewster and Wheatstone on Vision*, op. cit., p.27; 322.