A new approach for non-linear analysis of power amplifiers

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A NEW APPROACH FOR NON-LINEAR ANALYSIS OF POWER AMPLIFIERS

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Abstract – This paper presents a new approach for non-linear analysis of power amplifiers based on the n-order polynomial modelling. A general expression for the fundamental frequency output is developed in this paper. It allows considering any order of polynomial model and obtaining analytical expression for the fundamental-frequency output without complex computations, using developed formulas. This model significantly simplifies non-linear analysis of power amplifiers and the process of designing linearizers. This paper also presents a simple approach for obtaining polynomial model from a frequency response of power amplifier. The proposed method is verified by experiments. Obtained polynomial model shows simulation results similar to the measurements for real amplifier. Developed n-order output model and proposed model extraction method can significantly simplify, optimize and enhance power amplifiers’ non-linear analysis and behavioural modelling. Proposed modelling approach can be also used to design linearizers.

I. Introduction

New demands to communication systems require enhancement of power amplifiers' performances, such as widening dynamic range and even operation in saturation mode without distortion. To satisfy these requirements, linearizers are used [1]-[2]. While designing a linearizer for particular amplifier, non-linear analysis and output distortion modelling are carried out. Therefore, simple, quick and accurate methods for behavioural modelling and non-linear analysis are required.

Usually power amplifiers (PA) are represented by a Winner or Hammerstein model, which consist of a non-linear memoryless block and a linear FIR filter [3]. Filters are used to represent memory effects and polynomials in the memoryless blocks denote PA non-linear distortions. The higher order of the polynomial is considered, the more accurate PA behaviour model is, and the more efficient and precise linearizer can be designed. Therefore, there is a growing demand of straightforward methods for obtaining output signal analytical model using high-order polynomial representation of PAs. Also, simple methods are required for extracting a PA polynomial model from the real measurements.

This paper presents a new approach for PA non-linear analysis, aimed at obtaining analytical expression for the output signal using high-order polynomial model. In literature 3rd and 5th-order polynomials are usually used for representing PA distortion [1]-[2]. Therefore, analytical model for output distorted signal for the 3rd and 5th orders are well-known. However, there is a lack of such knowledge for the high-order models. This paper aims to compensate for this shortage and develops a general n-order polynomial model for the PA output, where n can be as high as necessary for the required accuracy. The precise formulas up to 10th-order are presented in the paper.

Moreover, this paper describes a simple and accurate approach for obtaining PA polynomial model from its frequency response. This approach is verified by experiments and shows good performances. The obtained model produces simulation results close to the measured PA performances.

II. Proposed Approach for Polynomial Model Extraction

PA non-linear output is usually represented by a polynomial expression [1]-[3]:

\[ V_{out}(t) = \sum_{n=1}^{n} g_n V_{in}^n(t) \]  

(1)

Where \( V_{in} \) is the input voltage of PA and \( g_1, g_2, g_3... \) are coefficients of the nonlinear terms. The model (1) allows obtaining mathematical formulas for distortion components of a signal, when PA operates in a non-linear mode.

Consequently, for the non-linear analysis, behavioural modelling and linearizing of PA, g-coefficients need to be extracted. This section describes a developed approach for obtaining polynomial model for a power amplifier from its frequency response.

For a tone input signal \( V_{in}(t) = V \cdot \cos(\omega t) \). Considering the first three elements in (1) after applying trigonometric relations, PA output looks like:

\[ V_{out}(t) = g_1 V^2 + (g_2 V^3 + \frac{3}{4} g_3 V^5) \cos(\omega t) \]

\[ + \frac{g_1}{2} V^2 \cos(2\omega t) + \frac{3g_3}{4} V^5 \cos(3\omega t). \]

According to (2) the output signal includes second and third harmonics apart from the fundamental tone. An example of such response is plotted in Figure 1.

![Figure 1. Typical frequency response of a PA in saturation](image)

Using this graph and expression (2), it is easy to determine g-coefficients for the polynomial model (1). From the frequency response (Fig. 1) amplitudes of the harmonics can be obtained: \( V_{fund}, V_{sec}, V_{third}... \). Then, using (2), the system of equations can be written:
From (3) the coefficients \( g_1, g_2, g_3 \ldots \) are obtained.

### III. Proposed n-order Polynomial Output Modelling

Real power amplifiers can be presented by polynomial models, which have larger number of elements. This makes theoretical analysis of the distorted output products extremely complicated. In literature 3rd and 5th-order polynomial models are usually considered [1]-[2]. Therefore, analytical expressions for the 3rd and 5th-order of PA outputs are widely known. But there is a lack of such knowledge for the high-order polynomial models.

This section presents a developed general analytical expression for the fundamental-frequency output of n-order PA polynomial model.

The modulated input signal with variable amplitude \( V_{\text{IN}}(t) \) and phase \( \phi(t) \) can be written as:

\[
V_{\text{IN}}(t) = V_s(t) \cdot \cos(\omega t + \phi(t))
\]

Using I and Q components, (4) looks like:

\[
V_{\text{IN}}(t) = V \cdot (I \cdot \cos(\omega t) - Q \cdot \sin(\omega t))
\]

Where: 
\[
I = \frac{V_s(t)}{V} \cdot \cos(\phi(t)), \quad Q = \frac{V_s(t)}{V} \cdot \sin(\phi(t)), \quad V = \text{average}(V_s(t))
\]

Therefore, average \((I^2 + Q^2) = 1\).

After substituting (5) into (1) and applying trigonometric relations, the PA output looks like:

\[
V_{\text{OUT}}(t) = \frac{V^2 g_1}{4} \cdot I^2 + \frac{V^2 g_2}{2} \cdot I^2 \cdot Q^2 + \frac{3V^2 g_3}{4} \cdot (I \cos \omega t - Q \sin \omega t) \cdot \cos(\omega t)
\]

\[
+ \frac{V^2 g_4}{2} \cdot (I^2 - Q^2) \cos(2\omega t) - V^2 g_5 I Q \sin 2\omega t
\]

\[
+ \frac{V^2 g_6}{4} \cdot (I^2 - Q^2) \sin 3\omega t
\]

Here, DC, second and third harmonics are not interesting for consideration because they can be easily filtered at the PA output. However, fundamental frequency distortion is very important:

\[
V_{\text{OUT}}(t) = V g_1 \cdot I^2 + V g_2 I Q \sin 2\omega t
\]

In (7) distortion components produced by the even terms of the PA model are not present, therefore distortion at the fundamental frequency is caused only by the odd terms. (7) can be rewritten in another way:

\[
V_{\text{OUT}}(t) = V g_1 \cdot I^2 + V g_2 I Q \sin 2\omega t
\]

Where:

\[
V_{\text{OUT}}(t) = \frac{3V^2 g_3}{4} \cdot I^2 \quad V \in (t)
\]

Because only odd-terms produce distortion at the fundamental frequency, the even ones will not be considered further. Carrying out similar mathematical calculations for the 5th-order polynomial model, it is possible to obtain the next term:

\[
V_{\text{OUT}}(t) = \frac{5V^4 g_5}{8} \cdot I^2 + Q^2 \quad V \in (t)
\]

The 7th-order term will produce:

\[
V_{\text{OUT}}(t) = \frac{35V^6 g_7}{64} \cdot I^2 \quad V \in (t)
\]

Having considered the third (9), fifth (10) and seventh-order (11) distortion components, it is possible to find regularity in their expressions. Using mathematical induction the general formula can be found:

\[
V_{\text{OUT}}(t) = \sum_{k=1}^{n} V_{2k+1} \cdot V_{n+1} (I^2 + Q^2)^k \cdot V \in (t)
\]

Here \( k = 1, 2, 3 \ldots \) and \( b_{2k+1} \) is a member of the mathematical series, which will be described below.

Using (8)-(12) the accurate formula for PA output at the fundamental frequency can be found for the n-order polynomial model:

\[
V_{\text{OUT}}(t) = V g_1 \cdot I^2 + V g_2 I Q \sin 2\omega t
\]

Here, DC, second and third harmonics are not interesting for consideration because they can be easily filtered at the PA output. However, fundamental frequency distortion is very important:

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\[
V_{\text{OUT}}(t) = V g_1 \cdot I^2 + V g_2 I Q \sin 2\omega t
\]

Here, the first term is undisrtorted linear output and the second one represents distortion. \( b_{2k+1} \) is a mathematical series of coefficients near the corresponding terms. Using Matlab the first nineteen members of the \( b \)-series has been found. Tab. I presents the odd terms. Series \( b_{2k+1} \) and formula (13) allow considering as many polynomial elements as necessary for the required accuracy.

### IV. Experimental Verification of the Proposed Approach

In order to verify accuracy of the proposed modelling method, experiments with power amplifier in saturation using harmonic test and 16-QAM digital modulated signal were carried out. Experimental equipment, shown on

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|c|}
\hline
\text{Member} & b_1 & b_3 & b_5 & b_7 & b_9 & b_{11} & b_{13} & b_{15} & b_{17} & b_{19} \\
\hline
\text{value} & 3 & 5 & 35 & 125 & 6435 & 12155 & 46189 & 200818 & 8-12 September, Sevastopol, Crimea, Ukraine. 2008: CriMiCo'2008 Organizing Committee; CsSTC. ISBN: 978-966-335-166-7. IEEE Catalog Number: CFP08788
\end{array}
\]
Initially, an RF tone with frequency 500 MHz has been generated and passed thought the PA. According to the PA data sheet, ZFL500 has 21 dB gain and a 1-dB compression point at 10 dBm output.

In order to obtain g-coefficients for the amplifier model, a frequency response has been generated (Figure 3). Measured amplitudes of the harmonics for different power levels are presented in Table II. From Table II the initial coefficients for the amplifier model were obtained according to procedure described above (3):

\[ g_1 = 10,5; \quad g_3 = -4,5; \quad g_5 = -1. \]

At fundamental frequency for the 5th order polynomial model, (13) gives general expression for the output:

\[ V_{\text{OUT}}^{\text{FUND}}(t) = g_1 \cdot V_{\text{IN}}(t) + \frac{3}{4} g_3 \cdot V_{\text{IN}}^3(t) + \frac{5}{8} g_5 \cdot V_{\text{IN}}^5(t) \]  

Therefore, output distortion model for ZFL500 looks like:

\[ V_{\text{OUT}}^{\text{FUND}}(t) = 10.5 \cdot V_{\text{IN}}(t) - 3.3 \cdot V_{\text{IN}}^3(t) - 0.6 \cdot V_{\text{IN}}^5(t) \]  

This model has been verified by comparison of the measured and modelled characteristics of the power amplifier: transfer function (Figure 4) and the 16-QAM output spectrum (Figures 5-6).

**Table II**

<table>
<thead>
<tr>
<th>$P_{\text{INPUT}}$</th>
<th>$V_{\text{FUND}}$</th>
<th>$V_{\text{SEC}}$</th>
<th>$V_{\text{THIRD}}$</th>
<th>$V_{\text{FOURTH}}$</th>
<th>$V_{\text{FIFTH}}$</th>
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<td>10.6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-5</td>
<td>15.5</td>
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<td>0</td>
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<td>-23</td>
<td>0</td>
<td>0</td>
</tr>
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<td>3</td>
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<td>-15</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
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<td>24.8</td>
<td>-3</td>
<td>-8</td>
<td>-24</td>
<td>0</td>
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<tr>
<td>7</td>
<td>26.0</td>
<td>4</td>
<td>-2</td>
<td>-16</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>26.5</td>
<td>6</td>
<td>0</td>
<td>-14</td>
<td>-22</td>
</tr>
</tbody>
</table>

**Figure 2.** Experimental test bench

**Figure 3.** Measured frequency response of ZFL500 at 8 dBm output

**Figure 4.** Transfer characteristic for ZFL500: measured and obtained from the developed model

**Figure 5.** Simulated 16-QAM output spectrum for the developed ZFL500 model

Simulations of the PA model (Figures 4, 5) have been carried out using Matlab-ADS co-simulation system. Here, proposed PA model was implemented in Matlab and then called from ADS during the simulation. Comparison of the simulated and measured results (Figures 4-6) allows making a conclusion that the developed polynomial behavioural model of power amplifier is accurate and can be used for simulations in place of the real device.

Figure 6. Measured 16-QAM output spectrum for ZFL500

V. Conclusion

This paper presented a new approach in PA behavioural modelling and non-linear analysis. The simple and reliable method of polynomial model extraction from a power amplifier frequency response has been presented. A new approach in modelling PA output has been developed. General n-order polynomial model for the output signal has been presented. The obtained expression allows making quick, simple and accurate analytical prediction of the output distortion at fundamental frequency by analytical formulas, which can be implemented to any power amplifier and any order of polynomial model. The new mathematical series \( b_n \), representing coefficients in output signal series has been developed. Obtaining its general or recursive formula will be one of the directions for future work. This series can be used in a non-linear analysis of PA. It gives straightforward prediction of the PA output signal in a saturation mode. The proposed modelling approach has been verified by experiments. The experimental results for a tone input signal as well as 16-QAM modulated signal proved the accuracy of the proposed method.

VI. References