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**León, Á. and Níguez, T.M.**

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# Copula methods for evaluating relative tail forecasting performance

Ángel León\*

Dept. Fundamentos del Análisis Económico, Universidad de Alicante,  
Campus San Vicente del Raspeig, Alicante 03080, Spain.

Email: aleon@ua.es

Trino-Manuel Níguez†

School of Organisations, Economy and Society  
Westminster Business School, University of Westminster,  
35 Marylebone Road, London NW1 5LS, UK.

Email: T.M.Niguez@wmin.ac.uk

## Abstract

In this paper we illustrate the use of conditional copulas for identifying differences in alternative portfolio performance strategies. We analyze which portfolios are capable of providing superior performance to those based on the Sharpe ratio.

**Keywords:** Conditional copula; Conditional performance measures; Equity-screening; GJR; SNP distribution

**JEL classification codes:** C22, G11.

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†Corresponding author

# 1 Introduction

The copula approach has been widely used as a flexible tool for modeling tail dependence between financial time series. See, for instance, Cherubini et al. (2004) and Patton (2012) for a review of copula models for economic time series. We propose copula methods to evaluate models' performance differences. In particular, we use conditional copula for studying the tail dependence patterns of portfolio return series obtained through equity-screening based on performance measures (PMs). This approach is useful to identify differences in performance related to the tails of a distribution. We analyze the relationship between the portfolio return distributions obtained under alternative conditional PMs with regard to the benchmarking conditional Sharpe ratio (SR), see Sharpe (1994). For example, if two conditional PMs lead to similar (different) stock screenings, then the corresponding bivariate return distribution would exhibit a very high (low) dependence according to copula methods. We adopt different tail dependence patterns as specified by the following copula models: Gaussian; symmetrized Joe-Clayton (SJC); Gumbel and Clayton.

The conditional PMs are closed-form expressions based on the semi-nonparametric (SNP) distribution of Gallant and Nychka (1987) and obtained in León and Níguez (2020). Indeed, our portfolio returns series data come from PMs based on asymmetric reward/risk measures with respect to those from the SR. The alternative conditional PMs considered are the following: (a) The skewness and kurtosis ratio (SKR), see Watanabe (2006). (b) PMs based on partial moments, such as (i) the Farinelli-Tibiletti (FT) ratio, which nests the popular Omega and Upside potential ratios, see Farinelli and Tibiletti (2008), and (ii) the Sortino ratio, see Sortino and Van der Meer (1991). (c) Quantile-based PMs, such as the Rachev or expected tail ratio (ETR), and the Value-at-Risk ratio (VaRR); see Biglova, Ortobelli, Rachev and Stoyanov (2004) and Caporin and Lisi (2011), respectively.

The remainder of the paper proceeds as follows. Section 2 describes the empirical application on portfolio composition through equity screening under alternative PMs. Section 3 shows conditional dynamic correlations based on the conditional Gaussian copula. Section 4 presents copula methods applied to evaluate tail dependence and models' performance differences. Section 5 provides a summary of the conclusions.

## 2 Modeling Portfolio returns

### 2.1 Database description

We use a total of thirteen daily portfolio return series borrowed from León and Níguez (2020). These portfolios were constructed from selecting stocks that were constituents of the S&P 100 index in October 2017. The data series correspond to the period from December 8, 2009 to October 18, 2017, a total of  $T = 1,980$  daily percentage log-return observations. Each portfolio return series is obtained according to an equity-screening procedure based on a particular PM, which is described below, under a weekly rebalancing horizon and assuming the reward-to-risk (RRT) weighting scheme; see Kirby and Ostdiek (2012). We are interested in the PM portfolio return behavior respecting the SR one. Figure 1 (upper panel) provides a boxplot comparison of spread series from the alternative PMs for cumulative returns. The spread is obtained

as the difference between a specific PM and the SR cumulative return series. We have a total of 12 series, each denoted with the selected PM. Note that the ETR(99,1) exhibits highest gains respecting the SR. Figure 1 (lower panel) shows the plots of cumulative return spread series from some selected portfolios.

## 2.2 Performance measures

Note that the name of each data series (a total of thirteen) comes from the PM used to rank the individual stocks from the S&P 100 index by only selecting the best ten stocks ( $j = 1, \dots, 10$ ) according to the highest PM values out of 90 stocks (after some restrictions) under a weekly rebalancing period and the RRT weighting scheme:  $w_{j,t} = (\mu_{j,t}^+ / \sigma_{j,t}^2) / \sum_{j=1}^{10} (\mu_{j,t}^+ / \sigma_{j,t}^2)$ , where  $\mu_{j,t}^+ = \max(\mu_{j,t}, 0)$  with  $\mu_{j,t}$  and  $\sigma_{j,t}^2$  denoting the conditional mean and variance. The PMs based on partial moments, quantiles and tail measures are closed-form expressions by using the conditional density of the individual stock returns. For more details, see León and Níguez (2020).

From now on, we denote any individual stock return as  $r_t$ . We start with the conditional Sharpe ratio defined as

$$SR_t(\theta) = \frac{\mu_t - \theta}{\sigma_t},$$

where  $\theta$  is the return threshold (e.g., risk-free rate, zero return,...). Second, the PM that aims to explicitly adjust for skewness and kurtosis by using the skewness-kurtosis ratio:

$$SKR_t = \frac{s_{r,t}}{k_{r,t}}.$$

Third, PMs based on the conditional upper/lower partial moments. The lower and upper partial moments of order  $m$  and threshold  $\theta$  are defined, respectively, as

$$LPM_t(\theta, m) = \int_{-\infty}^{\theta} (\theta - r_t)^m f(r_t | I_{t-1}) dr_t$$

and

$$UPM_t(\theta, m) = \int_{-\infty}^{\theta} (r_t - \theta)^m f(r_t | I_{t-1}) dr_t,$$

where  $f(r_t | I_{t-1})$  is the conditional density of returns. The Sortino ratio is obtained as

$$Sortino_t(\theta) = \frac{\mu_t - \theta}{\sqrt{LPM_t(\theta, 2)}}$$

The following two PMs come from the Farinelli and Tibiletti family defined as

$$FT_t(\theta, q, m) = \frac{\sqrt[q]{UPM_t(\theta, q)}}{\sqrt[m]{LPM_t(\theta, m)}}$$

with  $q > 0$  and  $m > 0$ . The higher the value for  $q$  the greater the investor's preference for expected gain, and the higher the value for  $m$  the greater the investor's dislike of expected losses. The Omega ratio and the Upside potential ratio are defined, respectively, as  $FT_t(\theta, 1, 1)$  and  $FT_t(\theta, 1, 2)$ . We have set  $\theta = 0$  for all the previous PMs.

Fourth, a class of PMs similar to the FT replaces partial moments with reward and risk measures based on quantiles or tail measures. The VaRR which is the ratio of the upper and lower quantiles given the stock

return distribution:

$$VaRR_t(\alpha) = \left| \frac{VaR_t(1-\alpha)}{VaR_t(\alpha)} \right|,$$

where  $VaR_t(\alpha)$  and  $VaR_t(1-\alpha)$  are, respectively, the conditional lower and upper quantiles of  $r_t$  with  $\alpha$  set equal to 1%, 5%, 10% and 20%. The ETR defined as

$$ETR_t(\alpha) = \left| \frac{E_{t-1}(r_t | r_t \geq VaR_t(1-\alpha))}{E_{t-1}(r_t | r_t \leq VaR_t(\alpha))} \right|,$$

where the numerator (denominator) is the reward (risk) measure corresponding to the right-tail (left-tail) of the return distribution. These PMs are denoted, for instance, as  $VaRR(95,5)$  and  $ETR(95,5)$  such that  $1-\alpha = 95\%$  (upper quantile for VaRR or right tail for ETR) and  $\alpha = 5\%$  (lower quantile for VaRR or left tail for ETR).

### 2.3 The C-GJR-SNP model

First, we estimate the different PM return series according to the conditional variance model suggested by Glosten et al. (1993), and denoted as the GJR model, with constant mean and the SNP distribution for the innovations or standardized returns, see León et al. (2009). Let  $r_t$  be the portfolio return process characterized by the sequence of conditional densities  $f(r_t | I_{t-1}; \mathbf{\Upsilon})$ , where  $I_{t-1}$  denotes the information set available prior to the realization of  $r_t$ ,  $\mathbf{\Upsilon} = (\mu, \boldsymbol{\theta}, \mathbf{v})$  is the vector of unknown parameters with  $\mu$  as the constant (C) mean of  $r_t$ ,  $\boldsymbol{\theta}$  is the subset characterizing the conditional variance of  $r_t$ , and  $\mathbf{v} = (v_1, v_2)$  characterize the shape of the standardized SNP distribution for the innovations, denoted as  $z_t$ , which are i.i.d. distributed with  $E(z_t) = 0$  and  $E(z_t^2) = 1$ . Thus, the asset return model is as follows:  $r_t = \mu + \varepsilon_t$  with  $\varepsilon_t = \sigma_t(\boldsymbol{\theta}) z_t$  such that  $\sigma_t^2 = E[\varepsilon_t^2 | I_{t-1}]$  is the GJR model and  $z_t \stackrel{iid}{\sim} SNP(0, 1; \mathbf{v})$ . The GJR model is defined as

$$\sigma_t^2 = \alpha_0 + \beta \sigma_{t-1}^2 + \alpha_1^+ (\varepsilon_{t-1}^+)^2 + \alpha_1^- (\varepsilon_{t-1}^-)^2, \quad (1)$$

such that  $\alpha_0 > 0$ ,  $\beta \geq 0$ ,  $\alpha_1^+ \geq 0$ ,  $\alpha_1^- \geq 0$ , and consider  $\varepsilon_t^+ = \max(\varepsilon_t, 0)$ ,  $\varepsilon_t^- = \min(\varepsilon_t, 0)$ .

Note that  $z_t$  is a linear transformation of the random variable  $x_t$  with pdf given by the SNP distribution,

$$z_t = a(\mathbf{v}) + b(\mathbf{v}) x_t, \quad b = 1/\sigma_x, \quad a = -b\mu_x, \quad (2)$$

where  $\mu_x = E(x_t)$  and  $\sigma_x = \sqrt{V(x_t)}$  are, respectively, the mean and the standard deviation of  $x_t$  with density function transformed according to the Gallant and Nychka (1987) method:

$$q_n(x_t) = \frac{\phi(x_t)}{\mathbf{v}'\mathbf{v}} \left( \sum_{k=0}^n v_k H_k(x_t) \right)^2, \quad (3)$$

where  $\mathbf{v} = (v_0, v_1, \dots, v_n)' \in \mathbb{R}^{n+1}$ ,  $\phi(\cdot)$  denotes the pdf of a standard normal random variable and  $H_k(\cdot)$  are the orthonormal Hermite polynomials, which can be obtained recursively for  $k \geq 2$  as  $H_k(x) = \frac{xH_{k-1}(x) - \sqrt{k-1}H_{k-2}(x)}{\sqrt{k}}$  with initial conditions  $H_0(x) = 1$  and  $H_1(x) = x$ . We set  $v_0 = 1$  to solve the scale of indeterminacy since (3) is homogeneous of degree zero in  $\mathbf{v}$ . Here, we implement the SNP pdf for the case

of  $n = 2$ , and denoted to simplify as  $q(\cdot)$ . An alternative expression of  $q(\cdot)$  by solving the square in (3) is as follows:

$$q(x_t) = \phi(x_t) \left( 1 + \sum_{k=1}^4 \gamma_k(\mathbf{v}) H_k(x_t) \right), \quad (4)$$

such that

$$\begin{aligned} \gamma_1(\mathbf{v}) &= \frac{2v_1(1 + \sqrt{2}v_2)}{\mathbf{v}'\mathbf{v}}, & \gamma_2(\mathbf{v}) &= \frac{\sqrt{2}(v_1^2 + 2v_2^2 + \sqrt{2}v_2)}{\mathbf{v}'\mathbf{v}}, \\ \gamma_3(\mathbf{v}) &= \frac{2\sqrt{3}v_1v_2}{\mathbf{v}'\mathbf{v}}, & \gamma_4(\mathbf{v}) &= \frac{\sqrt{6}v_2^2}{\mathbf{v}'\mathbf{v}}, \end{aligned} \quad (5)$$

where  $\mathbf{v}'\mathbf{v} = 1 + v_1^2 + v_2^2$ . The first four noncentral moments of  $x_t$  with pdf in (4) are:

$$\begin{aligned} E(x_t) &= \gamma_1(\mathbf{v}), & E(x_t^2) &= \sqrt{2}\gamma_2(\mathbf{v}) + 1, \\ E(x_t^3) &= \frac{6v_1(1 + 2\sqrt{2}v_2)}{\mathbf{v}'\mathbf{v}}, & E(x_t^4) &= \frac{12(v_1^2 + 3v_2^2 + \sqrt{2}v_2)}{\mathbf{v}'\mathbf{v}} + 3. \end{aligned}$$

Hence,  $\mu_x = E(x_t)$  and  $\sigma_x^2 = E(x_t^2) - \mu_x^2$ . The pdf of  $z_t$  is obtained as  $g(z_t) = \frac{1}{b(\mathbf{v})} q\left(\frac{z_t - a(\mathbf{v})}{b(\mathbf{v})}\right)$ . The third and fourth moments of  $z_t$  in (2) are directly the skewness and kurtosis:

$$E(z_t^3) = a^3 + 3a^2bE(x_t) + 3ab^2E(x_t^2) + b^3E(x_t^3), \quad (6)$$

$$E(z_t^4) = a^4 + 4a^3bE(x_t) + 6a^2b^2E(x_t^2) + 4ab^3E(x_t^3) + b^4E(x_t^4). \quad (7)$$

Henceforth, the above process for  $r_t$  is referred to as C-GJR-SNP. More properties about this model can be seen in León and Níguez (2020).<sup>1</sup> We estimate this model for each return series by maximum likelihood (ML). The parameter estimates, exhibited in Table 1, show that all return series present significant skewness and kurtosis; they present statistically significant constant mean; and their conditional variances are highly persistent and respond asymmetrically to positive and negative shocks.

### 3 Conditional dynamic correlations

Next, we proceed to analyze the behavior of the daily conditional correlations between the PM portfolio returns and the SR ones. We apply the conditional Gaussian copula, see Patton (2006). The copula dependency parameter (or conditional correlation in this particular case),  $\rho_t$ , is driven by an ARMA(1,q)-type process:

$$\rho_t = \Lambda_{(-1,1)} \left( \gamma_0 + \gamma_1 \rho_{t-1} + \gamma_2 \frac{1}{q} \sum_{j=1}^q \Phi^{-1}(u_{1,t-j}) \Phi^{-1}(u_{2,t-j}) \right), \quad (8)$$

where  $\Lambda_{(-1,1)}(x) = (1 - e^{-x})(1 + e^{-x})^{-1}$  is the logistic transformation that keeps  $\rho_t$  within  $(-1, 1)$ , and  $u_{i,t} = F_i(r_{i,t} | I_{t-1})$   $i = 1, 2$  such that  $F_i(\cdot | I_{t-1})$  denotes the conditional distribution for the C-GJR-SNP model for the return  $r_{i,t}$ . We set  $q = 8$  in equation (8) which is a common value adopted in some studies as e.g. Reboredo (2011). The parameters  $\gamma_j$  in (8) are also estimated by ML where the inputs are the estimates of  $u_{i,t}$ , denoted as  $\hat{u}_{i,t}$ . In short, the parameters of our bivariate distribution are estimated in two stages.

<sup>1</sup>See Del Brio et al. (2020) for an EGARCH specification with SNP innovations.

In a first stage, we estimate all parameters implied in the conditional marginal distributions for  $r_{i,t}$  and in a second stage those for the copula model. The same procedure is applied in the next section.

Figure 2 exhibits the time series of (8) for the different PMs. Note that the daily conditional correlations are very high for Sortino, Omega, Upside Potential and most PMs based on VaRR. Finally, those portfolios based on ETR and SKR exhibit remarkably low correlations respecting the SR portfolio, which enhance the difference between the latter and the former PMs. These results are also in line with those about equity screening by León et al. (2019). Because of these findings, in the following section we explore the behavior of the upper/lower tail of the bivariate distribution of SR and every other PM portfolio so as to highlight possible differences in simultaneous occurrence of large/small PM portfolio returns.

## 4 Tail dependence analysis

In this section, we focus on the tail dependence measuring the probability that two variables are either in the lower or in the upper joint tails. Specifically, we study the propensity of two portfolio returns, from a given PM and SR strategies, to upward or downward comovements. This behavior is explained through the upper and lower tail dependence parameters denoted by  $\lambda_U \in [0, 1]$  and  $\lambda_L \in [0, 1]$ , respectively. Larger values of  $\lambda_U$  ( $\lambda_L$ ) indicate greater trend of the portfolio returns to cluster in the upper (lower) tail of a bivariate distribution. In such a case, the returns are said to be upper (lower) tail dependent. More precisely,  $\lambda_U$  ( $\lambda_L$ ) measures the probability that a random variable –defined as a PM portfolio return– is above (below) a high (low) quantile, given that a second random variable –defined as the SR portfolio return– is above (below) a high (low) quantile. This dependence structure is modeled through copula functions.

Note that the Gaussian copula does neither capture upper nor lower dependence where the extreme tails of the distribution of the variables are independent, i.e.  $\lambda_U = \lambda_L = 0$ . Thus, we implement alternative copula models allowing for both/either upper or lower tail dependence. Namely, among the wide range of copula functions, we use the SJC, Gumbel and Clayton copulas. The SJC has both upper and lower tail dependence parameters, whilst Gumbel (Clayton) gathers only upper (lower) tail dependence. The SJC is defined directly in terms of the above probabilities. Nonetheless, both Gumbel and Clayton copulas are defined in terms of the parameters  $\gamma_G > 1$  and  $\gamma_C > 0$ , respectively. Hence, the corresponding probabilities are given by  $\lambda_U = 2 - 2^{(1/\gamma_G)}$ ,  $\lambda_L = 0$  for the Gumbel copula and,  $\lambda_U = 0$ ,  $\lambda_L = 2^{-(1/\gamma_C)}$  for the Clayton copula, see Patton (2006).

Table 2 reports the probability estimates of the previous time-invariant copula models. We obtain the following conclusions. Firstly, for the SJC copula it is found a statistically significant and higher asymmetry value on the lower than on the upper tail, mainly for both SKR and ETR. Note that the estimates of  $\lambda_L$  double those of  $\lambda_U$  for the latter two strategies. Secondly, for Sortino, Omega, Upside potential and most VaRR cases both SJC probability coefficients are similar in magnitude as well as higher than the SKR and ETR counterparts. This means that the former PMs exhibit higher upper tail dependence respecting the SR than the latter. Thirdly, according to both Clayton and Gumbel copulas, it can be shown that both SKR and ETR exhibit statistically significant and lower values for both  $\lambda_L$  and  $\lambda_U$  than the other PMs.

This evidence is in accordance with the previous results under SJC. Summing up, these findings support the superior performance of both SKR and ETR.

In order to reinforce the previous results, we estimate the time-varying SJC copula for the different PMs with respect the SR portfolio under weekly rebalancing with the RRT scheme. Following Patton (2006), the dynamics of both  $\lambda_L$  and  $\lambda_U$  under the conditional SJC copula are specified as

$$\lambda_{L,t} = \Lambda_{(0,1)} \left( \omega_L + \beta_L \lambda_{L,t-1} + \alpha_L \frac{1}{q} \sum_{j=1}^q |u_{1,t-j} - u_{2,t-j}| \right), \quad (9)$$

$$\lambda_{U,t} = \Lambda_{(0,1)} \left( \omega_U + \beta_U \lambda_{U,t-1} + \alpha_U \frac{1}{q} \sum_{j=1}^q |u_{1,t-j} - u_{2,t-j}| \right), \quad (10)$$

where  $\Lambda_{(0,1)}(x) = (1 + e^{-x})^{-1}$  is the logistic transformation that keeps  $\lambda_{L,t}$  and  $\lambda_{U,t}$  within  $(0, 1)$ . According to the Akaike information criterion –not exhibited here–, the time-varying SJC estimations (see Figures 3 and 4) provide better fit than their corresponding time-invariant versions (see Table 2), except for Omega, VaRR (95,5) and VaRR (80,20) portfolios. Note that the averages of plot series in Figures 3 and 4 (red and blue horizontal lines, respectively) are rather close to the unconditional SJC estimates of  $\lambda_L$  and  $\lambda_U$  in Table 2. This new evidence corroborates the results previously found under time-invariant SJC modeling.

## 5 Conclusions

We have applied conditional copula methods to study the behavior of portfolio returns constructed with different PMs and compare with the benchmark SR portfolio. The portfolio return series we use are obtained in León and Níguez (2020). We assume different (conditional) copula models for the bivariate distributions of the PM return series. We estimate the univariate series by using the GJR-SNP model.

Our results show that under the Gaussian copula, both ETR and SKR portfolios exhibit remarkably low correlations respecting the SR portfolio. This means that these two portfolios are different respecting the SR one. We also find that copulas which focus on either the upper tail (Gumbel) or the lower tail (Clayton) render significant differences. In short, our copula analysis is useful to understand what kind of equity-screening strategy based on its corresponding PM performs better in relation to the SR portfolio. In particular, results from SJC, Clayton, Gumbel as well as time-varying SJC copula show that Sortino, Omega, Upside potential and VaRR equity-screening strategies render portfolio returns which are rather more similar to those under the SR than the returns obtained under PMs based on SKR and ETR.

Several interesting avenues for further research would be the following. First, application of the copula quantile regression as in Bouyé and Salmon (2009) for our tail dependence analysis between the portfolio returns series according to alternative PMs. They estimate several distinct non-linear quantile regression models implied by their copula specifications and obtain closed-form expressions of the quantile curve for several copulas. See, for instance, Koenker (2005) for a review of quantile regression. Second, extensions to the multivariate SNP framework as in Jiménez et al. (2020). Finally, a robustness analysis about our tail dependence results based on both alternative GARCH-family models and different distributions for the innovations corresponding to the first estimation stage in Table 1. See, among others, Del Brio et al. (2014, 2020).

# Tables

Table 1: C-SNP-GJR model estimation results

	$\alpha_0$	$\beta$	$\alpha_1^+$	$\alpha_1^-$	$\mu$	$\nu_1$	$\nu_2$
SR	0.029*** (0.007)	0.870*** (0.015)	0.036*** (0.013)	0.185*** (0.023)	0.040** (0.020)	0.577*** (0.036)	0.300*** (0.033)
SKR	0.022*** (0.005)	0.894*** (0.012)	0.023* (0.012)	0.152*** (0.019)	0.055*** (0.019)	0.564*** (0.044)	0.257** (0.036)
Sortino	0.028*** (0.006)	0.866*** (0.015)	0.044*** (0.012)	0.188*** (0.023)	0.042** (0.020)	0.590*** (0.035)	0.311*** (0.033)
Omega	0.029*** (0.007)	0.869*** (0.015)	0.040*** (0.012)	0.184*** (0.023)	0.042** (0.020)	0.587*** (0.036)	0.306*** (0.034)
Upside P	0.019*** (0.005)	0.893*** (0.013)	0.033*** (0.011)	0.159*** (0.020)	0.045** (0.020)	0.585*** (0.036)	0.304*** (0.033)
VaRR(99,1)	0.006*** (0.003)	0.925*** (0.007)	0.047*** (0.010)	0.103*** (0.012)	0.053*** (0.019)	0.588*** (0.032)	0.299*** (0.032)
VaRR(95,5)	0.020*** (0.005)	0.898*** (0.013)	0.032*** (0.011)	0.148*** (0.019)	0.049** (0.020)	0.587*** (0.036)	0.300*** (0.035)
VaRR(90,10)	0.025*** (0.006)	0.877*** (0.015)	0.044*** (0.013)	0.172*** (0.021)	0.044** (0.020)	0.567*** (0.035)	0.294*** (0.033)
VaRR(80,20)	0.035*** (0.007)	0.851*** (0.017)	0.054*** (0.015)	0.199*** (0.025)	0.033* (0.020)	0.580*** (0.036)	0.306*** (0.033)
ETR(99,1)	0.018*** (0.004)	0.889*** (0.013)	0.033** (0.014)	0.171*** (0.021)	0.047* (0.019)	0.570*** (0.038)	0.288*** (0.033)
ETR(95,5)	0.040*** (0.008)	0.855*** (0.018)	0.019 (0.013)	0.208*** (0.029)	0.042* (0.019)	0.544*** (0.035)	0.283*** (0.030)
ETR(90,10)	0.044*** (0.008)	0.844*** (0.017)	0.033** (0.013)	0.209*** (0.029)	0.036* (0.020)	0.499*** (0.044)	0.259*** (0.032)
ETR(80,20)	0.031*** (0.006)	0.860*** (0.016)	0.003 (0.012)	0.213*** (0.026)	0.031* (0.017)	0.488*** (0.061)	0.213*** (0.040)

Model:  $r_t = \mu + \varepsilon_t$ ,  $\varepsilon_t = \sigma_t(\boldsymbol{\theta})z_t$ ,  $\sigma_t^2 = \alpha_0 + \beta\sigma_{t-1}^2 + \alpha_1^+(\varepsilon_{t-1}^+)^2 + \alpha_1^-(\varepsilon_{t-1}^-)^2$ ,  $z_t \stackrel{iid}{\sim} SNP(0, 1; \mathbf{v})$ ,  $\mathbf{v} = (v_1, v_2)$ .

This table presents ML estimates of the C-SNP-GJR parameters for the portfolio returns obtained under alternative PMs ( $T = 1,980$  obs.). Heteroscedasticity-consistent standard errors are in parentheses below the parameter estimates. (\*\*\*) indicates significance at 1% level; (\*\*) indicates significance at 5% level and (\*) indicates significance at 10% level.

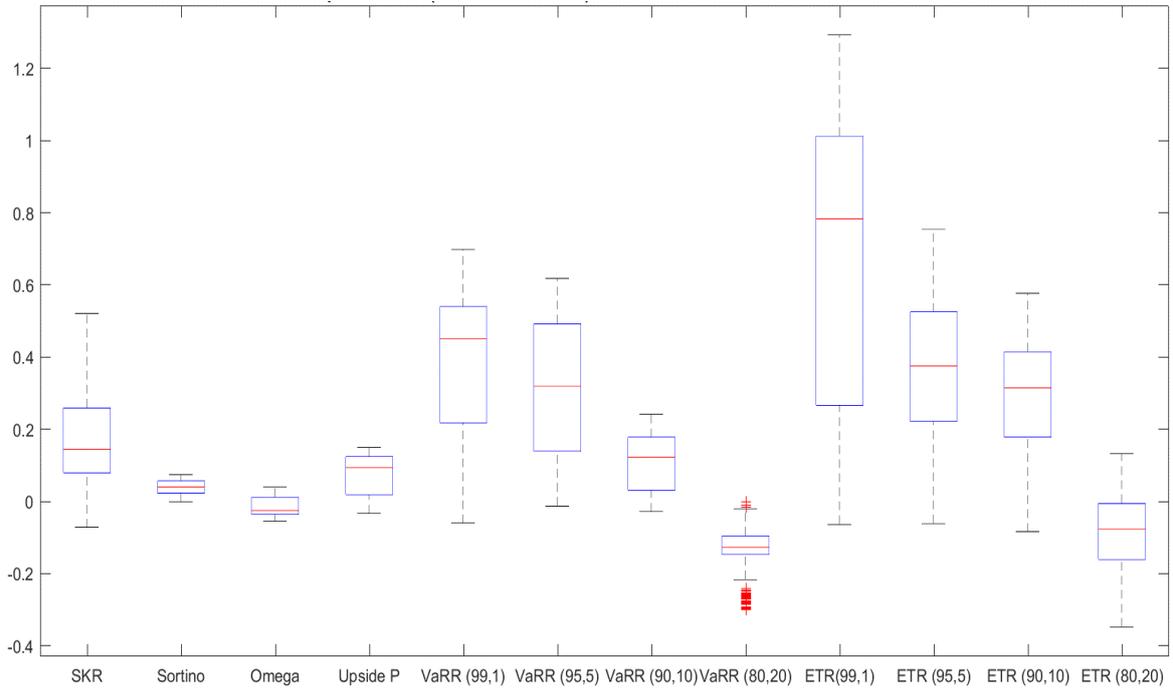
Table 2: Estimates for copula models (PM-SR)

PM	$\lambda_U$ (SJC)	$\lambda_L$ (SJC)	$\lambda_L$ (Clayton)	$\lambda_U$ (Gumbel)
SKR	0.38**	0.65**	0.69**	0.59**
Sortino	0.72	0.72	0.98**	0.97**
Omega	0.71	0.78	0.98**	0.93**
Upside P	0.73**	0.74**	0.95**	0.93**
VaRR (99,1)	0.58**	0.76**	0.81**	0.74**
VaRR (95,5)	0.74	0.78	0.92**	0.88**
VaRR (90,10)	0.74**	0.77	0.95**	0.93**
VaRR (80,20)	0.73**	0.78**	0.96**	0.94**
ETR (99,1)	0.37**	0.65**	0.69**	0.59**
ETR (95,5)	0.39**	0.65**	0.68**	0.59**
ETR (90,10)	0.38**	0.67**	0.70**	0.59**
ETR (80,20)	0.34**	0.61**	0.64**	0.55**

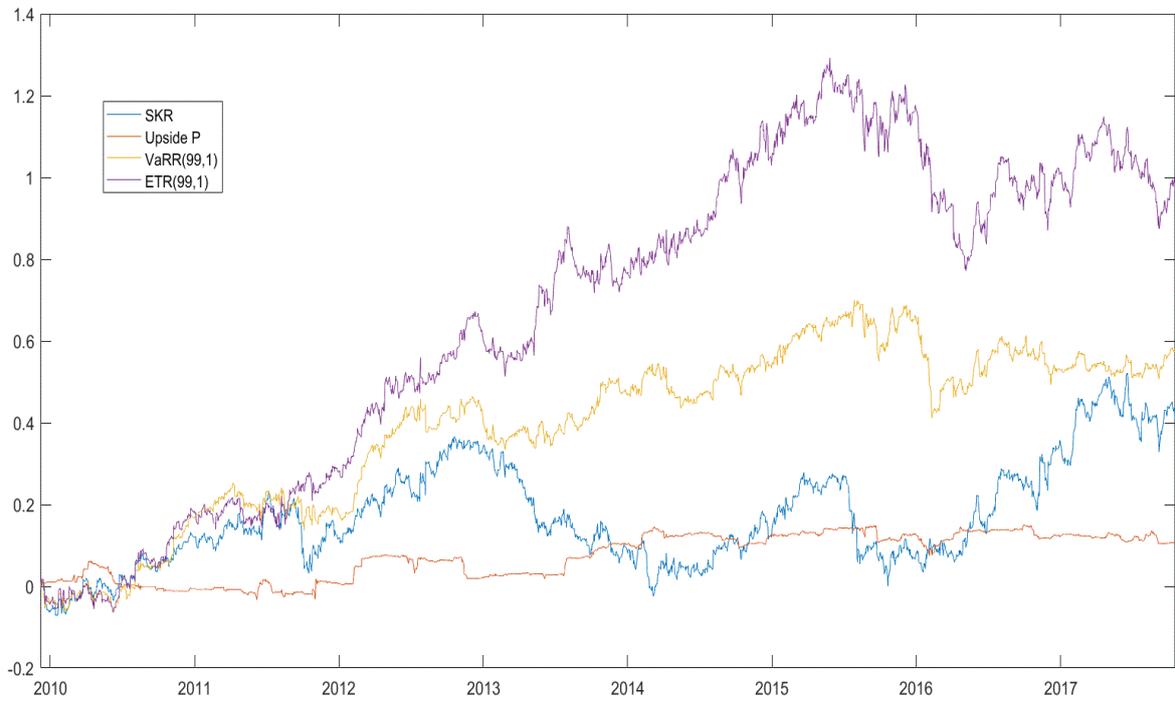
This table presents probability estimates of the parameters  $\lambda_U$  and  $\lambda_L$  (i.e., upper and lower tail dependence) for the time-invariant SJC, Gumbel and Clayton copulas to model the bivariate PM-SR. (\*\*) indicates significance at the 5% level for the implied parameters ( $\gamma_G$  for Gumbel,  $\gamma_C$  for Clayton and both  $\lambda_U$  and  $\lambda_L$  for SJC).

# Figures

Figure 1: Spread series from different PMs with respect to SR

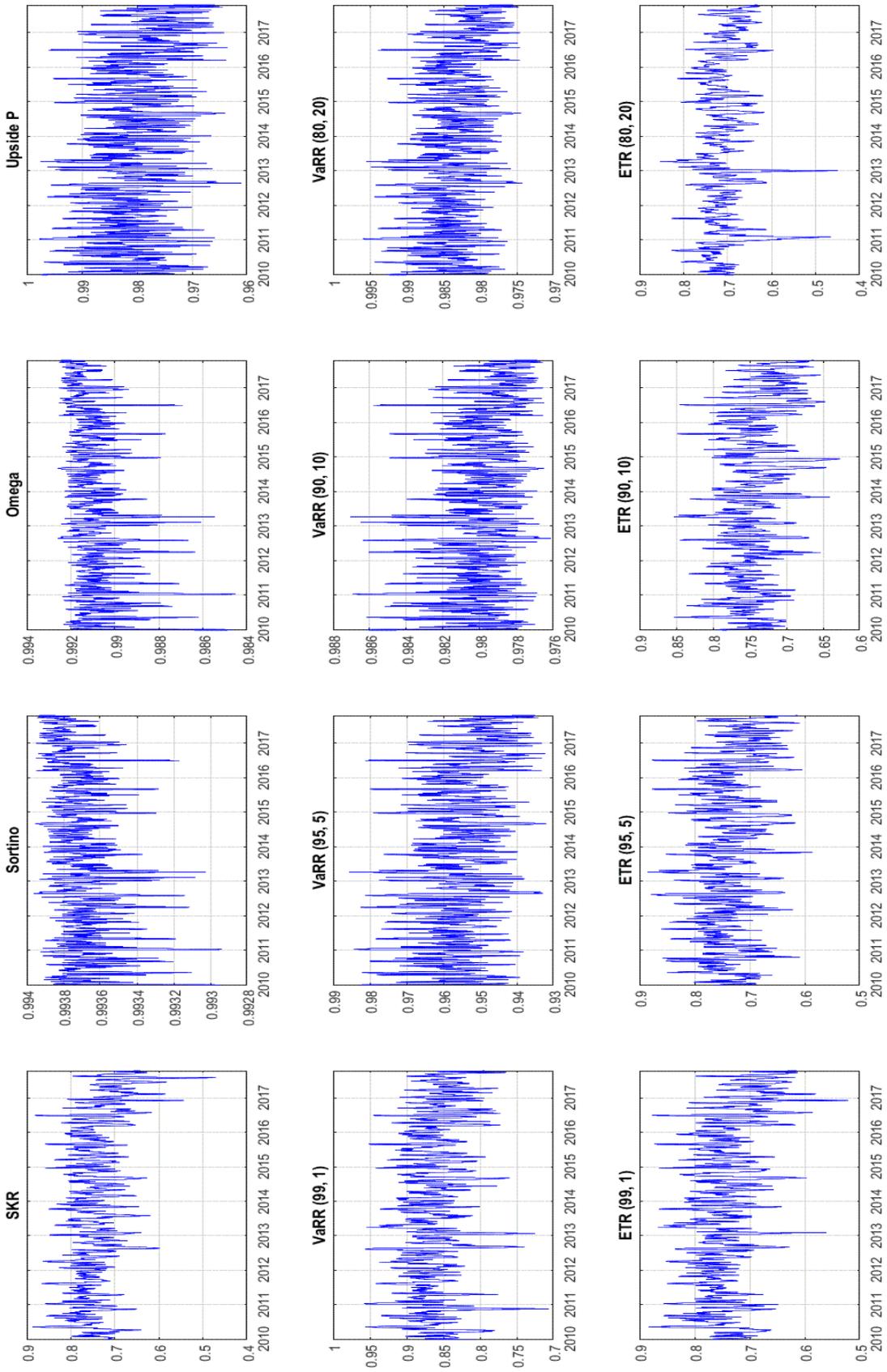


Boxplots of spread series (cumulative returns) from alternative PMs.



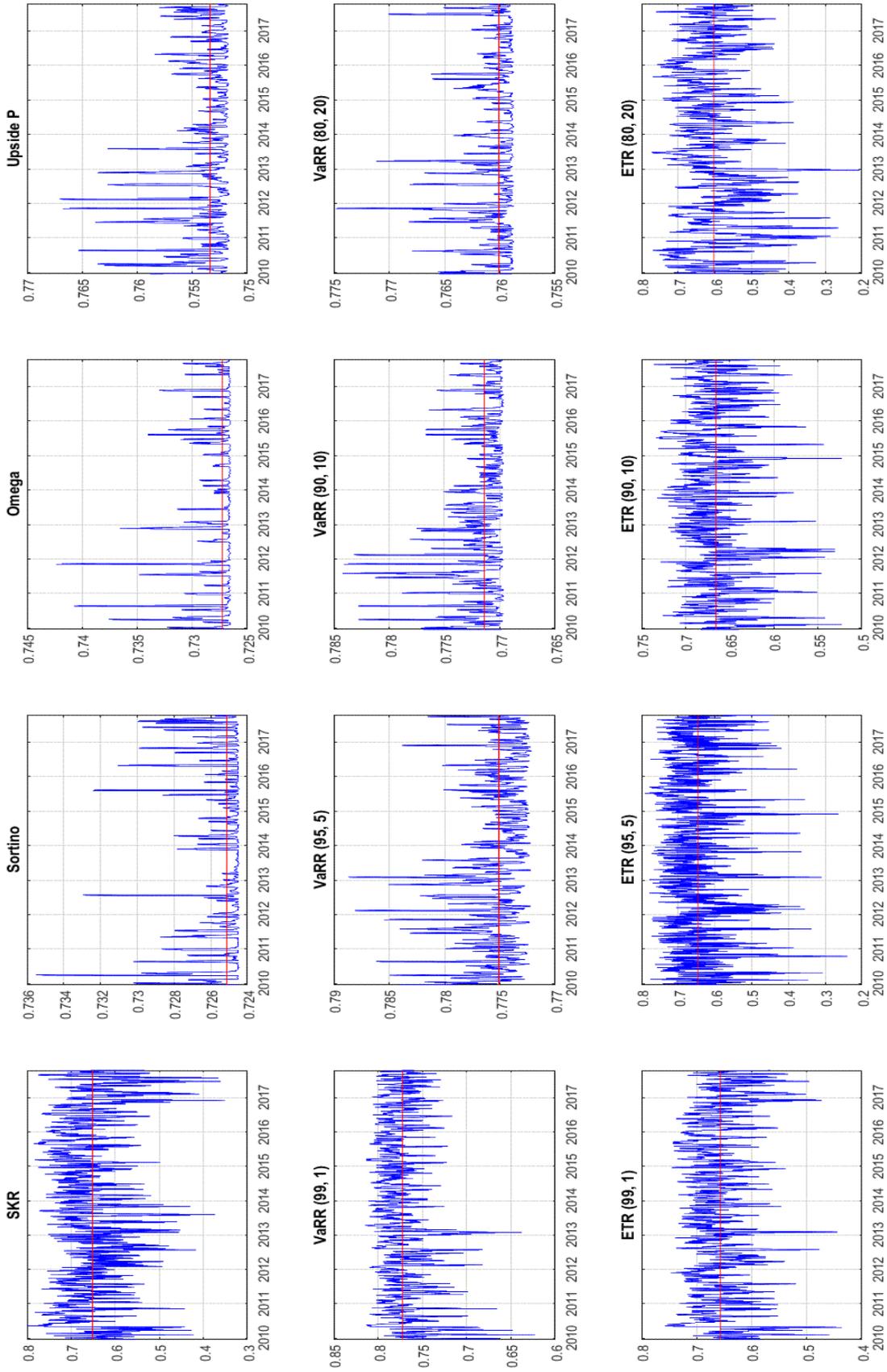
Cumulative return spread time series.

Figure 2: Conditional Gaussian copula correlation between PM and SR



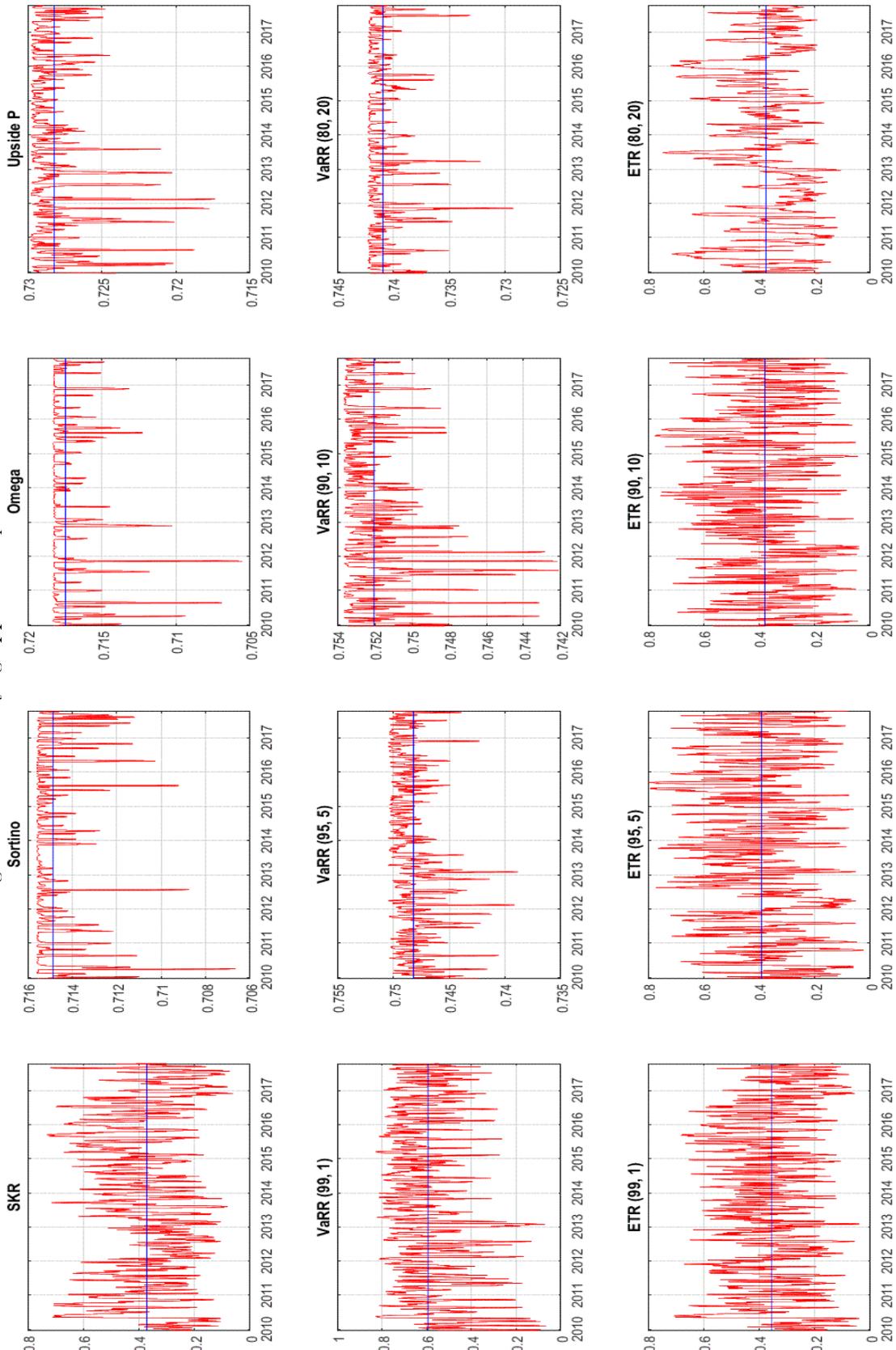
Plots of daily correlation between PM and SR from conditional Gaussian copula.

Figure 3: SJC time-varying lower tail dependence



Plots of time-varying lower tail dependence,  $\lambda_{L,t}$ , for PM and SR from SJC copula. The red line represents the sample mean of the  $\lambda_{L,t}$  time series.

Figure 4: SJC time-varying upper tail dependence



Plots of time-varying upper tail dependence,  $\lambda_{U,t}$ , for PM and SR from SJC copula. The blue line represents the sample mean of the  $\lambda_{U,t}$  time series.

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