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Recent trends in firm-level total factor productivity in the UK: new measures, new puzzles

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Abstract
Understanding the poor productivity performance of the UK economy since the financial crisis is complicated by the well-known challenges in estimating total factor productivity (TFP) using only revenue data. We develop a structural framework to infer quality-adjusted TFP from an estimated firm-level revenue function. We use microdata for two sectors previously identified as being significant contributors to the UK's productivity growth slowdown—manufacturing and ICT—from 2008 to 2019. The revenue function is estimated using the Blundell–Bond System GMM estimator. We also use an alternative cost-shares approach to identifying and measuring TFP. For both methods, we find an overall fall in TFP levels in manufacturing and a rise in ICT. We find a striking decline of between 13% and 18% in the level of within-firm manufacturing TFP, and of between 11% and 16% in ICT, although with reallocation effects differing between the two sectors. The finding of declining within-firm TFP is robust, although the magnitude varies between methods. We discuss a possible explanation for this extended UK productivity puzzle based on the relative underperformance of UK firms in international markets.

1 | INTRODUCTION

By the end of 2019, nearly eleven years after the financial crisis, aggregate labour productivity in the UK was about a fifth lower than if the 1990–2007 trend had continued (Office for National Statistics (ONS) 2023). The slowdown has been more pronounced in the UK than in other OECD economies. Several authors (e.g. Coyle and Mei 2023; Goldin *et al.* 2024; Fernald and Inklaar 2022) find that productivity growth slowdowns in certain sectors (parts of manufacturing, information and communications technology (ICT), electricity, transportation, and finance) can largely account for the aggregate slowdown. Others highlight increasing heterogeneity among firms' productivity performance, in the UK and other OECD countries, with the most productive pulling increasingly far ahead of the remainder (Andrews *et al.* 2019; Coyle *et al.* 2022). The considerable heterogeneity across firms along dimensions such as size, use of digital technology, R&D performance, and export intensity, indicates that exploring the UK productivity puzzle in the post-2008 era requires combining firm-level evidence with sectoral insights.

This paper provides new measures of firm-level total factor productivity (TFP) for two sectors—manufacturing and ICT—that have been found to be disproportionate contributors to the UK's productivity slowdown. To develop these new measures, we use microdata from the UK's Annual Business Survey for the period 2008 to 2019. In addition to showing the sectoral evolution of a revenue-share-weighted index of firm-level TFP, we also decompose it into the effects of the evolution of within-firm TFP and the effects of reallocation across firms in each sector.

Recovering production function parameters faces two main challenges in most empirical applications. First, the researcher typically has access to firm-level revenues rather than prices and quantities. The estimation of a revenue function rather than a production function leads to well-known biases in recovering production function parameters (Klette and Griliches 1996; De Loecker 2011; Bond *et al.* 2021). Second, even with data on prices and quantities, the researcher faces the problem of simultaneity bias in recovering input parameters in the presence of unobserved productivity shocks (Griliches and Mairesse 1995; Olley and Pakes 1996; Blundell and Bond 1998, 2000; Levinsohn and Petrin 2003; Ackerberg *et al.* 2015).

In this paper, we first address these challenges by: (i) developing a structural model that, in principle, allows for the recovery of the objects of interest, including output elasticities and measures of TFP from the estimated revenue function; and (ii) using the Blundell–Bond System GMM estimator to estimate the parameters of the revenue function. For a given 2-digit industry, our baseline structural model imposes a non-varying demand-side elasticity of substitution and output elasticities. We also allow for product quality to vary across firms in an industry. We show that our method recovers firm-level estimates of quality-adjusted TFP, which we label TFPQ*.

One way to relax the assumption of non-varying output elasticities within an industry is to use the cost-shares approach to calculate time-varying firm-level output elasticities directly, if we are willing to assume constant returns to scale (CRS). To implement the cost-shares method, we start by testing the CRS assumption in our baseline model, taking into account that we are estimating a revenue function rather than a production function. We find the CRS assumption to be consistent with the data for most 2-digit industries within the manufacturing and ICT sectors. After inferring the output elasticities from the cost shares, we then re-estimate an appropriately restricted revenue function to obtain an estimate of the industry-specific elasticity of substitution and alternative measures of TFPQ*.

We show the sectoral implications of our two measurement methods by deriving the implied indexes of revenue-weighted TFPQ* for manufacturing and ICT. We then decompose the sectoral index into the product of an index capturing the evolution of within-firm TFPQ* and one capturing broad reallocation effects across firms in each sector.

We find evidence of a fall in the revenue-share-weighted average level of TFPQ* in manufacturing, and a rise in ICT. However, our most striking finding is of large declines in the within-firm measure of TFPQ* in both sectors, which is robust across the baseline and cost-shares methods. The reallocation effect is positive and larger than the within effect in ICT; for manufacturing, the size of the reallocation effect is sensitive to the approach used.

These findings raise new puzzles regarding the UK's recent productivity performance. In the literature to date, the puzzle has been framed as the need to explain the slowdown in the UK's productivity *growth* since the financial crisis—a slowdown that has taken place despite the apparent rapid technological changes associated with digitalization. Our findings show an even more puzzling fall in the *level* of within-firm productivity in manufacturing and ICT. The within-firm findings point to outright regression in some mix of the product quality and technical efficiency of UK firms. Although resolving these puzzles is beyond the scope of this paper, we suggest that a possible explanation lies in adverse movements in the *relative* product quality and technical efficiency of UK firms in international competition.

Section 2 provides a brief overview of the literature on the UK productivity slowdown. Section 3 sets out our framework for inferring TFPQ* and our estimation methodology. Section 4 describes the data. Section 5 presents the new TFPQ* measures for UK manufacturing and ICT. Section 6 explores a possible explanation for the puzzling findings on within-firm productivity. Section 7 concludes.

2 | UK PRODUCTIVITY SLOWDOWN: RELATED LITERATURE

Slower TFP growth is a common issue for other OECD economies (Fernald and Inklaar 2022) but is particularly pronounced in the UK. Most of the literature on the UK's stagnant productivity focuses on variables that affect productivity performance. One approach is to consider that in a perfectly competitive environment with wages reflecting marginal productivity, returns on capital and labour are indicative of their productivity at the aggregate level. Thus Pessoa and Van Reenen (2014) identify poor productivity as the outcome of weak growth in the effective capital–labour ratio. Accordingly, real wages have fallen dramatically, while the real cost of capital has increased, most likely due to the 2008 global financial crisis. Goodridge *et al.* (2013, 2018) apply growth accounting methods to national accounts data and find that the poor performance of TFP in the UK results from sluggish labour productivity. On this account, the observed TFP slowdown can be attributed to low labour productivity. The natural question then arises as to what drives low real wages. As Blundell *et al.* (2014) do not find any change in the compositional quality of labour, they conclude that the decrease in nominal wages may have been caused by a disproportionate (compared to previous recessions) increase in labour supply.

Barnett *et al.* (2014) challenge this diagnosis. They point instead to a serious problem of capital misallocation across sectors. The problem of resource misallocation is also raised in Bloom and Van Reenen (2010), which is more likely a structural factor reflecting *inter alia* poor management practices that persist in the UK (Bryson and Forth 2016). Goodridge *et al.* (2013) emphasize the importance of intangible assets and demonstrate that real value-added growth is understated by 1.6% because intangible assets are omitted. Incorporating intangibles can account for about one-third of the gap between observed and prior trend productivity.

Recent studies by Coyle and Mei (2023) examining the slowdown in UK labour productivity growth from 2008 to 2019 with sectoral data suggest that the primary cause of the post-2008 stagnation is a within-industry slowdown in certain sectors, including manufacturing and ICT. Focusing on the UK productivity puzzle between 2008 and 2012, Harris and Moffat (2017) find that the service sector's poor performance accounted for the largest part of the UK TFP slowdown in 2008.

Our contribution here is that, unlike most of the previous literature, we measure quality-adjusted TFP at the firm level for the UK. Our approach is conceptually similar to Forlani *et al.* (2023) and Jacob and Mion (2023). The former study separates firm-level price and quantity data to develop a novel framework that recovers heterogeneity in demand and quantity TFP across Belgian firms. They find that physical TFP and demand are negatively correlated. Jacob and Mion (2023) define a revenue measure of TFP as the product of physical TFP and price index. Total revenue is defined as this product times an input index. Looking at the weak UK productivity performance since 2008, Jacob and Mion (2023) find demand and decreasing physical TFP as the key determinants that push down revenue TFP (and labour productivity).

3 | TFP ESTIMATION FRAMEWORK

3.1 | A structural approach to estimating TFP

Our starting point reflects the well-known challenges involved in recovering the relevant parameters using deflated firm revenue and input data. We adopt a structural approach to estimating the relevant parameters and a (quality-adjusted) measure of TFP that we label TFPQ*. Details of our structural model are provided in Appendix Subsection A.1. Here, we summarize the main elements of the model and the final functional form of the firm revenue function that provides our baseline estimating equation.

Consumers maximize a nested utility function with Cobb–Douglas preferences over indexes of industry aggregates and constant elasticity of substitution (CES) preferences over the quality-adjusted products within each index. The elasticity of substitution η_j between quality-adjusted products within the index for industry j is constant over time. On the production side, heterogeneous firms have a Cobb–Douglas production function with firm- and time-varying Hicks-neutral technical efficiency, but common and constant output elasticities with respect to inputs of labour, capital and materials.

As detailed in Appendix Subsection A.1, utility and profit maximization imply a firm-level revenue function. Written in logs, this revenue function takes the form

$$r_{ijt} - p_{jt} = \frac{1}{\eta_j}\left(r_{jt} - p_{jt}\right) + \frac{\left(\eta_j - 1\right)\beta_j^l}{\eta_j} l_{ijt} + \frac{\left(\eta_j - 1\right)\beta_j^k}{\eta_j} k_{ijt} + \frac{\left(\eta_j - 1\right)\beta_j^m}{\eta_j} m_{ijt} + \frac{\left(\eta_j - 1\right)}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt}\right). \tag{1}$$

Firms are indexed by i, industries by j, and time by t. Here, r_{ijt} is the log of firm i revenue, r_{jt} is the log of industry revenue for industry j, and p_{jt} is the log of the (ideal) price index for industry j, which is affected by both price and quality changes. Also, l_{ijt}, k_{ijt} and m_{ijt} are the logs of firm-level labour, capital and material inputs, respectively; and β_j^l, β_j^k and β_j^m are the associated output elasticities of these inputs. The final term in equation (1) is a revenue-based measure of a firm's TFP that we label TFPR. Given an estimate of η_j, we can recover an estimate of a firm's TFPQ*, where the log of TFPQ* is the sum of a product quality component $\left(\lambda_{ijt}\right)$ and a technical efficiency component $\left(\omega_{ijt}\right)$. Critically, an estimate of η_j can be recovered from the estimated coefficient on deflated industry revenue in the firm-level revenue function (Klette and Griliches 1996; Melitz 2000).

In addition to our functional form assumptions, the key identifying assumptions are that the demand-side elasticity of substitution and the output elasticities are common across firms and constant over time in defined industry segments. Importantly, however, this does not necessarily imply that firm *markups* are similarly firm- and time-invariant within an industry. This will be true under monopolistic competition, where industry markups are equal to $\eta_j/\left(\eta_j - 1\right)$, thus will be invariant across firms in an industry and time for a given value of η_j. However, common and constant markups are not implied under other market structures. For example, under differentiated-product Cournot or Bertrand competition, the optimal markup will depend on the firm's revenue share within its industry for a given value of η_j (Atkeson and Burstein 2008). In our application, we do not make a specific assumption on the market structure, and the revenue function follows only from the functional-form assumptions and utility and profit maximizing behaviour.

Of course, common and constant output elasticities within narrowly defined industries are strong assumptions in this baseline model. We explore one way to relax this assumption by using the cost-shares approach to directly calculate firm- and time-varying output elasticities instead of inferring these elasticities from the estimated revenue function.[1] The equation of cost shares and output elasticities depends on the alternative identifying assumption of CRS in the production function (see Appendix Subsection A.2). We first test for CRS, which we find to be a reasonable restriction for most 2-digit industries, before implementing this cost-shares approach. Treating CRS as a reasonable restriction, we then use the cost shares to calculate the output elasticities, and re-estimate a restricted revenue function to obtain alternative estimates of η_j for each industry. We then use this revenue function to provide a set of alternative measures of firm-level TFPQ*.

3.2 | Econometric estimation

Our empirical setting allows for adjustment costs in all inputs,[2] serially correlated TFPR/TFPQ* shocks (following an AR(1) process), and unobserved heterogeneity in TFPQ* across firms. However, along with the adjustment costs, we allow input choices to respond to contemporaneous TFPQ* shocks, so consistent estimation of the revenue function faces the challenges of both unobserved heterogeneity and simultaneity that are common in the production function estimation literature (see, for example, Griliches and Mairesse 1995).

Letting $\theta_{ijt} = \left[(\eta_j - 1)/\eta_j\right]\left(\lambda_{ijt} + \omega_{ijt}\right)$, we thus assume

$$\theta_{ijt} = \theta_{ij} + v_{ijt}, \tag{2}$$

where

$$v_{ijt} = \rho_j v_{ij(t-1)} + \xi_{ijt}. \tag{3}$$

Here, ξ_{ijt} is a zero mean random shock that is potentially correlated with input choices, and we assume $0 <| \rho_j |< 1$. Lagging equation (1) by one period, multiplying the resulting equation through by ρ_j, and subtracting the result from equation (1), gives the quasi-differenced equation

$$\begin{aligned} r_{ijt} - p_{jt} = {} & \rho_j\left(r_{ij(t-1)} - p_{j(t-1)}\right) + \frac{1}{\eta_j}\left(\left(r_{jt} - p_{jt}\right) - \rho_j\left(r_{j(t-1)} - p_{j(t-1)}\right)\right) + \frac{\left(\eta_j - 1\right)\beta_j^l}{\eta_j}\left(l_{ijt} - \rho_j l_{ij(t-1)}\right) \\ & + \frac{\left(\eta_j - 1\right)\beta_j^k}{\eta_j}\left(k_{ijt} - \rho_j k_{ij(t-1)}\right) + \frac{\left(\eta_j - 1\right)\beta_j^m}{\eta_j}\left(m_{ijt} - \rho_j m_{ij(t-1)}\right) + \left(1 - \rho_j\right)\theta_{ij} + \xi_{ijt}. \end{aligned} \tag{4}$$

The presence of a firm fixed effect leads to a correlation between the lagged dependent variable and the error term ξ_{ijt} (Nickell 1981). Input variables in the revenue equation will also be correlated with the error term, where there are contemporaneous input responses to TFPQ* shocks. One option for consistently estimating equation (4) is to take first differences and instrument for potentially endogenous right-hand-side variables. Blundell and Bond (1998, 2000) identify relatively mild initial conditions that allow lagged levels of endogenous variables to be valid instruments for the endogenous first differences. However, Blundell and Bond (2000) find that lagged levels provide weak instruments in a production-function-estimation setting. Alternatively, they suggest estimating a System GMM that includes the estimating equation in first differences and that equation in levels. Moreover, they provide mild initial conditions under which lagged first differences are valid instruments for the endogenous variables in the levels equation. They show that the System GMM provides more efficient estimates than a single equation approach. Given the demonstrated good performance of this estimator in the context of production function estimation (Blundell and Bond 1998, 2000), we use it to estimate the parameters of the revenue function.[3]

3.3 | Decomposition of observed trends in TFPQ*

To show the implied evolution TFPQ* at the sectoral level, we present an index of revenue-share weighted TFPQ* for both manufacturing and ICT. Following De Loecker *et al.* (2020), we can decompose this sector-specific index into the product of an index of within-firm TFPQ*, an index of reallocation effects, and an index of entry and exit effects.[4]

Letting x_{ct} represent the log of the sectoral revenue-share-weighted geometric mean of the corresponding firm-level variable, where $c \in$ (Manufacturing, ICT), we have

$$x_{ct} = \sum_{i \in c} x_{ijt} s_{ict}, \tag{5}$$

where s_{ict} is the share of firm i in the total revenue of the sector at time t, and x_{ijt} is the log of measured firm-level TFPQ*. Using the De Loecker *et al.* (2020) decomposition, we can write the growth rate of the sector aggregate as

$$\Delta x_{ct} = \sum_{i \in c} \Delta x_{ijt} s_{ic(t-1)} + \sum_{i \in c} \hat{x}_{ij(t-1)} \Delta s_{ict} + \sum_{i \in c} \Delta x_{ijt} \Delta s_{ict} + \sum_{i \in Entry|c} \hat{x}_{ijt} s_{ict} - \sum_{i \in Exit|c} \hat{x}_{ij(t-1)} s_{ic(t-1)}, \tag{6}$$

where $\hat{x}_{ijt} = x_{ijt} - x_{c(t-1)}$ and $\hat{x}_{ij(t-1)} = x_{ij(t-1)} - x_{c(t-1)}$.[5]

The first term on the right-hand side of equation (6) is the effect of within-firm productivity growth on the sector aggregate growth rate. The next two terms capture reallocation effects between firms in the sector, and the final two terms capture the effects of firm entry and exit, respectively. We term the sum of the second and third terms the reallocation effect, and the final two terms the entry/exit effect. The final four terms taken together can be collectively thought of as a broad reallocation effect. Finally, setting the relevant level of the index equal to 1 in the first year of the sample, we use the relevant calculated weighted growth rates to infer the evolution of the level of the index over the remainder of the sample period. We show the evolution of the within-sector index and the broad reallocation index as well as the overall sector index. We present these separately for the manufacturing and ICT sectors.

4 | DATA

We construct a firm-level dataset that includes non-financial business firms in the UK in the ONS Annual Business Survey (ABS), covering the period 2008–19. The ABS covers approximately two-thirds of UK non-financial businesses, including firms' revenue, employment costs, capital expenditure and purchases of intermediates (materials).

To build the dataset, we implement the lowest local unit[6] in the data. We checked for duplication and removed 94 units from the sample. Building on Coyle and Mei (2023), we focus on the two sectors that made the biggest contribution to the post-2008 productivity growth slowdown in a sectoral decomposition: manufacturing (19 SIC2 subsectors with 148,962 observations), and information and communication (6 SIC2 subsectors with 112,503 observations). This gives us an unbalanced panel with 261,465 observations for 2008–19.

For each firm, there are data on total revenue, total employment, capital stock and purchases of inputs. As all monetary values are in nominal terms, we employ the 2-digit industry-level ONS producer output price deflator and input price indices (manufacturing PPI and non-manufacturing SPPI) to deflate the nominal values to 2015 prices (in £ thousand).

We construct firm-level capital stocks using the perpetual inventory method. One approach to identifying the initial level of capital stock for each firm is to use an estimate for total capital stock in the initial year, and allocate it according to firm-level revenue shares. However, this approach is problematic in our application because our dependent variable is a measure of firm-level revenue (Haskel and Martin 2002; Harris and Moffat 2017). We instead initialize the capital stock using the assumption that observed firm investment (measured net of disposals) is growing at the same rate prior to the appearance of a firm on our sample as we observe it to grow during the period it is observed in the sample. The initial capital stock K_{ij0} for firm i in industry j is then a depreciation-rate adjusted sum of all prior investments, and I_{ij0} is the (net) investment level of firm i in the first year that the firm appears in the sample. The initial capital stock is then given by the infinite series

$$K_{ij,0} = \frac{I_{ij0}}{\left(1+\bar{g}_{ij}+\delta_j\right)} + \frac{I_{ij0}}{\left(1+\bar{g}_{ij}+\delta_j\right)^2} + \frac{I_{ij0}}{\left(1+\bar{g}_{ij}+\delta_j\right)^3} + \cdots, \tag{7}$$

where $\bar{g}_{ij}$ is the firm-specific average growth rate of investment observed in the data, δ_j is the industry-specific depreciation rate, and $\bar{g}_{ij} \times \delta$ is assumed to be a very small number and is ignored.[7] Multiplying both sides of equation (7) by $1/\left(1+\bar{g}_{ij}+\delta_j\right)$ and subtracting the resulting equation from equation (1), we obtain

$$K_{ij0} = \frac{I_0}{\bar{g}_{ij}+\delta_j}. \tag{8}$$

To estimate the capital stock for subsequent periods, we use the difference equation (consistent with equation (1))

$$K_{ij(t+1)} = K_{ijt}\left(1-\delta_j\right) + I_{ijt}. \tag{9}$$

The industry-specific value of δ_j is obtained from EU KLEMS.[8] To obtain an estimate of $\bar{g}_{ij}$ used in the calculation of the initial capital stock for a given firm, we use the first and last investment levels observed in the sample, which we label I_{ij0} and I_{ijd}, respectively, where the number of periods between the first and last observation is τ_{ij}. The average growth rate is then calculated as

$$\bar{g}_{ij} = \frac{\ln I_{ijd} - \ln I_{ij0}}{\tau_{ij}}. \tag{10}$$

5 | RESULTS

5.1 | Baseline results

Table 1(a) shows the results from our structural model estimating the baseline revenue function for 2-digit manufacturing industries; Table 1(b) shows the corresponding results for 2-digit ICT industries. Column (1) records the coefficient on the deflated industry revenue variable, where the inverse of the coefficient provides an estimate of the demand-side elasticity of substitution in the industry. The unweighted average implied elasticity of substitution is 10.4 in manufacturing and 19.4 in ICT. Columns (2)–(4) record the estimates of the revenue elasticity with respect to each input. Columns (5)–(7) then record the implied output elasticities with respect to these inputs, where the output elasticities are inferred given the estimate of the elasticity of substitution derived from column (1). The final column shows the implied estimate of the returns to scale. Overall, the baseline revenue function appears to produce sensible estimates of the output elasticities, with the returns to scale close to 1 for most industries, and textiles showing the strongest evidence of increasing returns to scale. The unweighted average of returns to scale parameters is 1.01 for manufacturing and 0.90 for ICT.

Obtaining realistic estimates of output elasticities with respect to capital is often challenging in production function estimation. For manufacturing, the unweighted average of output elasticities with respect to capital is 0.17, which compares with 0.48 for labour and 0.31 for materials. The unweighted average output elasticity with respect to capital is lower in ICT (0.06), which compares with 0.59 for labour and 0.26 for materials. Overall, the relatively reasonable output

TABLE 1(a) Estimation of baseline revenue function for 2-digit manufacturing industries.

	$1/\eta_j$ (1)	$((\eta_j-1)/\eta_j)\beta_j^l$ (2)	$((\eta_j-1)/\eta_j)\beta_j^k$ (3)	$((\eta_j-1)/\eta_j)\beta_j^m$ (4)	β_j^l (5)	β_j^k (6)	β_j^m (7)	$\overline{\beta}_j$ (8)
SIC10	0.10***	0.18***	0.21***	0.45***	0.20	0.23	0.51	0.94
(Food Products)	(0.01)	(0.05)	(0.04)	(0.05)				
SIC11	0.04**	0.39***	0.29***	0.31***	0.41	0.30	0.33	1.04
(Beverages)	(0.02)	(0.10)	(0.08)	(0.04)				
SIC13	0.07	0.47*	0.52*	0.24	0.52	0.57	0.26	1.35
(Textiles)	(0.04)	(0.24)	(0.21)	(0.15)				
SIC16	0.08	0.45***	0.13	0.24	0.50	0.14	0.26	0.90
(Wood Products)	(0.09)	(0.13)	(0.12)	(0.36)				
SIC17	0.09	0.65***	0.25*	0.04	0.72	0.28	0.05	1.05
(Paper Products)	(0.07)	(0.18)	(0.14)	(0.12)				
SIC18	0.07***	0.66***	0.07	0.13*	0.71	0.08	0.14	0.93
(Printing & Reproduction)	(0.02)	(0.08)	(0.04)	(0.08)				
SIC20	0.11***	0.07**	0.29***	0.41***	0.08	0.32	0.46	0.86
(Chemicals)	(0.01)	(0.03)	(0.05)	(0.09)				
SIC22	0.03***	0.31***	0.05	0.73***	0.32	0.05	0.76	1.13
(Rubber & Plastic)	(0.007)	(0.11)	(0.03)	(0.09)				
SIC23	0.06	0.50***	0.01***	0.52***	0.53	0.02	0.56	1.11
(Non-Metallic Mineral)	(0.05)	(0.03)	(0.008)	(0.03)				
SIC24	0.07**	0.29*	0.29***	0.35***	0.31	0.31	0.38	1.00
(Basic Metals)	(0.03)	(0.15)	(0.12)	(0.14)				
SIC25	0.08***	0.56***	0.03	0.35***	0.61	0.04	0.38	1.03
(Fabricated Metal)	(0.01)	(0.09)	(0.02)	(0.07)				
SIC26	0.11***	0.46***	0.16***	0.19***	0.52	0.18	0.12	0.82
(Comp., Elec. & Optical)	(0.03)	(0.12)	(0.03)	(0.08)				
SIC27	0.09***	0.44***	0.10	0.34***	0.49	0.11	0.37	0.97
(Electrical Equipment)	(0.02)	(0.13)	(0.07)	(0.08)				
SIC28	0.09***	0.53***	0.10***	0.30***	0.59	0.11	0.33	1.03
(Machinery and Eqp. n.e.c.)	(0.01)	(0.06)	(0.03)	(0.05)				
SIC29	0.02**	0.41***	0.06**	0.53***	0.42	0.06	0.54	1.02
(Motor Vehicles)	(0.01)	(0.06)	(0.03)	(0.07)				
SIC30	0.04***	0.58***	0.04	0.42***	0.61	0.05	0.43	1.09
(Other Transport Eqp.)	(0.001)	(0.07)	(0.03)	(0.06)				
SIC31	0.03	0.45***	0.09	0.41	0.46	0.09	0.42	0.97
(Furniture)	(0.08)	(0.16)	(0.08)	(0.13)				
SIC32	0.03	0.67	0.13	0.20	0.70	0.13	0.21	1.04
(Other Manufacturing)	(0.01)	(0.06)	(0.03)	(0.05)				
SIC33	0.06***	0.27***	0.09***	0.55***	0.29	0.09	0.60	0.98
(Repair and Installation)	(0.01)	(0.04)	(0.01)	(0.04)				

Notes: $\overline{\beta}_j = \beta_j^l + \beta_j^k + \beta_j^m$. Robust standard errors are in parentheses.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

TABLE 1(b) Estimation of baseline revenue function of 2-digit ICT industries.

	$1/\eta_j$ (1)	$((\eta_j-1)/\eta_j)\beta_j^l$ (2)	$((\eta_j-1)/\eta_j)\beta_j^k$ (3)	$((\eta_j-1)/\eta_j)\beta_j^m$ (4)	β_j^l (5)	β_j^k (6)	β_j^m (7)	$\overline{\beta}_j$ (8)
SIC58	0.07***	0.55***	0.01*	0.31***	0.59	0.01	0.34	0.94
(Publishing Activities)	(0.01)	(0.06)	(0.006)	(0.05)				
SIC59	0.01	0.71***	0.02**	0.04	0.73	0.03	0.05	0.81
(Motion Pictures)	(0.01)	(0.07)	(0.006)	(0.04)				
SIC60	0.01	0.57***	0.17***	0.08	0.58	0.18	0.08	0.84
(Programming & Broadcasting)	(0.10)	(0.16)	(0.03)	(0.34)				
SIC61	0.07***	0.56***	0.03***	0.34***	0.61	0.03	0.37	1.01
(Telecommunications)	(0.003)	(0.02)	(0.01)	(0.02)				
SIC62	0.08***	0.38***	0.05***	0.43***	0.42	0.06	0.46	0.94
(Computer Programming)	(0.004)	(0.04)	(0.009)	(0.04)				
SIC63	0.07***	0.55***	0.04***	0.23***	0.60	0.04	0.24	0.88
(Information Service)	(0.006)	(0.03)	(0.005)	(0.02)				

Notes: $\overline{\beta}_j = \beta_j^l + \beta_j^k + \beta_j^m$. Robust standard errors are in parentheses.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

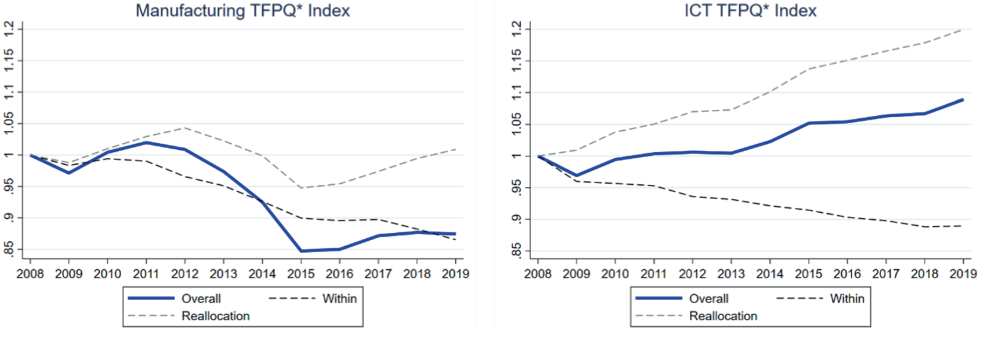

FIGURE 1 Baseline decomposition of TFPQ* index (2008 = 1). *Notes*: The TFPQ* index shows the evolution of a revenue-share weighted index of TFPQ* between 2008 and 2019, where the value of the index is set equal to 1 in 2008. The revenue shares are calculated at the sectoral level. The decomposition of the index is based on equation (6). We have combined the reallocation and entry/exit terms into a single broad reallocation index.

elasticities for capital—especially for manufacturing—lead us to favour the direct structural estimation method rather than the cost-shares method in making our choice of baseline; these latter estimates are reported below.

Figure 1 shows the evolution of the revenue-share weighted TFPQ* index at the sectoral level, where the index is set equal to 1 in 2008. The proportional change in the index relative to 2008 for each year can then be read easily from the figure. The index shows contrasting evolutions for manufacturing and ICT, with a cumulative fall of approximately 12% for manufacturing, and a rise of approximately 9% for ICT.

As described in Section 3, we decompose the overall index into the within-firm effect and a broad reallocation effect. The most striking finding is the fall in the within-firm measure of TFPQ* between 2008 and 2019 for both manufacturing (−13%) and ICT (−11%). The two sectors show opposing trends for the reallocation term, with an adverse reallocation effect further worsening the aggregate measure of TFPQ* for manufacturing, but a positive reallocation effect more than outweighing the adverse within-firm effect for ICT.

Figure 2 Compares the distributions of estimated log TFPQ* for the firms in the sample in the first year (2008) and the last year (2019). There is evidence of a leftward shift in the distribution for manufacturing, and a rightward shift for ICT.

Figure 3 Provides further evidence on the shift in the distribution by looking at different deciles of the TFPQ* distribution, where the deciles are identified in terms of cumulative market share. To construct the deciles, we first order the firms in a sector in terms of lowest to highest measured TFPQ*, and record their market shares. For the first decile, we then identify the firms in the ranking that have a cumulative market share of 10%, which gives us the cut-off

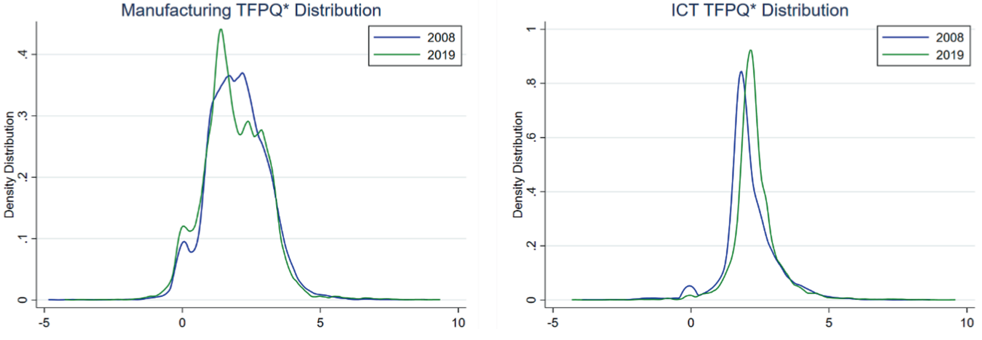

FIGURE 2 Comparison of log TFPQ* distributions in 2008 and 2019 (baseline method).

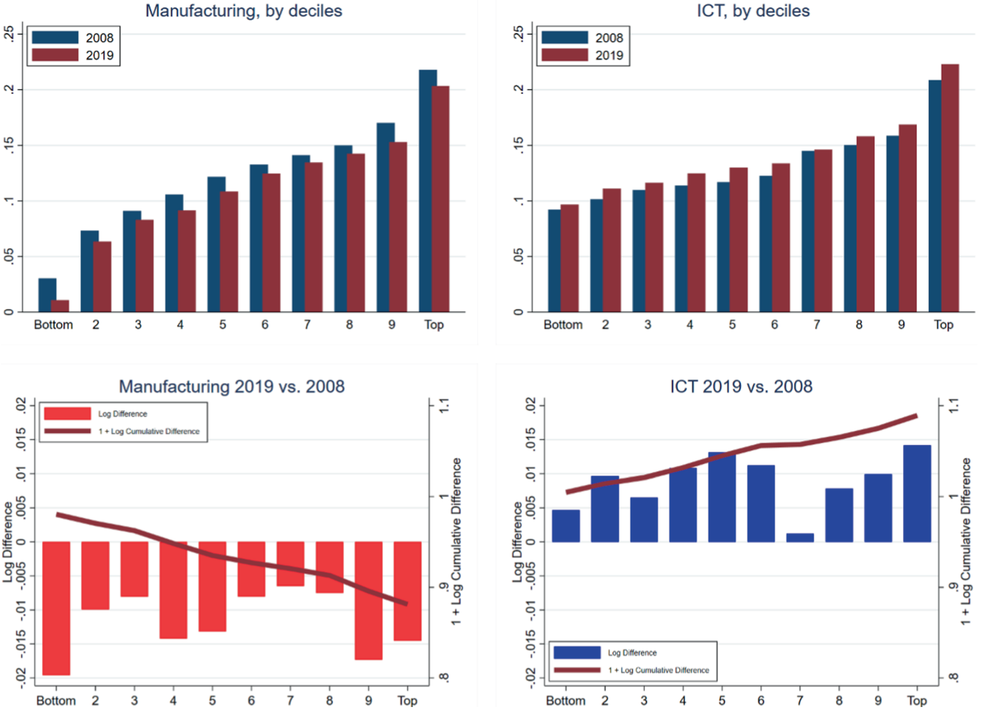

FIGURE 3 Measured TFPQ* across deciles—baseline. *Notes*: Deciles based on the cumulative market share (see text and Appendix Subsection A.3 for details). The figure shows the revenue-share weighted average TFPQ* for the firms in the decile. In the lower panels, 1 + cumulative log difference matches the overall change in the index of TFPQ* between 2008 and 2019 shown in Figure 1.

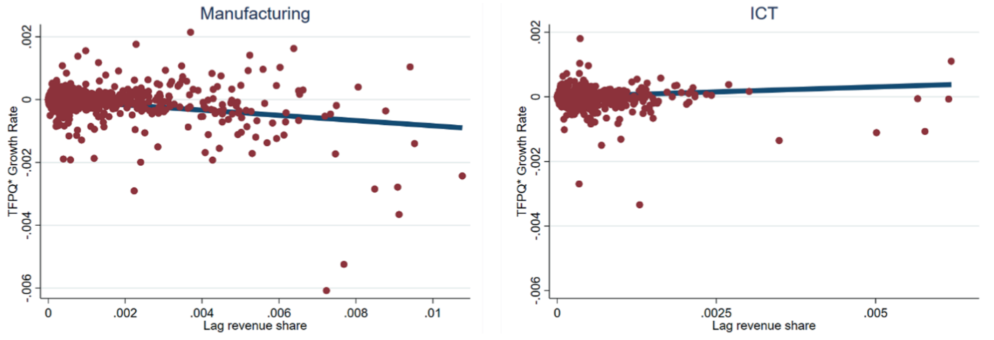

FIGURE 4 Understanding the decline in within-firm TFPQ*: scatterplot of TFPQ* growth and lagged revenue (baseline method). *Notes*: Outliers including TFPQ* growth rate below −0.006 and lag revenue share above 0.008 are trimmed (ICT only) for presentation purposes.

TFPQ* for the decile. We then calculate the revenue-share weighted TFPQ* for firms in the decile. This quantity is shown by the height of the first bar in Figure 3. We repeat this procedure for the other deciles, where each decile represents 10% of the total market share by construction. We record the results for the first (2008) and last (2019) years of the sample. In the lower panel, we show the log difference in the decile measures between 2018 and 2019. By construction, 1 + cumulative log difference is equal to the value of the overall index for 2019 (see Appendix Subsection A.3). For manufacturing, the results show a fall in TFPQ* for each decile, with the largest falls observed for the bottom and 9th deciles. In contrast, overall the change TFPQ* is positive for each decile in ICT, reflecting a positive reallocation effect that outweighs the negative within-firm effect.

To obtain more insight into the negative within-firm effects, Figure 4 examines a scatterplot of the growth in firm-level TFPQ* against the lagged firm-level revenue share in the index. Note from equation (6) that the within-firm aggregate measure is equal to the sum of the products of the firm-level TFPQ* growth rates and their lagged revenue shares, so the scatterplot is a useful way to examine the elements of this sum. Notably, a large fraction of firms experience negative year-on-year growth rates—driving down the within-firm index. Moreover, there is evidence for manufacturing that the growth rate falls with the firm's share in aggregate revenue. The combination of these findings suggests what we think of as a 'convergence to mediocrity' effect in manufacturing, with lower revenue-share firms experiencing some catch-up, but to a declining mean level of TFPQ*.[9] For ICT, there is some evidence that higher revenue-share firms experienced somewhat higher TFPQ* growth rates, although there is again clear evidence that many firms saw their TFPQ* decline over the sample.

Therefore beyond the puzzle of a slowdown in productivity growth that has been the focus of the literature to date, our findings point to an even greater puzzle of declining *levels* of within-firm quality-adjusted TFP in UK manufacturing. We discuss a possible explanation for this puzzle in Section 6.

5.2 | Testing and imposing CRS on the production function

The alternative approach of using cost shares to estimate firm- and time-varying output elasticities requires us first to test the appropriateness and implications of imposing CRS on our baseline estimation. We denote the returns to scale by $\overline{\beta_j}$, such that $\overline{\beta_j} = \beta_j^l + \beta_j^k + \beta_j^m$, and hence $\beta_j^l = \overline{\beta_j} - \beta_j^k - \beta_j^m$. We have CRS when $\overline{\beta_j} = 1$. We can conveniently write the coefficient on the labour input variable as

$$\frac{(\eta_j - 1)\beta_j^l}{\eta_j} = \frac{\eta_j - 1}{\eta_j}\left[1 - \left(1 - \overline{\beta_j}\right) - \beta_j^k - \beta_j^m\right]$$
$$= 1 - \frac{1}{\eta_j} - \frac{(\eta_j - 1)}{\eta_j}\left(1 - \overline{\beta_j}\right) - \frac{(\eta_j - 1)}{\eta_j}\beta_j^k - \frac{(\eta_j - 1)}{\eta_j}\beta_j^m. \quad (11)$$

We can therefore distribute the labour term in the firm revenue function to yield a re-parametrized revenue function:

$$r_{ijt} - p_{jt} - l_{ijt} = \frac{1}{\eta_j}\left(r_{jt} - p_{jt} - l_{ijt}\right) - \frac{(\eta_j - 1)}{\eta_j}\left(1 - \overline{\beta_j}\right)l_{ijt} + \frac{(\eta_j - 1)\beta_j^k}{\eta_j}\left(k_{ijt} - l_{ijt}\right)$$
$$+ \frac{(\eta_j - 1)\beta_j^m}{\eta_j}\left(m_{ijt} - l_{ijt}\right) + \frac{\eta_j - 1}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt}\right). \quad (12)$$

Given our assumption that the demand-side elasticity of substitution is strictly greater than 1 $\left(\eta_j > 1\right)$, a test for CRS is that the estimated coefficient on the labour variable is not significantly different from zero (i.e. we cannot reject the null that $1 - \overline{\beta_j} = 0$). This test is performed as a standard t-test on the significance of coefficient on the labour variable in equation (12). The results are shown in Tables 2(a) and 2(b). The assumption of CRS is not rejected by the data for most 2-digit industries.

To impose CRS, we impose the restriction that $1 - \overline{\beta_j} = 0$ and estimate the restricted revenue function:

$$r_{ijt} - p_{jt} - l_{ijt} = \frac{1}{\eta_j}\left(r_{jt} - p_{jt} - l_{ijt}\right) + \frac{(\eta_j - 1)\beta_j^k}{\eta_j}\left(k_{ijt} - l_{ijt}\right)$$
$$+ \frac{(\eta_j - 1)\beta_j^m}{\eta_j}\left(m_{ijt} - l_{ijt}\right) + \frac{\eta_j - 1}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt}\right). \quad (13)$$

The results are shown in Tables 3(a) and 3(b), and the decomposition of the implied TFPQ* index in Figure 5. In Tables 3(a) and 3(b), we also show the implied output elasticities, where the output elasticity of labour input is inferred as $\beta_j^l = 1 - \beta_j^k - \beta_j^m$. Overall, the implied time paths for revenue-shared weighted TFPQ* index and its components are similar to those found under the baseline method without the restriction of CRS, albeit the overall fall in TFPQ* is somewhat lower in manufacturing, and there is no overall increase in ICT, as compared with our baseline results (see Figure 5). Both sectors continue to show significant decreases in the within-firm measures.

5.3 | Estimation of TFPQ* using the alternative cost-shares approach under CRS

The finding that CRS represents a reasonable restriction for most 2-digit industries suggests using the cost-shares method as an alternative approach to estimating the output elasticities of the various inputs used in production. As noted in Section 3, an advantage of this approach over our baseline is that it allows for firm- and time-specific output elasticities, which compares to the non-varying elasticities within an industry that are imposed under the baseline method. Under the baseline method, these elasticities are inferred from the relevant estimated coefficient in the estimated revenue function and the implied estimate of η_j from the coefficient on the deflated

TABLE 2(a) Testing for CRS in 2-digit manufacturing industries.

	$1/\eta_j$	$((\eta_j-1)/\eta_j)\left(1-\overline{\beta}\right)$	$((\eta_j-1)/\eta_j)\beta_j^k$	$((\eta_j-1)/\eta_j)\beta_j^m$
	(1)	(2)	(3)	(4)
SIC10	0.12***	0.02	0.22***	0.46***
(Food Products)	(0.02)	(0.02)	(0.05)	(0.05)
SIC11	0.05**	0.05	0.38***	0.37***
(Beverages)	(0.02)	(0.03)	(0.08)	(0.05)
SIC13	0.05**	0.20**	0.45***	0.19
(Textiles)	(0.02)	(0.10)	(0.12)	(0.14)
SIC16	0.16***	−0.02	0.12***	0.23***
(Wood Products)	(0.02)	(0.04)	(0.03)	(0.07)
SIC17	0.09***	0.09***	0.28***	0.05
(Paper Products)	(0.02)	(0.02)	(0.03)	(0.03)
SIC18	0.08***	0.03	0.09**	0.16**
(Printing & Reproduction)	(0.01)	(0.02)	(0.04)	(0.06)
SIC20	0.15***	−0.02	0.29***	0.48***
(Chemicals)	(0.02)	(0.02)	(0.05)	(0.08)
SIC22	0.03***	0.07***	0.003	0.72***
(Rubber & Plastic)	(0.008)	(0.02)	(0.03)	(0.09)
SIC23	0.07***	0.07***	0.01**	0.52***
(Non-Metallic Mineral)	(0.0006)	(0.004)	(0.005)	(0.13)
SIC24	0.05	0.003	0.26***	0.40***
(Basic Metals)	(0.04)	(0.03)	(0.09)	(0.11)
SIC25	0.10***	0.04	0.04***	0.35***
(Fabricated Metal)	(0.01)	(0.03)	(0.01)	(0.08)
SIC26	0.16***	−0.03	0.19***	0.15**
(Comp., Elec. & Optical)	(0.06)	(0.06)	(0.05)	(0.08)
SIC27	0.13***	−0.01	0.11***	0.37***
(Electrical Equipment)	(0.02)	(0.02)	(0.09)	(0.08)
SIC28	0.11***	0.05	0.10***	0.28***
(Machinery and Eqp. n.e.c.)	(0.01)	(0.03)	(0.03)	(0.05)
SIC29	0.04***	0.05	0.06**	0.56***
(Motor Vehicles)	(0.01)	(0.03)	(0.03)	(0.07)
SIC30	0.04***	0.12***	0.07**	0.44***
(Other Transport Eqp.)	(0.01)	(0.02)	(0.03)	(0.05)
SIC31	0.05***	0.02	0.10***	0.43***
(Furniture)	(0.01)	(0.02)	(0.04)	(0.07)
SIC32	0.05***	0.06**	0.13***	0.18***
(Other Manufacturing)	(0.01)	(0.03)	(0.03)	(0.04)
SIC33	0.08***	0.03	0.11***	0.58***
(Repair and Installation)	(0.008)	(0.02)	(0.02)	(0.04)

Notes: $\overline{\beta}_j = \beta_j^l + \beta_j^k + \beta_j^m$ is the measure of returns to scale. Given $\eta_j > 1$, a test for CRS (i.e. $1-\overline{\beta}=0$) can be conducted by testing for the statistical significance of the log labour variable in the regression (see column (2)). Robust standard errors are in parentheses. ***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

TABLE 2(b) Testing for CRS in 2-digit ICT industries.

	$1/\eta_j$ (1)	$((\eta_j-1)/\eta_j)\left(1-\overline{\beta}\right)$ (3)	$((\eta_j-1)/\eta_j)\beta_k$ (3)	$((\eta_j-1)/\eta_j)\beta_m$ (4)
SIC58	0.10***	0.02	0.02***	0.30***
(Publishing Activities)	(0.01)	(0.02)	(0.006)	(0.05)
SIC59	0.05***	−0.05	0.04***	0.19***
(Motion Pictures)	(0.01)	(0.03)	(0.006)	(0.05)
SIC60	0.10***	−0.06	0.17***	0.06
(Programming & Broadcasting)	(0.03)	(0.04)	(0.03)	(0.04)
SIC61	0.07***	0.008	0.05***	0.32***
(Telecommunications)	(0.005)	(0.007)	(0.005)	(0.02)
SIC62	0.11***	−0.01	0.05***	0.43***
(Computer Programming)	(0.01)	(0.01)	(0.01)	(0.04)
SIC63	0.09***	0.02**	0.06***	0.28***
(Information Service)	(0.007)	(0.01)	(0.007)	(0.02)

Notes: $\overline{\beta}_j = \beta_j^l + \beta_j^k + \beta_j^m$ is the measure of returns to scale. Given $\eta_j > 1$, a test for CRS (i.e. $1 - \overline{\beta} = 0$) can be conducted by testing for the statistical significance of the log labour variable in the regression (see column (2)). Robust standard errors are in parentheses.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

industry variable. Under CRS, the firm- and time-specific output elasticity with respect to a given input can be inferred from the input's share in total firm costs. Importantly, the equivalence of the output elasticities and the cost shares under CRS requires only the assumption of cost minimization and is invariant to the assumptions on demand and market structure (see Appendix Subsection A.2).

In addition to data available in the ABS microdata file, calculating cost shares requires data on the user cost of capital. We label this user cost per pound of capital for a firm in industry j as P_{jt}^k. The ONS provides estimates of the total user cost of capital at various industry levels of industry disaggregation. To obtain the user cost per £ of capital at a particular level of disaggregation, we divide the total user costs by the ONS estimate of the total capital stock in the industry at the 2-digit level of disaggregation.[10] Both labour and material costs are obtainable directly from the ABS data. The output elasticity of a given factor input F, for $F \in (L, K, M)$, is then

$$\frac{P_{jt}^f F_{ijt}}{\sum_{F\in(L,K,M)} P_{jt}^f F_{ijt}} = s_{ijt}^f = \beta_{ijt}^f, \tag{14}$$

where P_{jt}^f is the price of input F in industry j at time t, F_{ijt} is the quantity of input F used by firm i in industry j at time t, s_{ijt}^f is the share of the input in the firm's total cost, and β_{ijt}^f is the implied output elasticity of the input. We record the implied average cost share/output elasticities at the 2-digit level in Tables 4(a) and 4(b) (columns (1)–(3)). Overall, the implied output elasticities of capital diverge a lot from those implied by estimation of the baseline revenue function, both with and without the imposition of CRS. In general, the implied capital shares/output elasticities are clustered around 0.02–0.03 for manufacturing (with average 0.023), which is significantly lower than our priors, and generally lower than inferred from the baseline estimated revenue function. In contrast, the estimated capital shares are significantly higher in ICT, with average 0.380—which is much higher than average of 0.06 that we found under our baseline method.

To produce estimates of TFPQ* under the alternative cost-shares approach, the next step is to re-estimate the revenue function with the output elasticities given by the cost shares. The

TABLE 3(a) Estimation of revenue function for 2-digit manufacturing industries with CRS.

	$1/\eta_j$ (1)	$((\eta_j-1)/\eta_j)\beta_j^k$ (2)	$((\eta_j-1)/\eta_j)\beta_j^m$ (3)	β_j^l (4)	β_j^k (5)	β_j^m (6)
SIC10	0.13***	0.20***	0.48***	0.22	0.23	0.55
(Food Products)	(0.02)	(0.05)	(0.05)			
SIC11	0.08***	0.34***	0.37***	0.23	0.37	0.40
(Beverages)	(0.02)	(0.08)	(0.04)			
SIC13	0.10***	0.30	0.22*	0.42	0.34	0.24
(Textiles)	(0.02)	(0.18)	(0.13)			
SIC16	0.15***	0.13***	0.23***	0.57	0.15	0.28
(Wood Products)	(0.03)	(0.04)	(0.06)			
SIC17	0.13***	0.25***	0.07**	0.63	0.29	0.08
(Paper Products)	(0.02)	(0.04)	(0.03)			
SIC18	0.09***	0.07	0.17***	0.73	0.08	0.19
(Printing & Reproduction)	(0.01)	(0.04)	(0.06)			
SIC20	0.14***	0.32***	0.48***	0.07	0.37	0.56
(Chemicals)	(0.02)	(0.05)	(0.08)			
SIC22	0.05***	0.08**	0.75***	0.12	0.09	0.79
(Rubber & Plastic)	(0.01)	(0.03)	(0.08)			
SIC23	0.07***	0.02	0.54***	0.40	0.02	0.58
(Non-Metallic Mineral)	(0.006)	(0.07)	(0.03)			
SIC24	0.05	0.26**	0.40***	0.30	0.27	0.43
(Basic Metals)	(0.03)	(0.11)	(0.12)			
SIC25	0.11***	0.02	0.35***	0.58	0.02	0.40
(Fabricated Metal)	(0.01)	(0.02)	(0.08)			
SIC26	0.14***	0.22***	0.15***	0.58	0.25	0.17
(Comp., Elec. & Optical)	(0.03)	(0.07)	(0.09)			
SIC27	0.12***	0.13	0.38***	0.44	0.14	0.42
(Electrical Equipment)	(0.01)	(0.10)	(0.08)			
SIC28	0.14***	0.09***	0.28***	0.58	0.10	0.32
(Machinery and Eqp. n.e.c.)	(0.01)	(0.03)	(0.05)			
SIC29	0.06***	0.03	0.59***	0.34	0.03	0.63
(Motor Vehicles)	(0.01)	(0.03)	(0.08)			
SIC30	0.07***	0.05	0.45***	0.46	0.05	0.49
(Other Transport Eqp.)	(0.01)	(0.03)	(0.05)			
SIC31	0.05***	0.09**	0.45***	0.44	0.09	0.47
(Furniture)	(0.01)	(0.04)	(0.07)			
SIC32	0.06***	0.07	0.18***	0.73	0.08	0.19
(Other Manufacturing)	(0.01)	(0.04)	(0.05)			
SIC33	0.09***	0.10***	0.58***	0.27	0.10	0.63
(Repair and Installation)	(0.01)	(0.02)	(0.04)			

Notes: $\beta_j^l = 1 - \beta_j^k - \beta_j^m$ under CRS. Robust standard errors are in parentheses.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

TABLE 3(b) Estimation of revenue function for 2-digit ICT industries with CRS.

	$1/\eta_j$ (1)	$((\eta_j-1)/\eta_j)\beta_j^k$ (2)	$((\eta_j-1)/\eta_j)\beta_j^m$ (3)	β_j^l (4)	β_j^k (5)	β_j^m (6)
SIC58	0.10***	0.01	0.29***	0.65	0.02	0.33
(Publishing Activities)	(0.01)	(0.01)	(0.05)			
SIC59	0.03***	0.05***	0.18***	0.76	0.05	0.19
(Motion Pictures)	(0.005)	(0.007)	(0.06)			
SIC60	0.07***	0.21***	0.03	0.74	0.23	0.03
(Programming & Broadcasting)	(0.01)	(0.05)	(0.05)			
SIC61	0.07***	0.04***	0.32***	0.60	0.05	0.35
(Telecommunications)	(0.005)	(0.005)	(0.02)			
SIC62	0.10***	0.06***	0.45***	0.43	0.07	0.50
(Computer Programming)	(0.004)	(0.01)	(0.03)			
SIC63	0.09***	0.05***	0.27***	0.64	0.06	0.30
(Information Service)	(0.006)	(0.008)	(0.02)			

Notes: $\beta_j^l = 1 - \beta_j^k - \beta_j^m$ under CRS. Robust standard errors are in parentheses.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

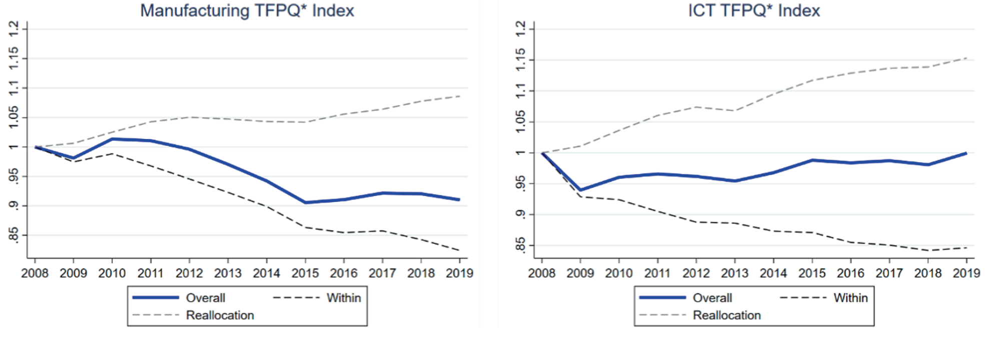

FIGURE 5 Decomposition of TFPQ* index (2008 = 1) imposing CRS. *Notes*: The TFPQ* index shows the evolution of a revenue-share weighted index of TFPQ* between 2008 and 2019, where the value of the index is set equal to 1 in 2008. The revenue shares are calculated at the sectoral level. The decomposition of the index is based on equation (6). We have combined the reallocation and entry/exit terms into a single broad reallocation index.

estimating equation then becomes

$$r_{ijt} - p_{jt} = \frac{1}{\eta_j}\left(r_{jt} - p_{jt}\right) + \frac{\eta_j - 1}{\eta_j}\left(\beta_{ijt}^l l_{ijt}\right) + \frac{\eta_j - 1}{\eta_j}\left(\beta_{ijt}^k k_{ijt}\right) + \frac{\eta_j - 1}{\eta_j}\left(\beta_{ijt}^m m_{ijt}\right) + \frac{\eta_j - 1}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt}\right), \tag{15}$$

where the terms in parentheses—that is, the products of the output elasticities and the log of the input variables—are the new explanatory variables, and the final term is the residual of the revenue function. An obvious problem with directly estimating this equation is that η_j is *over*-identified, with potentially four separate estimates if no restrictions are imposed. However, we can impose the restriction implied by the theory that the values of η_j inferred from the four explanatory variables are all equal by subtracting $r_{jt} - p_{jt}$ from both sides of equation (15). After some cancellation and rearrangement, we obtain the required restricted revenue function:

$$r_{ijt} - r_{jt} = \frac{\eta_j - 1}{\eta_j}\left(\beta_{ijt}^l l_{ijt} + \beta_{ijt}^k k_{ijt} + \beta_{ijt}^m m_{ijt} - \left(r_{jt} - p_{jt}\right)\right) + \frac{\eta_j - 1}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt}\right). \tag{16}$$

As before, following estimation of equation (16), our estimates of the firm- and time-specific TFPQ* values are then obtained from the residuals and the inferred industry-specific value of η_j.

TABLE 4(a) Implied cost shares/output elasticities and implied elasticity of substitution under the cost-shares method—2-digit manufacturing industries.

	Cost shares/Output elasticities				
	β_j^l (1)	β_j^k (2)	β_j^m (3)	$(\eta_j - 1)/\eta_j$ (4)	Implied η_j (5)
SIC10	0.25***	0.025***	0.72***	0.81***	5.2
(Food Products)	(0.01)	(0.003)	(0.01)	(0.02)	
SIC11	0.18***	0.034***	0.78***	0.85***	6.7
(Beverages)	(0.02)	(0.003)	(0.02)	(0.05)	
SIC13	0.30***	0.016***	0.68***	0.87***	7.5
(Textiles)	(0.03)	(0.003)	(0.03)	(0.04)	
SIC16	0.27***	0.016***	0.72***	0.87***	8.2
(Wood Products)	(0.01)	(0.004)	(0.01)	(0.04)	
SIC17	0.23***	0.03***	0.74***	0.67***	3.1
(Paper Products)	(0.01)	(0.002)	(0.01)	(0.07)	
SIC18	0.36***	0.016***	0.62***	0.73***	3.7
(Printing & Reproduction)	(0.02)	(0.003)	(0.03)	(0.05)	
SIC20	0.21***	0.022***	0.77***	0.76***	4.2
(Chemicals)	(0.02)	(0.003)	(0.02)	(0.05)	
SIC22	0.26***	0.022***	0.71***	0.77***	4.4
(Rubber & Plastic)	(0.02)	(0.003)	(0.02)	(0.04)	
SIC23	0.19***	0.021***	0.78***	0.96***	29.7
(Non-Metallic Mineral)	(0.02)	(0.008)	(0.03)	(0.01)	
SIC24	0.24***	0.015***	0.74***	0.87***	8.1
(Basic Metals)	(0.02)	(0.005)	(0.02)	(0.05)	
SIC25	0.35***	0.03***	0.63***	0.91***	11.2
(Fabricated Metal)	(0.01)	(0.005)	(0.02)	(0.02)	
SIC26	0.33***	0.021***	0.65***	0.81***	5.3
(Comp., Elec. & Optical)	(0.02)	(0.003)	(0.02)	(0.07)	
SIC27	0.29***	0.03***	0.68***	0.81***	5.4
(Electrical Equipment)	(0.02)	(0.001)	(0.02)	(0.05)	
SIC28	0.31***	0.02***	0.67***	0.86***	7.5
(Machinery and Eqp. n.e.c.)	(0.02)	(0.003)	(0.02)	(0.02)	
SIC29	0.23***	0.02***	0.74***	0.91***	11.8
(Motor Vehicles)	(0.01)	(0.002)	(0.01)	(0.04)	
SIC30	0.29***	0.031***	0.67***	0.97***	40.9
(Other Transport Eqp.)	(0.01)	(0.004)	(0.01)	(0.02)	
SIC31	0.31***	0.018***	0.67***	0.93***	14.4
(Furniture)	(0.02)	(0.005)	(0.03)	(0.02)	
SIC32	0.33***	0.017***	0.65***	0.97***	34.9
(Other Manufacturing)	(0.02)	(0.004)	(0.02)	(0.03)	
SIC33	0.34***	0.025***	0.63***	0.87***	8.2
(Repair and Installation)	(0.02)	(0.005)	(0.02)	(0.02)	

Notes: Standard errors are in parentheses. The reported cost shares are within-industry averages. The cost-shares and standard errors for capital shares are reported at three digits due to the small size of the standard errors.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

TABLE 4(b) Implied cost shares/output elasticities and implied elasticity of substitution under the cost-shares method—2-digit ICT industries.

	Cost shares/Output elasticities				
	β_j^l (1)	β_j^k (2)	β_j^m (3)	$(\eta_j - 1)/\eta_j$ (4)	Implied η_j (5)
SIC58	0.52***	0.41***	0.07***	0.90***	10.2
(Publishing Activities)	(0.04)	(0.03)	(0.02)	(0.02)	
SIC59	0.64***	0.30***	0.06***	0.77***	4.4
(Motion Pictures)	(0.05)	(0.05)	(0.01)	(0.03)	
SIC60	0.56***	0.38***	0.05***	0.73***	3.7
(Programming & Broadcasting)	(0.03)	(0.04)	(0.01)	(0.06)	
SIC61	0.70***	0.23***	0.06***	0.88***	8.7
(Telecommunications)	(0.04)	(0.02)	(0.02)	(0.01)	
SIC62	0.46***	0.48***	0.05***	0.85***	6.8
(Computer Programming)	(0.01)	(0.02)	(0.02)	(0.02)	
SIC63	0.42***	0.48***	0.09**	0.83***	5.9
(Information Service)	(0.04)	(0.04)	(0.01)	(0.02)	

Notes: Standard errors are in parentheses. The reported cost shares are within-industry averages.
***, **, * denote statistical significance at the 1%, 5%, 10% levels, respectively.

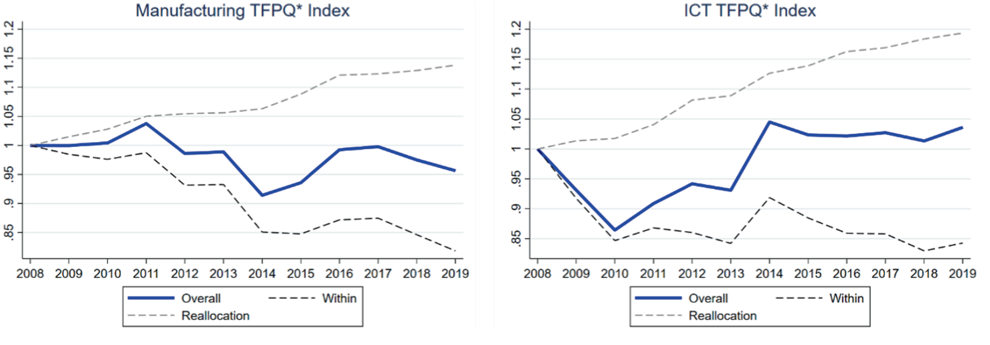

FIGURE 6 Decomposition of TFPQ* index (2008 = 1) using the alternative cost-shares method. *Notes*: The TFPQ* index shows the evolution of a revenue-share weighted index of TFPQ* between 2008 and 2019, where the value of the index is set equal to 1 in 2008. The revenue shares are calculated at the sectoral level. The decomposition of the index is based on equation (6). We have combined the reallocation and entry/exit terms into a single broad reallocation index.

Tables 4(a) and 4(b) also record the estimated values of $(\eta_j - 1)/\eta_j$ and the consequent implied values of η_j for each 2-digit industry in manufacturing and ICT. There is a wide range in the estimated demand-side elasticities of substitution across industries. For manufacturing, these estimates range from a low of 3.1 in Paper Products (SIC17) to a higher of 40.9 in Other Transport Equipment (SIC30). For ICT, the range of estimates is lower, with a low of 3.7 in Publishing and Broadcasting (SIC60) to a high of 10.2 in Publishing Activities (SIC58).

Figure 6 Shows The decomposition of the revenue-weighted index of TFPQ* for the two sectors. These alternative estimates also show declines in the within-firm index over the the sample of 18% in manufacturing and 16% in ICT—which are even higher than we find under our baseline method.

However, a more notable difference between these alternative estimates and our baseline estimates is seen for the reallocation effect, which is now positive over the sample period for manufacturing. As before, the reallocation effect is positive for ICT. This suggests that the

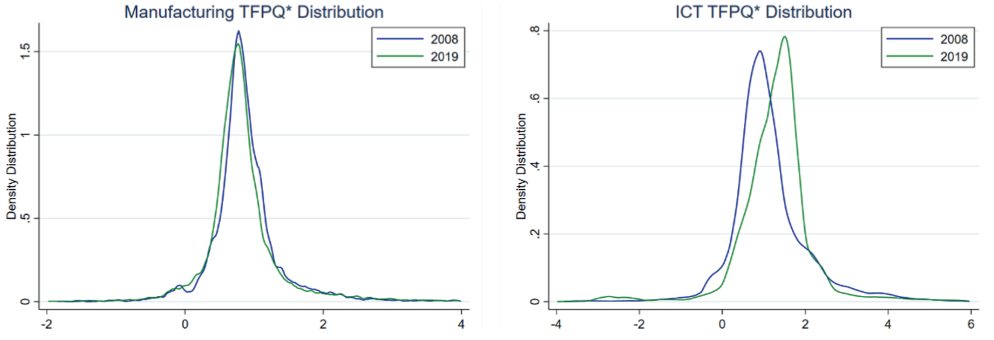

FIGURE 7 Comparison of log TFPQ* distributions in 2008 and 2019 (cost-shares method).

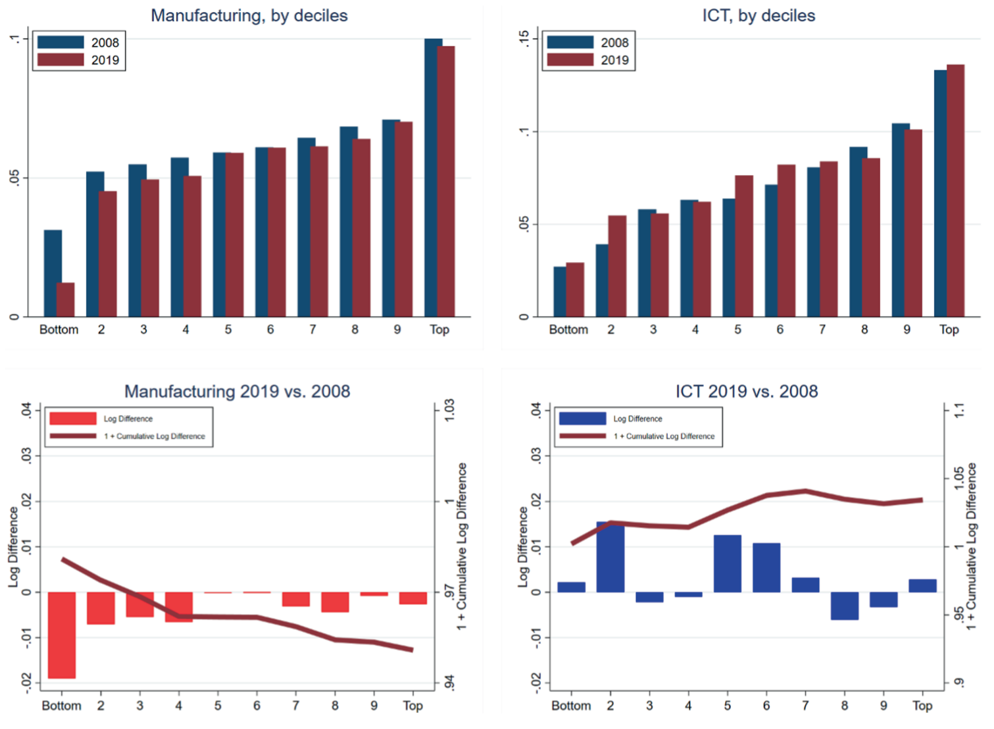

FIGURE 8 Measured TFPQ* across deciles—cost-shares method. *Notes*: Deciles based on the cumulative market share (see text and Appendix Subsection A.3 for details). The figure shows the revenue-share weighted average TFPQ* for the firms in the decile. In the lower panels, 1 + cumulative log difference matches the overall change in the index of TFPQ* between 2008 and 2019 shown in Figure 6.

estimation of the distribution of TFPQ* is quite sensitive to the method used. A comparison of Figures 2 and 7—shown in Figure 7 for the start and end years of our sample—confirms this. Figure 8 repeats our decile analysis, which again shows evidence of falling dispersion across all deciles for manufacturing, but a more mixed picture across the deciles in ICT.

Repeating the scatterplot analysis of the relationship between the firm-level growth rate of estimated TFPQ* lagged sectoral revenue shares (Figure 9), we again find evidence that a large majority of firms in both sectors experienced negative year-on-year growth rates, with the largest revenue-share firms in manufacturing appearing, as with the baseline method, to experience that largest deteriorations in their measured TFPQ*.

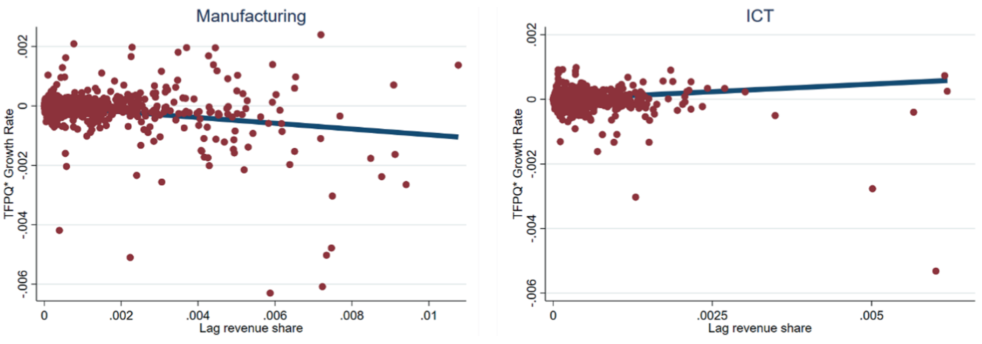

FIGURE 9 Understanding the decline in within-firm TFPQ*: scatterplot of TFPQ* growth and lagged revenue (cost-shares method). *Notes*: Outliers including TFPQ* growth rate below −0.006 and lag revenue share above 0.008 are trimmed (ICT only) for presentation purposes.

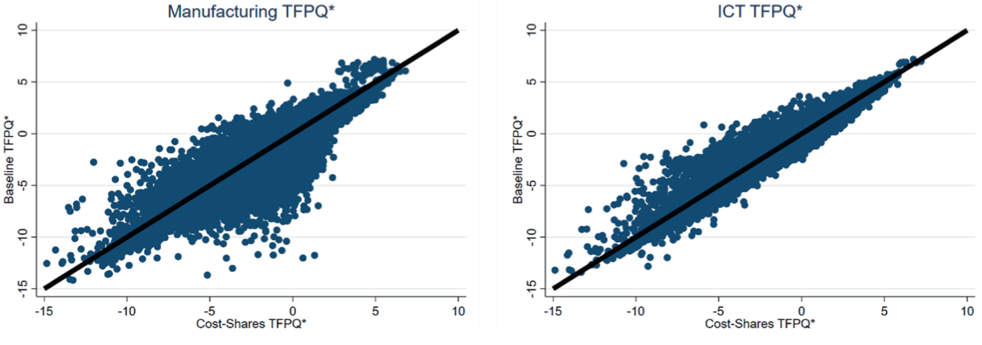

FIGURE 10 Comparison of measured log TFPQ* between the baseline and cost-shares methods. *Notes*: We show measured log TFPQ* for each method normalized by the method-specific revenue-share weighted mean TFPQ* for the sample. The correlations between the methods are 0.94 for manufacturing and 0.96 for ICT.

Finally, given the differences in the implied TFPQ* distributions across the two methods, we attempt to gain more insight into these differences using Figure 10, showing a scatterplot of the two measures of log TFPQ* across the entire sample, where each measure is first normalized by the revenue-share weighted mean TFPQ* in the sample. The correlation coefficient between the measures is 0.94 for manufacturing and 0.96 for ICT, which suggests that the measures produce reasonably similar although not identical results, but with some notable outliers (especially for manufacturing). Overall, the figure reveals that the differences between the methods are more pronounced for manufacturing than ICT. The sensitivity of the implied distributions to the precise assumptions made in the calculation of a measure indicates that we need to be cautious in drawing strong conclusions from any specific measure. Nevertheless, the finding of a significant fall in the within-firm measure of the level of TFPQ* in both manufacturing and ICT appears to be robust across methods. We next turn to a possible explanation for this striking—but puzzling—finding.

6 | WHAT EXPLAINS THE PUZZLING FALL IN MEASURED WITHIN-FIRM TFPQ*?

The title of our paper refers to new measures and new puzzles. As the phrase is typically used, the ‘productivity puzzle’ refers to the disappointing productivity growth performance of advanced economies. These economies have also seen deterioration in other aggregate variables, such as the average markup, the labour share and business dynamism—effects that have previously been associated with increasing firm heterogeneity (Elsby *et al.* 2013; Karabarbounis and Neiman 2014; De Loecker and Eeckhout 2018; Autor *et al.* 2020; De Loecker *et al.* 2020; Decker *et al.* 2020; Diez *et al.* 2022). As noted in the Introduction, the 'productivity puzzle' has been particularly pronounced for the UK since the financial crisis (for recent discussions, see De Loecker *et al.* 2022; Van Reenen and Yang 2023; Goodridge and Haskel 2023).

We have provided new measures of firm-level TFPQ* that show, on average, a decline in the level of within-firm TFP for UK manufacturing. Although the slower growth rate of TFP in the UK since the financial crisis has been well documented, such pronounced falls in its *level* are even more puzzling. On its face, this suggests outright falls in the quality of products produced by UK firms or cost-increasing technological regress, both of which seem implausible.

A possible explanation for this new puzzle stems from the closed economy nature of the model that we use to derive the revenue function that we apply to the UK microdata. Consider instead a global economy version of the model in the spirit of Krugman (1980), where the relevant manufacturing index includes all firms operating in the global market, and we abstract from transportation costs. We continue to assume that heterogeneous firms compete with differentiated products with an elasticity of substitution in demand equal to η_j. However, we assume that the quality-adjusted price index used in the estimation is (as in our application) specific to UK firms. Moreover, assume that the relationship between the true global price index P^*_{jt} and the applied domestic price index P_{jt} is given by

$$P^*_{jt} = B_{jt} P_{jt}, \tag{17}$$

where B_{jt} can be interpreted as the bias in using the domestic price index in place of the true global price index. All else equal, relative improvements in the quality of foreign products in the index will lower the value of B_{jt} (i.e. the true global price index is lower for the given value of the domestic price index).

A second potential source of bias in our country-specific estimated equation enters through the use of domestic industry revenue rather than global industry revenue in our estimated revenue function. Letting R^*_{jt} represent global industry revenue, R_{jt} domestic industry revenue, and S_{jt} the share of domestic industry revenue in global industry revenue, we have

$$R^*_{jt} = \frac{R_{jt}}{S_{jt}}. \tag{18}$$

If the correct deflated revenue function includes the global price index and global industry revenue, then the revenue function that we use for the estimation can be seen to take the form

$$\frac{R_{ijt}}{B_{jt}P_{jt}} = \left(\Lambda_{ijt}\Omega_{ijt}\right)^{(\eta_j-1)/\eta_j} \left(\frac{R_{jt}}{S_{jt}B_{jt}P_{jt}}\right)^{1/\eta_j} L_{ijt}^{\beta_j^l(\eta_j-1)/\eta_j} K_{ijt}^{\beta_j^k(\eta_j-1)/\eta_j} M_{ijt}^{\beta_j^m(\eta_j-1)/\eta_j}. \tag{19}$$

Writing in logs and rearranging, we can then write the revenue function as

$$\begin{aligned} r_{ijt} - p_{jt} = {} & \frac{1}{\eta_j}\left(r_{jt} - p_{jt}\right) + \frac{\left(\eta_j - 1\right)\beta_j^l}{\eta_j} l_{ijt} + \frac{\left(\eta_j - 1\right)\beta_j^k}{\eta_j} k_{ijt} \\ & + \frac{\left(\eta_j - 1\right)\beta_j^m}{\eta_j} m_{it} + \left[\frac{\left(\eta_j - 1\right)}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt} + b_{jt}\right) - \frac{1}{\eta_j} s_{jt}\right], \end{aligned} \tag{20}$$

where $b_{jt} = \ln B_{jt}$ and $s_{jt} = \ln S_{jt}$. The term in square brackets is the residual of the closed-economy revenue function that is used to infer log TFPQ*. Given the relatively high elasticity of substitution, the aggregated domestic revenue-share effect should be small. However, a downward trend in b_{jt}, due to some combination of the deteriorating *relative* quality of UK products and the deteriorating *relative* technical efficiency of UK firms competing on international markets, could show up as a significantly declining trend in the log of measured TFPQ*. Both effects would lead to a decline in the true global price index relative to the domestic price index, reducing the demand for UK products and consequently firm revenues. A full investigation of this explanation for the puzzle of declining within-firm TFPQ* would require a consideration of non-UK firms in addition to domestic firms, which is beyond the scope of the present paper. However, we believe that consideration of such relative quality/technical efficiency effects in the context of international competition provides the most promising avenue for explaining the puzzling falls in measured within-firm TFPQ* in manufacturing that we document in this paper.

7 | CONCLUSION

This paper has developed a framework that allows estimation of quality-adjusted TFP (TFPQ*) from firm-level revenue and input expenditure data, given an estimate of the demand-side elasticity of substitution. We derive an estimate of this elasticity from the coefficient on a deflated industry revenue variable in the revenue function. We address endogeneity in the firm-level revenue function by using the Blundell–Bond System GMM to obtain consistent estimates of the relevant elasticities. This estimated revenue function can then be used to derive measures of TFPQ*, even though separate data on firm-level prices and quantities are unavailable. By making an additional assumption of constant returns to scale, we produce estimates at firm level of the output elasticities, and use this cost-shares method to provide alternative measures of TFPQ*. This second approach confirms the robustness of our findings.

These findings are striking. For manufacturing, we find that annual firm-level TFPQ* fell for a majority of firms, including a 13–18% decline in the within-firm measure of TFPQ* at the industry level between 2008 and 2019; for ICT, we find a decline of between 11% and 16%. These falls in within-firm TFPQ* are the main finding in the paper, particularly given the robustness of the results across the two alternative methods of measuring TFPQ*. These findings extend the UK productivity puzzle.

The reason for the within-firm declines is beyond the scope of the present analysis and represents an important area for future research. We conjecture that this trend reflects relative product quality and technical efficiency effects in international competition, rather than outright quality/technological regression, but testing this potential cause of within-firm declines in productivity in such key sectors of the UK economy requires extension to an open economy framework.

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This work uses data from the ONS. This does not imply endorsement by the ONS in relation to the interpretation or analysis of the statistical data. This work uses research datasets that may not exactly reproduce published National Statistics aggregates.

ENDNOTES

[1] We emphasize that we use here the cost-shares approach to infer measures of a 'fundamentals'-based measure of TFP. Following Foster *et al.* (2008) and Hsieh and Klenow (2009), a large literature has used a revenue-based measure of TFP to infer a measure of 'distortions' or allocative inefficiencies. Blackwood *et al.* (2021), Foster *et al.* (2016) and Haltiwanger (2016) discuss how the type of residual-based measures of (revenue) TFP that we use in this paper are properly interpreted as measures of fundamentals rather than allocative distortions. They also caution that the interpretation of various measures of TFP depend on the 'devilish details' of the definitions and assumptions underlying the measures.

[2] See Bond and Söderbom (2005).

[3] In production function estimation, the control function approach provides the main alternative to dynamic panel estimation (Olley and Pakes 1996; Blundell and Bond 1998, 2000; Levinsohn and Petrin 2003; Ackerberg *et al.* 2015). De Loecker and Syverson (2021) provide an excellent overview of the two approaches in different settings. In addition to its good performance in previous production function estimation settings, the flexibility of a dynamic panel estimation in a revenue function estimation setting leads us to choose it for our empirical analysis. A common challenge for all methods is in obtaining reasonable output elasticities for capital. As discussed further in Section 5, we find that the System GMM estimator produces reasonable output elasticities of capital in our setting.

[4] See Foster *et al.* (2001) and Blackwood *et al.* (2021) for discussion of the decomposition and comparisons with alternative approaches.

[5] Following Haltiwanger (1997) and De Loecker *et al.* (2020), we de-mean by the appropriate aggregate (revenue weighted) level in order to correctly identify the reallocation term.

[6] This follows a strand of literature, including Oulton (1998), Griffith (1999), Harris (2002), Harris and Robinson (2005), and Harris and Moffat (2015, 2017).

[7] Although this approach assumes that the firm exists in perpetuity, the effects of historically distant investments have negligible effects on the estimate of the initial capital stocks due to the growth rate and depreciation assumptions.

[8] We implement depreciation rates provided by the EU KLEMS database (from the additional variables column); see http://www.euklems.net (accessed 19 June 2024).

[9] However, as our data are at the plant level, care must be taken in inferring that larger revenue-share enterprises have lower growth rates. A given enterprise might be comprised of multiple plants, each with relatively low revenue shares at the industry level. It is possible, then, that a given large enterprise comprising multiple smaller plants exhibits high TFPQ* growth.

[10] The ONS capital services estimates can be accessed at https://www.ons.gov.uk/economy/economicoutputandproductivity/output/datasets/capitalservicesestimates (accessed 19 June 2024). We apply the user costs data from lines 15 and 19 on the content page.

[11] Quality change thus enters the utility function in a 'better is more' form (for a related analysis in the context of combining different vintages of capital in a capital aggregate, see Fisher 1965).

[12] There are several definitions of TFPR in the literature. In the context of a model without quality change, Hsieh and Klenow (2009) define TFPR as TFP multiplied by price, and show that under certain conditions, TFPR does not vary with TFP. Blackwood *et al.* (2021) employ the intuitive approach of identifying TFPR as the residual from an estimated revenue function, and label this TFP^{rr} (where rr denotes regression residual), where the relevant elasticities used to identify TFP^{rr} are revenue elasticities and not output elasticities. We adopt the regression residual approach for measurement of TFPR in this paper.

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APPENDIX A

A.1 Derivation of the revenue function

A.1.1. Consumer preferences

Consumers are assumed to engage in two-stage budgeting whereby they first allocate expenditure shares to specified expenditure aggregates and then decide on their choices of the goods within those aggregates. More specifically, consumers are assumed to have a CRS Cobb–Douglas utility function over CES indexes Z_{jt} of the goods produced by J industries:

$$U_t = \prod_{j=1}^{J} Z_{jt}^{\alpha_j}, \quad \text{where } \alpha_J = 1 - \sum_{j=1}^{J-1} \alpha_j. \tag{A1}$$

As the nested utility function is homothetic, we can sum equation (A1) over consumers to get the aggregate output index Y_t, and can define the aggregate price index as

$$\sum_{j=1}^{J} P_{jt} Z_{jt} = P_t Y_t, \tag{A2}$$

where the price of a unit of the Z_{jt} index is P_{jt}. Maximizing their utility, the representative consumer allocates their nominal income over the two aggregates, to yield the expenditure shares.

$$P_{jt} Z_{jt} = \alpha_j P_t Y_t, \tag{A3}$$

where $P_t Y_t$ is nominal income.

Aggregate industry j consumption/output is a (homothetic) CES function of the quality-adjusted goods produced by the N firms in the industry:

$$Z_{jt} = \left[\sum_{i=1}^{N} \left(\Lambda_{ijt} Q_{ijt} \right)^{(\eta_j - 1)/\eta_j} \right]^{(\eta_j - 1)/\eta_j}, \tag{A4}$$

where Λ_{ijt} is a measure of the quality of the good produced by firm i in industry j at time t, Q_{ijt} is the volume output produced by firm i in industry j at time t, and η_j is the elasticity of substitution between the N goods in the index for industry j. We thus incorporate both a representative consumer with a preference for variety and vertical differentiation based on quality between products that enter into the industry output index. We denote quality-adjusted output as $Q^*_{ijt} = \Lambda_{ijt} Q_{ijt}$.[11] We assume that $\eta_j > 1$ and that each firm produces a single product variety.

A.1.2. Firm production functions

We next derive the demand curve facing an individual firm i in industry j at time t producing a good with the quality level Λ_{ijt}. Given the allocation of income to manufacturing goods, we can use standard results in the literature to derive the demand function facing a firm with quality level Λ_{ijt} as

$$Q_{ijt} = \Lambda_{ijt}^{\eta_j - 1} \left(\frac{P_{ijt}}{P_{jt}} \right)^{-\eta_j} Z_{jt} = \Lambda_{ijt}^{\eta_j - 1} \left(\frac{P_{ijt}}{P_{jt}} \right)^{-\eta_j} \frac{\alpha_j P_t Y_t}{P_{jt}}, \tag{A5}$$

where the price index for the industry, P_{jt}, is given by

$$P_{jt} = \left[\sum_{i=1}^{N} \left(\frac{P_{ijt}}{\Lambda_{ijt}} \right)^{\eta_j - 1} \right]^{1/(\eta_j - 1)}. \tag{A6}$$

From equation (A6), we can see that quality improvements are reflected in a lower industry price index. Moreover, the effect of a change in quality on the cost of achieving a particular level of Z_{jt} is equivalent to a price change of equal proportion but opposite in sign.

Turning to the production function for a firm i in industry j at time t, we assume that each firm has the Cobb–Douglas production function

$$Q_{ijt} = \Omega_{ijt} L_{ijt}^{\beta_j^l} K_{ijt}^{\beta_j^k} M_{ijt}^{\beta_j^m}, \tag{A7}$$

where Ω_{ijt} is a (firm-specific) measure of Hicks-neutral technical change, L_{ijt} is labour, K_{ijt} is fixed capital, and M_{ijt} is materials. The industry-specific output elasticities for labour, capital and materials are given by β_j^l, β_j^k and β_j^m, respectively.

A.1.3. Firm revenue functions

To derive the firm revenue function, we first write the demand function (A5) in inverse form as

$$\frac{P_{ijt}}{P_{jt}} = \Lambda_{ijt}^{(\eta_j-1)/\eta_j} Q_{ijt}^{-1/\eta_j} \left(\frac{\alpha\, P_t Y_t}{P_{jt}} \right)^{1/\eta_j}, \tag{A8}$$

where the quality indicator, Λ_{ijt}, is a shift factor for the inverse demand function.

Using equations (A2), (A7) and (A8), total deflated firm revenue is

$$\frac{R_{ijt}}{P_{jt}} = \frac{P_{ijt} Q_{ijt}}{P_{jt}} = \left(\Lambda_{ijt}\Omega_{ijt}\right)^{(\eta_j-1)/\eta_j} \left(\frac{R_{jt}}{P_{jt}} \right)^{1/\eta_j} L_{ijt}^{\beta_j^l(\eta_j-1)/\eta_j} K_{ijt}^{\beta_j^k(\eta_j-1)/\eta_j} M_{ijt}^{\beta_j^m(\eta_j-1)/\eta_j}, \tag{A9}$$

where industry revenue is $R_{jt}=P_{jt}Z_{jt} = \alpha_j\, P_t Y_t$.

From equation (A9), total revenue varies with the increased use of factors of production for two reasons. First, an increase in the use of a factor of production (say labour) leads to an increase in physical output. Second, the firm must lower its price to sell this increased level of output given that it faces a downward-sloping demand curve. The coefficient on each input is the revenue elasticity of the input, $\left((\eta_j - 1)/\eta_j\right)\beta_j^f$ for $f \in (l, k, m)$, where the revenue elasticity will be lower than the output elasticity given our assumption that $\eta_j > 1$.

We next define our measure of revenue-based total factor productivity (TFPR).[12] We denote TFPR for firm i in period t as Ψ_{ijt}. Analogously with the representation of technical progress in a production function, we identify Ψ_{ijt} based on a general multiplicative form of the revenue function:

$$R_{ijt} = \Theta_{ijt}\, G\left(R_{jt}/P_{jt}, L_{ijt}, K_{ijt}, M_{ijt}\right). \tag{A10}$$

Given the specific form of the revenue function (A9), we can thus identify TFPR as

$$\Theta_{ijt} = \left(\Lambda_{ijt}\Omega_{ijt}\right)^{(\eta_j-1)/\eta_j}. \tag{A11}$$

Taking natural logs of equation (A9) and rearranging, we obtain

$$r_{ijt} - p_{jt} = \frac{1}{\eta_j}\left(r_{jt} - p_{jt}\right) + \frac{(\eta_j - 1)\beta_j^l}{\eta_j} l_{ijt} + \frac{(\eta_j - 1)\beta_j^k}{\eta_j} k_{ijt} + \frac{(\eta_j - 1)\beta_j^m}{\eta_j} m_{ijt} + \frac{(\eta_j - 1)}{\eta_j}\left(\lambda_{ijt} + \omega_{ijt}\right), \tag{A12}$$

where lowercase letters represent the natural log of a variable. A convenient feature of equation (A12) is that identification of η_j is possible from the estimated coefficient on the deflated-industry-revenue variable in the estimated firm revenue function (Klette and Griliches 1996). Using this estimate of the elasticity of substitution, the output elasticities β_j^l, β_j^k and β_j^m can then be obtained from the estimated coefficients on l_{ijt}, k_{ijt} and m_{ijt}, respectively.

The natural log of TFPR is identified as

$$\theta_{ijt} = r_{ijt} - p_{jt} - \left(\frac{1}{\eta_j}\left(r_{jt} - p_{jt}\right) + \frac{\left(\eta_j - 1\right)\beta_j^l}{\eta_j} l_{ijt} + \frac{\left(\eta_j - 1\right)\beta_j^k}{\eta_j} k_{ijt} + \frac{\left(\eta_j - 1\right)\beta_j^m}{\eta_j} m_{ijt} \right). \quad \text{(A13)}$$

Finally, the natural log of TFPQ* (which includes both product quality and technical efficiency components) is

$$\lambda_{ijt} + \omega_{ijt} = \frac{\eta_j}{\eta_j - 1}\theta_{ijt}. \quad \text{(A14)}$$

Given an estimate of η_j, it is therefore possible to identify both TFPR and TFPQ* from revenue and input data, given our structural assumptions. It is also possible to obtain estimates of the relevant output elasticities for the different inputs from the estimates of the revenue elasticities of those estimates and an estimate of the elasticity of substitution. From equations (A13) and (A14), note that the estimated value of TFPR will be close to the estimated value of TFPQ* when the estimated elasticity of substitution η_j is high. As we typically find a high estimate of η_j in our empirical application, we report only the results for TFPQ* in our main analysis.

A.2 Equivalance of output elasticities and cost-shares under CRS

In this subsection, we show how output elasticities can be inferred from cost shares under CRS. Importantly, this depends only on cost minimization, thus is independent of the specifications of demand and market structure.

We first specify the Lagrangian for the cost-minimization problem:

$$\mathcal{L}_{ijt} = P_{jt}^l L_{ijt} + P_{jt}^k K_{ijt} + P_{jt}^m M_{ijt} + \chi_{ijt}\left(\overline{Q}_{ijt} - F\left(L_{ijt}, K_{ijt}, M_{ijt}\right)\right), \quad \text{(A15)}$$

where P_{jt}^l, P_{jt}^k and P_{jt}^m are the prices of labour, capital and materials, and χ_{ijt} is the Lagrangian multiplier, which is also equal to marginal cost. Importantly, under CRS, marginal cost is equal to average cost.

The first-order condition with respect to an input F_{ijt}, where $F_{ijt} \in \left(L_{ijt}, K_{ijt}, M_{ijt}\right)$, is given by.

$$\frac{\partial \mathcal{L}_{ijt}}{\partial F_{ijt}} = P_{jt}^f - \chi_{ijt}\frac{\partial Q_{ijt}}{\partial F_{ijt}} = 0 \quad \Rightarrow \quad \frac{\partial Q_{ijt}}{\partial F_{ijt}} = \frac{P_{jt}^f}{\chi_{ijt}}. \quad \text{(A16)}$$

Now noting that $\chi_{ijt} = MC_{ijt} = TC_{ijt}/Q_{ijt}$ under CRS, we can rewrite equation (A16) as

$$\frac{\partial Q_{ijt}}{\partial F_{ijt}} = \frac{P_{jt}^f}{TC_{ijt}/Q_{ijt}}. \quad \text{(A17)}$$

Finally, multiplying both sides by F_{ijt}/Q_{ijt}, we obtain.

$$\frac{\partial Q_{ijt}}{\partial F_{ijt}}\frac{F_{ijt}}{Q_{ijt}} = \frac{P_{jt}^f}{TC_{ijt}/Q_{ijt}}\frac{F_{ijt}}{Q_{ijt}} \quad \Longrightarrow \quad \beta_{ijt}^f = \frac{P_{jt}^f F_{ijt}}{TC_{ijt}} = s_{ijt}^f \quad \text{(A18)}$$

where β_{ijt}^f is the output elasticity of input F, and s_{ijt}^f is the input F share in total cost. Thus, under CRS, cost minimization is sufficient for the inference of output elasticities from the relevant input's share in total cost.

A.3 Decomposing the overall change in TFPQ* by decile

We start with the definition of the log of the revenue-share weighted geometric mean of TFPQ* used in equation (5):

$$x_{ct} = \sum_{i=1}^{N_{ct}} s_{ict} x_{ict}, \tag{A19}$$

where there are N_{ct} firms in sector c at time t. The change in x_{ct} between period 1 and period T (our measure of the sector-level growth rate between those periods) is then

$$\Delta x_{cT} = \sum_{i=1}^{N_{jT}} s_{icT} x_{ijT} - \sum_{i=1}^{N_{j1}} s_{ic1} x_{ic1}. \tag{A20}$$

For each period, we can rank the firms from lowest to highest TFPQ*. Let firm i's rank in a given period t be given by r_{ict}, where $t \in (1, T)$. We denote the log TFPQ* of a firm at rank r_{ict} in sector c at time t as $x_{r_{ict}}$, and a firm's share of total sector revenue at rank r_{ict} as $s_{r_{ict}}$.

Reordering the sums in terms of ranks, we can thus rewrite equation (A20) as

$$\Delta x_{cT} = \sum_{r_{iT}=1}^{N_{cT}} s_{r_{icT}} x_{r_{icT}} - \sum_{r_{i1}=1}^{N_{c1}} s_{r_{ic1}} x_{r_{ic1}}. \tag{A21}$$

For each period, we now identify the deciles such that the firms in a decile constitute 10% of total sector revenue. The cut-off rank for a given decile d at time t is r^*_{dct}, where $d = 1, \ldots, 10$. We can thus sort the ranked firms in any given time period so that

$$\Delta x_{cT} = \sum_{d=1}^{10} \left(\sum_{r_{icT}=r^*_{(d-1)cT}+1}^{r^*_{dcT}} s_{r_{icT}} x_{r_{icT}} - \sum_{r_{ic1}=r^*_{(d-1)c1}+1}^{r^*_{dc1}} s_{r_{ic1}} x_{r_{ic1}} \right), \tag{A22}$$

where we note that $r^*_{(d-1)ct} = 0$ when $d = 1$. Effectively, we are just sorting the firms into deciles, but for a given time period, each firm is included somewhere, so the overall revenue-weighted geometric mean for a given time period must remain unchanged. Thus the change in the revenue-share weighted geometric mean can be written as the sum of the respective changes over the 10 deciles, where each decile accounts for 10% of the total sector revenue by construction.