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Modelling the Demand for Long-term Care to Optimise Local Level Planning

Philip James Richard Andrew Worrall

A thesis submitted in partial fulfilment of the requirement for the degree of Doctor of Philosophy

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Modelling the Demand for Long-term Care to Optimise Local Level Planning

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To my parents and family

Abstract

Long-term care (LTC) includes the range of health, social and voluntary support services provided to those with chronic illness, physical or mental disability. LTC has been widely studied in the literature, in particular due to concerns surrounding how future demographic shifts may impact the LTC system's ability to cater to increasing amounts of patients not withstanding what the future cost impact might be. With that said, few studies have attempted to model demand at the local level for the purposes of informing local service delivery and organisation. Many developing countries with mature and developed systems of LTC in place are under pressure to reduce health care spend, whilst delivering greater value for money. We suggest that the lack of local studies in LTC stems from the lack of a strong case for the benefits of demand modelling at the local level in combination with low quantity and incomplete social care data. We propose a mathematical model to show how savings may be generated under different models of commitment with third party providers. Secondly, we propose a hybrid-fuzzy demand model to generate estimates of demand in the short to medium term that can be used to inform contract design based on local area needs – such an approach we argue is more suited to problems in which historic activity is incomplete or limited. Our results show that commitment models can be of great use to local health care planners with respect to lowering their care costs, at the same time our formulation had wider generic applicability to procurement type problems where commitment size in addition to the timing of commitments needs to be determined.

To my friends and family

Declaration

I declare that all the material contained in this thesis is my own work and that where any material could be construed as the work of others, it is fully cited and referenced, and/or with appropriate acknowledgement given

– Philip Worrall

Acknowledgements

Completion of my PhD would simply not have been possible without the help, support and guidance of a number of important people. Firstly, I would like to thank and express my sincerest gratitude to my supervisor Professor Thierry J. Chaussalet. Thierry, I am eternally grateful for all your support and the patience you have shown me during my years at Westminster. I could not have asked for a better advisor in all matters professional and personal alike, I consider myself very privileged.

I especially thank my mum and dad. They have done so much for me and been an immense tower of strength when times were tough. It has been very hard being away from you both and I would have given up long ago if it were not for you.

I thank fellow past and previous research group members, Salma Chahed, Saiful Muhammed Islam and Sarah Dalton, for their expertise in specific areas of the thesis and making the office an enjoyable place to work and study. To my head of department, Patrick Leas, I would like to say thank you for the professional courtesy you have shown me. I have always been appreciative of your impartial and honest advice.

I would also like to take the opportunity to thank my uncle Mike for his countless words of encouragement, financial support and the many midnight conversations we had – they really helped to take my mind off things.

Zara, I wholeheartedly wished we had more time together. I will never forget you, may you rest in peace.

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Nomenclature

ACF	Autocorrelation function
ADF	Augmented dickey-fuller
ADL	Activities associated with daily living
AGO	Accumulated generating operation
ARIMA	Autoregressive integrated moving average
BC	British Columbia
BI	Barthel index
BMA	British Medical Association
CASSR	Councils with adult social service responsibilities
CCG	Clinical commissioning group
CHC	Continuing health care
CI	Cognitive Impairment
DoH	Department of health
EMI	Elderly mentally infirm
EU	European Union
FNC	Funded nursing care
FY	Fiscal year
GAD	Government actuaries department
GDP	Gross Domestic Product

GP	General practitioner
GPO	Group purchasing organisation
HSCIC	Health and social care information centre
IADL	Instrumental activities of daily living
IAGO	Inversed accumulated generating operation
IQR	Interquartile range.
JHSCNA	Joint health and social care needs assessment
KPSS	Kwiatkowski–Phillips–Schmidt–Shin
LA	Local authority
LTC	Long-term care
LTPM	Long-term policy model
MAPE	Mean absolute percentage error
MDT	Multi-disciplinary team
MH	Mental health
MIPP	Mixed Integer Programming Problem
MP	Mathematical program
MVC	Model-view-controller
NGO	Non-government organisation
NH	Nursing homes
NHS	National health service
OECD	Organisation for economic co-operation and development
OLS	Ordinary least squares
ONS	Office for National Statistics
PACF	Partial autocorrelation function
PCT	Primary care trust

PSSRU	Personal Social Services Research Unit
RC	Residential care
RH	Residential homes
RMSE	Root mean squared error
SF	Self-funded
SHA	Strategic health authority
SIH	Severe ill health
SQL	Structured query language
SSM	Soft systems methodology
SSP	Supplier selection problem
STOM	Short-term operational model
UN	United nations
US	United States of America

Chapter 1

Introduction

Long-term care (LTC) is an umbrella term that refers to a range of treatment and support services provided to those that experience difficulty performing activities associated with daily living (ADL). For instance, an individual may be unable to physically feed, bathe, go to the toilet, take medication or dress without assistance. A key aspect of LTC is that it crosses both the health and social care domains, in that an individual may require health treatments to manage chronic illnesses or disability, whilst at the same depend on domestic and supportive assistance. As a result, the needs of LTC patients are met through collaboration between different local government services, the health service, the voluntary sector and family members. In contrast to other health services, the general premise of LTC is not to cure but to help an individual to both obtain and maintain an optimal level of functioning throughout the remainder of their life.

1.1 LTC delivery methods

Although the need for LTC could have arisen at any point in one's life, for instance an individual could become physically frail as a result of a road traffic accident, it is typically provided to those aged 65 or over, who have an estimated 40% chance of entering a NH (Medicare 2009). Diseases and illnesses that are frequently associated with LTC include dementia, cancer, Parkinson's disease and Huntington's disease.

Within LTC we can distinguish between two main types of care, namely informal and formal care. Informal care is the care provided at home by friends or relatives of patients, whilst formal care includes care provided by qualified health and social care professionals. Whilst both types of care incur costs, formal care is directly paid for whereas informal care costs are more closely linked with the opportunity cost of a relative or family member not working. In a number of cases, patients will use receive both formal and informal care throughout their time in LTC. For example, patients receiving formal care might have their care stepped down temporally during holidays and weekend periods, when perhaps relatives or family members are in a position to take over care responsibilities. Similarly, family members that normally provide LTC may ask for more formal assistance from time to time to help reduce the burden.

1.2 The nature of LTC

Depending on factors such as the individual's needs, level of mobility and preferences, LTC can take place in a range of different settings. It can be provided at home, in nursing homes (NH), residential homes (RH), community centres, assisted living accommodation as well as in hospices (Medicare 2010). Although in the UK system there has been a move away from making a distinction between NH and RH, the general premise is that NH have larger numbers of qualified nursing staff and thus more likely to cater for patients with higher levels of health related, rather than socially related, care needs. Hospices, on the other hand, are more likely to cater for those in need of specialist palliative support and pain management.

LTC is a highly labour intensive form of care. The complex needs of LTC patients, combined with the high resource requirements and range of services necessary to manage LTC conditions can result in high care costs. In London alone the cost of LTC is attributed to one twelfth of the NHS non-pay spent - circa £320 million per annum (London Procurement Programme 2009). In the United States (US) around 10% of the patients in nursing homes stay for 5 years or longer (Medicare 2009) thus representing persistent and on-going costs. Yet even when LTC is provided largely by informal means, the total care costs might not truly reflect the total societal cost of care. In particular, there is an on-going opportunity cost associated with providing informal care. Such costs are most often borne by family and relatives, including for instance the loss of earnings.

1.3 Funding for LTC

In England and Wales, LTC is funded in a variety of different ways depending on the extent to which an individual's need for LTC is due primarily to an underlying health condition. If an individual has a greater proportion of health care needs they will be more likely to qualify for fully funded LTC, also known as NHS Continuing Health Care (NHS CHC) (Department of Health 2007). If an individual's needs are less health related then the responsibility for providing care rests with local authorities (LA). In this case the LA, corresponding to where a person lives, will contribute all or the majority of the funds necessary to cover care costs - subject to means-testing.

In recent times, systems of LTC have received increasing attention from policy makers (Martini, et al. 2007, Brau and Bruni 2008). In part, this appears largely due to the belief that changes in population demographics this century, as a result of high birth rates in the post-war period, together with an increasing probability of surviving into older age (Tamiya, et al. 2011), will further increase the burden on healthcare systems to provide LTC to elderly patients. Furthermore, a decrease in the ability of family-support networks to provide informal care has been cited as additional pressure for a potentially already overstretched system (Pavolini and Ranci 2008).

Not surprisingly, a number of studies have therefore proceeded to pose serious questions surrounding both the ability of existing LTC systems to cope with sharp increases in the number of elderly patients (Peng, Ling and Qun 2010) and the implications for cost. Clearly there is a need to accurately gauge the future pattern of demand to assess what impact, if any, such effects are likely to have. The potential future cost of running LTC systems is particularly of interest to countries like the US, Germany, UK, Sweden, Netherlands and Taiwan, who currently run either a fully public funded system of LTC or a hybrid public-private funding programme. To date, the majority of studies have focused on national rather than regional level issues, thus the needs of those often tasked with the operation of the local LTC system have not fully been considered. At this stage it is not yet clear how conclusions draw at the national level translate to the local level.

In this respect, we have identified a number of concerns with current methodologies that have been used to explore such issues and generate reliable estimates of LTC demand, the impact and ultimately the cost for local LTC planners. Issues of particular interest include:

- Differences in quality and comprehensiveness of local LTC datasets,
- The validity of results based on short term forecast horizons and,
- How such forecasts may support and increase efficiency of local LTC planning.

Answers to these issues should facilitate a greater understanding of the LTC demand process at the local level and provide opportunities to explore areas where cost savings and improved outcomes can be achieved.

1.4 Aims of the thesis

The thesis aims to provide an investigation of these issues from a local planning perspective. More specifically, it:

- Develops a number of modelling approaches, which systematically tackle the issue surrounding the appropriate choice of model for such short-term problems.
- Constructs a forecasting framework using routinely available data to illustrate the ability of limit historic patient data to predict future care costs, duration and future spend at the local level. The final modelling framework enables both LA and NHS planners to more efficiently plan their future LTC spend and, through web-

enablement, provide a unique ability to compare projections of costs across different health care regions in London.

- Proposes a mathematical formulation that can be used to determine optimum allocations of provider contracts given robust estimates of patient demand and provider discounts for specific volume or time-based commitments.

Although the focus of our work is in the study of LTC, in principle it could be extended to model other health care processes where forecasting at a local level may be desirable. For instance, our approach could be adapted to study mental health (MH) services, where the duration of care is typically long and care costs are high. At the same time, our mathematical formulation, which harnesses the derived forecasts to generate cost savings, may also have applications in more general procurement problems in which time or volume based discounting occurs.

1.5 Contributions

This research makes a number of unique contributions relating to forecasting LTC patient demand at the local level and optimisation of the commitment volume held by LTC planners. In this thesis we use the example of health care in England, specifically London, to illustrate our findings. A number of specific contributions are summarised as follows:

- The research provides a novel approach in modelling patient flow into LTC, the duration of their stay and cost based on routinely available data on previous LTC placements. The novelty lies in the intuitive adaptation/extension of a hybrid grey-fuzzy regression methodology.
- It focuses on initiating and developing a methodology that has rarely been used in patient demand forecasting. The usefulness of the methodology will be exposed to the academic community, and to health and social care planners operating at the local level, which will lead to many interesting investigations and potentially an application to other areas of the health care system.

- Together with the forecasting framework, we propose a more general formulation of the LTC contracting process, whereby increased information about future demand can be modelled and incorporated into the decision making process surrounding the number and duration of placements to purchase from external care-providing organisations.
- The contracting methodology proposed considers the possibility of contracts being formed that have variable durations and staggered start and end dates – something that has not been addressed within existing studies.
- Our formulation of the contracting process has sufficient generality to allow extensions and applications to other procurement problems when contract choice or the decision to make a monetary commitment to a provider service is involved.
- The research is expected to contribute to the academic community; operational researchers; and to the community of health and social care planners since the methodology can help support more effective decision making in LTC allocation, budgeting and purchasing decisions.
- The web-based tool that was developed specifically to disseminate the research within the health care sector makes contributions in the areas of software design methodologies for health care planning systems and the integration of different data formats used to populate the tool with patient level data is useful for those involved with the development of such systems.
- The methodology is transferable in that it can be expanded to consider problems in similar domains, particularly where the availability of long periods of historic data for model building is limited and/or incomplete or demand is slow moving.

1.6 Collaborator

The patient level data used to support the research problem was collected and provided by the NHS London Procurement Programme on behalf on several NHS Primary Care Trusts

(PCTs), including: NHS Havering, NHS Islington, NHS Hammersmith and Fulham, NHS Croydon, and NHS Bexley.

1.7 Outline of the thesis

Figure 1.1 illustrates the flow of the remaining chapters and their relations. These chapters, presented in sequence, are grouped into topics of background, literature reviews, theoretical concepts and contributions.

Chapter 2: An overview of long-term care

In this chapter, we present an extensive overview of LTC and describe in detail the functioning of the system of LTC in England and Wales. We examine the historical milestones in the development of the LTC system and perform a simple cross country comparison. Thought briefly, we discuss the market for LTC services, different systems of funding and outline the various purchasing options for health and social care commissioners.

Chapter 3: Literature review

In this chapter, we present an extensive literature review on the current state of research in LTC activity and cost modelling. The scope of this thesis, namely, the identification of a lack of modelling framework for forecasting demand and cost of LTC at the local level, together with the theoretical basis for using such predictions in planning decisions, is derived from this chapter.

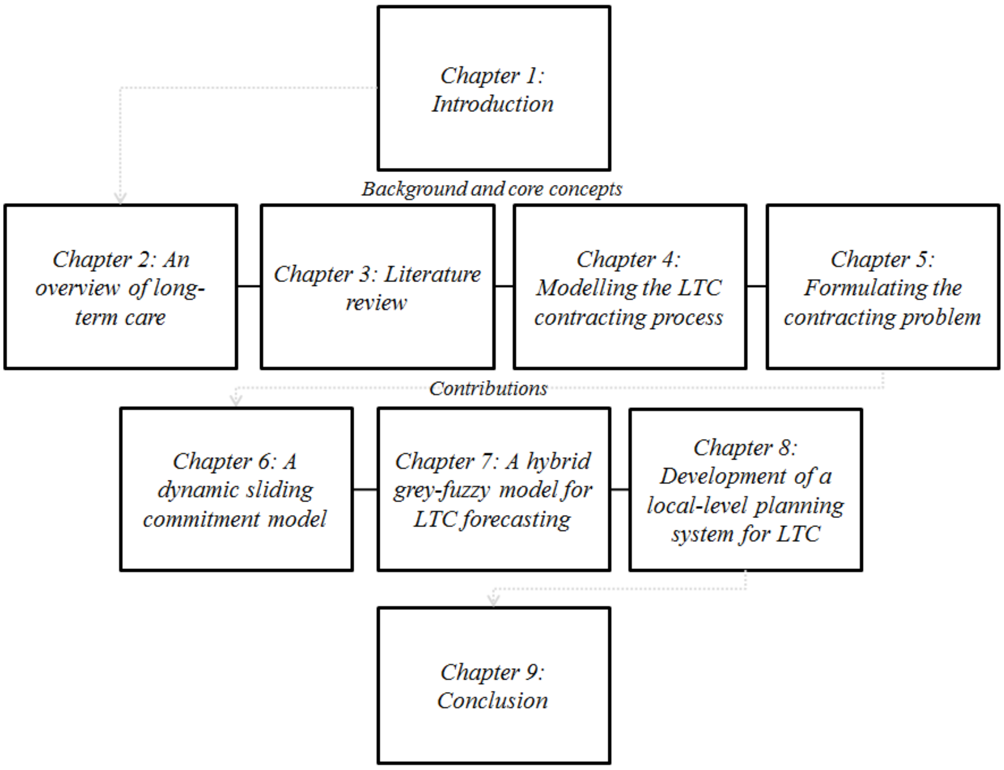

Figure 1.1– Overview of Thesis

Chapter 4: Modelling the LTC contracting process

In chapter 4 we argue that a key barrier to the adoption of local level LTC forecasting stems from the assumption that little relevant insight can be gained. Here we explore the use of contracting to generate cost savings, contracts that require LTC planners to make a commitment to using a particular provider. Prior to formulating the contracting problem facing local commissioners we examine available data on LTC activity to identify what data is readily available to support the contracting process.

Chapter 5: Formulating the contracting problem

Extensive studies on contract design have been carried out but few mathematical models of contract formulation for service type goods have been proposed. To date, much of the mathematical modelling work has centred on production or material goods, for which production quantities are consumed by demand. In order to utilise methodologies for the purposes of contract design, we study the similarities and differences between production and service orientated models to identify what adaptations, if any, need to be considered before demonstrating a simple min cost formulation of the problem facing LTC commissioners.

Chapter 6: A dynamic sliding commitment model

Having stated the theoretic basis for allocating patients to LTC, we model the provider commitment decision faced by local LTC planners. We formulate the problem as a mathematical program (MP) and consider optimal contract choice under different commitment scenarios. In contrast to previous work, we allow for the possibility of sliding contracts and control over the maximum market share awarded to private sector providers.

Chapter 7: A hybrid grey-fuzzy model for LTC forecasting

Chapter 7 gives a brief account of Grey Systems Theory, a methodology originally proposed by Deng (Deng, Control problems of Grey Systems 1982) that models interactions in complex systems where the information that describes the underlying processes is poor, uncertain and incomplete. A variant of this methodology is used and hybridised with the fuzzy regression approach to address the real-world needs and constraints facing LTC planners when estimating future activity.

Chapter 8: Development of a local-level planning system for LTC

Contemporary health and social care commissions use a variety of reporting software and different patient software management tools to support day-to-day decision making and strategic planning. In chapter 8, we explore the possibility of implementing our forecasting and contracting framework using the model-view-controller (MVC) paradigm to enable commissions to make use of our modelling approach. Specifically, we study the implications of different data formats used to report LTC activity and how such data can be integrated and assembled so as to make more effective planning decisions.

1.8 Summary

In this chapter, we provided a brief background of long-term care, along with a summary of the aims of this thesis and its contributions. In the next chapter, we present a more detailed review of the UK system of LTC including, giving a brief account of its development, the provider market and funding arrangements. The purpose of the next chapter is to help us identify our research problem within the wider context of LTC.

Chapter 2

An overview of long-term care

2.1 Introduction

LTC includes the range of services and treatment options provided to those with chronic illness, mental or physical disability. Despite LTC in the UK being host to a number of reforms it remains key part of the UK health and social care system, both in terms of expenditure and the volume of people receiving such services. The UK is not unique in having a LTC system, a number countries including: Japan, US, Netherlands, Taiwan, France, Spain and Germany have similar systems in place. LTC systems differ in a number of ways, one fundamental difference relates to how they are funded and the amount of contribution individuals make towards their care costs.

In this chapter, we will step through historic developments in LTC in the UK, with a particular emphasis on changes to funding. In section 2.3 we state the role of NHS Continuing Healthcare (CHC), a form of LTC provided by the NHS in England and Wales. In section 2.4, we will compare the structure of the UK system of LTC with other international systems before addressing the LTC planning process.

2.2 Brief history of LTC in the UK

The current incarnation of LTC in the England and Wales is based upon a dual system of health and social care, with local authorities (LAs) providing means-tested social care and the NHS providing health services, including funded nursing, that is free at the point of use. Needs that are not primarily due to an underlying health condition are met by LAs whereas, those needs that have arisen due to chronic illness or disability are met by the NHS. In practice, most individuals will fall between these two extremes and the challenge is in deciding the individual responsibilities of both the LA and NHS. In England and Wales, this situation is often referred to as joint funding.

In the mid-1990s, following a number of welfare reforms and the enactment of the Health Service and Community Care Act 1990, the system of LTC in the UK was radically overhauled. LAs, previously responsible for the organisation and funding of LTC, were encouraged to rely more heavily on the voluntary sector. At the same time, all service users would now make a contribution towards their care costs (Thane 2009). This quite different from early policy in which the NHS contributed funding for joint health and social care projects with LAs. After the reforms, LAs tended to focus more on meeting the needs of those with the highest levels of needs so as to make the most effective use of budgets. Following the reforms, access to social services and funding for LTC became highly variable.

2.3 National framework for NHS continuing healthcare

NHS Continuing Healthcare (NHS CHC) relates to LTC that is wholly funded by the NHS and is a key area for which our collaborating organisation, the NHS London Procurement Programme, is responsible for securing commercial advantage. To this effect our collaborating organisation has provided the study with data on NHS CHC activity to support the development of our methodology. In this section we describe the role of NHS CHC and its associated processes.

In 2007, a national framework for LTC was introduced in England and Wales by the Department of Health (DoH) (Department of Health 2007). This was in response to numerous legal cases that had been brought to the attention of the courts surrounding funding decisions for LTC been made by various LAs. The aim of the framework was to introduce a standard way of deciding whether an individual would be eligible for NHS support towards care costs to prevent service disparity. Rather than each of the 28 strategic health authorities (SHAs) having their own rules and processes for determining eligibility for LTC there would be a single national policy that all NHS organisations - including NHS Primary Care Trusts[1] (PCTs) who at the time were chiefly responsible for funding and organising NHS CHC - would have to adhere to.

Since the introduction of the framework the number of people receiving NHS CHC had increased by 67% from 27,822 at the end of September 2007 to 46,599 at the end of March 2009 (Alzheimers Society 2009). By late 2011 this number had reached 53,466 in the UK or 108 people per 100,000 (Department of Health 2009).

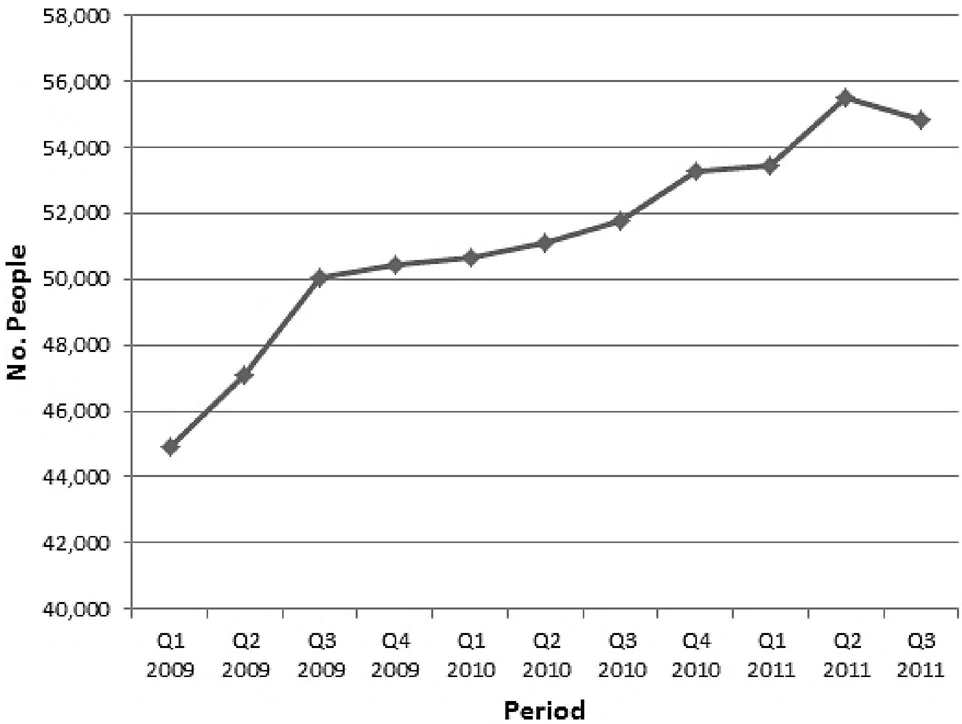

Figure 2.1– Number of people in receipt of NHS CHC

[1] Since April 2013 Primary Care Trusts (PCTs) have been replaced by Clinical Commissioning Groups (CCGs) in England.

The framework provided two tools that could be applied to determine an individual's eligibility for either (i) NHS CHC or (ii) funded nursing care (FNC). While NHS CHC included both the medical and non-medical costs of LTC, FNC was limited to covering the cost of a NHS nurse. The checklist tool was used to quickly determine whether in principle an individual would be eligible while the more rigorous decision support tool (CHC DST) would be used to determine ultimate eligibility.

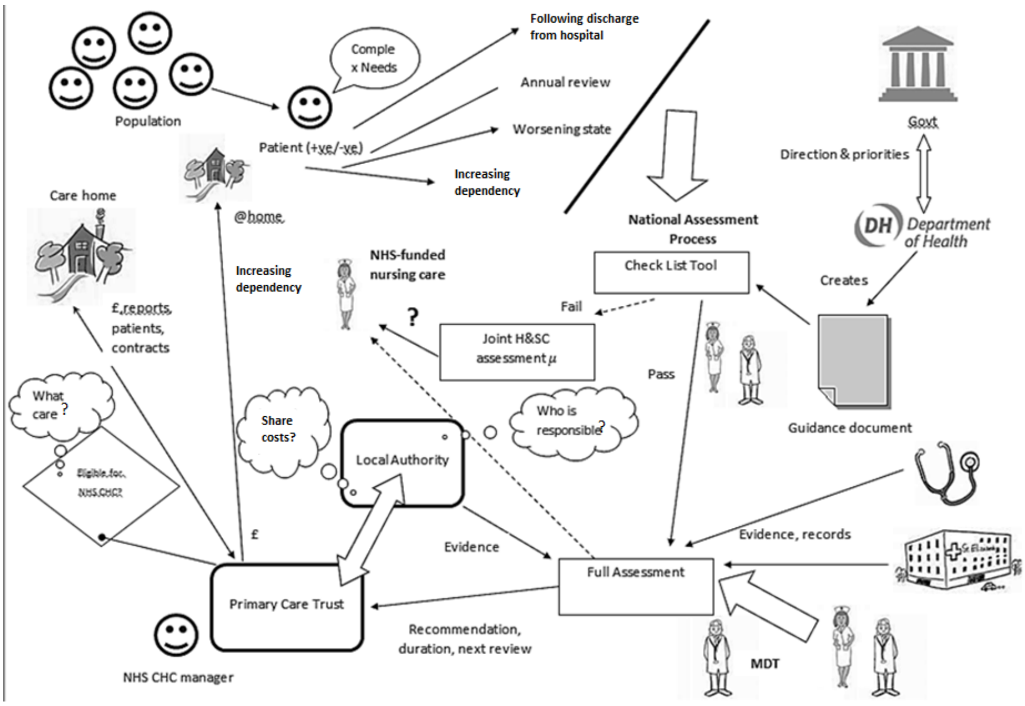

Figure 2.2– Rich picture of key processes in LTC

Figure 2.2 presents a rich picture representation of the NHS LTC situation in England and Wales following the introduction of the national framework in 2007. This picture was drawn in 2009 using Soft Systems Methodology (SSM) using feedback from LTC commissioners and review of health legislation (Checkland 1998).

From Figure 2.2 we observe that the DoH submits a guidance document that forms the basis of the national assessment process for CHC. Individuals begin the assessment process when they are identified as having a worsening state, perhaps following a GP appointment or hospital attendance. Similarly, family or relatives of an individual may also initiate the assessment process by contacting their LA and providing them with details surrounding the individual in question.

2.3.1 Check list tool

The DoH issued check list tool serves to provisionally determine whether an individual would benefit from some form of LTC service. As such, it is chiefly used as a basis for determining whether the individual should go through a full CHC assessment. The check list tool requires the practitioner, either a social care worker or medical practitioner, to indicate whether the patient appears to exhibit difficulties in one or more key areas. Such areas include but are not limited to: level of mobility, ability to consume adequate food and drink, breathing ability and ability to take medication.

If an individual does not exhibit severe difficulties in any of the areas set out in the checklist tool, they will not be considered suitable for the full assessment. Although there is no strict limit as to the number of times the checklist tool can be applied, in practice individuals will likely be advised to consider reassessment on an annual basis or when their circumstances change dramatically. In special cases the use of the checklist tool may be sidestepped if the individual is directly referred to the assessment phase by a clinician or social worker – commonly this is known as a direct referral to CHC.

2.3.2 Joint health and social care assessment

If an individual does not meet the requirements of the check list tool then they will not be considered for fully funded NHS CHC. In such cases the individual's care needs are assumed to not be primary due to an underlying health condition and are more closely related to social care needs. However, a joint health and social care assessment (JHSCA) may still be carried out to determine what social care the LA may need to put in place to support an individual's ongoing needs, and what, if any, support is required from the NHS in meeting specific health needs. For example a patient may require visits from a NHS funded nurse to administer specialist medication. It should be noted that whilst the NHS is always required to meet the medical needs of individuals, social services provided by the LA may be subject to means-testing and individuals may thus be responsible for meeting some or all of their care costs.

2.3.3 Full assessment

The role of the full assessment is to determine edibility for NHS CHC. The assessment itself is carried by a multidisciplinary team (MDT) of health and social care professionals, including: clinicians, GPs, social workers and community nurses. The goal of having a MDT is to help determine the full extent of an individual's health and social care needs, mitigate any potential basis and to facilitate a consistent evidence-based decision making process.

As part of the assessment process, the MDT has to complete the DoH's decision support tool (DST CHC) by providing responses to key questions surrounding the circumstances of the patient, relevant history, what health interventions are currently in place to help the patient manage their condition and what could reasonably be added, whether or not the patient's care needs are episodic or require continuous long-term support, their mental capacity and a description of their daily routines. Before full assessment can take place, the individual must consent to being assessed and be given an opportunity to voice their concerns and opinions surrounding possible improvements to their care situation.

To support the patient during the assessment process any carers currently working with the individual are invited to participate in the assessment and voice any concerns that they have. Similarly family members may express their views to the MDT to be taken on board if consent for this is given by the individual. The LA contributes to the assessment process by providing evidence as to an individual's current situation. Such evidence could include:

records of visits the LA has mad, any current or previous social care arrangements. This information is considered alongside medical history and details of recent A&E attendances to support the MDT make a more informed recommendation.

The MDT is responsible for completing a care domain assessment scorecard, detailed in Figure 2.3, which indicates the extent to which support is required in each of the 12 NHS CHC care domains. The scorecard serves to summarise the overall content of the assessment whilst forming an evidence basis for the MDTs final recommendation. The column headers P, S, H, M, L, and N correspond with priority, serve, high, medium, low and no level of need respectively. For each care domain the MDT must indicate the extent to which the individual requires support in this area.

Care Domain	P	S	H	M	L	N
Behaviour						
Cognition						
Psychological Needs						
Communication						
Mobility						
Nutrition – Food and Drink						
Continence						
Skin (including tissue viability)						
Breathing						
Drug Therapies and Medication						
Altered States of Consciousness						
Other significant care needs						
Totals						

Figure 2.3– Decision Support Tool (DST) Scorecard

For some care domains it is noted that it is impossible for the patient to be allocated a priority or severe need, as for instance shown by the greyed out boxes for P and S under the care domain cognition. This is not to say that the patient's care needs for these care domains are not a priority, or in the case of S severe, but instead refers to the fact that within the context of LTC a mark in this box alone would not constitute a high enough a level of need for fully funded LTC. On the contrary, difficulty breathing alone would be considered a priority and hence, even in the absence of needs in each of the 11 remaining care domains, would be sufficient for NHS CHC to be awarded.

Once the assessment is complete the final step is for the MDT to make a recommendation as to what support services the patient requires and decide whether their needs are fundamentally due to an underlying health condition. Prior to 2013, this recommendation was submitted to the PCT (Primary Care Trust) to which the patient's registered GP belonged. Since 2013, this recommendation is now forwarded to the patients local CCG (Clinical Commissioning Group). Typically, a CHC panel will meet once per month to ultimately decide whether to award CHC funding to each individual on a case by case basis taking into account the recommendation of the MDT and the output from the assessment process.

Only rarely will the CHC panel disagree with the MDT unless it can find fault with how the assessment has been carried out or where there are significant disagreements between different members of the MDT as to the precise needs of the patient. In the case that NHS CHC is awarded the CCG will be responsible for arranging and managing the services that will be provided as part of the care package, taking into account any preferences or opinions of the individual, in addition to letting them know which provider organisations will be involved and where care will be provided. If the decision is not to fund the individual's LTC under NHS CHC then the patient may be referred to their local authority to consider other forms of means-tested social support and or NHS funded nursing care.

2.3.4 Fast-tracked assessment

Despite no specific time limit for the assessment process most assessments should take less than one month to be completed and at most one month before a decision is reached. The exact duration depends on a number of factors including the date upon which the CHC assessment panel meets each month relative to when the assessment is in fact submitted. In addition, delays can be incurred if members of the MDT cannot find suitable times in which they can all simultaneously meet. Furthermore, depending on an individual's situation, the CCG will usually try to invite potential MDT members with specific expertise or experience that may be relevant to an individual's situation. Depending on practitioner availability and expertise required this can be a time-consuming process. Even where a decision is reached quickly there is usually an additional delay of up to one month to allow for allocation and arrangement of the services required.

In cases where an individual has a rapidly deteriorating condition the process of assessment and allocation can be sidestepped through the Fast-Track option. Under the fast-track route, patients are given priority access to care treatments and support such that their care commences almost immediately following a clinician's recommendation. The key criterion is that a patient's condition is rapidly deteriorating and the condition may be entering a terminal phase. Furthermore, the nature of needs of the patient are beyond what a social services authority reasonably be expected to provide. Within the context of LTC, fast tracked patients represent a small minority of all allocation decisions and most often relate to individuals with terminal illnesses, such as cancer, where the care provided largely deals with the management of the patients pain.

2.3.5 Allocation to care

Once an individual has become eligible for NHS CHC it remains to allocate them to a suitable care packages. Figure 2.4 presents a graphical overview of the assessment process prior to allocation, starting at the point whereby an individual is identified as potentially being in need of LTC. Only if LTC is awarded, either by the CHC panel or through a fast track process, do individuals in fact enter into the CCG commissioner's allocation decision. This is shown on the diagram by the dashed dotted lines.

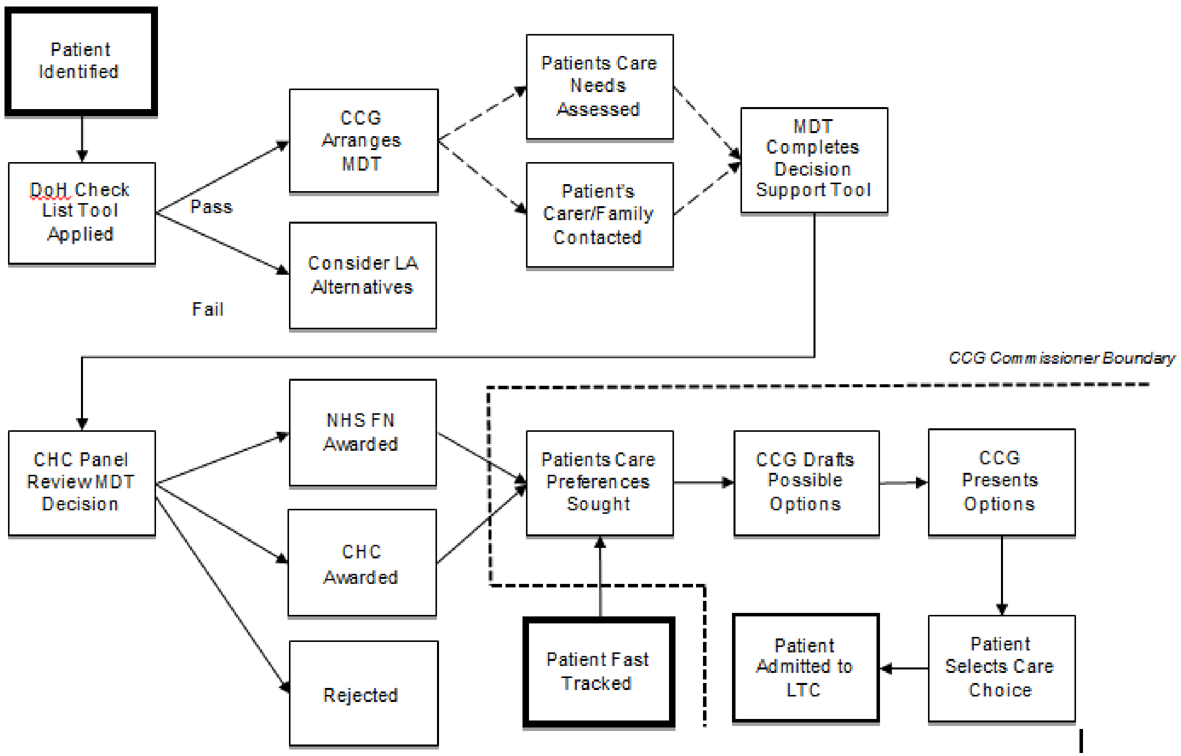

Figure 2.4– Graphical Representation of CHC Assessment Process

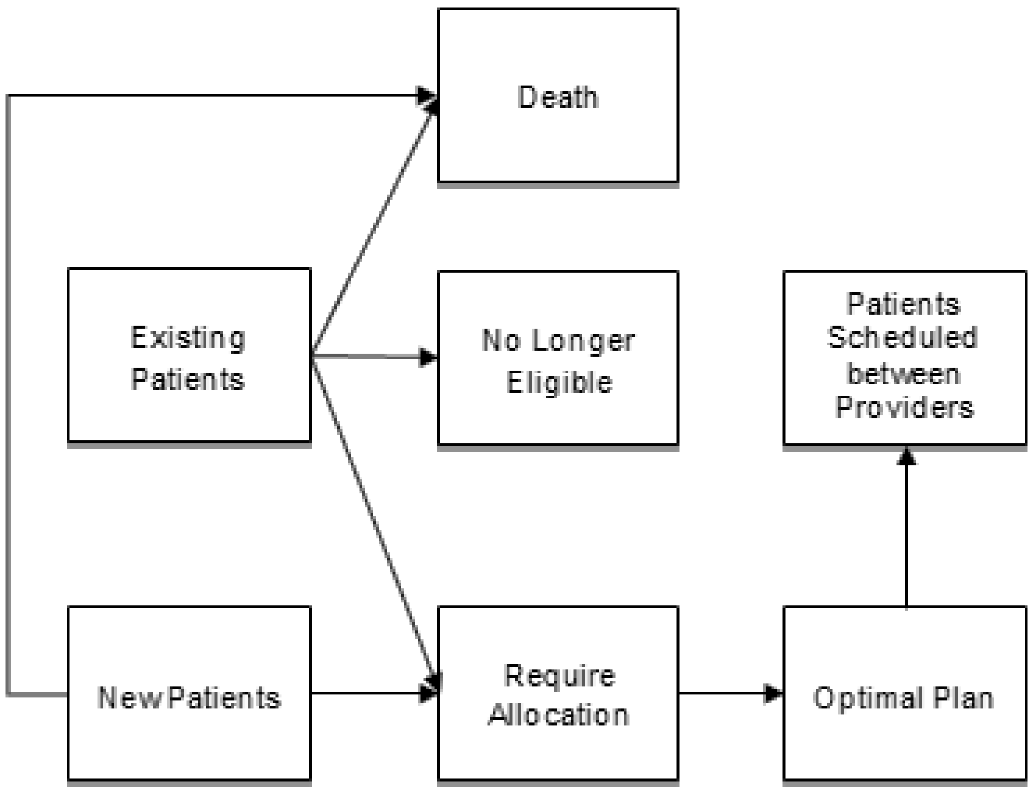

Figure 2.5– Graphical Representation of CHC Allocation Process

Figure 2.5 breaks down the allocation process further and follows on from the decision to award LTC being made. The total number of patients requiring allocation to LTC is given by the sum of those currently in care that continue to be eligible and that don't leave due to death plus the total number of new patients. Based on these two elements of demand the CCG allocates patients between available care providers to devise an optimal plan of care such that each patient is allocated to an appropriate care setting in light of their needs and to meet specific budget requirements for the CCG as a whole. We note that individuals may leave NHS CHC due to death or because they become no longer eligible. Given that the condition of a LTC patient will more than likely worsen over time, typically a patient will only ever become ineligible for care if it is later found that they did not require fast tracking or in cases where it was discovered that there were mistakes made in the assessment process.

2.3.6 Subsequent revisions to the national framework

The 2007 framework left open the possibility for subsequent review and in 2009 the DoH issued a revised version (Department of Health 2009). While much of the framework remained the same, an attempt was made to clarify several key definitions. Moreover, additional emphasis was placed on the involvement of the LA and the patient during the assessment process. Time allowed for communicating funding decisions was also extended from 14 to 28 days on the understanding that this would permit collecting input from all stakeholders and thus increase rigorousness.

Following the reorganisation of the NHS in England that came into force on the 1st of April 2013, the responsibility for determining eligibility for NHS CHC and the management of existing patients was transferred from PCTs to Clinical Commissioning Groups (CCGs)[2]. At the same time, strategic health authorities (SHAs), previously responsible for overseeing funding decisions made by the NHS organisations, were abolished and replaced by a national commissioning board – NHS England. A revision of the 2009 national

[2] For more information about the structure of the NHS and its historic changes http://www.nhs.uk/NHSEngland/thenhs/about/Pages/nhsstructure.aspx

framework was then enacted, providing a means for the statutory responsibilities of PCTs and SHAs to be transferred to their contemporary counterparts (Department of Health 2013).

A new feature of the revised 2009 framework was the establishment of a right for individuals to request a personal health budget (PHB). Such a budget would empower individuals to manage the provision of their care - albeit with input from health and social care professionals, including their GP.

2.4 International perspectives on LTC

Internationally, LTC systems exist in variety of forms and stark contrasts can exist in several key areas (Alzheimer Europe 2009), not least in terms of terminology used, access to funding, available services and the role of informal care. As a consequence, cross country evaluation of two or more LTC systems can be problematic. Compared with the funding situation in England, it does tend to be the case that a greater proportion of expenditure on LTC from countries outside of the UK is met through the private sector either through insurance schemes or personal contributions.

A common feature across several LTC systems is the move by policy makers to rethink the role and structure of LTC provision in light of expectations of future demand. In Organisation for Economic Co-operation and Development (OECD) countries, recognition of the growing interest in LTC was highlighted when LTC was included for the first time in the annual Health at a Glance report in 2011 (OECD 2011). One concern is the anticipated growth rate of the population aged 65 and over. Assuming existing LTC service patterns and current trends continue such growth has the potential to significantly increase the demand for LTC and total expenditure. In the EU, a formalisation of LTC through legal policy has been one of the biggest drivers of increased public demand (European Commission 2010) and the concern is that demand may outstretch the pace of expansion of many LTC systems. Before turning our attention to current expenditure on LTC, we summarise prevalent LTC systems in Europe and other OECD countries.

2.4.1 Hospital bed usage

In England, hospitals historically played a much greater role in the provision of LTC. Since the 1980s there has been a gradual shift away from caring for LTC patients in large long-stay geriatric wards and specialist hospitals towards smaller community orientated facilities. The rationale for this change in policy was in part due to concerns surrounding the quality of existing long-stay provision and the view that hospitals may not be an appropriate place for LTC patients – given their needs often met through social support (BBC 2003). The last hospital providing LTC in England was closed in mid-2009 (Mencap 2009, Disability News Service 2009).

Unlike the UK, most other OECD countries dedicate a moderate proportion of hospital beds to LTC provision. Despite there being no general consensus on which method of LTC provision provides the most suitable environment, the benefit of having LTC hospital provision appears to relate to the typically higher concentration of specialist medical practitioners familiar with LTC illnesses and their complications compared with other institutional settings. On the other hand, LTC patients often a higher proportion of non-medical related needs they may be more effectively managed outside of the hospital setting and at lower cost. Other drawbacks which may also be applicable to general long-term hospital stay relate to the perceived lack of long-term privacy in a hospital setting, barriers to ordinary activity, lack of independence, lack of choice of care provision other than particular hospital and potentially more distant patient-staff relations (Perring 1998). In our analysis it was found that that Japan (28%), Korea (29%) and Ireland (30%) have the highest proportion of LTC hospital beds among OECD members, while the UK belongs to a minority group (including Turkey, Greece, Denmark and Portugal) that sets aside few or no beds to long-term care (OECD 2011).

2.4.2 Informal LTC

Whilst many countries have formalised their provision of LTC gradually over time, informal care remains an integral part of many LTC systems, including countries where a comprehensive system of LTC exists (OECD 2011). The proportion of informal care that takes place varies according to the overall societal and cultural attitudes towards care of the elderly and the role of the family unit in supporting relatives. Across the OECD more than one in ten adults provides informal care giving or assistance in performing ADL. In the EU, it has been noted that in the Nordic-style countries, where state provision of institutional care is high, informal care is of less importance. In contrast, for most Mediterranean countries informal care plays a much greater role.

European continental countries, including the UK, tend to sit somewhere between these two extremes (Styczynska and Sowa 2011) and informal care complements formal LTC provision to a greater or lesser extent. In the US in 2009, it was found that approximately 87% of Americans in need of LTC receive care from informally (The Scan Foundation 2012) whilst for Canada it was found to be 80% (Canadian Life and Health Insurance Association 2012) .

2.4.3 Funding LTC

Funding for LTC is made problematic due to the number of different funding programmes that exist (OECD 2011). In universal systems, funding for LTC is provided to all individuals that are deemed eligible through a single system. Universal systems can either be funded through general taxation or through a separate pubic long-term insurance programmes that are mandatory for those in employment. A universal system does not necessarily cover all care costs; individuals may be required to contribute towards their care costs if their income is above a certain threshold or to access non-standard services. Such additional contributions are known as co-payments.

Mixed systems are distinguished by the fact that individuals access LTC financing through a several different benefit and insurance schemes rather than through a single system of entitlement. Here, one aspect of care might be funded through a public insurance scheme, yet individuals will need to apply to other programmes if they want to access other services. As with universal systems of funding, there may be elements of means testing and depending on coverage; individuals may need to personally meet the costs of services that are not included.

The UK is an example of a mixed system in that different types of services are covered within different funding systems. While individuals access social and means-tested residential care through their local authority, nursing care is provided either through the national NHS continuing care framework or NHS funded nursing schemes.

2.4.4 Expenditure on LTC

In England, expenditure on LTC is substantial despite the number of people in receipt of such care being relatively small. This is due to the average cost per patient being both high and on-going. LTC is labour intensive due to services being provided on a one-to-one basis between patient and care worker. Such services are not easily automated or subject to the same types of innovations or technological advancements that have helped gradually lower other health care costs. Secondly, LTC costs may include the cost of accommodation in a NH or RH, food and other domestic costs. Thirdly, due to the on-going nature of LTC costs they often persist for many months or years.

Measuring exact expenditure on LTC in England, particularly at the patient level, is made problematic by the fact that several agents may contribute towards the care costs of individuals – including the patient, the LA, the NHS, friends and relatives and the voluntary sector. At the same time, data availability and patient confidentiality make linking individual patient records across different organisations a technical and legal challenge. The way in which LTC cost is reported within more general adult social services budgets can also lead to it being understated. For the NHS, overlap between mental health services and those with mental disorders funded under the umbrella of NHS CHC is also a challenge for deriving NHS expenditure. As we have alluded to earlier, informal care is often omitted from LTC expenditure reports despite in many ways representing true societal cost.

According to the Health and Social Care Information Centre (HSCIC), councils with adult social services responsibilities (CASSRs) in England in 2012/13 were reported to have spent £8.79 billion on social care for the elderly. Furthermore and over the same period, the average cost per adult in supported social care, including in residential care (RC) or intensively at home, was estimated at £599 per week (HSCIC 2013).

Table 2-1 summarises the reported real expenditure on LTC by councils in England since 2007. From the table we note that average NH and RH weekly care costs are very close over the period considered. In practice, NH placements should be higher than for RH due to NHs having a higher proportion of clinical staff and the fact that such institutions manage with patients with more complex needs. On the other hand, we recognise that a limitation of the data reflects the fact that LAs largely refer to these two distinct types as care homes and thus it is difficult to retrospectively attribute expenditure to the precise type. An interesting observation is that self-funded RC placements were found to be consistently more costly than either RH or NH, for example 178% more expensive in 2011/12 compared with RH placements, despite self-funding individuals having lower care needs. We postulate that care providers are more likely to grant discounts to LTC commissioners as a result of their greater buying power compared with self-funding individuals.

Table 2-1 – Social Care Expenditure by Councils in England

Year	Average Cost per Week (Nursing, Residential or at Home)	Total Expenditure (£ Billions) on Those Aged 65+	Average Cost per Week in Nursing Homes**	Average Cost per Week in Residential Homes	Cost per Week for Self-Funded Place in Residential Care	Source
2012/13	599	8,730	N/A	N/A	N/A	(HSCIC 2013)*
2011/12	609	8,920	519	522	934	(HSCIC 2012)
2010/11	623	9,440	534	522	895	(HSCIC 2012)
2009/10	609	9,390	510	520	895	(HSCIC 2011)
2008/09	593	9,080	493	498	824	(HSCIC 2010)
2007/08	559	8,770	467	465	716	(HSCIC 2009)

Whilst the NHS in England spends far less on LTC for the elderly compared with the aggregate amount spent by councils, expenditure on LTC represents a sizable proportion of the overall NHS budget. In nominal terms, the NHS spent £4.81 billion in 2010/11 on LTC services - not including services indirectly related to LTC such as accident and emergency (A&E) attendances following falls or burns. Furthermore, since 2003 the percentage of the

** The similarity in average weekly nursing home and residential home can in part be attributed to difficulty in LA attribution of expenditure between nursing homes and residential homes.

* Since 2013 the average cost per week is no longer reported by care location.

NHS budget in England used to fund LTC has increased from around 2.19% in 2003/04 to 3.9% in 2010/11 – representing an increase of more than 78%.

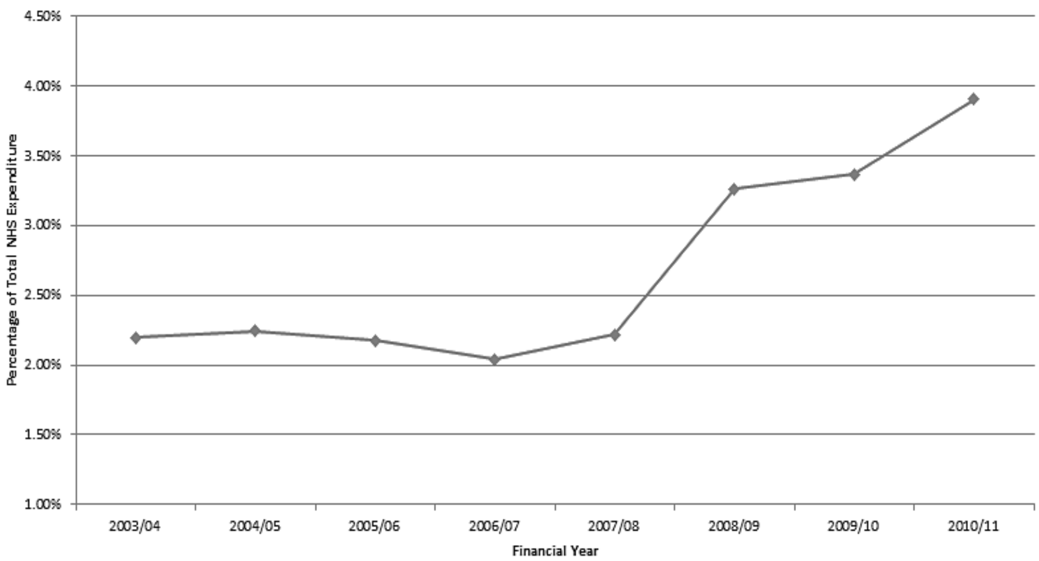

Figure 2.6– Expenditure on Social Care in England as Percentage of Total NHS Expenditure

As in England, many international health care systems report that expenditure on LTC as a whole, including contributions from the private sector, is massive. In the case of the US in 2000, 65% of the total expenditure on LTC (US$ 123 billion) was met through the Medicaid and Medicare federal state based health programs (Freedman, Martin and Schoeni 2002). By 2004, expenditure on LTC in the US had risen to US$ 134.9 billion nationally, with Medicaid accounting for 35.1% of the cost, despite the US government's overall share of the total expenditure falling by 5.7% to 59.3% (Congressional Budget Office 2004). A report in 2009 for FY2008 found that LTC spending through Medicaid alone had passed the US$ 106 billion mark (Burwell, Sredl and Eiken 2009). In Japan, the LTC expenditure for FY2006 was US$ 54.7 billion and represented a doubling of the LTC budget since 2000, following an overhaul in the system of funding (Olivares-Tirado, et al. 2011).

In the Netherlands, “the first country to introduce a universal and mandatory insurance program for LTC”, expenditure on LTC in 2007 was €17.6 billion (Van Den Berg and Schut 2010) (approximately US$ 24.27 billion as of November 2013) with 65% of the total expenditure allocated to the support of the elderly and chronically ill. On the other hand in Hong Kong, where no formal LTC system exists, the nation as a whole was estimated to have spent around 1.4% of its GDP on long-term related care in 2004 (Chung, et al. 2009).

A report into expenditure on LTC in 2000 within OECD countries found that, although there were large variations in spending as a percentage of GDP, public and private sector combined spending accounted for an average of 1.21% of GDP across the OECD with an interquartile range of 0.70% (Haynes, Hill and Banks 2010). By 2009 the average spend in the OECD had risen to 1.3% of GDP (OECD 2011). It should be noted that, during this period several LTC systems shifted their focus towards meeting the needs of most complex cases.

2.5 The market for LTC in England

The majority of LTC provision in England was once owned and run by public sector organisations, including LAs and NHS Trusts. Since the 1980s there has a shift towards increasing amounts of private sector provision (Laing and Buisson 2005). In 2012, it was reported that of those living in LTC institutions, only 1 in 10 were residing in NHS or LA owned institutions (The Independent 2012). As such, the market for LTC has moved from a social to a more mixed market good (Deloitte 2008). The majority of formal care is now provided by a small number of private sector firms, including: Bupa[3], Care UK[4] and Southern Cross[5]. Whilst several independent and specialist providers exist, they mostly focus on meeting the needs of those with specific diseases and or religious preferences.

[3] http://www.bupa.co.uk/care-homes
[4] http://www.careuk.com
[5] http://www.schealthcare.co.uk

For example, Jewish Care[6] runs 70 centres across London and the South East and recognises traditions, beliefs and cultures shared by Jewish People (Jewish Care 2013).

Under this new landscape, LAs and the NHS purchase care from private sector providers, This subtle difference means that, more often than not, commissioners are required to enter into contracts with the private sector on behalf of patients so as to put in place the required services. Whilst the type of contract formed will depend on various factors, including whether there is an existing relationship with the provider; the main contract types include spot, block and framework contracts.

2.5.1 Spot contracts

A spot contract purchases LTC as a "one of" or without any long-term commitment. Such contracts may be are used when the purchaser of care places an individual with a provider they don't routinely use. This can occur when a regular provider is at capacity or because the individual has very specific needs. Other situations that may require the use a spot contract include when a patient wishes to be placed outside of their borough, perhaps due to them wishing to remain closer to family and friends that live further away. Spot contract care costs are normally more expensive compared with both block and framework agreed contracts due to the lack of commitment on behalf of the purchaser.

2.5.2 Block contracts

A block contract consists of a fixed number of care packages purchased in advance by the LA or NHS for a set duration, most commonly between 1-5 years. In this way, the LA or NHS pays a fixed regular amount to the provider on the basis that it has access to the specified number of places defined in the block contract.

[6] http://www.jewishcare.org

Although block contracts have the potential to reduce care expenditure as a result of provider discounting, their use leads to reduced flexibility on behalf of the purchaser and the potential for inefficiency, especially when they are not fully utilised.

2.5.3 Framework contracts

In many ways, framework contracts represent a middle ground between spot and block based contracts. Framework contracts, like block contracts, fix the cost of care for a set period yet as in a spot contract there is no commitment. In the context of LTC, frameworks are created when providers that are party to the framework agree to provide care for a fixed price for the duration the framework is in place. Providers may submit different prices for different services and for different groups of patients, yet the prices submitted must be kept the same until the framework is either: overridden by a new framework agreement; new terms or prices are agreed; or the framework expires and is no longer in operation.

One major benefit of framework contracts is that they can overcome the problem of price disparities, a situation whereby a commissioner pays a different rate to the same provider as another commissioning organisation despite the care being highly similar. Furthermore, less time is spent negotiation price and commissioners can more easily compare prices across providers in an open and transparent way. Standards of care may also be defined within the overall framework agreement which can help encourage commissioners to utilise a provider that they little or no prior experience with.

The disadvantage of framework contracts relates to the strict legal process[7] surrounding their formulation. In addition, owing to the fact that frameworks may be put in place for several years, providers have are incentivised to set higher initial prices than compared with an equivalent spot contract in the starting period to take into account that, except for some exceptions, it can be extremely problematic to adjust them once the framework is operational.

[7] In the UK the relevant law concerning procurement of services supplied to public bodies http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2004:134:0114:0240:EN:PDF

2.5.4 Mini competitions and tendering

Although not a type of contract in their own right, mini competitions may be used as a basis to which block contract prices are agreed and established between care commissioners and providers. During a mini competition, several providers may be invited to take part in a tendering exercise whereby they bids are submitted for consideration. The goal of such exercises is often to put in place a block contract at an optimum price with specific quality requirements. As with framework agreements, various procurement laws have to be followed to ensure a fair and open contest. The smaller scale of mini completions allows them to be finalised in significantly less time compared with the time required to establish a new framework agreement.

2.6 Summary

LTC represents a sizable proportion of total GDP for a significant number of countries. In §2.4 we noted that internationally a key concern of those involved in the organisation and management of LTC systems related to the growth in both the nominal size and relative proportion of the elderly population. At the same time, significant formalisation of LTC systems has taken place such that a number of countries have begun to rethink existing models of funding. In the case of the UK, we documented some of the evolution of UK policy in §2.2 and §2.3.

Within §2.5 a summary of the different methods by which LTC may be purchased, using an example from the UK system, was presented to demonstrate how cost savings could generated by purchasing LTC in a fixed arrangement – in the case of the UK the terminology used is block contract. The potential for savings to be made under this method of funding assumes that care purchasers have an understanding of future demand for LTC services. When such information is known, it remains to decide the optimal contract size and duration so as to minimize overall care costs. At this stage it is not clear whether this type of analysis is currently carried out and which methodologies might best be suited to the underlying demand forecasting problem.

In addition to cost savings, we also noted that there is an opportunity to increase care standards and the quality of the care delivered by working with a small set of providers as compared to a large group owing to the fact that monitoring their performance uses less resources and greater opportunity exists to tailor services to the needs of LTC patients. Given the importance of having a clearer picture of demand for LTC services, for the aforementioned reasons, we will now review recent literature surrounding LTC modelling and the factors that drive demand for LTC.

Chapter 3

Literature review

3.1 Introduction

In recognition of the uncertainty with respect to the potential future demand and cost of LTC, several studies have modelled its operation with the intention of exploring both the number of future users and associated cost. However, we would tend to agree with other studies in that, for a number of reasons, producing accurate forecasts of LTC remains a challenging area of research (De Block, et al. 2010, M. Lagergren 2005). In this chapter, we explore literature surrounding LTC demand modelling and related issues. Our goal is to draw out factors related to the demand for care, both at the micro and macro levels and identify recent developments in LTC forecasting methodologies. We begin by firstly examining current research themes in LTC.

3.2 Research themes in LTC

Within the body of LTC studies found we identified 4 key contemporary research themes, including: future demand, funding and access to services, reform of the LTC system and health of the LTC population. We derived these classifications based upon an examination of the core purpose of each study in terms of stated aims, approach and findings for commonalities. In some cases it was necessary to classify a study in more than one category where sufficient overlap was found. Table 3-1 provides a summary of our results.

Overall we found that a significant body of current research had been undertaken in the area of reform of the LTC system, for example changes to the operation of care homes (Levenson 2009, Mukamel, et al. 2008) , public perception of the LTC system (Blackstone 2008, C. A. De Meijer, et al. 2009, Munn and Adorno 2008) and gaining support for reforms in LTC from the public (Chappell and Penning 2001). Studies that investigated of changes in demand (Eskildsen and Price 2009, Coleman 2002) and forecasting (Caprio, et al. 2008, Murphy, Shea and Cooney 2007) made up the second largest body of research. Papers in the funding category were more concerned with changes in funding models (Asahara, Momose and Murashima 2003) and the use of fee-for-service funding (Bartels, Levine and Shea 1999). Dental care (Wyatt 2009, Pruksapong and Macentee 2007), anaemia (Sabol, et al. 2010) and malnutrition (Dunne and Dahl 2008) were among some of the illnesses and diseases found to be prevalent in the LTC population.

Table 3-1 – Core Research Themes in Long-term Care

Category	**Theme**	**Studies**
1	Future demand	(Campbell, Ikegami and Gibson 2010, Kaye, Harrington and Laplante 2010, Stevenson, et al. 2010, Karlsson, Mayhew, et al. 2006, Macdonald and Cooper 2007, Hancock, et al. 2003) (Eskildsen and Price 2009, Coleman 2002) (Costa-Font, et al. 2008) (Newcomer, et al. 2001) (Harrington, et al. 2008) Damiani et al. (2009)
2	Funding & access to services	(Campbell, Ikegami and Gibson 2010, Kaye, Harrington and Laplante 2010, Stevenson, et al. 2010, Karlsson, Mayhew, et al. 2006, Macdonald and Cooper 2007, Hancock, et al. 2003) (Bartels, Levine and Shea 1999) (Asahara, Momose and Murashima 2003)
3	Reform of LTC system (including management practices)	(Ng, Harrington and Kitchener 2010, Smith and Feng 2010, Booth, Miller and Mor 2008) (Caprio, et al. 2008, Murphy, Shea and Cooney 2007) (Stone and Newcomer 2009, Bolda, et al. 2006, Booth, Miller and Mor 2008) (Levenson 2009, Mukamel, et al. 2008) (Stone and Newcomer 2009, Bolda, et al. 2006) (Chappell and Penning 2001) (Blackstone 2008, C. A. De Meijer, et al. 2009, Munn and Adorno 2008)
4	Disease prevalence in the LTC population, monitoring of the LTC population, health promotion	(Stone and Harahan 2010) (Wyatt 2009, Pruksapong and Macentee 2007) (Dunne and Dahl 2008) (Sabol, et al. 2010) (Campbell, Ikegami and Gibson 2010, Kaye, Harrington and Laplante 2010, Stevenson, et al. 2010, Karlsson, Mayhew, et al. 2006, Macdonald and Cooper 2007, Hancock, et al. 2003)

3.3 Factors related to demand

In terms of the drivers of LTC demand, there has been an increase in studies that have incorporated factors other than ageing to explain fluctuations in demand and cost of LTC (Fukuda, et al. 2008). In such studies, we have identified two distinct themes - those which aim to relate aggregate demand and cost with socio-economic variables, so called macro-level drivers of demand, and those which aim to understand the type and or level of LTC consumed by an individual patient – micro-level factors. The degree to which studies have incorporated these factors varies considerably. Studies that have focused on measuring the amount of LTC resources consumed by individuals or cohorts of patients often place greater emphasis on factors driving individual patient need. On the other hand, those which quantify the number of future patients pay closer attention to aggregate health and social trends.

Macro-level factors

In the case macro-level drivers of LTC, factors that were found to be related to overall LTC demand included: prevalence rates of disease (Macdonald and Cooper 2007); rates of mortality (Comas-Herrera, et al. 2007); cultural attitudes towards care of the elderly (Kim and Kim 2004); future levels of educational attainment[8] (Batljan, Lagergren and Thorslund 2009); eligibility criteria for government LTC funding (Reschovsky 1998); availability of free LTC services (Wittenberg, Malley, et al. 2006); future patterns of care and general improvements in the level of health (Karlsson, Mayhew, et al. 2006); and living status (Martikainen, et al. 2009).

Micro-level factors

[8] Several studies have found an associated between mortality and educational attainment. In the case of LTC, we assume education to be both a micro and macro level driver of LTC since higher education has been linked with greater socioeconomic status, lifestyle behaviour (e.g. cigarette smoking and exercise), higher self-reported health status in old age and size of social support network.

Compared with macro-level drivers of LTC demand, the literature surrounding micro-level drivers of LTC demand is arguably far more extensive. Factors that have been reaffirmed by multiple studies include: proximity to death (Murphy and Martikainen 2010, Weaver, et al. 2009, C. De Meijer, et al. 2011); type and number of diagnoses (Huang, Lin and Li 2008); level of disability (C. A. De Meijer, et al. 2009, Imai and Fushimi 2011); and marital status (Woo, et al. 2000, Wong, et al. 2010).

Miller and Weissert (2000), in their review of predictors of nursing home placement, found that the most significant factors associated with being placed in a nursing home included: living status; level of family support; personal control; having informal care; homeownership; supply of beds; prior hospital use; prior nursing home stay; number of medications; and need factors. Woo et al. (2000) found that age, being female, being single, not having a formal education, cognitive impairment, physical dependency, and the presence of depressive symptoms were factors predisposing to institutionalisation. Other studies have also supported similar conclusions (Tomiak, et al. 2000).

In Karlsson et al. (2006) study of LTC demand, it was found that demand was linked with future levels of health in the population, which could help to offset some of the increases in expected demand for more formal types of care. When a comprehensive investigation into the relationship between age and LTC care costs was carried out by Zhang and Imai (2007), it was found that there were considerable differences between care costs for males compared with females as they aged. Thus proximity to death more appropriately explained increases in cost compared with ageing alone. Similar conclusions have been made in later studies (Weaver, et al. 2009, Forma, et al. 2009).

Asakawa et al. (2009) present the results of a logistic regression model developed to identify important predictors of admission to institutional care. In their study, they used data from a Canadian Health Survey and found that age, number of chronic conditions and education were statistically significant factors.

Kaplan et al. (2014) investigated the effect of alcohol use and LTC placement among older Canadians. Their study used data from the longitudinal Canadian Notional Population

Health Survey (NPHS) covering the years 1994-2009 covering a sample of 5404 participants aged 50 years or older. Their model investigated the association between alcohol use and subsequent placement in LTC using a Cox proportional hazard model after adjusting for age, gender, marital status, education, household income, smoking, no. of life-threatening illnesses and chronic illnesses. The authors found that abstainers were more than twice as likely to be placed in LTC as moderate drinkers. Former and infrequent drinkers were also at a higher risk of placement compared to moderate drinkers. Heavy drinkers were not significantly different from the moderates in terms of the risk of being placed in a LTC facility. Overall this study would tend to reaffirm earlier findings that alcohol use can in fact reduce the risk of LTC institutionalisation (McCallum, et al. 2005).

Ono et al. (2014) carried out a retrospective survey of dementia patients in day care over a two year period to identify factors associated with the long-term use of such services in Fukui, Japan. The survey included 162 participants whom were divided into three groups according to the duration they had used the service. For reference, the highest length group contained individuals that had received care for 5 years or more. Ultimately using cognitive status as their target variable, defined as the Hasegawa Dementia Scale-Revised (HDS-R)[9], the authors used a series of non-parametric tests to evaluate differences in cognitive status between the three groups. The study found that the HDS-R score significantly deteriorated during the study period except for the HDS-R score of the 3-year group. Higher age was associated with a shorter period of day care service attendance; where as being cared for by a daughter-in-law was associated with the long-term use of day care services. Whilst being cared for by a son was found to be related to using more day care, the effect was less than for daughters-in-law and less statistically significant ($P = 0.002$ vs. $P = 0.047$).

Hung et al. (2013) estimated the LTC needs of stroke patients by examining a sample of 16,043 hospital patients that had had their first stroke during 1995-2010 and extrapolating

[9]The HDS-R score is calculated based on the ability of an individual to provide answers to several general knowledge questions. See: http://dtsc.com.au/download/hierarchic-dementia-scale-revised-hds-r-score-sheet/

their relative proportions of different disabilities over time to derive their LTC needs. The authors recruited individuals that had their first stroke and been admitted to the National Taiwan University Hospital (NTUH). The authors found that the type of stroke experienced was important in predicting future LTC needs given that specific stroke subgroups - namely, cardio-embolic infarct and ICH - led to the longest durations of severe functional disability. It is important to note however, that this study only investigated the physical needs of patients, ignoring other types of support often provided - most notably with respect to cognition and speech.

Having considered more general research themes in LTC, together with both micro and macro level drivers, we now consider proposed forecasting methodologies. We begin by describing the formal process used to identify relevant literature.

3.4 Methodology

The procedure and reporting of our review is broadly inspired by the Preferred Reporting Items for Systematic Reviews and Meta-Analysis (Moher, et al. 2009). The goal of the literature search was to identify papers which primarily focused on modelling the demand for LTC. We searched for papers that modelled LTC at the national or regional level, regardless of whether a formal LTC system was in place and the mode of funding for care.

Papers published before 2005 together with those papers not available in English were excluded so as to limit the scope of the review to the most recent methodological developments. An initial screening of the papers found using some of the keywords used revealed a number of research models that largely focused on determining the demand for LTC insurance or the willingness of individuals to pay for LTC. Whilst forecasting demand for LTC insurance is clearly a related problem, we were more interested in models which provided insight into demand for tangible LTC services and hence such papers were not included.

3.4.1 Search strategy

To identify relevant works we searched PubMed (including MEDLINE) and ISI Web of Knowledge. In addition to these databases we also searched government websites and sites related to health care policy for documents related to future LTC policy, including: the DoH; OECD; Medicare; and BMA. As LTC is referred to by different names around the world we used a wide range of different terms when carrying out our search in addition to the policy names of the most widely known funding programs for LTC, including NHS Continuing Healthcare in the UK.

3.4.2 Inclusion criteria

Articles found within the search results were screened according to their title and abstract. The full text of the original article would be requested if and only these data items were believed to fall into the scope of the review. For each article in the search we reviewed the introduction, results and discussion as a basis for deciding whether the paper was suitable for inclusion in the analysis. The data abstracted from the studies which met the inclusion criteria included: the stated aims and objectives of the paper; source of data used for model development; country of origin; methodology; categories of patients modelled; findings and results; presence of any bias in the studies and stated level of forecast error.

We included papers published in peer-reviewed journals or published as a full paper in conference proceedings provided they contained (1) a model in which an attempt was made to predict the future number of arrivals into LTC or incidence of LTC needs or (2) a model of future expenditure on LTC or a related service or (3) a model of patient progression through the LTC system and (4) the topic or setting related to population health or health service delivery.

3.5 Results

Our search of ISI Web of Knowledge across all keywords identified 9,526 potential papers, 3,439 of which were published in 2005 or after. By applying an initial screening test of title and abstract we disregarded 2,922 papers that were believed not to be relevant as demand modelling was not mentioned.

We found that a large proportion of the papers screened contained a short review on previous modelling work and we have therefore made an attempt to summarise their general findings within the background of this review. In addition, we found 4 papers where no English translation was available. We next screened the discussion and results section of the remaining articles to check whether the paper made a methodological contribution, in terms of a theoretical development or industrial application, which left us with 92 papers for which the entire article would be requested and analysed for potential inclusion.

The search of PubMed (including Medline) found a total of 15,629 papers across all keywords used. Using a date filter 10,281 papers were removed because they were published before the first of January 2005. Screening of abstract and title removed a further 8,019 papers. We next screened articles by their discussion and results section, to see whether each paper made an attempt to model the LTC demand process in some way, which left us with 288 papers for which the full article would be requested.

Across both databases we retrieved and considered 380 articles, 9 of which were removed due to being duplicates and 350 that did not fall into the scope of the review when the full description was considered. This left us with 21 papers that met our inclusion criteria and would therefore be included in the final structured review. A summary table detailing the studies included in our review can be found in Appendix A.1.

3.5.1 General observations

From a methodological standpoint, the most frequent way in which LTC demand and cost projections have been derived is through discrete time simulation modelling. Out of the 21 papers included in our review 7 (33%) used either micro or macro simulation as a basis for making their LTC forecasts. We also found several other methodologies that have been adapted to model LTC demand, including: trend extrapolation, markov chains and grey systems theory. Before discussing the methodologies used to date we summarise some general features of the studies under consideration.

Time horizon

Across all papers we found that the majority used a forecasting time horizon of several decades, with the average, median and standard deviation in years equal to 31.25, 21 and 14.168 respectively. The longest forecasting horizon within our review was 51 years (Wittenberg, Comas-Herrera, et al. 2004) whilst the shortest was 5 years (Ker-Tah and Tzung-Ming 2008, Manton, Lamb and Gu 2007).

Whilst studies of the UK system of LTC represented the largest proportion of the research literature, the international interest in LTC modelling was evident. Non-UK studies included: United States (Manton, Lamb and Gu 2007); Sweden (Batljan, Lagergren and Thorslund 2009); Canada (Hare, Alimandad, et al. 2009); Finland (Hakkinen, et al. 2008); Japan (Fukawa 2011); Taiwan (Ker-Tah and Tzung-Ming 2008); Hong Kong (Chung, et al. 2009); and China (Peng, Ling and He 2010). We found only two papers that modelled and compared the projections of LTC cost and demand across multiple countries (Comas-Herrera, Wittenberg, et al. 2006, Costa-Font, et al. 2008).

Study objectives

Whilst nearly all studies shared a common aim of modelling the impact of changes in demographics on LTC we found that, using a fairly broad definition, studies fell into one of three categories. The largest category contained studies that modelled the LTC system under different demographic or socioeconomic scenarios (Hare, Alimadad, et al. 2009, Wittenberg, Comas-Herrera, et al. 2004, Comas-Herrera, Wittenberg, et al. 2006, Karlsson, Mayhew, et al. 2006, Caley and Sidhu 2011, Costa-Font, et al. 2008, Hakkinen, et al. 2008, Peng, Ling and He 2010). An second class of papers placed more emphasis on investigating the impact of changes in non-demographic factors related to LTC, including those found in §3.3 (Malley, et al. 2011, Manton, Lamb and Gu 2007, Batljan, Lagergren and Thorslund 2009, Ker-Tah and Tzung-Ming 2008, M. Lagergren 2005). Finally, a third group of papers analysed the demand or cost of a specific LTC service or set of diseases associated with a corresponding need for LTC treatment (Comas-Herrera, et al. 2007, Hare, Alimadad, et al. 2009, Comas-Herrera, Northey, et al. 2011, Macdonald and Cooper 2007).

Data sources

In the majority of cases, studies incorporated data on population projections from their respective national bodies, including the UK's Office of National Statistics (ONS)[10], Statistics Canada, Statistics Sweden, and the US Census Bureau. In the case of the UK, prior to 2007, population projections were the responsibility of the Government Actuary's Department (GAD) and hence a number of papers in our study refer to their 2005 projections.

Whilst the United Nations (UN) population projections are commonly used in other areas of healthcare policy research, only one paper in our review used the UN worldwide population projections. One explanation is that the UN's population projections are not sufficiently broken down according to the demographic age profiles typically used in LTC modelling. Furthermore, the only other papers to use population projections that were not produced by their respective national agencies were those that made an attempt to compare forecasted costs across different EU member states. In such cases the European Eurostat population projections were used so as to provide a fair basis for comparison. One additional reason for studies using their own nation's population projections could be due to the UN projections using very general assumptions about keys trends, such as fertility

[10] Since 2007 population projections in the UK are the responsibility of the Office for National Statistics

rate being the same across Europe, that empirical evidence disputes (Office for National Statistics 2012).

Population projections often supplemented with additional data sources from public sector bodies and research institutes. Such data sets included: projected or current rates of disability (Ker-Tah and Tzung-Ming 2008), household composition (Comas-Herrera, et al. 2007), historic LTC care costs (Karlsson, Mayhew, et al. 2006) and hospital registers (Hakkinen, et al. 2008). We could only find two studies which gathered their own data from primary sources, including a paper which used telephone surveying of care home residents was carried out to gauge the incidence of Dementia (Macdonald and Cooper 2007) and one in which a Delphi process was used to gather expert opinion (Comas-Herrera, Northey, et al. 2011).

3.5.2 Modelling approaches

Simulation modelling

Simulation modelling concerns the creation of a digital representation of a system of interest using parameters that are obtained by close observation of the system or via expert judgment (Morgan 1984) Through reconfiguration of the parameters the operation of the actual system, together with its behaviour, can be inferred (Maria 1997).

Commas-Herrera et al (2006) developed separate cell-based macro-simulation models using a common structure for each of the four EU countries, namely UK, Germany, Spain and Italy, to project future expenditure on LTC services. Each cell represented a cohort of individuals by well-defined age-gender characteristics. Modelling the situation in this way appeared to stem from the observation that the LTC systems of interest exhibited substantial differences, including: the level of means-testing for services, amount of resources targeted to specific categories of dependency, the composition of care services offered and indeed the definition of dependency.

In their model, the authors represented systems of LTC delivery from an initial need for LTC through to service delivery and on-going treatment. The aims of the work were stated in terms of being able to increase understanding of the sensitivity of LTC expenditure in Europe with respect to changes in different socio-economic factors. Projections of expenditure were made according to different assumptions about the future population composition and how other key trends may evolve. It was found that expenditure projections were highly sensitive to anticipated unit costs of care and availability of informal care services. Other factors found to be significant included the future number of older people and dependency rate.

Simulation modelling of LTC demand using the cell-based approach, a design originally inspired by the work of the Personal Social Services Research Unit (PSSRU) also known as the PSSRU LTC model, is a recurrent theme in current LTC demand forecasting. Indeed it has been the basis of a number of related models. For instance, the demand for LTC services as a result of cognitive impairment was reported by Commas-Herrera et al. (2007) based on the PSSRU approach. In this case each forecasted cell corresponded to the number of people by cognitive impairment and disability specific cells Compared with the PSSRU model they used population projections from the UK Government's Actuary's Department for 2005 on the number of older people until 2031, future marital status and projections of rates of cohabitation and prevalence of cognitive impairment taken from a cognitive function and ageing study carried out in 1998. As in (Comas-Herrera, Wittenberg, et al. 2006) the authors reported that such projections were highly sensitive to assumed growth rates in real unit costs of care and the future availability of informal care.

Closer inspection of the PSSRU model's projections under different official population projections and demographic scenarios was carried out by Costa-Font et al. (2008). In their study, variability in expenditure projections we calculated by running each country specific model on both the Eurostat 1999 based population projections for the UK, Germany, Italy and Spain, together with official statistics from each of their respective national bodies. Different demographic scenarios including levels of future fertility, which might influence the number of informal care givers, together with migration estimates[11] and mortality data were analysed. For Germany and the UK, the difference in projected expenditure for LTC constituted 1% of GDP under the low and high population estimates. Except for Germany, the projected numbers of elderly people exhibited little deviation between national projections and the model's projections using the Eurostat data.

Chung et al. (2009) adapted the PSSRU model further to help understand the factors that drive individual need for LTC services and estimate LTC expenditure in Hong Kong. In contrast to the PSSRU model, they used separate logistic regression models to derive the probability of individuals within each age-gender cell requiring a LTC service defined in the Thematic Household Survey 2004. The regression model was based on historic data obtained from the Hong Kong domestic accounts from 1989-2002, in conjunction with Hong Kong specific population projections from 2007-2032 and the Hong Kong annual digest of statistics. The probabilities obtained for service usage within each cell was then calibrated according to current observed levels of LTC usage before being multiplied by future population projections in each cell to obtain usage in future years. Unlike previous simulation models, costs were reported as a percentage of real GDP, adjusted according to different real annual growth rates in unit costs of care.

The authors' key findings were that demographic changes were more significant in explaining changes in LTC expenditure compared with real unit rises in the cost of care. It was also found that the expenditure on institutional care could rise from 37% in 2004 to 46% in 2006 if existing patterns of service continued, although expenditure could be contained within 2.3-2.5% of total GDP in 2036 if some institutional care could be substituted by home and day care services.

Whilst the parameters used in the PSSRU model and its derivatives were largely driven by historic data, Comas-Herrera et al. (2011) have also explored the incorporation of expert opinion during parameter estimation. In this case, a variant of the PSSRU model called the PSSRU CI model was developed to test the PSSRUs original projections for a specific

[11] Changes in migration was assumed to affect the future supply of caregivers

class of patients – namely those with cognitive impairments (CI). The authors used a Delphi-style approach to gauge the opinions surrounding future incidence of CI and related patterns of care from 19 experts in the field of dementia and Alzheimer's disease.

In contrast with previous work, the responses collected favoured a slight fall in the incidence of dementia over the next 50 years and a freeze in the numbers of people in care homes. The result would be an increase in the numbers cared for at home or in the community, which would be met by an increase in the qualifications and pay of care assistants. Overall this led the projection model to the conclusion that although expenditure on this group of patients will rise as a result of increases in wages to between 0.82% and 0.96% of GDP in 2032, the effect is less so than in the base case whereby expenditure could be as much as 0.99% of GDP at the end of the period.

A related problem to estimating expenditure on LTC is determining the shares of total cost met by different economic actors. Outside of the UK, the extent to which an individual has to contribute towards their care costs can vary widely as can the services covered by government funded schemes. In their paper Malley et al. (2011) extended the PSSRU model to partition the expenditure projection for each cell according to different sources of funding. This was achieved by combining the results of an earlier model called CARESIM (Hancock, et al. 2003), a related simulation model which specifically models the future income and assets of older people, with the demand projections obtained by the PSSRU model. The benefits related to being able to model not only demand but how different demand cohorts were able to meet LTC care costs.

While static macro-simulation models, in which assumptions are constant throughout the projection period, have been the most prevalent type of simulation models in LTC, Fukawa (2011) has shown how a more dynamic methodology can be used to add additional realism. Using an initial set of simulated data on household composition, households rather than individuals were transitioned according to the probabilities of specific live changing events, which for instance included death, marriage and divorce, to arrive at the number of persons with specific attributes in each year. At the end of the period, this information was used to calculate the expected long-term care costs for each household according to how many elderly people were present and their level of disability. Unlike earlier studies, annual changes in key socioeconomic variables were incorporated through adjustment of the relative household transitional probabilities.

Conclusions drawn from the study included the observation that future LTC expenditure was heavily dependent on future service usage by dependency level. Furthermore, according to the model the proportion of the elderly population that stay in LTC institutions will increase. The expectation that the fertility rate will stay constant at 1.3 throughout the period has the implication of increasing the ratio of parents to adults aged 40 and above. This study has therefore highlighted the possibility of more extensive informal care provision by younger relatives of LTC patients.

Grey Theory

In our structured review, we found only a single paper using grey theory as its core methodology. In essence grey theory is a methodology that can be used to approximate the relationships between variables in conditions of incomplete or very limited information. Grey models take the following general form, GM(n, m), where n represents the order of differencing used to smooth the data series and m the total number of predictors (Yao, Forrest and Gong 2012).

Ker-Tah & Tzung-Ming (2008) used a grey-inspired methodology, specifically a GM(1,1) model which represented a forecasting framework to estimate the disability rate for the aged section of Taiwanese population using time as the independent variable and one level of differencing. Under the assumption that the LTC population of Taiwan was equal to the disabled proportion of the elderly population, they forecasting future values of the disability rate and multiplied it by the expected elderly population in future years to obtain future demand.

Although the GM(1,1) model can appear somewhat naive in its assumptions, given the short length of time of the forecast, the fact that aggregate yearly data on expenditure was used and the overall aim of the model it represented a reasonable choice. Unlike previous work it more closely resembled the observation that the rate of disability in the population is variable and, in Taiwan's case, steadily increasing over time. Furthermore, the data demands of this approach are relatively smaller and hence it would tend to suit the real-world situation in LTC. Compared with historical values of LTC expenditure, the average absolute percentage error was found to be 7.27% under the grey model and hence demonstrated reasonable fit with the underlying data. At the end of the data period the grey model showed that LTC in Taiwan could increase from 38,805 individuals receiving care to in 1991 to 606,305 by 2011, primary as a result of an increase in the disability rate for the elderly population.

Markovian and transitional models

Markov chains belong to a broader class of stochastic modelling methodologies than can be used to model the behaviour of a stochastic process at discrete-time intervals. Essentially, they allow for the next realisation of a variable in a sequence to estimated based on a stationary set of probabilities associated with the likelihood of the variable assuming a particular future value (Winston 1993)

Karlsson et al. (2006) analysed the sustainability of expenditure on LTC in the UK in light of expected changes in health status among the elderly population. The methodology was based on an extension of an earlier disability model, proposed by Rickayzen & Walsh (2002), whereby cohorts of individuals by age and level of disability are transited in time, according to a markov process, into steadily worsening levels of disability. Crucially in this study, the transition probabilities were calculated initially using current disability-free life expectancy and other related mortality data - updated at each period according to perceived trends in healthy life disability. To generate total future expenditure on LTC and the associated resource need, levels of care and services used were estimated for each cohort and multiplied by the respective costs so as to arrive at the total resource requirements.

The authors considered the integration of different assumptions surrounding mortality, levels of disability in the elderly population and the speed at which disability worsened by adjusting the respective values in the transition matrix. It transpired that as in previous LTC studies, assumptions of future disability were critical to the overall projections of both cost and service use. An additional result was that that if female care-giving patterns converged to those of males then under the baseline health improvement scenario there could be a shortage of between 10 and 20 million hours of LTC care giving per week in the UK by 2040.

Hare et al. (2009) studied the future number of LTC patients among different home and community care categories in British Columbia (BC) using a deterministic multi-state markov model. In this methodology, 10 care categories were defined across home and community care, 8 of which represented publically funded packages whilst the remainder represented care funded by private means.

Estimates of the number of people in each age range specific care category, together with the transitional probabilities for individuals moving between different packages of care were then estimated using historic data on service usage. Even though data on publically funded care were available from the BC Ministry of Health, little was available for non-publically funded care and so the authors used a telephone survey of usage across all care home facilities in BC as an approximation.

Using the ratio of publically funded to non-publically funded care packages, the total number of patients transitioning between different packages of care were calculated before being partitioned between the publicly funded and non-publically funded packages. Transitional probabilities were assumed to be fixed over the forecast range and estimates of future service usage were obtained by adding the incremental addition in the forecasted population at the beginning of each period. One weakness of this approach was that it largely based the transitional probabilities on historic data, including a period where demand for LTC in BC far outstripped supply, and that the model performed poorly when the numbers of privately funded cases were removed owing to the fact that a large proportion of LTC patients use a mixture of both publically and privately funded services.

Unlike previous studies that have used medical diagnosis and the extent to which a person needs assistance with activities of daily living (ADL) as a basis for estimating level of individual disability, Peng et al. (2010) used self-rated health status collected from a sample of elderly people aged 80+ from the Chinese Longitudinal Healthy Longevity Survey in 1998, 2000 and 2002. In this case the transition between worsening levels of health across 5 different age bands between 80 and 100+ was modelled as a non-homogeneous Markov process, one for each of the genders and for each initial starting state of self-reported health status. They considered that a response of “poor” health would identify a person as having a need for LTC, although individuals in the study also had an option of selecting “very good, “good” and “fair”. The basis for this choice was because the relative risk of mortality was greatest, by the Mantel-Haenszel test statistic, between the fair and poor groups in the majority of the gender-age cohorts studied.

For a given start and end period, the authors transitioned individuals through time and noted the overall time each person spent in the “poor” health state. At the end of each period, the difference between their age when they entered the poor state and their estimated life expectancy was considered the number of years of unhealthy life expectancy - where LTC would be needed. By multiplying by the average annual LTC cost in China for an individual they arrived at the projection of total LTC costs.

The study highlighted how for men in China with very good or good reported self-health, the probability of them maintaining their health status or changing to very good health is higher than that of women, but the result is the opposite when men are in fair or poor health. One issue is that by using self-reported health status the percentage of the oldest Chinese requiring LTC was estimated at 44% while if defined by the notion of ADL then the proportion fell to 32%, given that care is provided on the later basis it could quite overstate true costs. Furthermore, the authors also assumed that transition rates between worsening states were constant throughout the period and thus may offer less precise results if there are underlying changes in the health status of the Chinese population.

Chahed et al (2011) used data from NHS continuing care patients in London between 2005 and 2008 to estimate the survival pattern and movement of patients in LTC. In this case, a continuous time markov model is used to capture the flow of patients between different care states and overall time in care, with the final state corresponding to death of the patient. Demand projections were produced by considering the number of patients still likely to be in one of the non-death states at a given future time horizon in light of the fitted transition probabilities. In their approach the authors proposed using three distinct care states to represent the LTC system whilst in practice several different care pathways were known to exist. Similarly, the small sample size of certain categories of patients limited their application to just two groups of LTC patients - namely physically frail and palliative patients.

Extrapolative models

By an extrapolative methodology, we are referring to a model whereby the principal method of generating forecasts of LTC demand or cost is through the application of historic trends to future population projections.

In Lagergren (2005) the ASIM-III model was proposed, a model which contains both a retrospective and prospective component to predict LTC usage across Sweden. The retrospective component, described in (M. Lagergren 2005), although linked to LTC demand forecasting focuses on establishing the level of LTC need by population subgroup by studying its historic consumption. The prospective part, which is the attention of our review, addresses the need to understand how such consumption may vary in the future given specific assumptions about prevailing health trends that may be relevant. A key feature of the research is the recognition that future LTC need depends largely on the extent to which systems of informal care can be relied upon is highlighted.

Using the underlying simulated estimates of LTC consumption by gender, age group, civil status and degree of health the author obtained usage rates of three tiers of LTC services, including 3 levels of home or community help and a single institutional category. In this case, the levels of community support were defined by the number of hours of assistance required per day. The author then applied population projections, obtained from Statistics Sweden, covering the years 2005-2030 for each cohort and by multiplying with the corresponding estimate of LTC usage by group in 2000 obtained forecasts of the numbers of people requiring LTC. Although marital status has been shown to be a relevant factor in driving need for LTC, the authors were unable to obtain population projections by marital status and estimated this by linear extrapolation per 5 year age group and gender in the period 1985-2000.

In order to assign costs to the number of people requiring care in each subgroup, the authors used logarithmic extrapolation to derive levels of ill health and the associated level of LTC service usage based on survey data from the Swedish National Survey of Living Condition 1975-1997 and using fixed prices of care at 2000 levels. Different assumptions surrounding how levels of ill-health may improve or worsen can be incorporated by adjustment of the probabilities of different levels of ill-health across subgroups of the population, in the base case the authors assumed continued improvements in ill-health until 2020 where based on expert judgment it was believed to remain constant until the end of the forecast horizon.

A related methodology that also used survey data to obtain estimates of the incidence of disability was carried out by Macdonald & Cooper (2007). In this research, the focus was much narrower in the sense that only future costs and demand for home care placements by those suffering from dementia were considered. In this study, the authors used the findings from a survey which reported the results of a mental state examination from a sample of 445 residents across 157 non-EMI (non-elderly mentally infirm) care homes in the south-east of England. The incidence of dementia among elderly patients (here aged 60 and above) from the survey was then linked to the total number of older people in care homes and the overall prevalence of dementia across the UK. The resulting age and gender specific incidence rates were then applied to future population projections provided by the Government Actuary's Department (GAD) population projections.

Weaknesses of this particular study related to the fact that incidence for the UK was estimated on the basis of a survey carried out in a single region of the UK, the results of which may not be comparable with other areas of the UK where differences in funding arrangements or the supply of available places may exist. Indeed given supply constraints for LTC in the UK, such incidence rates may more closely resemble historic activity and not the underlying demand for dementia related care.

Manton, Lamb, & Gu (2007) investigated the observed decline in the disability rate for the US population and implications for LTC spending using data from enrolees in the US Medicare programme. In their work, samples of people aged 65 and above were taken from several National Long-Term Care Surveys between 1982 and 1999, surveys which directly draw samples from computerized Medicare enrolment files. Not only did each survey detail the costs and services delivered to each individual, they also contained a set of measures relating to the extent to which each person required help to perform six ADLs and 10 instrumental activities of daily living (IADL). To this data, several additional variables describing the level of difficulty with physical performance of certain tasks and sensory limitations were also added.

An issue incorporating the disability data into the forecasting model related to the observation that many such indicators were correlated with each other and that the matrix of all disability measures, where each row represented an individual's patient, was sparse. The authors used latent class models (LCM) to reduce the disability measures into 7 distinct and homogeneous groups. Using the prevalence of these 7 disability groups estimated at each yearly interval, future Medicare costs are projected for 2004-2009 using age specific population projections applied to the estimated cost of care in each of the disability groups.

Owing to the fact that individuals may not be present in care for the entire year, perhaps due to death, the authors used an inverse survival function to weight their costs appropriately. Several variations were considered, including where the LCM of disability was taken for a specific year and used to estimate costs in the future assuming the disability rate would be constant in future years. A more dynamic approach used the changes in the LCM model between two time periods to model future costs.

Hakkinen et al. (2008) played more attention to the proximity to death in estimating the future care costs of the elderly where it was found that 55.2% of total health expenditure on those 65+ in Finland was due to LTC. Data used comprised of a 40% sample of the Finish population linked to hospital registers, death registers, social insurance and the Finish hospital benchmarking project. Although their projection of future care costs was not limited to LTC, they estimate costs due to LTC and non-LTC separately by firstly calculating the likelihood than an individual is a LTC patient. This was achieved using a logit model with age, gender, days from 31st December 1998 until death and an indicator if they died period to the end of 2002. Variants of this model included additional socio-economic data, such as income and region. A second model, using ordinary least squares, was then fitted to the resulting LTC costs of care over the period relating to the each individual patient.

The results of the model fitting showed that time to death and age were more significant in explaining LTC costs compared to just age on its own. Population projections by age-gender were obtained from Statistics Finland and used to extrapolate expenditure on LTC for the years 2016 to 2036 using the obtained gender-specific age-expenditure profiles and proximity to death. The authors found that for the year 2036, compared with an approach that didn't take into account proximity to death, total health care expenditure in Finland would 12% higher.

Weaknesses in the study related to the fact that LTC patients include only those that have been in receipt of care for at least 3 months. As a result, it may fail to capture costs due to respite and or palliative services. Furthermore only services provided by 24-hour institutions were considered and no attempt was made to break down the costs of LTC into their various components.

In neighbouring Sweden Batljan, Lagergren, & Thorslund (2009) studied the link between educational status of the elderly and the need for LTC. Using the Swedish national survey of living conditions (SNSLC) carried out in the period 1975-99, they classified the educational status of the elderly population into one of three groups. In this case the low group represented those with less than 10 years of education whilst for the high group it was more than 11. Logistic regression were then fitted to estimate differences in the prevalence of severe ill health (SIH), specifically a health state that would require LTC, by different age, gender and educational level cohorts. The importance of including education level was stated in terms of being able to incorporate different mortality and morbidity differentials according to changing educational level.

By applying demographic extrapolation and taking into account educational level they developed several models, each representing a different scenario as to future overall levels of mortality and morbidity. A separate model for both males and females was used, to aid the alignment of results with how Swedish population projects are provided, and for each gender separate models were created reflecting improvements in mortality and declining mortality for both sexes. The authors also assumed that by age 35 the education level of an individual was fixed.

Their key finding was that severe ill health among higher levels of educational level was less than for lower levels. Dramatic increases in the educational level of the population between 2000, 2020 and 2025 will place a greater proportion of the population in higher levels of education. Specifically the percentage of women in the low category of education level will fall from 60% in 2000 to around 16% by 2025. Given that higher levels of educational level coincide with a decreased observed likelihood of severe ill-health, the effect of including educational level acts to counterbalance the effect of ageing on LTC needs and in one cases reduces the percentage of those in serve ill-health to 18% of the level estimated when only age in taken into account assuming continuing downward trends in mortality. Even when mortality rates are assumed to rise, the effect of increasing educational level was shown to reduce the percentage of SIH to less than half that when using age alone by 2035.

Proximity to death and the effects of changing life expectancy on future LTC demand in the UK was investigated by (Caley and Sidhu 2011). In recognition of the limited availability of LTC data outside of the acute sector, they used published estimates of LTC by age provided by the Department of Health to generate estimates of total expenditure in light of future population projections. The effect of increases in life expectancy was considering by postponing the cost of LTC by expected increases in life expectancy (provided by the Office for National Statistics), whilst a third model took into account how much of the additional life expectancy was spent disability free. To relate these estimates to cost, the authors revised the future age bands to put it in terms of cost at the present time. For instance, if life expectancy in the 80 year old group was expected to rise by 5 years but only 1 of these years was expected to be disability free, they would represent the same cost in present terms as an 84 year old individual.

Even though all three of their models highlighted an expected increase in LTC related costs by the end of the period, the percentage increase in the second model was only 47% of the increase estimated in the first model whilst this figure was 57% in the case of the third. Ultimately therefore, the authors have illustrated the potential for LTC models to significantly overstate cost if changes in life expectancy and or the associated years of disability free life expectancy are not considered.

3.6 Discussion

Of the studies included in our review only a small subset projected that future expenditure on LTC would be less than or equal to the current levels. In such cases, it was postulated that changes in patterns in care and shifts from formal LTC arrangements to informal ones – a trend that has been witnessed to date – would largely offset the increases in expenditure due to increases in the elderly population adjusted for disability-free life expectancy. However, such containments in cost rely heavily on strong assumptions surrounding the availability of informal care and the substitutability of certain types of institutional care for more community orientated arrangements. In the case of the former, there is evidence to suggest that this might not be the case especially for western countries as participation in the labour force has increased for both sexes and in particular for women. One study finds that if informal care provided by women converges to that of males then there will be a significant shortage of informal care provision. In the case of the later, it remains for policy makers to provide clear evidence as to which LTC services can be effectively substituted with less intensive community services and what, if any, repercussions this may have.

Within current literature there has been a clear interest in linking the impact of non-age related drivers of LTC to future demand and expenditure. This may have been a result of more recent initiatives that call into question the reliability of projections based on using ageing alone as the core driver of LTC demand. According to one scenario, expenditure projections using ageing alone were estimated to overstate future expenditure by up to 12% annually. Indeed nearly all studies attempted to utilise a mixture of factors more closely related to individual disability to measure need, together with information about an individual's living arrangements to approximate their effective level of dependency. However, many studies have cited the difficulties in collecting data on some of these additional factors at the individual level. To counteract this limitation a small number of studies have therefore carried out their own surveys of the LTC population or used expert opinion – the overwhelming majority have however extrapolated data from national surveys. It is not clear that the later approach is inherently less reliable but as such surveys are likely conducted at 5 or 10 year intervals it does call into question how representative the data is with respect to the current population.

In our review we found that the most prominent model in LTC modelling was the PSSRU cell-based simulation model. Its strengths appear to relate to its ability to allow policy makers to experiment with different economic and social conditions to derive projections of LTC demand. The PSSRU model has seen multiple adaptations over the period to address specific limitations in its early design – most notably with respect to the fact that simple extrapolation is used to determine LTC need; the fact that the approach was validated using a single dataset and that all parameter estimates are based solely on historic data. However, despite several attempts to address related research questions with the

PSSRU model, later adaptations have not addressed a major concern relating to its somewhat naïve assumption surrounding the static nature of the LTC system. For example, under the PSSRU model, and its variants, many assumptions surrounding the system of LTC, socioeconomic variables and health trends are assumed to be constant. In reality, this assumption has been shown to be constantly disproven. With that said, a more recent direction has been to take the principles of the cell-based design of the PSSRU model and incorporate a more dynamic view of the LTC system. This has meant a change in the fundamental unit of the forecast, from cohorts of individuals to households containing LTC individuals, but does seem to more effectively capture the dependency element of LTC and recognise the importance of informal care provided by family members. A challenge remains however to estimate the propensity of family caregiving and in general how the informal care market will itself evolve.

Whilst a range of factors have been shown to important in gauging future demand for LTC, outputs of existing approaches have been found to be highly sensitive to the disability rate and specifically how the level of disability of a population is incorporated. In principle, those with greater disability should in principle require more care, but since there is no single measure of disability and indeed the extent to which an individual is disabled only makes sense in both the context of being able to carry out a specific task, it is one of a class of variables that practitioners have found increasingly hard to gauge at the individual patient level. One way in which the disability rate has been incorporated is through the examination of treatment patterns. Although such information can be obtained by health surveys it is not always optimal to assume the same level of disability for those receiving similar types of treatments given the high variability in care costs among those in similar treatment groups. An alternative approach, inspired by the use of latent class models, has showed promise by using a data-driven approach to categorise patients into a small number of groups based on their self-reported ability to carry out a range of IADL and ADL. This approach provides the benefit of grouping patients by their specific care needs and uses a data-driven rather than an arbitrarily defined definition of disability. Unfortunately, the only example of this approach in the literature identified a tendency of participants to overstate the assistance they required in carrying out a number of activities.

To date much of the research into LTC modelling has arguably been focused at the national or indeed international level. In some respects this may reflect how LTC models have historically been used thus far, as a means for policy makers and key stakeholders to test certain assumptions surrounding the impact of different scenarios on current service models. Similarly, as there continues to be debate surrounding how LTC will continue to be funded such models have been used as a way to test a range of different funding models, including those in which the private individual funds a greater proportion of their costs or public funding is concentrated in those with the highest level of need. We noted that LTC in the UK system of care, and indeed in many other countries, is coordinated at the local level yet few studies have modelled the intrinsic detail of the local LTC system for the purposes of modelling the impact of proposed changes surrounding LTC policy and optimising the efficiency of local care delivery. At this stage it remains unclear why few studies exist in the published literature, we expect that it might be that the methodologies used at the local level are less developed compared with those presented at the national level and thus go unpublished, there are few published examples of the benefits to local planners and that such models may be commercially sensitive.

3.7 Conclusion

The purpose of our literature review has been to address two key questions: what are the historical developments in LTC demand forecasting and what progress has been made towards developing a local level model of demand. To date, two broad categories of demand forecasting models have been proposed. The first studies demand from a national and long-term perspective, while the second studies demand at the regional or local level over a couple of years. We refer to these types of models as long-term policy models (LTPM) and short-term operational models (STOM) respectively.

Whilst LTPMs are numerous within the literature, providing both static and dynamic representations of the LTC system to aid policy makers, it is in fact at the local rather than national level where LTC is coordinated. Local planners, typically operating over one to two year time horizons, plan and organise the care to be delivered, liaising with private sector providers where necessary. Despite the benefit of the use of modelling in the planning of budgets; investigating scope for changes in patterns of service; and in the design of formal contracts with care providers, literature surrounding local level forecasting is limited. One challenge in developing STOMs for LTC is that local level data can lack sufficient quality, detail and volume to be able to generate reliable projections of patients and their future care needs. This stems from data covering social and informal care being characteristically difficult to obtain (Kinosian, Stallard and Wieland 2007) and indeed link to other health services. This can result in underestimation of cost due to the obscuring of patient progression through the system.

3.8 Summary

In this chapter we have performed a literature review surrounding the modelling of LTC. Through our review we have identified factors statistically significant in explaining the demand for care services, both at the micro and macro level, and highlighted developments in LTC forecasting methodologies since 2005. One observation is that few studies have attempted to model the LTC system at the local level, where organisation and coordination of care takes place. While macro models of LTC activity have been used extensively in the policy debate surrounding future funding for LTC, there are far fewer concrete examples of the impact of local level modelling. In the next chapter, we explore one such use of local level demand forecasting - for the purpose of contract design.

Chapter 4

Modelling the LTC contracting process

4.1 Introduction

A contract, as it is understood here, refers to an agreement that is formed by two or more parties that serves to standardise and reduce the complexity associated with the exchange of goods and or services (Collins 1999). Typically contracts will contain two important features (Macneil 1980): a discrete element which specifies key elements of the relationship formed between the two parties and, a time-based element that serves to bring the future environment into the present. Whilst the former may deal with the amounts of goods or services that will be exchanged, together with their cost, the later serves to specify the duration the agreed terms and conditions will be valid for. In essence, these features work to reduce the uncertainty associated with any future transaction between the contracting parties and minimise their respective risks (Friedman 1965).

4.2 Contracting within the health care sector

In the UK and indeed in other developed countries, there has been greater widespread use of contracting to both regulate and govern how health care services are delivered (Tynkkynen, Lehto and Miettinen 2012) (Heard, et al. 2011) (Glinos, Baetenb and Maarsea 2010): within the published literature several explanations for this apparent shift in strategy have been explored. One suggestion is that as health care systems have come under increasing financial pressure to provide services, they have employed contracting as a means to increase efficiency (Loevinsohn and Harding 2005) through a process of competitive bidding. Here the assumption is that the provider who is ultimately selected is the one who is judged best able to provide a given service, according to one or more cost or quality based metrics.

An additional motivation is linked to the move towards greater decentralisation of health care systems and the division of national and regional health care planning to local autonomous units. In the UK for instance, the high transactional costs associated with forming contracts with external organisations compared with internally formed ones, have often limited their appeal. However, as the NHS and in particular its commissioning arms have become more decentralised and autonomous in nature, the transactional costs of contracting internally with other NHS organisations has been brought more closely in line with those faced when contracting with third parties (Petsoulas, et al. 2011).

4.2.1 Design and implementation considerations

Despite the potential advantages of contracting, there remains debate surrounding whether the purported benefits are in fact ever realised for the health service (Liuemail, Hotchkiss and Bos 2007). Indeed a number of studies have been carried out to assess whether health care contracting increases efficiency and lowers expenditure on services across a wide range of domains. Such care domains include but are not limited to: HIV prevention services (Zaidi, Mayhew and Cleland 2012); primary health care (Liu, Hotchkiss and Bose 2008); cross-border service provision (Glinos, Baetenb and Maarsea 2010); and pharmaceuticals (Graf 2014). Even though the evidence to date has been mixed, the results gathered from these studies offer an insight into the issues and challenges that need to be addressed when contracts are both designed and implemented so as to maximise their benefit.

A study of the effects of health care contracting involving non-governmental organizations (NGOs) in Pakistan found that wide-scale contracting was beyond the institutional capacity of many local health care planners. In addition, the authors cited a lack of skills in writing and costing proposals, together with poor knowledge of the private sector market, as key reasons as to why demand and supply were often miss-matched (Zaidi, Mayhew and Cleland 2012). A review of the literature surrounding the contracting-out of primary care services in low to medium level income countries reported that contracting, whilst perhaps not offering clear cost savings, did appear to improve the level of access to services (Liu, Hotchkiss and Bose 2008).

In a review of large scale governmental contracting for HIV prevention services (Zaidi, Mayhew and Palmer 2011), the authors examined the process of contracting-out health service delivery for the purpose of identifying both technical and relational requirements of those ultimately responsible; both the negotiation and implementation of such contracts. One of the chief findings was that, due to purchasers often being divorced from the operational and clinical aspects of HIV prevention, purchasers relied heavily on submitted bidding paperwork and found identifying cases of overstatement of provider costs difficult. Furthermore, weak governance and a reliance on a small number of key individuals within purchasing teams gave rise to the slow implementation of contracts and a long drawn out bidding processes.

An investigation in to the use of contracting in the Australian health care system, using data collected from interviews with senior executives from major health funds, revealed that the process of contracting in itself may have benefits that go beyond the original planned exchange (Donato 2010). For example, contracting can help health organisations establish stronger inter-organisational ties with partner organisations and leveraged their mutual capabilities. In such cases an increased willingness to exchange data and information may foster increased innovation and ultimately lead to better outcomes for patients. With that said, the authors also highlighted that the widespread use of contracting, by forging closer ties, may raise challenges for future competition policy.

A study of cross-border health care contracting within Europe, with the stated aim of summarising the findings and outcomes of pan-European contractual arrangements, explores how the application of EU principles of free movement have been applied to health service provision (Glinos, Baetenb and Maarsea 2010). Their work is ultimately based on interviews with stakeholders at Belgian hospitals in addition to a literature review of cross-border patient mobility within the EU; including studies from Denmark, England, Germany, Ireland and the Netherlands. One important observation was that cross-country provision of services had a number of advantages for domestic purchasers of care, including: access to services that are practically and or financial advantageous; the ability to respond to unmet demand and to keep costs under control, for example by exploiting price differences among member states. It was also pointed out that the formulation of such contracts helped to strengthen purchaser power domestically: especially in countries where local market conditions gave rise to greater provider power in setting treatment and service prices.

An interesting example of the use of contracting to increase consumer surplus[12] was investigated in the German market for pharmaceuticals and other medical non-durables: a market for which spending accounted for 14.8% of total healthcare expenditure (Graf 2014). In the German market, medical supplies are frequently purchased through group purchasing organisations (GPOs), organisations that collect orders on behalf of their members and aggregate the demand to purchase in bulk from suppliers. A key aspect of such arrangements is the use of rebate clauses[13]. Despite the precise rebate terms varying according to which GPOs are willing to offer exclusivity or partial exclusivity, in the sense that they will buy from a single or at most two providers, the use of contracting in this fashion provides an industrial example of how contracting may serve to not only lower expenditure for health care purchases but indeed increase economic welfare.

[12] Consumer surplus refers to the differential between what consumers pay and what they are willing or able to pay.

[13] A rebate clause allows for consumers to claim a percentage of the cost of an item post-purchase directly from the supplier.

4.2.2 Contracting methodologies

Whilst the literature contains numerous examples of the outcomes of contracting with respect to health care purchasing decisions, details of the methodologies used and their quantitative underpinnings are often not clearly stated. We partially explain this observation by a suspected discontent with publishing such work given how tools and models used to evaluate bids in a public tendering process may be commercially sensitive in nature. At the same time as an area of research, contracting out health services remains in its infancy and thus far much of the work in this field has tended to more tightly focus on evaluating potential ways forward rather than establishing definitive modelling approaches. With this in mind we briefly turn to recent non-health related literature on contract design.

Yin and Nishi (2014) present a three-echelon supply chain optimization model under demand uncertainty and asymmetric quality information. The situation is modelled through a game theoretic approach and solved using Stackelberg equilibrium in which there is no-cooperation. In this case, the three-echelon aspect relates to the fact that there are three distinct entities in the model: N suppliers, a manufacturer and retailer. The manufacturer's problem is to determine the quantity of raw materials to purchase from each of the suppliers, which are assumed to complete with each other, based on an uncertain demand from the retailer and uncertain information about the quality of the raw material inputs from the suppliers. Whilst distinct from the problem of LTC contracting, the model and situation shares some similarities with the LTC in that: the suppliers could be exchanged for care home providers; the manufacturer could be replaced with the NHS commissioning organisation and lastly the retailer as the patient. In this case, as in LTC, the care home providers offer competing products and it remains for the planner to decide who to contract with so as to cater for patient demand.

Gilbert et al.(2015) describe a scenario in which an energy aggregator satisfies demand from the power grid for energy by entering into contracts with distributed energy generating firms. In this case, the demand for energy from the grid takes the form of a demand contract indexed by week number. The supply contracts, on the other hand, specify how long an available generating resource should be available. The decision of the energy aggregator is to select a set of contracts to form with third party generating companies so as to be able to satisfy its contracted demand commitments. Owing to uncertainties surrounding demand and maintenance periods of third party generating firms, the authors formulate their problem as a mixed integer stochastic problem.

Calfa and Grossmann (2015) investigated optimum contract design from the point of view manufacturers that can either choose to secure supply of raw materials through supplier contracts or use the more volatile spot market. Their proposed model consisted of a multi-period, multi-site stochastic programming production planning model. A novel feature was that, under the assumption that the manufacturer could determine the selling price of its products, it also considered optimisation of the selling price under both supply and demand uncertainty. Other authors (Nosoohi and Nookabadi 2015) have investigated the problem of manufactures forming contracts with suppliers where long lead-times exist. In such cases, the authors' note that manufacturers may face uncertain demand at the time of ordering but as time passes they are able to revise their estimate of demand. Their analysis compared the use of contracts that made define orders in addition to options contracts, which would allow the manufacturer to the option to purchase additional supplies after they had received update demand information. In order to contracting problem facing manufacturers the authors devised a mathematical programming model and solution methodology based on the process of backward induction.

4.3 Characteristics of the LTC contracting problem

In the non-constrained version of LTC allocation problem described in §2.3.5, individuals are assigned to a care location of their choice as they become eligible for funding. In particular, there is no intention by the CCG to minimise cost or maximise quality of care received, providers have infinite capacity, and all care homes cater for each and every type of patient. We assume in this case that CCGs are able to continuously solve the allocation problem and hence patients are perfectly substitutable between different providers of care. In practice, the decision to allocate patients is subject to a range of both linear and non-linear constraints as well as some additional considerations[14].

Patient preferences

The CHC framework allows for patient preferences to be expressed in a number of ways and throughout the assessment and allocation process. Patients may express a preference for being care for in a particular care home due to several reasons, including wanting to be closer to family and friends in the local area and perhaps because the patient perceives the quality of care in one home to be higher than that of another.

This is not to say that patients will always be given their first preference, indeed the CCG will be unwilling to pay for care that is significantly more expensive than is reasonable given the needs of the patient. While the CCG will take the preferences of the patient and family into consideration it is customary for the CCG to draft a list of potential homes, perhaps two or three that would be suitable, and for the patient to choose among these various options. Choosing outside of these three options would require the patient to make a strong case for being placed elsewhere and could slow the arrangement and commencement of their care.

Care quality

Quality of care is a term that features regularly within the literature surrounding LTC and indeed providing a good standard of NHS CHC is a statutory responsibility of CCGs (NHS England 2015). However, care quality in LTC is somewhat difficult to define and measure since it can be argued that it more closely resembles a perception of an individual to their care package and depends on an individual's own preferences.

One of the most prominent measures used by CCGs is the Care Quality Commission (CQC) rating[15]; this provides an assessment rating for each care home on the scale of 1-4

[14] Additional considerations can be understood to be soft constrains.

based on the extent to which the care homes services are safe, effective, caring, responsive and well led. Other measures that might be indicative of higher quality are linked to objectives set in domain 5 of the NHS Outcomes Framework (NHS England 2014). Such measures would include: the number of safety incidents reported the care homes; responsiveness of the care home to patients personal needs; the proportion of individuals that reported that they were treated with dignity and respect by the care home; the overall satisfaction of people who used the care home; the incidence of infections; and the availability of GP services at the care home.

Cost

We have already alluded to the fact that weekly care costs for LTC in an institutional setting can be significant. Although cost alone is rarely solely used to determine the exact allocation of patients, for instance rather we should think of cost as being in terms of cost per unit of care quality, it remains a key consideration for planners. We therefore use cost as a basis to constrain the problem such that we select an appropriate placement such that it costs at least no more than available alternatives for the same level of quality.

Time window

In our simple non-constrained problem it is assumed that the CCG continuously allocates patients among different care providers. While this may be true for newly eligible patients it's much less likely that the same is true for patients already in receipt of care. Furthermore, to be able to explore time based discounts the CCG will typically have to adhere to a minimum contract period in which the CCG cannot change its underlying allocation decision. We refer to this minimum contract term as the time window - that period of time where contracting decisions involving LTC providers remain fixed. Therefore, any potential allocation decision has to consider how stable a particular contracting decision is during the time window under consideration.

[15] http://www.cqc.org.uk/content/care-homes

It is important to note that whilst we assume that the CCG cannot change its contracting decision during the time window, the CCG can adjust the specific patients allocated to each slot in the contract such that they could, in principle, be moved and existing ones replaced as they leave LTC.

Provider capacity

In practice most providers are constrained by bed capacity. For small care homes the maximum number of patients that may be cared for at any one time can be as few as five. Larger providers on the other hand may have as many as 30-50 beds. Many care homes and specifically those on the borders of neighbouring boroughs may supply care home services to a number of distinct CCGs. Thus when determining the allocation of patients we need to take into account that available capacity in each care home may be less than the reported capacity given the demand from surrounding CCGs, self-funding individuals and LAs.

Provider specialty

Due to wide range of conditions within LTC not all providers are assumed to be able to cater to all individuals. For example, a provider may choose to specialise in a single LTC care category or a small subset so as to maximise the quality of care that it delivers and employ specialist staff with experience in managing specific conditions. In the same way providers, even though providing care services to particular care category, may not necessarily manage those with the most complex needs and hence prefer to care for those patients with low to medium levels of needs.

The implication of provider specialty is that when designing contracts the CCG may have to purchase the services from a range of different providers to ensure that it has sufficient free slots in each provider specific contract to manage the variety of conditions within its patient population.

Patient care needs

Related to provider specialty is the notion of patient care group. That is to say that each patient can be categorised into one of 6 care categories or care domains and that the care category for any given patient is known by the CCG. During allocation the CCG has to ensure that patients are allocated to a provider matching the care group of the individual and their level of needs.

The six care categories in the UK system of LTC include: palliative; physically frail, organic mental health, functional mental health, learning disability and physically disabled The palliative care group includes patients approaching end of life, organic mental health refers to individuals with diseases affecting the brain, in contrast with functional mental health which includes those who have experienced sudden rather than progressive physical damage to the brain. The physically disabled and physically frail categories include individuals that have been diagnosed with progressive physically disability as a result of old age or the diagnosis of one or more diseases affecting the structure and composition of the nervous system or skeleton. Learning disability covers individuals with cognitive learning disorders, such as dyspraxia and aphasia.

In addition to the care category deemed most appropriate for the patient, we also assume that commissioners are able to characterise the patient's level of need as being high, medium or low. Although in practice, each individual patient's level of need will lie on a continuous spectrum, we constrain our initial problem to three fixed levels so as to simplify the formulation whilst recognising that in practice providers often perform a similar simplification of their pricing policy.

Worsening state

A property of the illnesses and conditions associated with LTC include the fact that they are chronic and will worsen over time. We note that during the placement decision of patients, commissioners need to take into account that patients are likely to worsen over time and hence they should place individuals within a care location that is capable of managing their existing state with a clear view to the future. This may rationalise the decision of a commissioner whom places an individual in a care location that caters to both high and medium needs patients even when the current state of a patient gives rise to medium levels of need.

Respite care

Respite care concerns care that is provided on a short-term and temporary basis to patients who are normally cared for informally by family and friends but, perhaps due to taking a holiday, becoming ill themselves or other unforeseen circumstances, the usual career is unable to assume their normal care role. In addition, respite care may also include care provided to those that are awaiting more permanent allocation to care or to those that have been fast tracked. In many ways the respite care constraint forces CCGs to allow for some flexibility in their allocation decision so as ensure one-off or unforeseen situations can be accommodated.

Space sharing with local authorities

Space sharing of places refers the ability of the local health service to subcontract care home spaces that are currently under the control of a LA or the NHS. Such placements may be in LA owned homes or in homes to which the LA or NHS has a pre-existing block contract agreement with. Subleasing of such placements allows the LA and NHS to use any excess capacity them may have and provides an alternative means for the health service to purchase LTC care within an institutional setting. It should be noted that, owing to the nature of the needs of individuals that are the responsibility of LAs within the UK system of LTC, placements available from LA are normally restricted to those individuals with low levels of needs and those that are borderline between being primary the responsibility of the LA and the NHS. As such, local healthcare planners are constrained to using such placements for individuals with seemingly low levels of need.

Patient stickiness

Owing to the nature of a number of diseases and illnesses associated with LTC there is a marked tendency towards ensuring that patients in LTC are not routinely moved between placements, even where the cost savings may be substantial. For example, those with dementia and or other cognitive and mental disorders may benefit best from being in familiar surroundings with staff that are highly versed in the specific nature and state of their illness. The impact of this element of LTC may lead to individuals being cared for in a very limited number of care homes throughout their time in care if they are indeed moved at all. As a consequence, planners need to look to the long-term impact of their allocation decision and ensure that a stable optimal can be found even if in the short-run the decision may not be optimal from a cost standpoint or in light of their immediate care needs.

4.4 Data to support contracting decisions

The core of our dataset is based data collected from 27 out of the then 31 Primary Care Trusts (PCTs) within the Greater London region relating to NHS Continuing Healthcare activity. The data was collected jointly by the University of Westminster and Deloitte in 2009 as part of an earlier project that was funded by the NHS London Procurement Programme (LPP) to investigate CHC activity within London. As part of data collection, all 31 PCTs were each asked to complete a data collection template according to their recorded CHC activity.

Once individual PCT's responses had been collected they were merged into a single data file. The final merged data set consists of records relating to 13,700 individual patient assignments to long-term care: including those that are fully funded by the NHS under the umbrella of NHS CHC. The dataset covers cases of LTC that commenced from 1990 onwards and those have either ceased or remain in place as of 1st April 2009. In total 4 PCT's data are absent from the dataset due to a lack of response within the time period.

Despite the change in NHS structure from PCTs to CCGs from the 1st of April 2013 we do not expect a drastic impact on the design of CHC contracts going forward. As a result of government policy, PCT responsibility for CHC has been transferred to their respective CCGs. At the same time, CCGs in many cases cover a similar population size as PCT did previously and in some cases the population covered may be larger. Compared with PCTs, it could be argued that CCGs are under more pressure to show efficiency savings through the adoption of techniques such as those we propose.

4.4.1 Reported activity data

Appendix A.2 shows the names of the variables that were collected as part of the data request. Data quality was found to be highly variable among different PCTs, given that fields such as ***ethnicity*** and ***gender*** were largely not provided by PCTs we decided to remove these two columns from our analysis.

An issue highlighted during collection related to recording practices of LTC costs. For example, whilst PCTs pay for care costs on a weekly or monthly basis they are often reported in annual terms as there are a number of fixed costs often incurred during a person's care e.g. the cost of a specialist orthopaedic mattress, and costs can change depending on whether the individual's condition worsens. Such characteristics of the PCTs reporting practices culminated in a small number of cases having a very high weekly care cost: likely due to them being reported in annual terms.

Although to the best of our knowledge there is no commonly agreed cap on LTC care costs for NHS CHC, we observed through meetings with LTC commissioners that LTC care costs above £5,000 would typically be investigated as a matter of procedure. For this reason we set an upper bound of £5,000 on a weekly basis or £260,000 annually. Similarly, a number of individuals were recorded at zero weekly cost. We assumed that such figures represented costs associated with short respite care or potentially the fact that the individual was in a block contract and hence their cost was captured within an existing commitment. As such costs could have a damaging effect on our analysis we decided to set a lower bound for the weekly care cost of £112 – this corresponds with the average weekly cost of an NHS funded nurse[16] over the period considered.

[16] http://www.nhs.uk/chq/Pages/what-is-nhs-funded-nursing-care.aspx

In total we performed 11 additional data cleaning steps[17] including: removing data points with no care group specified, removing data with no provision type specified, removing data where weekly rate was greater than £5000 or less than £112, removing data where the provision type was not specified; and removing data where the funding band was not NHS CHC. Finally we inspected the start dates and end dates of care and removed inconsistent cases, those with ***provision start date*** after the ***provision end date,*** together with those with missing ***provision start date*** as we would not be able to identify for how long a patient's care package had been in place. The data cut of period for our analysis was the 1st of April 2009, as such individuals that had started care but not been given a provision end date were assumed to still be in receipt of NHS CHC at the end of the period. In total the 11 phases of our cleaning process removed a total of 8,152 (59%) cases resulting in 5,548 (39%) cases for analysis.

Data fields

Of the fields collected and available the following fields were selected for analysis: ***hostpct*** (Host PCT), ***commpct*** (Commissioning PCT), ***caregroup (***Care Group***), provisiontype (***Provision Type***), weeklyrate*** (Weekly Rate), ***prov_start_date*** (Provision Start Date) and ***prov_end_date*** (Provision End Date)***.*** To aid our analysis we have also included two computed fields, ***external*** (External) and ***days_in_care*** (Days in Care), which indicate respectively whether or not the care package is funded by the same PCT in which the care takes place and the total number of days in LTC: according to the difference between ***prov_start_date*** and ***prov_end_date***. Whilst ***external*** is a binary categorical variable, assuming the values 0 or 1, ***days_in_care*** is a positive integer.

Graphical overview

Table 4-1 and Table 4-2 provide a cross tabulation of activity by home care and institutional placements respectively. Abbreviations used for the six care groups are as follows; FMH (Functional Mental Health), LD (Learning Disability), OMH (Organic

[17] Full details of our data cleaning steps can be found in Appendix A.3

Mental Health), PAL (Palliative), PDA (Physically Disabled Adult) and PF (Physically Frail). Among the 5,548 care packages taking place 3,908 (~70%) took place within institutions compared with 1640 (~30%) taking place in the home. In the case of home care packages, a higher percentage were hosted within the PCT's catchment area (79.3%) compared with those taking place externally (20.7%). In contrast with those care packages taking place at home, intuitional placements were observed to slightly more evenly split between being hosted within the commissioning PCT's own borough (59.6%) compared with those hosted externally (40.4%).

In terms of the distribution of care groups among the provision type, 71.1% of care packages taking place at home were associated with patients in the PAL category. The second highest most prevalent care group in home care was PF (18.7%) followed by PDA (6.7%) in third. In contrast, while institutional placements were too associated with PF (39.9%) and PAL (25%), the ordering was the other way around and OMH (14.8%) played at greater role. FMH represented the least amount of activity taking place at home (0.1%); the same was true for LD (6.8%) under institutional placements.

In terms of the number of care days taking place, calculated by taking the difference between an individual's start and end date of care, Figure 4.1 and Figure 4.2 show the total numbers of days spent in NHS CHC by care group and provision type respectively. From Figure 4.1 we find that the PF care group account for the majority of NHS CHC care days (35%) followed by OMH (18%). From Figure 4.2 we find that institutional placements account for the overwhelming majority of NHS CHC care days (84%) versus 16% taking place in the home.

Table 4-1 - Cross Tabulation of Home Care Packages by Care Group

Home Care			External No	External Yes	Total
Care Group	FMH	Count	1	0	1
		% within CareGroup	100.0%	0.0%	100.0%
		% within External	0.1%	0.0%	0.1%
		% of Total	0.1%	0.0%	0.1%
	LD	Count	19	3	22
		% within CareGroup	86.4%	13.6%	100.0%
		% within External	1.5%	0.9%	1.3%
		% of Total	1.2%	0.2%	1.3%
	OMH	Count	26	8	34
		% within CareGroup	76.5%	23.5%	100.0%
		% within External	2.0%	2.4%	2.1%
		% of Total	1.6%	0.5%	2.1%
	PAL	Count	986	180	1166
		% within CareGroup	84.6%	15.4%	100.0%
		% within External	75.8%	52.9%	71.1%
		% of Total	60.1%	11.0%	71.1%
	PDA	Count	60	50	110
		% within CareGroup	54.5%	45.5%	100.0%
		% within External	4.6%	14.7%	6.7%
		% of Total	3.7%	3.0%	6.7%
	PF	Count	208	99	307
		% within CareGroup	67.8%	32.2%	100.0%
		% within External	16.0%	29.1%	18.7%
		% of Total	12.7%	6.0%	18.7%
Total		Count	1300	340	1640
		% within CareGroup	79.3%	20.7%	100.0%
		% within External	100.0%	100.0%	100.0%
		% of Total	79.3%	20.7%	100.0%

Table 4-2 - Cross Tabulation of Placements by Care Group

Placements			External No	External Yes	Total
Care Group	FMH	Count	182	89	271
		% within CareGroup	67.2%	32.8%	100.0%
		% within External	7.8%	5.6%	6.9%
		% of Total	4.7%	2.3%	6.9%
	LD	Count	234	31	265
		% within CareGroup	88.3%	11.7%	100.0%
		% within External	10.0%	2.0%	6.8%
		% of Total	6.0%	0.8%	6.8%
	OMH	Count	364	215	579
		% within CareGroup	62.9%	37.1%	100.0%
		% within External	15.6%	13.6%	14.8%
		% of Total	9.3%	5.5%	14.8%
	PAL	Count	677	299	976
		% within CareGroup	69.4%	30.6%	100.0%
		% within External	29.0%	19.0%	25.0%
		% of Total	17.3%	7.7%	25.0%
	PDA	Count	153	103	256
		% within CareGroup	59.8%	40.2%	100.0%
		% within External	6.6%	6.5%	6.6%
		% of Total	3.9%	2.6%	6.6%
	PF	Count	721	840	1561
		% within CareGroup	46.2%	53.8%	100.0%
		% within External	30.9%	53.3%	39.9%
		% of Total	18.4%	21.5%	39.9%
Total		Count	2331	1577	3908
		% within CareGroup	59.6%	40.4%	100.0%
		% within External	100.0%	100.0%	100.0%
		% of Total	59.6%	40.4%	100.0%

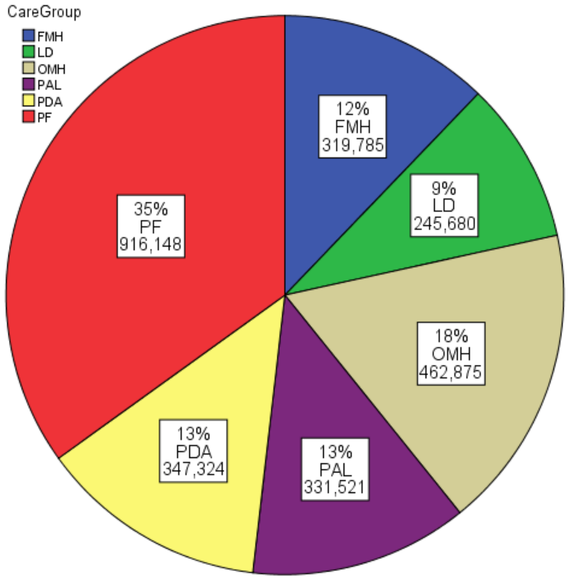

Figure 4.1– Days in Care by Care Group

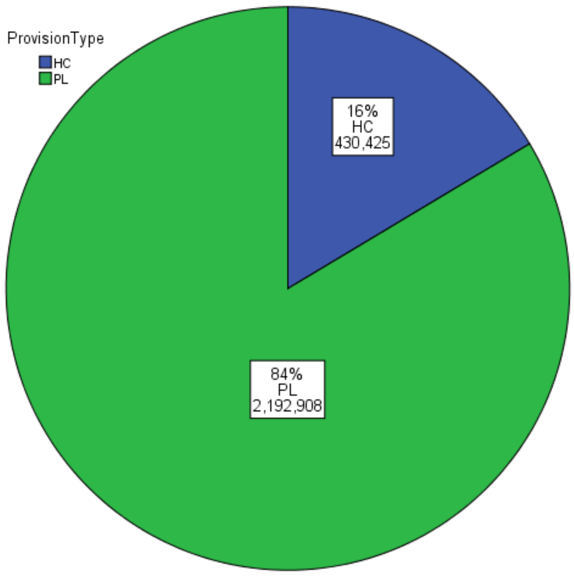

Figure 4.2– Days in Care by Provision Type

Figure 4.3 shows a histogram of weekly care costs weighted by the number of days in care. The average weekly care cost was found to be £1005.99 with a standard deviation of £701.905.

Figure 4.4 and Figure 4.5 provide the distribution of care costs for care days hosted externally and internally respectively. Whilst the standard deviation of weekly cost was roughly the same for both externally and internally hosted care, £700.223 and £701.322, the average weekly care cost was higher for externally hosted care packages (£977.19 versus £1054.37). One possible interpretation of this observation is that in cases where an individual has highly specialist or rare needs, needs that are typically more expensive to manage, a patient is more likely to be placed outside of the commissioning PCT's catchment area due to a lack of a capability on the behalf of the PCT. At the same time packages hosted externally may be provided by care providers for whom the PCT does not regularly use hence the PCT exhibits less ability to negotiate pricing discounts.

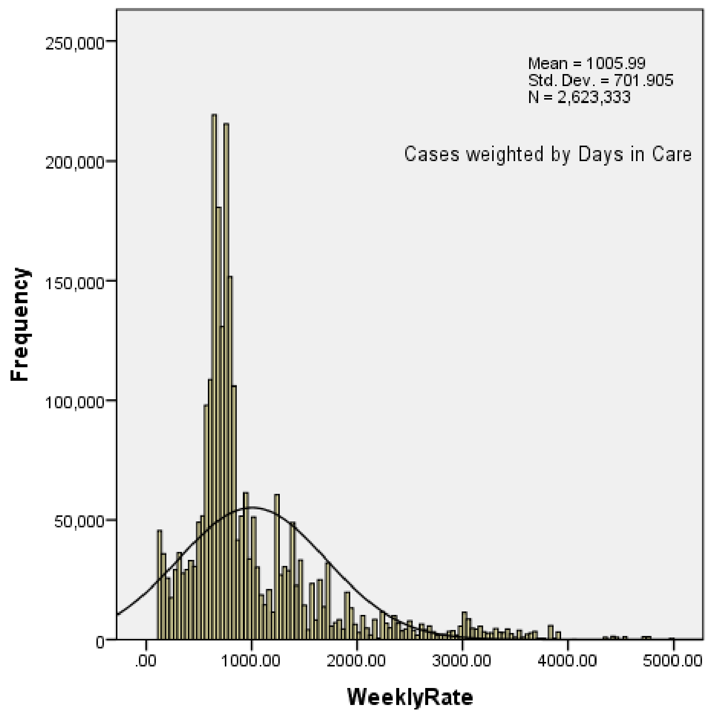

Figure 4.3– Distribution of weekly cost

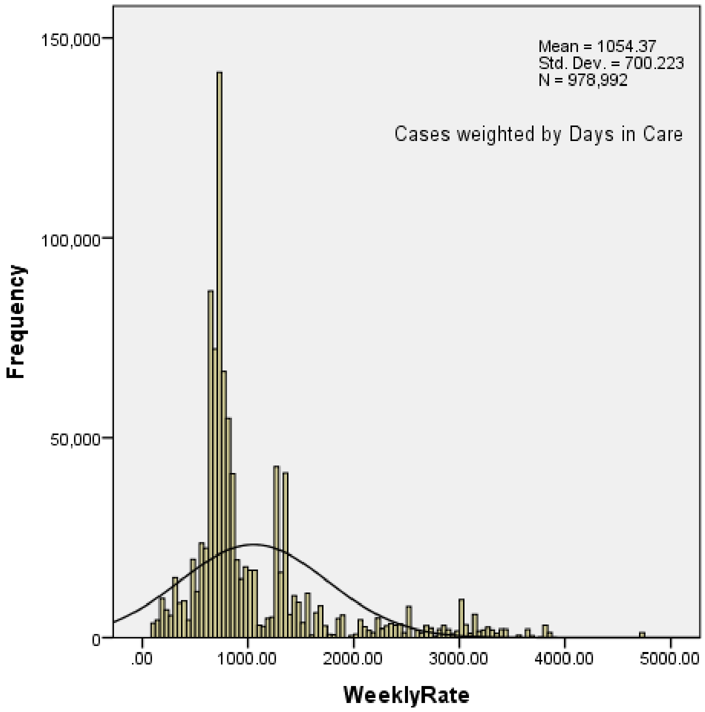

Figure 4.4– Distribution of weekly cost for externally hosted care

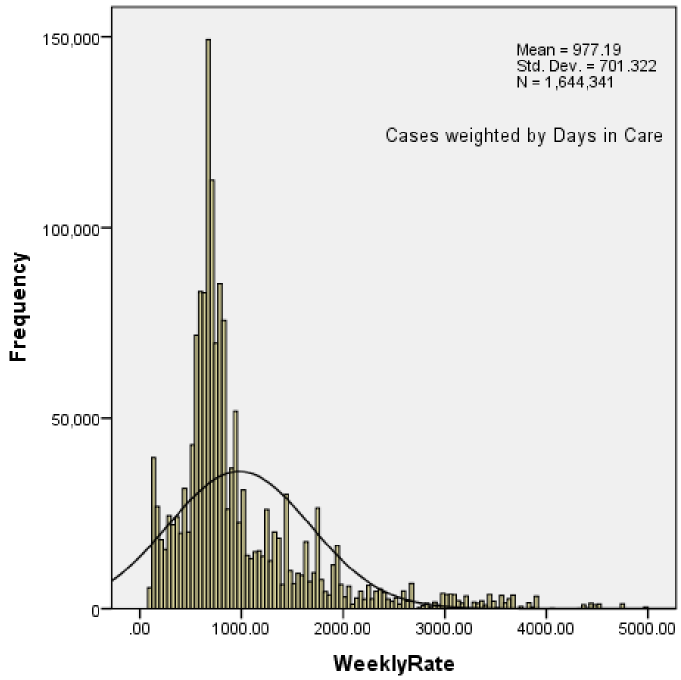

Figure 4.5– Distribution of weekly cost for internally hosted care

Figure 4.6– Weekly cost by care groupFigure 4.6 shows a breakdown of weekly care cost for different care groups. We observe that the median care costs for LD are higher than for other care groups. It is also the care group for which, except for outliers, the highest weekly care cost is recorded. In contrast, the median weekly cost of palliative care is found to be the lowest. In terms of spread of weekly care costs, FMH, LD and PDA share a similarly larger interquartile range (IQR) compared with the IQR for OMH, PAL and PF which is substantially smaller.

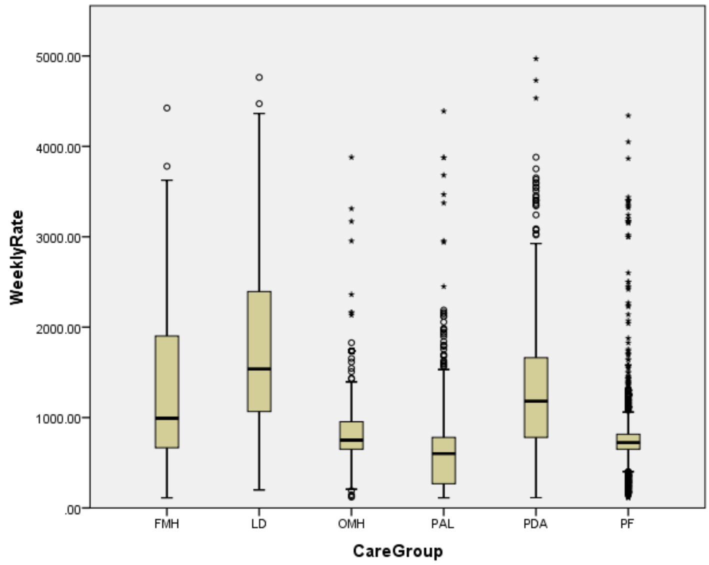

Figure 4.6– Weekly cost by care group

The distribution of days in care across all care groups and both provision types is shown in Figure 4.7. The mean stay in NHS CHC is found to be circa 472 days and general form of the distribution is characterised by a positive exponential shape that decays rapidly after 1,000 days in care – corresponding with circa 2.7 years in NHS CHC. The sharpest peak in activity is observed at between 0 and 90 days in care, closely resembling the typically stay of less than 3 months for palliative patients. Some of these packages may also relate to respite care. Figure 4.8 breaks down the number of days in care further by distinguishing between those days attributed to either home care or institutional placements. We observe that patients on average stay longer in institutional settings and that length of stay in care for home care provision is more homogeneous. Furthermore, a small number of individuals receiving LTC in institutions have been there for in excess of 5 years.

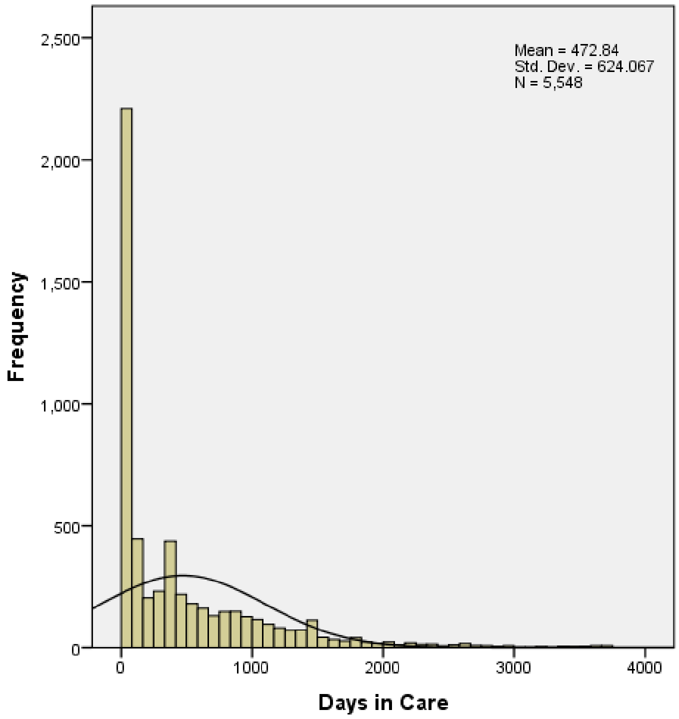

Figure 4.7– Distribution of days in care

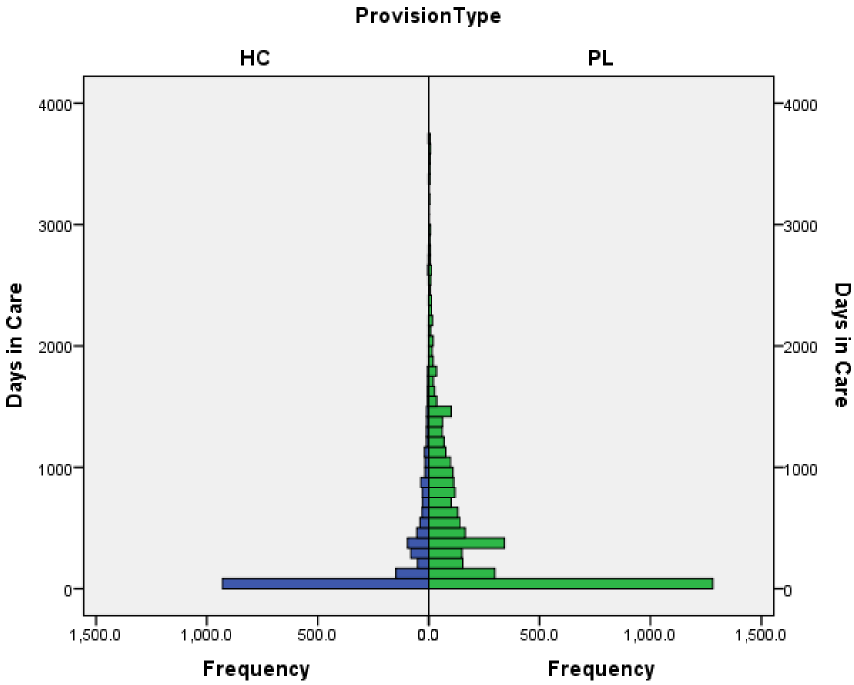

Figure 4.8– Distribution of days in care by provision type

By considering the start and end dates of care for each care package, whilst summing together packages of care that took place simultaneously, we estimated the total volume of daily LTC activity across London. Figure 4.9 reports our findings by showing an extract of LTC activity across London between the 1st of January 2005 and the 1st of January 2008. From the line graph we are able to observe a linear increase in reported daily activity over the period, rising from about 600 NHS CHC packages taking place in early January 2005 to just over 2,000 packages in early 2008. A notable feature is the slight levelling off in activity from mid-2007. Although we cannot offer a precise explanation, a partial explanation relates to the introduction of the 2007 NHS CHC Framework which standardised the application process by limiting NHS CHC to those whose need for care was based primarily on an underlying medical condition.

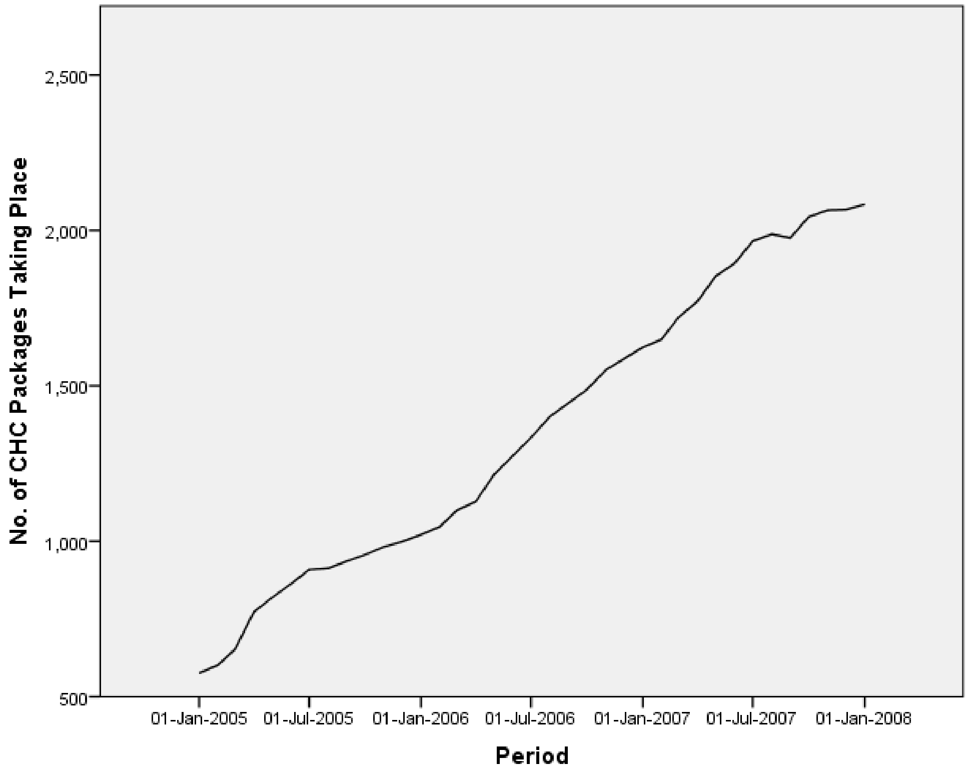

Figure 4.9– No. of LTC Packages Taking Place Over Time

4.5 Nursing home provider capacity

As we are interested in modelling the contracting decision facing LTC commissioners, we supplemented our dataset on recorded LTC activity with publically available data on nursing home supply. Whilst it was envisaged that provider-level data would be used to set appropriate constraints on the numbers of patients allocated to each care home under consideration, much of the nursing home capacity historically available to LTC commissioners has since been decommissioned and moved into the private sector: making obtaining specific details surrounding provider bed capacity much more problematic. However, since we are interested in developing a theoretical and illustrative approach to modelling such contracting decisions we are less reliant on obtaining exact values and instead more focused on using such data to set sensible assumptions. Similarly, even if such data were made available it would perhaps not include the capacity already in use or in the process of being purchased by other healthcare organisations.

Table 4-3 shows the numbers of registered nursing homes across different London boroughs as of November 2014 taken from the online care and nursing home search engine CareHome[18]: a service used in the UK to find potential care homes by both local authorities and private individuals. We observe that a high proportion of nursing home ownership lies within the private and voluntary sectors (83.98%), together with a tendency of boroughs further away from central London having a larger number of homes reflecting larger population size. To gauge capacity at individual nursing homes we randomly sampled the bed capacity of 25 nursing homes within the London region: the results of which are detailed in Table 4-4.

[18] http://www.carehome.co.uk/

Table 4-3 - Numbers of Registered Nursing Homes in London (2014)

London Borough	All	NHS	LA	Private/Voluntary
Barking & Dagenham Borough (7)	7	0	2	5
Barnet Borough (23)	23	0	2	21
Bexley Borough (9)	9	0	0	9
Brent Borough (14)	14	0	0	14
Bromley Borough (24)	24	0	1	23
Camden Borough (4)	4	0	2	2
Croydon Borough (32)	32	0	1	31
Ealing Borough (23)	23	0	1	22
Enfield Borough (13)	13	0	2	11
Greenwich Borough (17)	17	0	5	12
Hackney Borough (3)	3	0	1	2
Hammersmith & Fulham Borough (5)	5	0	2	3
Haringey Borough (2)	2	0	2	0
Harrow Borough (11)	11	0	7	4
Havering Borough (18)	18	0	0	18
Hillingdon Borough (18)	18	0	5	13
Hounslow Borough (10)	10	0	3	7
Islington Borough (9)	9	1	3	5
Kensington & Chelsea Borough (4)	4	1	0	3
Kingston upon Thames Borough (17)	17	0	5	12
Lambeth Borough (10)	10	0	0	10
Lewisham Borough (14)	14	0	0	14
Merton Borough (14)	14	1	2	11
Newham Borough (7)	7	0	0	7
Redbridge Borough (15)	15	0	0	15
Richmond Borough (8)	8	0	5	3
Southwark Borough (7)	7	0	1	6
Sutton Borough (23)	23	0	0	23
Tower Hamlets Borough (3)	3	0	0	3
Waltham Forest Borough (4)	5	0	5	0
Wandsworth Borough (14)	14	0	0	14
Westminster Borough (4)	4	0	2	2

	387	3	59	325
	100.00%	0.78%	15.25%	83.98%

Table 4-4 - Nursing Home Bed Capacity

Observation	Nursing Home Name	Postcode	Bed
1	Brook House Nursing Home	KT3 5EA	32
2	Barchester Thackeray House Care Home	CR0 5PH	39
3	The Chestnuts Nursing Home	E11 2PR	47
4	Abbey Cheam Centre	SM2 7QB	18
5	Parkview Nursing Home	SE19 3PY	64
6	The Cedars Care Centre	EN5 1SB	41
7	Abbey Ravenscroft Park Nursing Home	EN5 4ND	38
8	Barchester Cheverton Lodge Care Home	N19 3AY	52
9	Dulwich Care Centre	SW16 2JP	82
10	Muriel Street Centre	N1 0TH	63
11	Adelaide Care Home	DA7 4BE	76
12	Heatherbrook	RM7 7DT	45
13	Westcombe Park Nursing Home	SE3 7RZ	51
14	Clare House Nursing Home	UB8 1PP	39
15	Abbey Care Complex	IG2 7NE	47
16	Clavering	HA5 4HE	33
17	Kenwood Care Home	N12 8HG	32
18	Sundridge Court Care Home with Nursing	BR1 3NG	24
19	Southborough Nursing Home	KT6 6QL	46
20	Barchester Wilsmere House Care Centre	HA3 6UB	85
21	Maitland Park Care Home	NW3 2DU	60
22	Aspray House	E10 7EB	64
23	Elmstead House	NW4 3TH	50
24	Manor Farm Care Home	E6 3PD	72
25	Victoria Care Centre	NW10 7NS	115

Figure 4.10 provides a graphical overview of the distribution of bed capacity within our random sample. In our sample the smallest nursing home by bed capacity was the Abbey Cheam Center with 18 beds whilst the largest was the Victoria Care Centre with 155 beds. Average bed capacity was 52.6

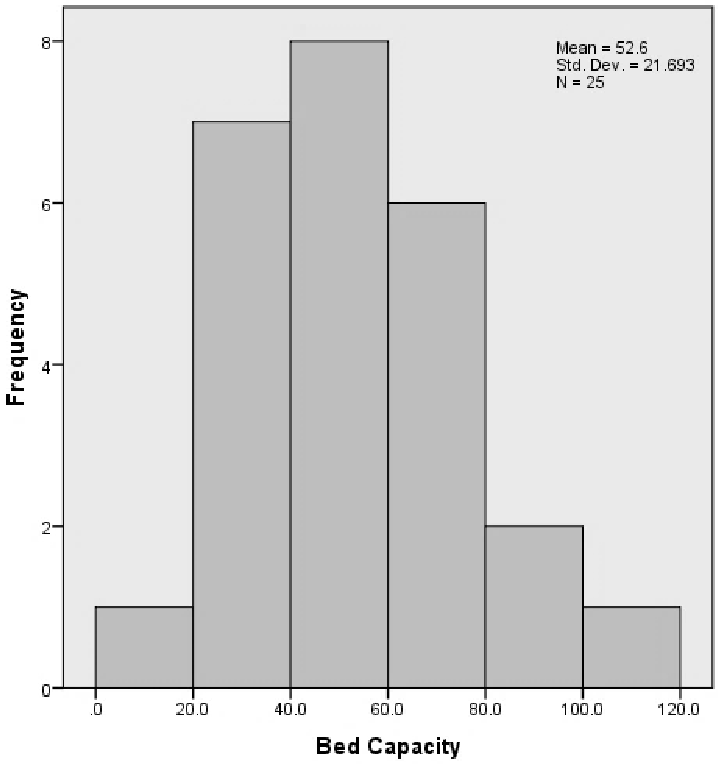

Figure 4.10– Sample Distribution of Bed Capacity of Nursing Homes in London

As nursing homes, together with more traditional residential homes, are periodically inspected and given a rating by the Care Quality Information we use this as a proxy as to the desirability of a care home by a newly eligible LTC patients. As of 2014 the three levels awarded are Good, Requires Improvement and Inadequate with the distribution of these levels for nursing homes in England currently reported at 56.6%, 28.3% and 15.1% respectively (Care Quality Commission 2014).

4.6 Discussion

By identifying key stages within the process of allocating patients to LTC, from the point of view of commissioners, we have sought to draw out key considerations and issues that may need to be addressed in any contract formulation. In particular, we alluded to potential problem constraints and saw how the fact that patients may have preference for a particular care provider may limit the ability of commissioners to make use of contracts that are already in place with other providers. Similarly, in addition to those patients that remain in care for long periods of time there are those that may be in receipt of care for very short periods, for example a couple of days or weeks, for which it may not be advantageous for a commissioner to contract out services for, given that discounts are more likely to be associated with longer time-based commitments.

Lastly, we have explored possible sources of data available to long-term care commissioners for the purposes of supporting contracting decisions. Whilst arguably not as rich or as comprehensive as data from other areas of the health care service, we have seen how valuable information surrounding expected lengths of stay in care can be deduced together with the amount of activity taking place by provision and care types to allow planners to better gauge local health demand.

Critical to the contracting decision is an understanding of provider ability to meet demand for a service. In the case of LTC this rests with private sector institutions external to the local health care planner, who we assume here is either unable or unwilling to provide the necessary information to allow optimization of the contracting decision. However, we have seen how publically available information on the supply of nursing home places can be obtained and, together with supplementary information from other public sector bodies, used to approximate nursing home supply.

4.7 Summary

In this chapter we defined what is meant by contracting with the health care setting and reviewed previous literature surrounding the use of contracting to support health care decision making. Despite the potential advantages of contracting out health services we noted that there are mixed opinions as to the potential benefits, including whether or not they are effective in improving services themselves or indeed helping health care agencies to lower costs. With that said several studies reflected on the fact that a lack of an understanding of both the demand and supply side processes at work may have been a key reason as to why previous attempts may have been less than successful. Indeed this is an area that we will attempt to address in our own application to long-term care.

Chapter 5

Formulating the contracting problem

5.1 Introduction

One an individual has become eligible for CHC; it is the responsibility of the NHS to arrange the necessary health and social services. As part of this process care planners are required to form care contracts with third party providers, subject to the constraints and issues raised in the previous chapter. While such contracts may be formed on an ad-hoc basis, planners have the option of making a contractual commitment to a given provider, so as to both secure supply and potentially earn quantity and or time based discounts. A challenge facing care planners is what such commitments, if any, should be made and indeed for how long. In this chapter we propose a simple formulation of the decision process, inspired by a related problem faced in production planning.

5.2 Production planning

Production planning, which incorporates the field of lot-sizing, involves determining how best to use resources in order to satisfy one or more production targets over a planning horizon. Such decisions may be of great strategic and organisational benefit to businesses: potentially allowing them to reduce the cost of production, maintain a set service-level target and or secure a competitive advantage through greater productivity. Problems involving production planning decisions can be characterised by the length of the planning horizon. Whilst short-term production planning decisions may involve determining day-to-day production requirements or employee scheduling or otherwise the efficient use of existing resources, more medium-term problems, where more factors of production can be adjusted, may involve determining the best production combination to satisfy a future pattern of demand or service-level target. In the longer-term, where it is argued all factors of production can be manipulated, production problems may involve the proposed relocation of production facilities themselves or the consideration of capital investment decisions, which may consider not just how much should be produced, but whether production should shift to a new product or service model entirely.

Lot-sizing problems represent a special type of production planning problem in which the objective is to determine how much of a product to produce in each period, or indeed whether to halt production, so as to meet demand in each respective period. While production could take place in each period it is generally assumed that each production-run, that is to say a period in which production starts and ends, has an associated setup cost. This cost is analogous with the cost of readying a machine for production and could include for instance the time taken to load the input materials.

Although a single production-run, in which sufficient production is made to satisfy all the demand over a period, may avoid multiple setup costs, producing large quantities can lead to stockpiling and thus raise inventory costs: costs associated with the storage of goods that are kept to satisfy demand in later periods. A commonly used example of an inventory cost is the cost of warehousing, refrigeration or interest rate. Lot-sizing problems therefore seek an objective way to minimise the overall costs of production taking into account any applicable setup and inventory costs.

Despite the classical use of lot-sizing models within the context of production optimisation they have also successfully be reframed to consider situations in which production per-se does not take place. In such cases, lot-sizing models have been applied to determine the optimum number of products to order, rather than produce, from one or more suppliers to meet demand. In this situation ordering costs replace costs associated with production setup, finished product costs replace the cost of raw materials, whilst inventory costs, and the more general mathematical formation, remain the same. In fact, it is this formulation of the lot-sizing problem that we suggest is analogous to the problem facing long-term care commissioners when deciding the number of contractual commitments to make.

5.2.1 Lot-sizing models

Whilst lot sizing is one of the most important problems concerning production and or order planning: it is also one of the most difficult to solve (Karimi, Fatemi Ghomi and Wilson 2003). The complexity of lot-sizing models can however vary according to the features taken account by a model, with one key distinction surrounding how the nature of demand itself behaves. In stationary lot-sizing models demand is assumed to be constant throughout the period, whilst more dynamic and arguably more realistic methodologies treat demand as more volatile. An additional distinction between stationary and dynamic models is that since demand must but specified for each period, dynamic models assume a finite time horizon whereas stationary models operate in continuous time.

Economic order quantity

Perhaps the earliest and most well-known example of a stationary lot-sizing model is the classical Economic Order Quantity (EOQ) model (Harris 1913). In the EOQ model the goal is to determine the optimal order or production quantify that minimises average inventory management cost per unit of time (Schwarz 2008) for a single item. Despite being relatively easy to compute, the simplistic assumptions (constant costs) of the EOQ model are restrictive and not frequently met in real-world applications. To address its shortcomings several other models have been proposed: one of the earliest extensions of the EOQ framework is the economic lot scheduling problem (ELSP).

Economic lot scheduling problem

The ELSP extends the EOQ model by allowing for the possibility of producing, or in fact ordering, several different items that will be made using a single machine: a common requirement in many real-world production processes (Holmbom and Segerstedt 2014). For example, on an assembly line an automobile manufacture might assemble cars in different trim levels or left-hand drive models and right-hand drive models.

The ELSP therefore involves determining an efficient production schedule, one that balances out the need to produce different types of products using a single machine, so that the customer demand for each product is always met (Chatfield 2007). As in the EOQ model it is assumed that each item has an associated unit price and that each item can be held in stock and carried over to the next period, subject to an item specific holding cost. Similarly there is a known and constant setup cost that is incurred when each production cycle begins. However, unlike the EOQ model no known deterministic solutions to the ELSP are currently available and the problem has been shown to be NP-Hard (Gallego and Shaw 1997).

The Wagner-Whitin model

Wagner & Whitin (1958) took a different approach to modelling the original EOQ problem and, though their assumptions surrounding the demand process, laid the foundations for more dynamic lot-sizing models. Under the Wagner-Whitin (WW) model, as in the EOQ model, demand is assumed to be known and the problem remains to decide upon the optimal inventory management scheme, which simultaneously satisfies demand whilst minimising total cost. In contrast to the EOQ model which solves the lot-sizing problem in continuous time, the WW model divides up the planning horizon into $\boldsymbol{N}$ discrete periods in which demand may vary.

Under the WW model, d_t is the amount demanded in the $\boldsymbol{t}$ -th period, $\boldsymbol{t}$ = 1, 2, … $\boldsymbol{N}$, i_t is the interest charge or holding cost per unit of inventory carried forward to period t + 1, s_t represents the ordering (or setup) cost and x_t is the amount ordered (or manufactured). Equation (5.1) represents the amount of inventory entering period t given by the starting inventory before any production takes place plus the difference between total production and the total demand up until period t.

$$I = I_0 + \sum_{j=1}^{t-1} x_j - \sum_{j=1}^{t-1} d_j \geq 0 \tag{5.1}$$

The minimal cost policy for periods t through N, given incoming inventory (5.2), is thus (5.3).

$$f_t(I) = \min_{x_t \geq 0,\ I + x_t \geq d_t} [i_{t-1} I + \delta(x_t) s_t + f_{t+1}(I + x_t - d_t)] \tag{5.2}$$

where

$$\delta(x_t) = \begin{cases} 0 \ if \ x_t = 0 \\ 1 \ if \ x_t > 0 \end{cases} \tag{5.3}$$

Wagner & Whitin (1958) showed that (5.2) could be solved using dynamic programing by calculating f_t, starting at $t = N$, as a function of I; ultimately deriving $f_1$thereby obtaining an optimal solution. To narrow the size of the search space and take advantage of the special properties of their formulation, Wagner & Whitin (1958) postulated four theorems. The theorems showed that the dynamic lot-sizing problem could be viewed as a series of separate sub-models that could be solved individually without foregoing optimality. Furthermore, the number of sub policies that would need to be explored to identify the optimal schedule would require investigation of $\frac{N(N+1)}{2}$ entries compared with 2^{N-1} possibilities.

The capacitated lot-sizing problem

Bitran & Yanasse (1982) extended the WW model by adding to an index $j \in \{1..M\}$, representing one of the M items to be produced or ordered. The addition of this index to each parameter in the model allowed for the possibility of producing multiple items as in the ELSP. Furthermore, the authors added capacity constraints such that no production in any period could exceed a known and fixed period-dependent production rate. Together these extensions combine to produce what is now known as the classical capacitated lot-sizing problem (CLSP).

Table 5-1 - Decision variables in the CLSP

Symbol	**Definition**
I_{jt}	Inventory for item i at the end of period t.
q_{jt}	Production quantity (lot-size) for item j in period t.
x_{jt}	Binary variable which indicates whether a setup for item j occurs in period t ($x_{jt} = 1$) or not ($x_{jt} = 0$)

Table 5-2 - Parameters for the CLSP

Symbol	**Definition**
C_t	Available production capacity in period t.
d_{jt}	Demand for item j in period t.
h_j	Non-negative per period holding cost of item j
I_{j0}	Initial starting inventory for item j
J	The number of items
p_j	Capacity needed for producing one unit of item j
s_j	Non-negative start-up costs for item j
T	Number of periods

The decisions variables and parameters of the CLSP are shown in Table 5-1 and

Table 5-2 respectively. Using this notation the CLSP, in which items are produced in a single production step, can be formulated as a mixed-integer programing problem (MIPP):

$$\text{Min Z} = \sum_{j=1}^{J} \sum_{t=1}^{T} \left(s_j x_{jt} + h_j I_{jt}\right) \tag{5.4}$$

subject to

$$I_{jt} = I_{j(t-1)} + q_{jt} - d_{jt} \quad (j = 1, \dots, J; \quad t = 1, \dots, T) \tag{5.5}$$

$$p_j q_{jt} \leq C_t x_{jt} \quad (j = 1, \dots, J; \quad t = 1, \dots, T) \tag{5.6}$$

$$\textstyle\sum_{j=1}^{J} p_j q_{jt} \leq C_t \quad (t = 1, \dots, T) \tag{5.7}$$

$$x_{jt} \in \{0,1\} \quad (j = 1, \dots, J; \quad t = 1, \dots, T) \tag{5.8}$$

$$I_{jt}, q_{jt} \geq 0 \quad (j = 1, \dots, J; \quad t = 1, \dots, T) \tag{5.9}$$

The objective (5.4) is to minimise the sum of setup and inventory holding costs over the time horizon. Equation (5.5) is the inventory balance constraint, is states that the amount of inventory carried to the next period is the difference between what is produced and available from the previous inventory, minus the demand in the current period. Equation (5.6) says that production in a period can only take place when setup costs associated with producing a particular item have been incurred. As capacity is limited, (5.7) is present to prevent production in a period exceeding the total capacity in each period, given the resource requirements of producing each item. The setup variables are defined to be binary (5.8) and (5.9) represents the non-negativity conditions imposed on the amount of inventory carried between periods together with the production quantities themselves.

Solutions to the CLSP

Solving[19] the classical version of the capacitated lot-sizing problem, with general (and not necessarily linear) cost functions, has been shown to be NP-Hard (Florian, Lenstra and Rinnooy Kan 1980) (Bitran and Yanasse 1982). Constraint (5.6) which links the fixed setup costs with production is usually modelled using Big M, yielding constraint (5.10) so as to allow relaxing of (5.8) such that $x_{jt} \in [0,1]$. In the CLSP, Big M could for instance be the

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[19] See Appendix Solution methods for the CLSPA.4 for alternative solution approaches

sum of the demand in all future periods. The introduction of Big M is important in forcing the now continuous variable x_{jt} to behave as if it were binary, whilst still allowing for production to take place. This approach allows the application of the simple algorithm to the resulting linear program to obtain lower bounds and to prune the search space (Alfieri, Brandimarte and D'Orazio 2002)[20].

$$q_{jt} \leq Mx_{jt} \quad (j = 1, \dots, J; \quad t = 1, \dots, T) \tag{5.10}$$

5.3 Provider selection and discounting

Despite aspects of the CLSP resembling elements of the problem facing LTC planners, a number of important characteristics of our contracting problem are not considered. For example, the classical CLSP does not allow for any form of discounting and says nothing about the selection of suppliers for whom which orders will be made. Such considerations have to date been modelled through extensions to the classical CLSP and are referred to as CLSP models with supplier or vendor selection.

The more general supplier selection problem (SSP) concerns three related components, that is to say: (1) which products should be ordered, (2) from which suppliers and (3) in what quantities. Historically, previous work surrounding the supplier selection problem has focused on analysing each of these different aspects in relevant isolation of one another. While (1) relates to strategic decisions that are made surrounding which products an organisation wishes to market and sell; (2) considers more the ability of sellers to meet shipment deadlines, the perceived quality of the products offered by different suppliers and the strength of relationship between purchaser and supplier; and (3) inventory management policies and sales forecasts. As we are interested in a very specific healthcare service, LTC, we consider only aspects (2) and (3). Apart from a few studies that consider purchasing

[20] The introduction of Big Ms into the model does not yield particularly sharp lower bounds, leading to some loss of precision, despite reducing the computational complexity.

decisions that form part of a services contract with a supplier, the vast majority of published works have investigated the SSP from the point of view of firms that intend to purchase raw materials (Aissaouia, Haouaria and Hassinib 2007).

Supplier narrowing

Earlier work in supplier selection placed more emphasis on choosing the initial suppliers to consider, perhaps prior to further negotiation of price, discounts and service level. The aim of such work is arguably to limit the number of suppliers for which it is possible to deliberate with, in cases where there are many, by eliminating suppliers according to either quantitative or qualitative metrics. One such approach was by (Timmerman 1986) who proposed a categorical ranking approach to sort suppliers into three classes; good, neutral or unsatisfactory, based on an evaluation of each supplier’s historic performance for different criterion. An approach that relied less on subjective opinion of supplier performance was proposed by (Hinkle, Robinson and E 1969) which used classification and clustering to identify groups of suppliers with similar performance characteristics. In this case, each supplier attribute was based upon a numerical performance indicator and the groupings could be used to identify groups of statistically related suppliers to consider.

Traditionally, once the supplier set has been narrowed sufficiently for further modelling, the supplier choice is then optimised so that the purchaser is able to minimise the total cost of ordering. However, given that a number of non-price based factors may also be Important in the purchasing decision, for example late delivery, quality of goods delivered and ability to consistently meet production, several researchers have developed methodologies to overcome these limitations and allow for some of these factors to be taken into consideration.

An approach that uses the total additional cost of purchasing from a supplier was proposed by (Roodhooft and Konings 1996) who added to the price of an item the expected total supplementary cost associated with using a given supplier’s materials. (Wind and Robinson 1968) proposed using a score card for each potential supplier under different criterion. For each criterion an appropriate weight could be assigned to reflect the importance the purchaser assigned to this particular aspect of the supplier. Based on the dot product of the score and weighting vectors an overall score could be obtained for each supplier and used to inform the decision making process. To overcome uncertainty in the criterion themselves, (Soukup 1987) has shown how the criterion weights may be represented by probabilities than can be adjusted to calculate a payoff matrix under different weighting scenarios.

Single and multiple sourcing models

Where a purchaser selects a single supplier from which to order the modelling approach is referred to as a single sourcing vendor selection model. One of the key approaches in this area was developed by (Morris 1959). In this case the purchaser must choose to purchase a product from one of several competing suppliers for the duration of the policy, during which time the price of a product is uncertain and modelled as a random variable. In this paper the problem is modelled using dynamic programming to analyse different purchasing strategies under price uncertainty. One of the many extensions to this approach was by (Polatoglu and Sahin 2000) whereby, in addition to future supplier price, demand for products in each period was modelled as a random variable dependant on selling price and the time period itself.

In contrast to single sourcing models, multiple sourcing models allow for the possibility of ordering from multiple suppliers. Reasons vendor selection models may be orientated around using several suppliers include being able to satisfy total demand where suppliers are capacity constrained and hence individually would be unable to satisfy total demand. (Hong and Hayya J 1992) have also suggested that the use of multiple suppliers in specific inventory management policies, including Just-in-Time (JIT), allows for greater opportunities to reduce overall inventory and purchasing costs. One of the first papers which report the use of a multiple sourcing model was by (Gaballa 1974) in which case a mixed integer programming formulation was used to select suppliers for the Australian Post Office.

Discounting

Two important extensions to multiple sourcing models have been made over the last few decades, the first of which concerns modelling the multiple supplier problem over multiple time periods and the second concerns modelling the discounting activity of suppliers. Discounting of items may take one of several forms, to date the key forms that have been modelled within the literature include: discounts based upon a price-break, whereby the per-item price falls when an order reaches a certain threshold (Chaudhry, Forst and Zydiak 1993); total volume discounts, where the discount granted is based upon the total volume of all orders (Sadrian and Yoon 1994); and bundling, where the price of an item depends on the quantities of other items a supplier sells (Rosenthal, Zydiac and Chaudhry 1995).

Other extensions

To date, few papers have addressed the problem of multi-period supplier selection and multi-item problems simultaneously (Lee, et al. 2013). A theoretical formulation of the use of discounting with regards to production constraints under multiple suppliers was presented by (Bender, et al. 1985) using mixed integer programming. A model by (Basneta and Leungb 2005) attempted to bridge the gap between the classical CLSP model with more recent supplier selection models using discounting, in which case a mathematical programming formulation was presented to select the optimum number of items to order from each supplier taking into account ordering costs, quantity discounts and holding costs. (Hassini 2008) has also considered the implication of limited supplier capacity and the discount rate to determine order quantity and frequency, in addition the cost of transporting products ordered to customers was also considered in the objective function.

Of the body of research that studies the supplier selection process, we find that the general direction has been in marrying the supplier selection decision with inventory planning models, including the CLSP, so that these two decisions can jointly be optimised. At the same time, while an increasing number of papers have investigated how features of the supplier selection and ordering process, for instance discounting, might be incorporated, a new wave of research has been directed towards defining and implementing more multi-objective style models and in treating demand for products, frequently taken as known and constant, in a more stochastic manner. The vast majority of existing research has also concerned the use of supplier selection and CLSP in production-type problems, those involving inventories and physical storage of goods, compared with for instance the optimum purchasing policy for services – items which cannot be stored or carried over to future periods.

5.4 Model I – A min cost flow model for spot contracts

We take the view that the problem facing commissioners resembles a CLSP-style of problem with elements of vendor selection and discounting. For example, demand for care is considered over multiple periods and, given that it is not assumed to be constant, it behaves in a dynamic fashion. Similarly, as in the CLSP providers are capacity constrained and hence demand in any given period may need to be serviced from one or more suppliers. In contrast to typical use cases of the CLSP, we are considering the purchasing of a service rather than a physical product and hence there are no direct holding costs since products, here LTC placements, that are not used in a period t cannot be transferred and made available in period t+1 or indeed any subsequent period. In this situation, the suppliers of LTC services are those care providers that are able to offer one or more types of care across each of the different care categories.

Formally, we wish to devise a model that can be used by LTC planners to procure care services at minimum cost, whilst taking into account a measure of the perceived quality of care at different providers. Such quality for example could relate to the CQC rating of the provider in question. In this situation the decision to be modelled is the number of places at a provider to purchase across the different care categories and care levels available. In practice, the decision will involve multiple providers from which to select and hence we allow for the possibility of distributing care places across different providers to satisfy demand.

Due to physical capacity limits, we assume that each provider can only cater for a limited number of individuals and hence there exists capacity restrictions[21]. In order to model the decision we make the assumption that the price of care across different providers, and for different levels of care, is known although it may not necessarily be constant. Although we recognise that each patient may have slightly different care needs, even when compared with those patients in the same category of care, we make the assumption that within a particular care group we can distinguish between those patients with LOW, MEDIUM and HIGH levels of need.

5.4.1 Relationship to the CLSP

Our problem resembles a procurement problem in that the decision involves the purchasing of services from an external contractor under specific terms, involving both quantity and quality considerations. Unlike the majority of procurement problems that have been presented in existing literature to date, we consider the impact of multiple periods in the problem formulation and are less concerned with the more complex legal process that may take place to negotiate the final decision. Our reasons for this are two-fold, firstly our intention is to investigate the suitability of using a variant of the CLSP for the purchasing of services in which significant existing work has identified possible solution methods and secondly because we envisage the purpose of the model to help guide and evaluate rather than necessary dictate the final procurement decision.

Assumptions

As part of model development we make the following assumptions;

1. Provider care group, care intensity costs, are known throughout the time period.
2. Demand for each care group, care intensity level, is known throughout the time period.

[21] A capacity restriction could take the form of the number of beds available or nursing staff available at each care home dedicated to a particular care group. In our model we assume the former as this is often publically available.

3. The planning horizon is fixed and each t in the horizon represents a fixed length period of time.
4. There are no competing purchasers of care, thus the purchaser is the sole buyer of LTC.
5. The price of care for each care group, intensity level and provider is known.
6. All prices are based on per period occupancy.
7. Supplier capacity throughout the time horizon is known and is based on the number of beds available at each care provider for different care groups.
8. Provider capacity is specified for a given care group across all intensity levels.
9. The prices offered are fixed for a given period and are not subject to any form of discounting.
10. The purchaser of care is able to assign a quality measure to each provider, the quality measure is assumed to be fixed throughout the time horizon and is based on the CQC rating of each provider.
11. Both the purchaser and the suppliers agree on the definition of the care intensity levels.

5.4.2 Graphical representation

We can visualise the problem using a series of figures to illustrate key concepts. Figure 5.1 represents a block of demand for a given time period t. Here our demand refers to the total number of care packages taking place in a period. As the demand in each period stems from demand for places in each of the different care categories, we have used blocks with different shading patterns to highlight the care categories under consideration. Notice how for a given time period demand across the different care groups may not be uniform, in that for instance the area of the block for our Organic Mental Health group is larger than that of the corresponding block for Learning Disability.

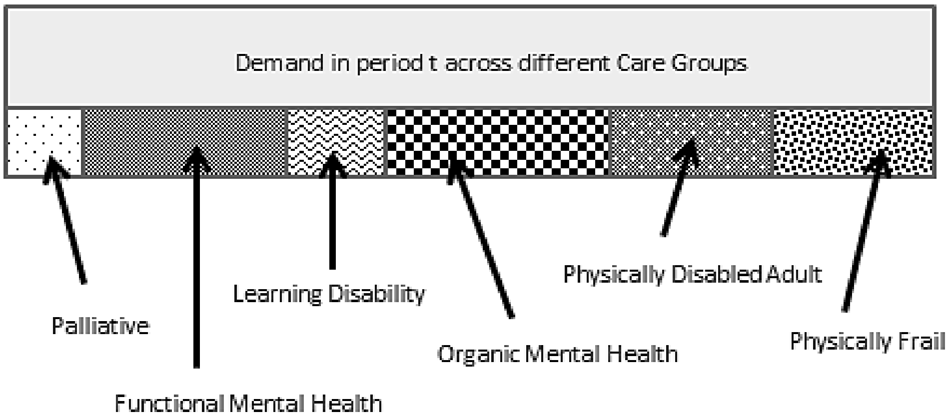

Figure 5.1– Demand block in period t

Figure 5.2 depicts an individual demand block at a moment in time t for a specific care group. We can see from the illustration that within a care group at time t, here palliative care, the amount of demand for a care group is divided between different care levels. These care levels corresponding with low, medium or high levels of need. This element reflects our recognition of different levels of need and intensity of care within the same care category.

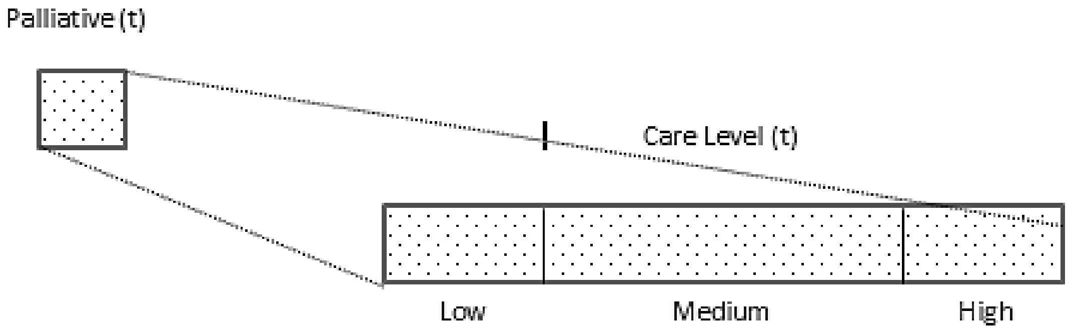

Figure 5.2– Distribution of care level within a palliative demand block

Figure 5.3 depicts the total care demand across each period. Each of the shaded bars represents total demand in a period, with individual demands for specific care groups shown in separate shaded regions. In this example, as in practice, we illustrate how demand across periods need not be constant and furthermore both the relative and absolute demand across different care groups may dynamically change from period to period.

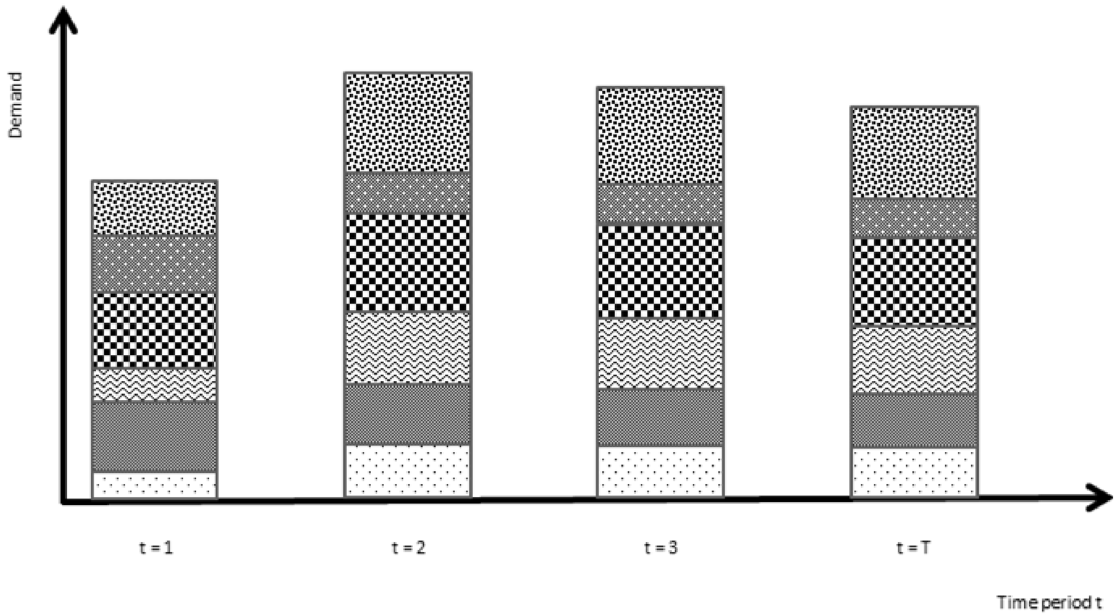

Figure 5.3– Demand in each period

Finally Figure 5.4 represents a graph of the capacity across different time periods at a specific provider k. In this instance, the provider is able to provide care for patients in the functional mental health group in periods 1 and 2 but by the end of the time horizon the provider, perhaps owning to expansion of their care services or merger, is now able to cater for OMH patients. The ability to allow for providers to modify capacity is taken into consideration in our model due to the length of the time horizon under consideration[22].

[22] Whilst LTC planning practices differ between NHS organisations, we assume that the majority of CCGs would at a minimum aim to budget for the next 1 to 2 years.

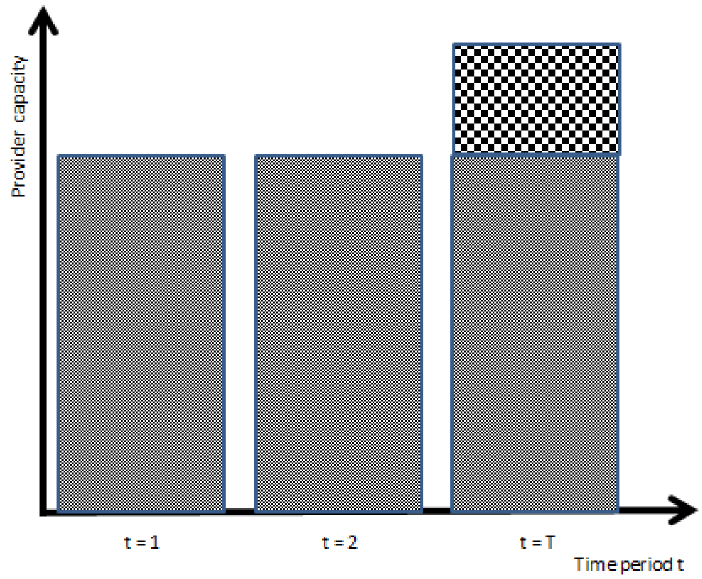

Figure 5.4– Demand block in period t

5.4.3 Mathematical formulation

We formulate the model as a mixed integer mathematical programing problem (MIPP). Our model represents a situation in which there are $i = 1,\ 2, \ldots, I$ care groups, each care group consists of different levels of care intensity $l = 1,\ 2, .., L$. There are $k = 1,\ 2, \ldots, K$ providers of LTC, each of which can supply care across $t = 1,\ 2, .., T$ time periods. Formally, we use the following notation in our formulation:

Table 5-3 - Definition of model 1 indices

Index	Definition
i	An index of care groups
l	An index of care intensity levels
t	An index of time periods
k	An index of care providers

Table 5-4 - Model 1 parameters

Symbol	Definition	Type
I	The number of care groups	INT
L	The number of care intensity levels	INT
T	The number of time periods	INT
K	The number of care providers	INT
$C_{k,i,t}$	The provider capacity for care group i in period t	INT
$d_{i,l,t}$	The demand for care group i at care intensity level l in period t	INT
$p_{i,l,k,t}$	The price of care group i at care intensity level i for provider k in period t	REAL
$q_{i,l,k,t}$	The purchase quantity of care group I, care intensity level l for provider k in period t	INT
α_k	The care provider quality rating	REAL

Our decision variable, $q_{i,l,k,t}$, represents the number of packages of care in care category i, for the intensity level l, from provider k that will be purchased in time period t. Our objective is to minimise the total purchasing cost over the period. The resulting mathematical programing model is as follows;

$$\text{Min}\, W = \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{t=1}^{T}(1-\alpha_k)p_{i,l,k,t}q_{i,l,k,t} \tag{5.11}$$

$$\sum_{k=1}^{K} q_{i,l,k,t} = d_{i,l,t} \quad (i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T) \tag{5.12}$$

$$\sum_{l=1}^{L} q_{i,l,k,t} \le C_{k,i,t} \quad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad t = 1, \dots, T)) \tag{5.13}$$

$$q_{i,l,k,t} \ge 0 \; (i = 1, \dots, I; \quad l = 1, \dots, L; \quad k = 1, \dots, K; \quad t = 1, \dots, T) \tag{5.14}$$

The objective function (5.11) is to minimise total quality-discounted cost across all time periods, care categories, care intensity levels and providers. The parameter α_k, where $-1 \leq \alpha_k \leq 0$, is the provider dependent quality rating which is used to revise prices offered by different providers according to a measure of quality. For values of α_k approaching 0 the provider quality-discounted price, across all care groups and intensity levels, approaches the true price. The effect is therefore to encourage more care packages to be purchased through this care provider. In contrast, as α_k approaches -1 the provider quality-discounted price is revised upwards leading to a negative penalty for purchasing care through this provider. The addition of α_k overall is therefore to help account for relative quality differences between competing providers where for instance price is otherwise equal.

Constraint (5.12) represents the demand constraint. That is to say that the amount of care ordered across different suppliers for a specific care group and intensity level must be equal to the demand for that care group and intensity level in the specified period. Constraint (5.13) represents the capacity restriction in that the total amount of care purchased across different care levels in a specific time period and for a given care group must not exceed the provider care category capacity in the specified time period. Finally, constraint (5.14) is our non-negativity condition to restrict the solution to non-negative purchase quantities.

5.4.4 Example

To illustrate the use of our model we created an example dataset representing a hypothetical situation in which there is two periods to consider. In this scenario there are two care providers, two care groups and two intensity levels (high and low) for each care group. The care quality provided by each of the providers is known and detailed in Table 5-5 – in this case provider A provides better quality care than provider B hence the value of α_k is closer to zero.

Table 5-5 – Quality ratings for providers A and B

Provider (k)	Quality rating (α_k)
A	-0.1
B	-0.15

Table 5-6 details the per period provider capacity for each care group, observe that the capacity is shared across care groups for different levels of intensity. Provider A has more capacity compared with provider B overall over both periods. While provider A adds additional capacity in period 2, provider B only switches some of its capacity from care group 2 to care group 1 between periods 1 and 2.

Table 5-6 – Per period provider capacity for each care group

Provider (k)	Time Period t = 1		Time Period t = 2	
	Care Group 1	Care Group 2	Care Group 1	Care Group 2
A	25	50	50	50
B	40	20	50	10

Table 5-7 displays the per-period demand in our example. Observe that demand for care group 2 places are constant throughout the period, across both intensities, whilst demand for care group 1 rises from 30 to 35 in period two for both intensity levels.

Table 5-7 – Per period demand for each care group and intensity level

Care Group	Low Intensity		High Intensity	
	Time Period t = 1	Time Period t = 2	Time Period t = 1	Time Period t = 2
1	30	35	30	35
2	30	30	30	30

Finally Table 5-8 provides per period prices for care at each provider across both care groups and intensity levels. Both providers charge a higher price for more intensive care with provider B offering lower prices for care group 1. The only case in which provider A is less expensive than provider B is for care group 2 and the high intensity level. In order to test the formulation[23] we modelled the example using LINGO[24] (Lindo Systems Inc. 2015) version 15 for 64-bit Windows. As a double check we also developed an equivalent Microsoft Excel 2010 model using the LP Solver add-in[25][26].

Table 5-8 – Per period provider price by care group and intensity level

		Provider A		Provider B	
Care Group	Intensity	Time Period t = 1	Time Period t = 2	Time Period t = 1	Time Period t = 2
1	Low	750	750	500	500
1	High	1200	1200	1000	1000
2	Low	750	750	400	400
2	High	1400	1400	1600	1600

Results

The model is solved to optimality in 0.03 seconds and finds that the minimum quality-adjusted cost of providing care is £249,875; this corresponds to a total nominal cost of £224,000. The total amount of demand allocated is 250, which is split 130 for provider A and 120 for provider B. The minimum cost solution, shown in Table 5-9, shows that due to quality-adjusted price differentials between provider 1 and 2, the solver favours allocating demand to provider B in period 1 and 2. Owing to the fact that provider 2 cannot satisfy demand exclusively, additional demand above what provider B can cater to is allocated to

[23] Details of our LINGO formulation can be found in Appendix A.5

[24] LINGO is a commercially available optimisation tool for linear, non-linear and integer programming problems that include a number of different solvers (http://www.lindo.com/)

[25] The LP solver add-in is part of Microsoft Excel and finds global optimums to LPs using simplex

[26] An Excel solution report can be found in Appendix A.6

provider A. This is represented by binding capacity constraints for provider B for both periods and slack capacity constraints for provider A. The reduction in capacity for care group 2 placements for provider B in period 2, from 20 to 10, leads to the solver relying less on provider B in period 2 to satisfy demand. However, owing to provider A offering lower quality-adjusted prices for care group 2 at the high level of intensity, the solver uses the capacity provided at B for care group 2 exclusively for low intensity demand.

Table 5-9 – Allocation for minimum cost solution

Care Group	Intensity	Provider	Period	Assigned	Cost
1	Low	A	1	0	0
1	Low	A	2	0	0
1	Low	B	1	30	15000
1	Low	B	2	35	17500
1	High	A	1	20	24000
1	High	A	2	20	24000
1	High	B	1	10	10000
1	High	B	2	15	15000
2	Low	A	1	10	7500
2	Low	A	2	20	15000
2	Low	B	1	20	8000
2	Low	B	2	10	4000
2	High	A	1	30	42000
2	High	A	2	30	42000
2	High	B	1	0	0
2	High	B	2	0	0

5.4.5 Application to the London LTC dataset

We now test our initial formulation using a subset of the data collected on actual LTC activity in London. As in our previous example we are considering the optimum allocation of spot contracts only.

Demand

Demand in our model is the number of placements required per care group, per period and per intensity level. We estimate demand based on the numbers of placements taking place between the 1st of January 2006 and the 1st of January 2008. In order to determine per period demand we need to consider how many care packages are active. Crucially, the choice of time period has an important effect on how demand is estimated and applied to our model. If the time period is small, for instance days, it will have the impact of introducing a large number of decision variables into our model. If the period considered is longer, for instance one year, then some granularity is lost. We therefore propose using a time period of one month such that $T = 24$.

Within a period we identify the care packages in our data set that are taking place within it by considering the start date and end date of each care package individually. A care package takes place in a period if its end date is on or after the start date of the period and at the same time the care package start date is on or before the end date of the period. Thus for period one, the start date is 01/01/2006 and the corresponding end date is 31/01/2006. If these two conditions are met then we know that a given care package contributed some demand to a particular period. The amount of demand in a time period for a given care package demands on how much time within the period the care package was active. We therefore have to consider how many days of overlap exist between an individual's care package and the time period under consideration. To do this we first assume an individual demanded care for the entire period, by inspection of the start and end dates of the care package we then revise the days spent if either of these two dates are not equal to the start and end dates of the period itself. Table 5-10 shows estimated demand per period using our chosen method.

Table 5-10 - No of Active Care Home Packages by Care group and Period

Period	Date	FMH	LD	PDA	OMH	PAL	PF	Period Total
1	Jan-06	52	39	43	121	102	153	510
2	Feb-06	55	40	42	125	92	150	504
3	Mar-06	56	39	42	125	91	162	515
4	Apr-06	47	40	45	124	102	147	505
5	May-06	46	40	46	127	103	165	527
6	Jun-06	47	39	46	133	114	174	553
7	Jul-06	48	37	45	133	116	182	561
8	Aug-06	47	37	52	130	130	187	583
9	Sep-06	44	38	50	133	134	180	579
10	Oct-06	44	38	51	133	137	190	593
11	Nov-06	44	38	51	137	132	201	603
12	Dec-06	44	37	51	145	131	197	605
13	Jan-07	45	38	52	147	122	192	596
14	Feb-07	45	39	49	143	119	182	577
15	Mar-07	45	39	54	151	124	180	593
16	Apr-07	41	42	52	141	137	182	595
17	May-07	41	44	51	144	171	182	633
18	Jun-07	27	44	54	142	166	184	617
19	Jul-07	28	44	53	143	165	181	614
20	Aug-07	25	46	53	138	157	168	587
21	Sep-07	23	46	50	137	143	162	561
22	Oct-07	23	47	44	124	139	160	537
23	Nov-07	22	49	38	127	129	143	508
24	Dec-07	21	48	30	119	110	129	457
	Care Group Total	960	988	1144	3222	3066	4133	

If an individual's care package started on the 08/01/2006 but ended on the 01/02/2006, the true days spent in period one is 23 days and in the case of period 2 it is 1 day. Aggregating the amount of days spent by each care package in each period provides the total monthly demand in days. From this we estimate the demand in total number of care packages by dividing this value by the length of the period. So as to obtain demand on a per care group basis, we perform this calculation several times using different filtering conditions for the individual care groups we wish to consider in the calculation.

Price

Price in our model, as with demand, is defined for a specific care group, intensity level and period. Once demand per period-per care group had been determined, it remained to apportion this demand between the different care levels and identify an appropriate price. Unfortunately, the dataset used contained no information on the intensity of an individual's care and furthermore prices were known to only be updated upon completion of an individual's care package. If for instance, an individual had been in care for 2 years we would only be able to observe the price paid per week in care at the end of year 2. We therefore proposed two main ways of dealing with these issues. Firstly, we assumed that prices were more reflective of the true price of care the closer they were to the end date of care. Secondly, we made the assumption that price in itself could be used as an indication of the level of intensity of an individual's care package.

We estimated the expected cost of care by care group and period by firstly considering only those care packages that completed in a year under consideration – this is known as our end year. We then calculated the difference between the start date of the year and the aforementioned end date for each care package, this yielded the number of days in care to which we expected the price entered to be reflective of the true cost of care. The maximum number of days of care at a given price was 365. This number of days in the end year was multiplied by weekly the care cost and then divided by the total number of care days across all care packages ending in the end year. Summation of this value across all care packages under consideration we obtained the expected weekly care costs for each care package in both 2006 and 2007 (Figure 5.5).

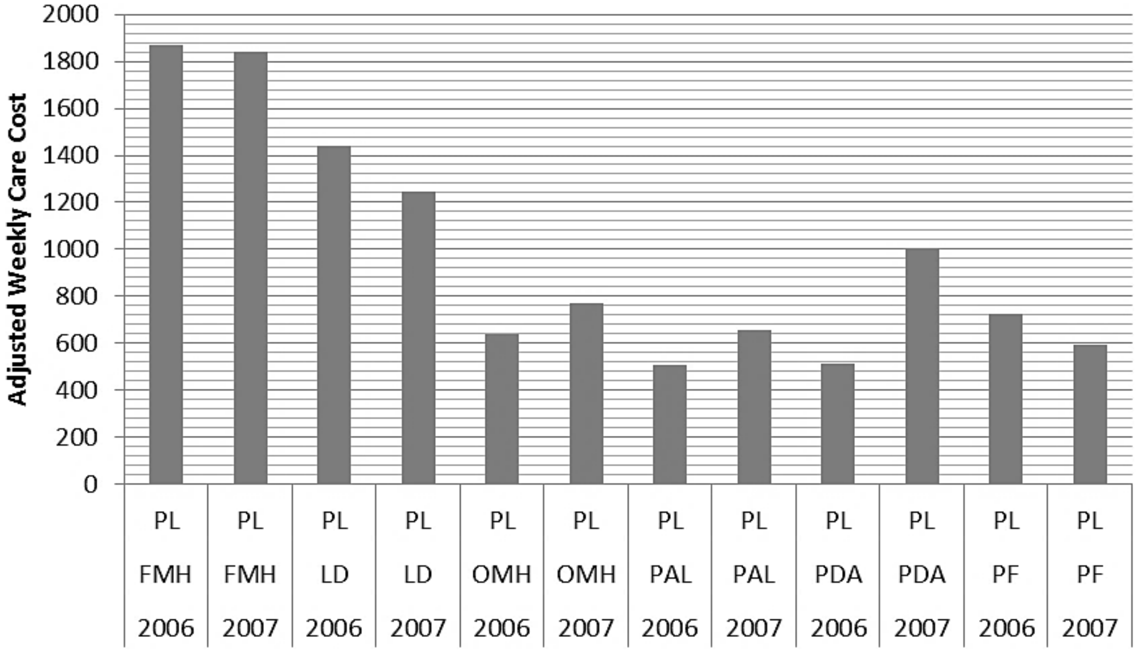

Figure 5.5– Adjusted Weekly Care Cost by Care Group, Year and Provision Type

While this method allowed us to derive the expected weekly cost, an inspection of the distribution of the weekly cost by end year showed that there was significant variation within each care group (Figure 5.6 shows an example for functional mental health). Rather than use the expected price in our model, and in recognition of the classification of care packages in reality, we therefore proposed classifying care costs into one of high, medium and low. In practice, regional planners would attempt to classify care costs so as to identify those care packages that are distinctly high, for the purpose of auditing, and those that are much lower to check whether the needs of the individual fall within the scope of LTC and could not be met by other services.

At the time of writing, the exact cost classification boundaries are not standardised among different LTC planners and may change according to both time period and care group. We therefore proposed using a data driven approach to identify three possible groups within our distributions of weekly cost, so as to infer the cost boundaries of low, medium and high cost categories themselves. At the same time, this classification would also provide us with potential classes to categorise different intensity levels assuming that those within higher cost groupings were incurring higher costs due to increased complexity of their condition.

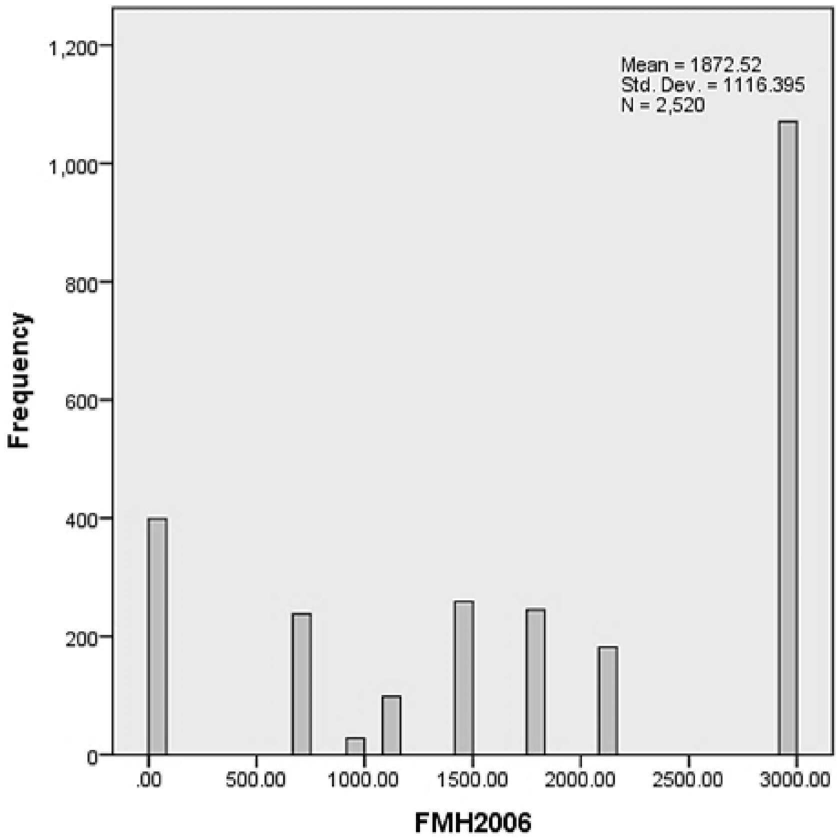

Figure 5.6– Distribution of Adjusted Weekly Care Costs for Functional Mental Health 2006

Our classification approach is based upon using two stages. In the first stage we use k-means clustering to identify expected weekly cost in each of the high, medium and low groups, whilst in the second stage we perform a visual inspection of the histogram to verify the selection and adjust the class boundaries where necessary based on expert judgement. K-Means clustering is a general purpose clustering algorithm that partitions data observations into k groups, where k is the desired number of groups to determine. Each group or cluster is defined by a centre point or centroid. The objective of the algorithm is to determine the centroids, or values for each cluster centre, so that the squared Euclidean distance between the data points and the centroid each data point is associated with is minimised (Jain 2010).

Table 5-11 - Mean Weekly Price of Care by Care Group and Year

		Group Mean Price		
	Year	**Low (Pr)**	**Medium (Pr)**	**High (Pr)**
PF	2006	180 (0.118)	760 (0.819)	3071 (0.063)
	2007	224 (0.06)	746 (0.845)	3221 (0.095)
PDA	2006	153 (0.743)	1058 (0.194)	3075 (0.063)
	2007	273 (0.564)	1354 (0.358)	3222 (0.077)
PAL	2006	168 (0.404)	760 (0.588)	3925 (0.008)
	2007	156 (0.388)	768 (0.569)	3909 (0.042)
OMH	2006	322 (0.86)	2054 (0.106)	4242 (0.034)
	2007	530 (0.842)	1577 (0.108)	3132 (0.049)
LD	2006	411 (0.481)	1859 (0.358)	3681 (0.16)
	2007	717 (0.242)	1203 (0.081)	1441 (0.677)
FMH	2006	302 (0.164)	1698 (0.353)	2975 (0.483)
	2007	365 (0.192)	1393 (0.341)	3364 (0.467)

Table 5-11 shows the final intensity clusters for each year and care group. For each group the centroid is used as the mean weekly cost of care for packages. Using information on the number of care packages that fall into a particular group divided by the total number of classified packages we estimate the probability of a given care package being either high, medium or low within its respective care group. This probability measure is then applied to the observed activity during each month in our time horizon to partition demand between high, medium and low intensity services.

Supply of places

To populate the variables in our model with data on the supply of places in nursing homes, so as to be able to determine appropriate capacity constraints, we made use of an online service (CareHome.co.uk 2015) which details registered care homes operating in England. As no information surrounding the name of the specific care provider for an individual care package was provided, we took a random sample of 20 nursing homes from the CareHome.co.uk site of those that were located within greater London. For each care home we recorded the total bed capacity, user rating, the care groups catered for, postcode, ownership type and name.

There were several ways to then integrate the provider information into our model depending on our assumption of what constituted a supplier of care in our model and in particular how we wanted to model the impact of geographical location. For example, if the user of our approach was a local commissioning unit operating at a local rather than regional level, the supplier units could be individual provider. Alternatively, individual care homes could be grouped according to the ownership of the supplier and then capacity would represent total capacity at the provider group level. This approach might then be more meaningful to larger regional planning units. In our final model we chose to model a supplier at the nursing home level, given that our intention was to keep the formulation generic and to illustrate the case for a local level provider.

For many providers of care the total bed capacity was not given per care group and so we made an assumption that the bed capacity would be divided between different care groups, according to the order in which each provider listed their specialist areas of care. Based on our understanding of the care needs of different types of patients it was assumed that in practice providers would have soft constraints on the numbers of patients that they supported in different care groups, owning to the different skill sets of staff that would be required. Hence, in the short run at least, a provider with a total of 50 beds and capable of supporting both functional mental health patients and those who were physically frail, would in practice share capacity between these two care groups rather than run at capacity under a single care group.

Quality rating

In the proposed model we allow for quality differences between providers using the vector α. The effect of the quality measure is to adjust prices so as to encourage the assignment of care packages to providers with higher quality for a given price. Our approximation of quality is based on using the CQC rating together with the user review score on the CareHome.co.uk site. The CQC score is measured from 1 to 4, with 4 being a care home that provides outstanding quality whereas 1 implies it is inadequate. The user review score is out of 10 and is based on public ratings. In our approach, the CQC rating is multiplied by 2.5 and then averaged with the user review score. This average is then divided by the maximum obtainable score of 10. For each provider we then calculate the percentage difference between this measure and the overall highest measure for all care homes under consideration to derive our approximate quality measure. The higher the quality measure deviates from the maximum obtainable quality measure across all homes, the higher the value of α_k and thus the greater the price penalty.

Supplier price differentials

To take into account price differentials between care homes we associated each provider with a price multiplier. Such price differentials were present to reflect different cost structures and management practices among providers. The price multiplier used was calculated by sampling from a continuous uniform distribution with minimum value -5 and maximum value 5. This random variable was then divided by 100 to convert to a percentage and added to 1 before being assigned to a provider for the remainder of the analysis[27].

Results

[27] Full details of the price indexes used in the test application are presented in Appendix A.7

As with the test instance, the data collected on demand together with the data surrounding provider capacity was entered into LINGO version 15 64-bit edition. Due to the size of the data and to ensure the relevant data was correctly entered into LINGO we wrote a Python script[28] to extract data and calculate the demand across each of the different intensity levels and adjust provider prices according to the price multipliers. The Python script outputted a set of LINGO data files, each containing the relevant matrix for each input data set across demand, capacity, quality and price.

An optimum feasible solution to the instance was solved using Lingo's branch-and-bound solver in 11.16 seconds, using a total of 8,640 integer variables and 11,953 constraints. The value of the objective function at the optimum solution, here the minimum total cost-quality purchase cost, was calculated as £51,032,730. This is compared with an upper bound of £65,979,260, when all orders are placed with the highest quality-cost provider, and the observed total cost of £58,847,017 calculated using exact costs from the recorded placement data itself.

5.5 Summary

Despite several examples of the use of contracting in the literature, there are very few examples of operational models directed at the health care sector. In this chapter we have shown how the allocation problem facing LTC planners can be viewed as a CLSP-style of problem, for which a significant body of research exists, with vendor selection and discounting. Using an example from LTC activity in London, we have formulated the LTC allocation problem for spot contracts using a more simplistic form of the CLSP and mathematical programming. Our formulation respects the fact that LTC is a service orientated good rather than an physical item that can be stocked as inventory and carried over to future periods.

[28] The version of Python used was 2.7 32-bit edition for Microsoft Windows.

Chapter 6

A dynamic sliding commitment model

6.1 Introduction

Whilst our previous model provides an illustration of the use of MIPP in the efficient allocation of care home places between different suppliers so as to minimise total overall cost, it deals only with spot placement arrangements. In practice, care planners may be willing to make longer term commitments with care providers if specialist terms, perhaps those involving the use of volume discounts, can be secured. Historically, block contracts, were used to secure these specialist terms but increasingly care planners have looked to ways to avoid large and lengthy block contract arrangements due to their inherent inflexibility. We now consider a novel approach balances the need to secure discounts with providers whilst respecting the aversion to establish large long-term commitments.

6.2 Rationale for our commitment model

Consolidation in the market for care home places has meant a reduction in the number of providers that operate independently. Thus has led to a gradual concentration of market share within a few large providers; providers that may operate hundreds of individual care homes. To a certain extent, this has strengthened the case for care planners to make use of a greater number of block contract arrangements across several suppliers so as to both leverage greater discounts, increasingly their allocation flexibility, all whilst reducing their dependence on any single provider. To address these issues we propose a second model, model II, which allows for the possibility of commitments being made towards a specific provider. Our methodology is based on an adaptation of the principles and formulation presented in a number of closely related works.

6.3 Related commitment models

Degraevea, Labrob and Roodhooftc (2001) consider a mathematical programming approach to optimise the cost of business travel by selecting between competing airlines according to the total cost associated with the cost of purchasing airline tickets to business destinations. In this case, the airlines offer volume based discounts when set thresholds relating to the sales volume of tickets are met. As in our problem, the objective is to choose from which suppliers to purchase and ultimately determine the market share that prevails for each provider. Furthermore, this study also acknowledges the fact that among existing research few papers have addressed the problem of contracting for a service compared with a physical product or material. In contrast to this paper, we want to consider the case where a commitment may be formed over the time horizon and thus the discount is based not only the quantity of services purchased but the duration for which such services are continually purchased.

An influential paper by in which the decision modelled is the amount of resources to commit to purchasing of a product at the start of a period so as to secure supply is presented by (Sadrian and Yoon 1994). Depending on the size of the commitment, or as referred to in the study the locked as-ordered quantity, greater discounts may be obtained according to the total amount of purchases from a particular vendor. More recently, this approach has been scaled to larger problems (Balakrishnan and Natarajan 2014) in which there may many hundreds of products to determine an efficient commitment for. To the best of our knowledge we can find no formulation of this model dealing with the commitment related to services. Furthermore, this particular methodology considers an all or nothing approach in which a commitment is either in place or not. If indeed a commitment is made then this commitment lasts for the entirety of the planning horizon.

A more modern formulation of a similar class of problem is presented in (Lee, et al. 2013) in which case a set of suppliers have to be selected and the order quantities determined, taking into account both incremental and all units discounts using price breaks. Thus, as a purchaser spends more with a supplier they may shift onto a different portion of the providers cost curve. Although the problem modelled considers goods rather than services, the model presented combines the time based dimension of ordering policy with multiple discounting policies.

Uniqueness of our problem

Overall we find that to the best of our knowledge no existing work combines the six essential properties of the contracting problem facing LTC commissions has been reported, namely time and volume based commitments; the ability to choose the length of the commitment in addition to the quantity associated with it; the ability to delay the commitment into some period after the starting period of the planning horizon; the ability to end the commitment on or before the end of the planning horizon; the ability to salvage some commitment quantity in cases where demand in a period may be less than the commitment quantity and finally the ability to simultaneously determine the market shares of the providers for whom commitment quantities are specified.

Having identified this gap within existing research we therefore present a mathematical programming formulation and, using an example for LTC, show how it can be applied to generate cost savings. Although we illustrate the case using LTC, we consider the model formulation suitable for any procurement problem involving the use of fixed commitments to generate price savings - in particular in situations where it may be desirable to have commitments that are not necessarily aligned with the start and end periods of the planning horizon.

6.4 Assumptions

Our assumptions for the revised model consist of a superset of the assumptions for model 1 and include the following additional considerations;

1. When a commitment is made it is subject to a one of negotiation cost which is known and constant throughout the period for all contracts.
2. Providers are willing to offer discounts based on the quantity-time value of a commitment. Thus a discount may be awarded in the case that: a low quantity commitment is made for several periods; a high quantity commitment is made for a short period; or some intermediate combination.
3. The discount is offered as a price break, thus once the value of the commitment reaches a certain threshold the discount is applied to all units in the commitment.
4. There are three discount thresholds, or price-breaks, which are known by the purchaser of care and all providers state their discount rate for each threshold.
5. The discount rate is non-decreasing with higher quantity-time thresholds and only one discount rate can be applied to a given commitment.
6. Demand for care can be satisfied from any mixture of spot and commitment orientated arrangements.
7. Excess commitment quantity can be salvaged by the purchaser of care by subletting the commitment to other organisations, for example the LA. When salvaging occurs the purchaser receives a salvage amount per period.
8. The salvage price is constant throughout the period for all care groups and intensity levels.
9. The commitment quantity and quantity purchased in spot contracts must respect each provider's known bed capacity constraints[29].
10. Commitments are subject to a minimum quantity
11. Commitments are subject to a minimum duration

[29] As with model I capacity is based on bed availability per care group in each time period for each provider. Capacity may be shared between different intensity levels only.

12. Commitments are subject to a maximum quantity
13. All commitment contracts must end on or before the time horizon
14. The purchaser of care specifies the maximum market share that each provider may hold in contracted placement quantity over the entire time period.

6.5 Mathematical formulation

As with model I, we formulate model II as a non-linear MIPP. Our model represents a situation in which there are $i = 1,\ 2, \ldots, I$ care groups, each care group consists of different levels of care intensity $l = 1,\ 2, .., L$. There are $k = 1,\ 2, \ldots, K$ providers of LTC, each of which can supply care across $t = 1,\ 2, .., T$ time periods. We introduce a set of price breaks $b = 1, 2, \ldots, B$. Formally, we use the following notation in our formulation:

Table 6-1 - Definition of indices for model II

Index	**Definition**
i	An index of care groups
l	An index of care intensity levels
t	An index of time periods
k	An index of care providers
b	An index of price breaks

Table 6-2 to Table 6-7 show the parameters and decision variables used in the formulation that are grouped according to the aspect of the model they relate to, for example core elements (demand and supply), commitment and discounting. Parameters are defined as inputs to the model whereas decision variables correspond to outputs that are generated as part of the solution process.

Table 6-2 - Definition of core parameters for model II

Symbol	**Definition**	**Type**
$c_{k,i,t}$	The provider capacity for care group i in period t	INT
$d_{i,l,t}$	The demand for care group i at care intensity level l in period t	INT
$p_{i,l,k,t}$	The price of care group i, care intensity level l, for provider k in period t	REAL
α_k	The care provider quality rating	REAL
M	A big number	BIGINT

Table 6-3 - Definition of core decision variables for model II

Symbol	**Definition**	**Type**
$q_{i,l,k,t}$	The purchase quantity of care group i care intensity level l for provider k in period t	INT

Table 6-4 - Definition of commitment parameters for model II

Symbol	Definition	Type
g	The negotiation cost associated with the formation of a contract	REAL
q^{smax}	The maximum market share each provider may have in contract placements	REAL
s^{min}	The minimum period in which a contract may start	INT
u^{min}	T e minimum duration of a contract	INT
u^{max}	The minimum duration of a contract	INT
q^{min}	The minimum size of a contract	INT
q^{max}	The maximum size of a contract	INT

Table 6-5 - Definition of commitment decision variables for model II

Symbol	Definition	Type
$o_{k,i,l}$	The cost of a contract with provider k, care group i and intensity level l.	REAL
$\hat{q}_{k,i,l,t}$	The contract quantity from provider k, care group i, intensity level l and period t	INT
$a_{k,i,l,t}$	A binary variable indicating whether a contract is active for provider k, care group I, care intensity level l in period t	BINARY
$\dot{q}_{k,i,t}$	The contract quantity from provider k, care group i, and period t	INT
$x_{k,i,l}$	A binary variable denoting whether a contract is in place with provider k, care group i and intensity level l.	BINARY
$\tilde{q}_{k,i,l}$	The size of the contract from provider k, care group i and intensity level l	INT
$s_{k,i,l}$	The starting period of the contract from provider k, care group i and intensity level l	INT

$e_{k,i,l}$	The end period of the contract from provider k, care group i and intensity level l	INT
$u_{k,i,l}$	The duration of the contract from provider k, care group i and intensity level l	INT
$\ddot{q}_{i,l,t}$	The total contract quantity for care group i, intensity level l and period t	INT
$\dddot{q}_k$	The total number of places purchased in contracts for provider k	INT

Table 6-6 - Definition of discounting and salvage parameters for model II

Symbol	Definition	Type
$\bar{v}$	The salvage price	REAL
h_b	The upper threshold for price break b	REAL
$r_{k,i,l,b}$	The discount rate offered from provider k, care group i, intensity level l for price break b	REAL

Table 6-7 - Definition of discounting and salvage decision variables for model II

Symbol	Definition	Type
$v_{i,l,t}$	The salvage quantity for care group i, intensity level l and in period t	INT
$\hat{v}_{i,l,t}$	A binary variable indicating whether salvage is allowed for care group i, care intensity level l in period t	BINARY
$\hat{r}_{k,i,l,b}$	A binary variable indicating whether the discount rate offered from provider k, care group i, intensity level l for price break b is used	BINARY
$\bar{r}_{k,i,l}$	The discount rate applied for the contract with provider k, care group i and intensity level l	REAL
$\bar{r}_{k,i,l,b}$	The discount rate applied for the contract with provider k, care group i, intensity level l for price break b	REAL

Table 6-8 - Definition of summations for model II

Symbol	Definition
TCS	The total committed spend across all providers, care groups, intensity levels and periods.
TSS	The total spend for spot placements across all providers, care groups intensity levels and periods.
TVQ	The total salvage quantity across all providers, care groups intensity level and periods.
TCQ	Total contract quantity across all providers, care groups intensity level and periods
TCF	Total number of contracts formed across all providers, care groups intensity level and periods

Our objective (6.1) is to minimise the total cost due to commitments, spot based allocations, the negotiation cost associated with the formation of contracts less the amount of revenue generated through subcontracting excess commitment capacity, for instance to local authorities or other LTC commissioning organisations.

$$\text{Min}\, W = TCS + TSS + gTCF - \overline{v}TVQ \tag{6.1}$$

The total commitment spend is represented by (6.2), here the amount spent by LTC commissioners depends on the cost of the equivalent number of spot placements for each contract, adjusted by the provider level of quality and discounted according to which price break the total contract cost falls into. Equation (6.3) is the total spot placement cost and takes the same form as in model I. The total number of contracts formed is the sum over the number of those formed (6.4) - this total is multiplied by the negotiation cost in (6.1) to represent the total overall cost due to contract formation.

Non-linearity enters into our model through (6.2), here the discount rate applied to the contract spend is multiplied by the total cost of the contract. Although the discount rates offered by the providers are known, it remains for the model to select the contract size and its duration so as to simultaneously determine the corresponding discount rate to use. Intuitively, this has the effect of causing changes in the slope of the total cost of contracts function in regions in which different discount rates are applied (i.e. as the total cost of the commitment breaks different discount thresholds).

$$TCS = \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L} o_{k,i,l}\,(1-\alpha_k)(1-\bar{r}_{k,i,l}) \tag{6.2}$$

$$TSS = \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{t=1}^{T}(1-\alpha_k)q_{i,l,k,t}p_{i,l,k,t} \tag{6.3}$$

$$TCF = \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{l=1}^{L} x_{k,i,l} \tag{6.4}$$

$$TVQ = \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} v_{i,l,t} \tag{6.5}$$

The total salvage quantity is represented by (6.5) whilst (6.6) is used to calculate the total quantity of placements taking place as part of a contract.

$$TCQ = \sum_{k=1}^{K}\sum_{i=1}^{I}\sum_{t=1}^{T} \ddot{q}_{i,l,t} \tag{6.6}$$

Equation (6.7) is the demand constraint, the quantity supplied in contracts less those placements that are salvaged plus those in spot placements must at least satisfy demand in a period for a particular care group and intensity level.

$$\ddot{q}_{i,l,t} - v_{i,l,t} + \sum_{k=1}^{K} q_{i,l,k,t} \geq d_{i,l,t} \tag{6.7}$$

$$(i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

The capacity constraint is given by (6.8) hence the volume in contracts and spot placements for a given care group cannot exceed the care group capacity at each provider.

$$\dot{q}_{k,i,t} + \sum_{l=1}^{L} q_{i,l,k,t} \leq c_{k,i,t} \tag{6.8}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad t = 1, \dots, T)$$

Constraint (6.9) determines the quantity in contracts taking place for a particular care group by summation of those across all its intensity levels.

$$\sum_{l=1}^{L} \hat{q}_{k,i,l,t} = \dot{q}_{k,i,t} \tag{6.9}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad t = 1, \dots, T)$$

The amount of capacity for a particular care group and intensity level for a specific period is calculated by summation across all providers (6.10).

$$\sum_{k=1}^{K} \hat{q}_{k,i,l,t} = \ddot{q}_{i,l,t} \tag{6.10}$$

$$(i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

Constraint (6.11) prevents salvage by forcing $\hat{v}_{i,l,t}$ when demand exceeds contract capacity in a period. Similarly, (6.12) forces the salvage quantity to the excess capacity in a period.

$$d_{i,l,t} - \ddot{q}_{i,l,t} < M(1 - \hat{v}_{i,l,t}) \tag{6.11}$$

$$(i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

$$\hat{v}_{i,l,t}\left(\ddot{q}_{i,l,t} - d_{i,l,t}\right) = v_{i,l,t} \tag{6.12}$$

$$(i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

The total number of placements in contracts for a provider over the time horizon is determined by (6.13). Equation (6.14) prevents any single provider from having more than a set market share in contracted placements.

$$\sum_{i=1}^{I}\sum_{t=1}^{T}\dot{q}_{k,i,t} = \dddot{q}_k \qquad (k = 1, \dots, K) \tag{6.13}$$

$$\dddot{q}_k \le q^{smax} TCQ \qquad (k = 1, \dots, K) \tag{6.14}$$

Constraint (6.15) forces a contract to be in place with a specific provider for a given care group and intensity level when its size is greater than or equal to one.

$$Mx_{k,i,l} \ge \tilde{q}_{k,i,l} \qquad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.15}$$

$$s^{min} x_{k,i,l} \le s_{k,i,l} \qquad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.16}$$

Constraint (6.16) forces a contract to start on or after the minimum start date if it is in place. Similarly (6.17) prevents contracts that are formed being smaller than the minimum size specified in the problem formulation, whilst (6.18) prevents individual contracts being formed that a bigger than a predetermined size.

$$q^{min} x_{k,i,l} \leq \tilde{q}_{k,i,l} \qquad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.17}$$

$$q^{max} x_{k,i,l} \geq \tilde{q}_{k,i,l} \qquad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.18}$$

Equation (6.19) says that a contract, if it is in place, must end on or before the last period in the time horizon.

$$T(x_{k,i,l}) \geq e_{k,i,l} \qquad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.19}$$

$$M x_{k,i,l} \geq s_{k,i,l} \qquad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.20}$$

Constraint (6.20) forces a contract to be in place if a starting date is specified. To prevent contracts from ending prior to their starting date we use (6.21). The duration of a contract is given by (6.22) and is limited to a minimum (6.23) and maximum value (6.24).

$$s_{k,i,l}x_{k,i,l} \leq e_{k,i,l} \quad (k = 1, \ldots, K; \quad i = 1, \ldots, I; \quad l = 1, \ldots, L) \tag{6.21}$$

$$x_{k,i,l}\left(e_{k,i,l} + 1 - s_{k,i,l}\right) = u_{k,i,l} \quad (k = 1, \ldots, K; \quad i = 1, \ldots, I; \quad l = 1, \ldots, L) \tag{6.22}$$

$$x_{k,i,l}\, u^{min} \leq u_{k,i,l} \quad (k = 1, \ldots, K; \quad i = 1, \ldots, I; \quad l = 1, \ldots, L) \tag{6.23}$$

$$x_{k,i,l}\, u^{max} \geq u_{k,i,l} \quad (k = 1, \ldots, K; \quad i = 1, \ldots, I; \quad l = 1, \ldots, L) \tag{6.24}$$

A necessary condition for a contract to be active in a specific period is that a contract is in principle in place (6.25). When a contract is in place it may or may not be able to service demand in a given period, this is given by (6.26) which multiplies the contract size by the binary variable indicating whether it is active in a given period.

$$a_{k,i,l,t} \leq x_{k,i,l} \tag{6.25}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L)$$

$$a_{k,i,l,t}\tilde{q}_{k,i,l} = \hat{q}_{k,i,l,t} \tag{6.26}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

Constraint (6.27) calculates the cost of a contract by considering the number of places it reserves over the planning horizon. As the care costs are period dependant they are multiplied by the quantity of contract places taking place in each specific period.

$$\sum_{t=1}^{T} a_{k,i,l,t}\tilde{q}_{k,i,l}p_{i,l,k,t} = o_{k,i,l} \tag{6.27}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L)$$

Equation (6.28) says that the sum of the binary variables indicating whether a particular contract is active must sum to the duration of the contract itself. To ensure the correct periods are set as having the contract in place (6.29) prevents a contract from being in place in a period when the period itself is later than the end date of the contract.

$$\sum_{t=1}^{T} a_{k,i,l,t} = u_{k,i,l} \tag{6.28}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L)$$

$$e_{k,i,l} + M(1 - a_{k,i,l,t}) > t \tag{6.29}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

Closely related to (6.29) is (6.30) which prevents contracts from being in place in periods prior to the contract start date by forcing the binary variable $a_{k,i,l,t}$ to take the value zero. Any period between the start and end date of a contract inclusive may have this active binary variable either set to zero or one, however the presence of (6.28) forces these variables to assume one.

$$s_{k,i,l} + M\left(a_{k,i,l,t} - 1\right) \leq t \tag{6.30}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L; \quad t = 1, \dots, T)$$

Constraint (6.31) says that if a contract is in place, at most one discount rate can be used.

$$\sum_{b=1}^{B} \hat{r}_{k,i,l,b} = x_{k,i,l} \tag{6.31}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L)$$

Constraints (6.32) and (6.33) set the correct discount rate. Firstly, we inspect the value of the contract and check whether for a given discount rate it is at least as large as the threshold set in the previous price break. If this is not the case then the discount rate cannot be applied. Secondly, we prevent discount rates being used where the amount spent is beyond the threshold for a given discount rate.

$$M(1-\hat{r}_{k,i,l,b})+o_{k,i,l}\geq h_{b-1}$$

$$(k=1,\ldots,K;\quad i=1,\ldots,I;\quad l=1,\ldots,L;\quad b=2,\ldots,B) \tag{6.32}$$

$$M\left(\hat{r}_{k,i,l,b}-1\right)+o_{k,i,l}<h_{b}$$

$$(k=1,\ldots,K;\quad i=1,\ldots,I;\quad l=1,\ldots,L;\quad b=1,\ldots,B) \tag{6.33}$$

The actual discount rate used (6.34) is the sum product of the possible discount rates offered by each provider for a given care group and intensity multiplied by the binary variable indicating which price break has been met. Finally constraints (6.35) through to (6.40) specify the non-negativity conditions.

$$\sum_{b=1}^{B}\hat{r}_{k,i,l,b}r_{k,i,l,b}=\bar{r}_{k,i,l} \tag{6.34}$$

$$(k=1,\ldots,K;\quad i=1,\ldots,I;\quad l=1,\ldots,L)$$

$$q_{i,l,k,t},\ \hat{q}_{k,i,l,t},\ a_{k,i,l,t}\geq 0 \tag{6.35}$$

$$(k=1,\ldots,K;\quad i=1,\ldots,I;\quad l=1,\ldots,L;\quad t=1,\ldots,T)$$

$$o_{k,i,l},\ \ x_{k,i,l},\ \ \tilde{q}_{k,i,l},\ \ s_{k,i,l},\ \ e_{k,i,l},\ \ u_{k,i,l},\ \ \bar{r}_{k,i,l}\ \geq 0$$
$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.36}$$

$$\dddot{q}_k\ \geq 0 \quad (k = 1, \dots, K) \tag{6.37}$$

$$v_{i,l,t},\ \ddot{q}_{i,l,t},\ \hat{v}_{i,l,t} \geq 0 \quad (k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L) \tag{6.38}$$

$$r_{k,i,l,b},\ \hat{r}_{k,i,l,b}, \bar{r}_{k,i,l,b} \geq 0$$
$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L; \quad b = 1, \dots, B) \tag{6.39}$$

$$\dot{q}_{k,i,t} \geq 0$$
$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad t = 1, \dots, T) \tag{6.40}$$

6.6 Application to the London dataset

To test the application of our model to the London dataset we entered the mathematical formulation from §6.4 into LINGO (Lindo Systems Inc. 2015). As in model I, we used LINGO version 15 64-bit edition on an Intel Core i7 system with 8GiB of memory. For the application of model II we used the same demand, quality and capacity information for each provider that we had determined previously for model I.

Discount thresholds

One parameter present in model II that remained to be determined was the provider discount rate under each of the price brakes. To simplify the application and also to better approximate the real world discounting function used by providers, we assumed that providers we each able to specify three different time and volume based commitment discounts. Rather than specify a per unit discount as is commonly used in production problems, this discount would be applied to the total cost of a contract which is dependant not only on its size but the period of time for which it is in place. The three price breaks are shown in Table 6-9 so that for instance price break 1 is applied when the total cost of an individual contract is up to £30,000 whilst price break 2 applies to spends greater than or equal to £30,000 but less than £50,000. Our third price break is applied to a contract spend of greater than or equal to £50,000.

Table 6-9 – Price break thresholds

Break	Threshold
1	30,000
2	50,000
3	50,000+

Discounts offered

Our model allows for different discount rates to apply to different care groups and intensity levels, reflecting the pattern of different cost structures for these different types of care that we had witnessed in the recorded activity data. A key assumption was that providers would be willing to grant higher discounts to those care groups that were subject to higher variability in their weekly cost[30]. The intuition for doing this related to the fact that higher

30 We recognise that the relationship could be in the opposite direction, i.e. lower variability could signal that cost is more standardised and thus more discount is possible, but for the purpose of model II we tend to favour the former assumption in practice.

variability in weekly cost would lead to more volatile revenues for the provider compared with costs that were more standardised – this would potentially be something that the provider would want to avoid. Furthermore, increased volatility could imply that there was more competition for places and hence greater price-based incentives might be needed. Due to these features we estimated the coefficient of variation (6.41) for each care group as shown in Table 6-10.

$$C_{v,i} = \frac{\sigma_i}{\mu_i} \qquad (i = 1, \dots, I) \tag{6.41}$$

Table 6-10 – Coefficient of variation by care group

Care Group	$\boldsymbol{C_{v,i}}$
FMH	0.7442
LD	0.2482
OMH	0.9035
PAL	1.2215
PDA	0.8278
PF	0.8244

For a given care group, each price break would have an increasing discount rate such that greater time-volume based commitments led to higher discounts. As a starting point, we estimated a base discount rate for each price-break by repeated sampling from a uniform distribution between 3% and 6% to determine the discount rate for the initial price break (smallest commitment). Subsequent discount rates (for increasing commitment) were calculated by adding the discount rate from the previous price break to a new discount rate drawn from the same uniform distribution. After repeated sampling we calculated the following base discount rates shown in Table 6-11.

Table 6-11 – Base discount rates

Break	Lower	Upper	Sampled ($base_b$)
1	0.03	0.06	0.05
2	0.06	0.09	0.08
3	0.09	0.12	0.11

The sampled base rates, which were common across all providers, care groups and intensity levels, would then need to be adjusted so as to take into account different provider discounting policies and the assumed tendency of providers to award higher discounts to care groups that experienced greater variability in weekly rate. Equation (6.42) shows how the provider, care group, intensity level specific discount was estimated by multiplication of the base discount multiplied by the care group specific variability measure. This value was then divided by the provider price index. Hence, if a provider was typically more expensive than others it would tend to offer less discount compared with providers which on average were less expensive. Recall that the price indexes were simulated values to reflect of different supplier cost structures and are detailed in Appendix A.7.

$$r_{k,i,l,b} = \frac{base_b C_{v,i}(1+l)}{priceIndex_k} \tag{6.42}$$

$$(k = 1, \dots, K; \quad i = 1, \dots, I; \quad l = 1, \dots, L; \quad b = 1, \dots, B)$$

Setup price

In recognition of the cost of contract formation and to penalise the model for making a large number of contracts we defined a setup price that would be incurred for each contract formed by the model. Under the CLSP this setup cost recognised the cost of ordering, perhaps including the delivery charge or downtime due to the cost of reconfiguring of production equipment to start producing a different product. In our procurement problem the setup cost is more closely related to negotiation costs. As such we set the setup cost to

£1500 which corresponds with approximately 2 weeks' salary of a senior health care planner - an amount we believe to be appropriate in terms of the time required to form a contract with a provider.

Salvage price

In our literature review we could find no existing methodologies taking into account salvaging within procurement problems, a process whereby unused capacity or materials may be leased or re-sold to a third party in the case that in a period capacity outstretches demand. However, we observe that in practice health care planners may be willing to lease excess capacity to local authorities for the purpose of delivering social care under special circumstances, e.g. short periods of respite. To take this feature of the problem into account we include a salvage price of £600, corresponding to 4 weeks of low intensity local authority care at £150 per week, which would help to lower total costs by allowing for health care planners to recoup some of the value of the contract commitment in periods in which capacity exceeds the demand for LTC. Here we have assumed that the salvage price is independent of care group and intensity level, furthermore the health care planner is able to perfectly salvage all excess capacity in a period.

Market share

Within out model the max market share parameter controls the combined contract quantities any given supplier may hold in relation to all contracts taking place. The rationale for this constraint is to prevent the model from assigning contracts to providers and becoming reliant on a single or small number of providers. Originally, the intention was to use a relatively small value for this parameters, for example 0.3 implying that at most a single provider may supply at most 30% of all contracted places, however after testing we observed that this often prevented the solver from finding a feasible solution using contracts. We therefore experimented with this value and observed the relative consequences for choosing values approaching 1.

Contract size and duration

When considering a 12 month period we limited the maximum length of a contract in duration to 12 months with a 1 month minimum term. Similarly, we set the approximate parameters such that no contract could be formed if its size was less than 5 or greater than 20. The first limit prevents contracts for usually low quantities, those unlikely to take place in practice, whilst the second helps to reduce the time to find a feasible solution by forcing the solver to not explore contracts bigger than those for which planners would likely implement.

6.6.1 Case I

12 Periods, 1 Care Group, 2 Intensities and 6 Providers

Our first experiment considers a situation in which there is a 12 month planning horizon, consisting of a single care group (FMH) with two care intensities (LOW and MEDIUM) to plan for. In this situation demand must be satisfied using 6 providers, each offering time and volume based discounts. After entering the formulation into LINGO we solved the model using the branch-and-bound solver under different maximum market share assumptions. The branch-and-bound solver is run until a local optimum solution is found. Once a solution had been found we recorded the output of the model, including the value of the minimum cost commitment plan, in a separate text file for further analysis.

Owing to the fact that LINGO generates a different initial solution for each run of the model based on random sampling we performed several runs of each model to provide a more accurate representation of average solution time and minimum cost plan. This is important since depending on the closeness of the initial solution to a local optimum, a higher quality solution may be found more quickly compared with other starting points. Figure 6.1 shows a graph of the minimum cost commitment plans obtained using maximum market share values between 90% and 40% for runs 1 and 2. From the graph we observe a general increase in the minimum cost commitment plan as the maximum market share allowed by any single provider is reduced. Although we experimented with using market shares of less than 40% this led to infeasibility of model and hence the results have been suppressed. The lowest cost commitment plan is found when the maximum market share was 70%, whilst the highest was found when the maximum market share was 40%, yielding costs of £1,291,977 and £1,469,640 respectively. This compared with a total cost of £1,623,689 under the spot contract only model presented previously.

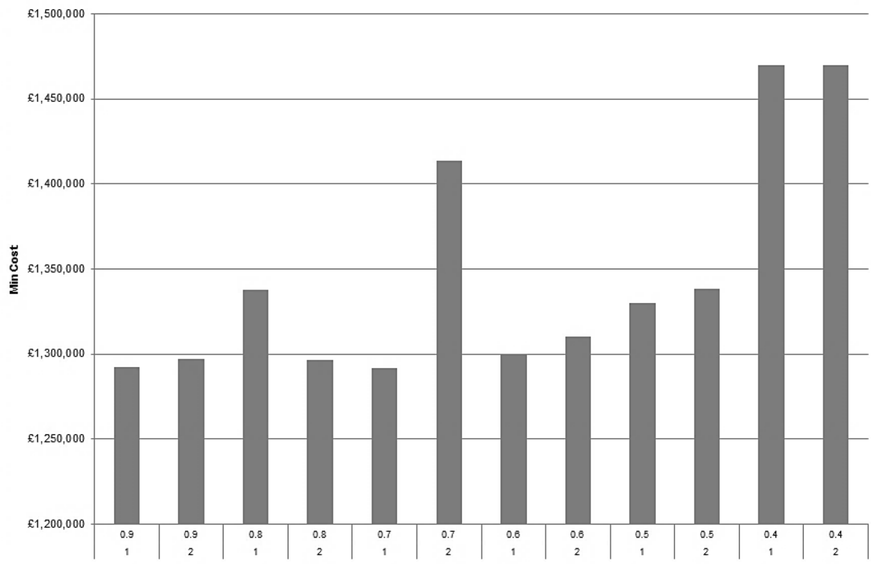

Figure 6.1– Case 1: Minimum cost commitment plan under different market share assumptions

Diving the maximum market share parameters into two groups, those with market share less than or equal to 60% and those 70% or more, and calculating the average total commitment plan across all runs we obtain the average minimum cost commitment plan for each group. We found that limiting the maximum market share to 60% leaded to an increase in cost of approximately £50,000 compared with allowing any one provider to have a market share of 70% or more. In terms of average computation time, we found that lowering the maximum market share parameter tended to increase the computation time from an average of 50.65 seconds, when market share may be greater than or equal to 70%, to 63.48 seconds when it must be less than or equal to 60%.

With regards to the proportion of spot placements as a proportion of all placements used we found that the second group of models, in which the maximum market share was allowed to be greater than or equal to 70%, lead to an overall average decrease in the proportion of spot placements. For the first group of models, in which market share was less than or equal to 60% an average of 15.96% of all care packages would use spot contracts compared with 8.12% for the less restricted second group.

Figure 6.2 shows the solution summary report given for the minimum cost commitment plan obtained in our experiments. Recall that this solution was found when the maximum market share for a provided was permitted to be no greater than 70%.

```
Local optimal solution found.
Objective value:                              1291977.
Objective bound:                              1291977.
Infeasibilities:                              781989.2
Extended solver steps:                            1435
Total solver iterations:                         62624
Elapsed runtime seconds:                         36.61

Model Class:                                      MIQP

Total variables:                  708
Nonlinear variables:              301
Integer variables:                420

Total constraints:               1232
Nonlinear constraints:            601

Total nonzeros:                  3917
Nonlinear nonzeros:               756
```

Figure 6.2– Solution summary report for the minimum cost commitment plan.

Table 6-12 details the contract commitment quantity for the care group FMH within the low intensity group under the minimum cost commitment plan. We observe that the total contract quantity per period has been optimised when it is set to 8. The demand pattern for this care intensity level is between 7 and 9 throughout the period, this leads to excess capacity in contracts during periods 4 through 12 in which case a salvage quantity of 1 is permitted. In terms of spot quantity, a single unit of care is purchased in periods 2 and 3 where demand rises to 9 and hence outstrips the available capacity in contracts.

Table 6-12 – Contract quantity for the minimum cost commitment plan

Period	Intensity	Demand	All Contract Quantity	Salvage Quantity	Spot Quantity
1	Low	8	8	0	0
2	Low	9	8	0	1
3	Low	9	8	0	1
4	Low	7	8	1	0
5	Low	7	8	1	0
6	Low	7	8	1	0
7	Low	7	8	1	0
8	Low	7	8	1	0
9	Low	7	8	1	0
10	Low	7	8	1	0
11	Low	7	8	1	0
12	Low	7	8	1	0

Table 6-13 shows the cost of the contracts and their size under the minimum cost commitment plan. Here we find that cost is minimised when 4 contracts are formed, 3 formed for the intensity medium and a single contract to cover low intensity care packages. Providers 2 and 4 each obtain a single contract whilst provider 5 has a contract for each level of care intensity. From the table we observe that it is the contract with provider 5 that is supplying capacity to our results in

Table 6-12. While the total contract cost of the 4 contracts alone exceed the total minimum commitment plan they represent costs prior to discounting.

Table 6-13 – Contract cost and size for the minimum commitment plan

Provider	**Care Group**	**Intensity**	**Contract Cost**	**Contract Size**
1	FMH	Low	0	0
1	FMH	Medium	0	0
2	FMH	Low	0	0
2	FMH	Medium	881989	10
3	FMH	Low	0	0
3	FMH	Medium	0	0
4	FMH	Low	0	0
4	FMH	Medium	356289	6
5	FMH	Low	130464	8
5	FMH	Medium	152820	5
6	FMH	Low	0	0
6	FMH	Medium	0	0

Recall that our methodology allows for the contracts to be staggered into the time horizon, which is in contrast to previous methodologies that assume a fixed duration or fixed position of the commitments. Table 6-14 shows the effect of this sliding contract principle in that for instance the contract with provider 5 doesn't start until the 9th period and ends on the 12th period whilst the contract with provider starts in period 1 but ends in period 8.

Table 6-14 – Contract start and end periods for the minimum cost commitment plan

Provider	Care Group	Intensity	Contract Start	Contract End
1	FMH	Low	0	0
1	FMH	Medium	0	0
2	FMH	Low	0	0
2	FMH	Medium	1	12
3	FMH	Low	0	0
3	FMH	Medium	0	0
4	FMH	Low	0	0
4	FMH	Medium	1	8
5	FMH	Low	1	12
5	FMH	Medium	9	12
6	FMH	Low	0	0
6	FMH	Medium	0	0

Finally Figure 6.3 depicts a graphical overview of the minimum cost commitment plan by per period quantities. The shaded areas represent either commitment or spot contracts that are in place. The numbers inside the shaded bars represent the size of the commitment quantity.

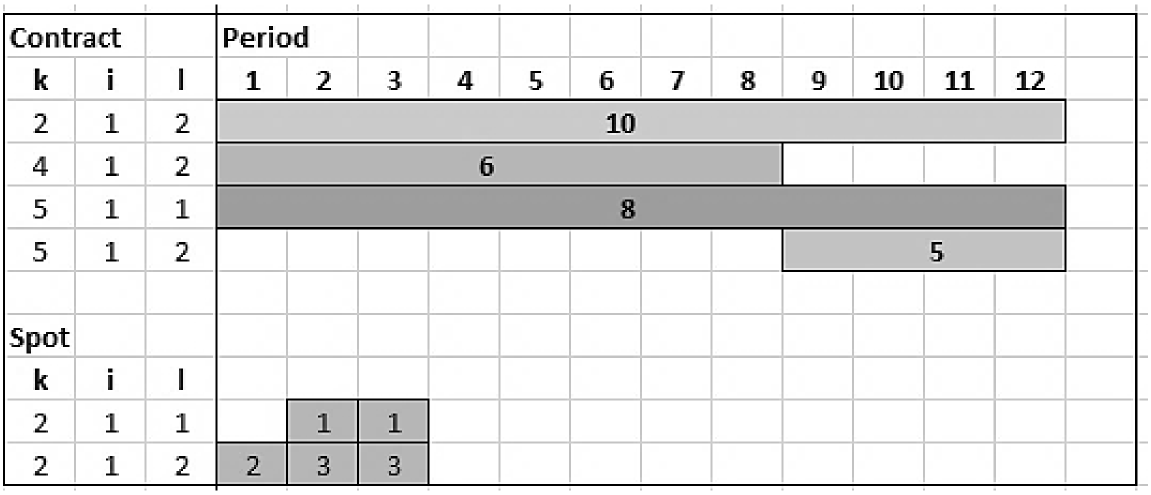

Figure 6.3– Optimal commitment quantities by period and commitment type.

6.6.2 Case II

12 Periods, 2 Care Groups, 2 Intensities and 6 Providers

To experiment with using the model on slightly larger instances we considered a second case in which there were now 2 care groups to plan for. Appendix A.8 and A.9 show that the total number of variables in the model increases from 708 to 1404 and in particular the number of nonlinear variables increases by 300.

The minimum cost plan under model I with spot only placements yielded total cost of £3,361,981. As with case 1 we experiment using different maximum market share assumptions to gauge the impact on the objective function for model II. Figure 6.4 presents the results of our findings for market share assumptions of between 90% and 60% for runs 1 and 2 of our model. As with case 1, we observe a general increase the minimum cost plan when market share is more restricted.

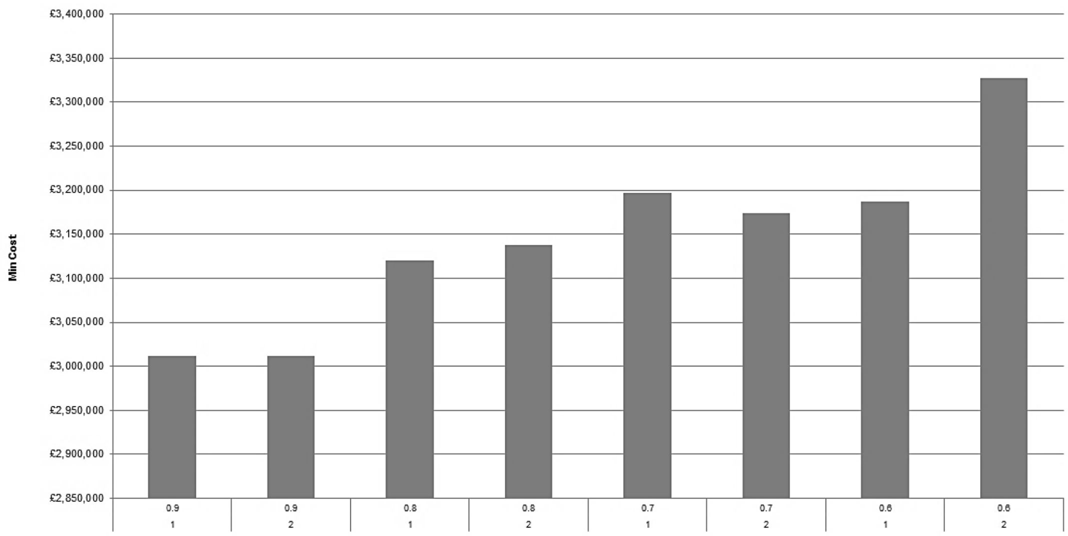

Figure 6.4– Case 2: Minimum cost commitment plan under different market share assumptions.

The lowest total cost was obtained when market shares could be as high as 90%, in which case the minimum total cost was £3,011,678 representing a saving of approximately 10.5% versus the spot placements only plan. The highest total cost plan obtained was for a maximum market share of 60%, in which case the total cost was found to be £3,327,020 – £315,342 more closely than when market share may be has high as 90%.

Figure 6.5 compares the cost of the minimum cost plan for different market share assumptions as a percentage of the cost obtained using model I, where only spot placements are considered. We find there to be fairly good linear relationship between the market share constraint and the cost savings versus the spot only placement plan. Increasingly more restrictive market share assumptions tend to give rise to lower overall cost savings.

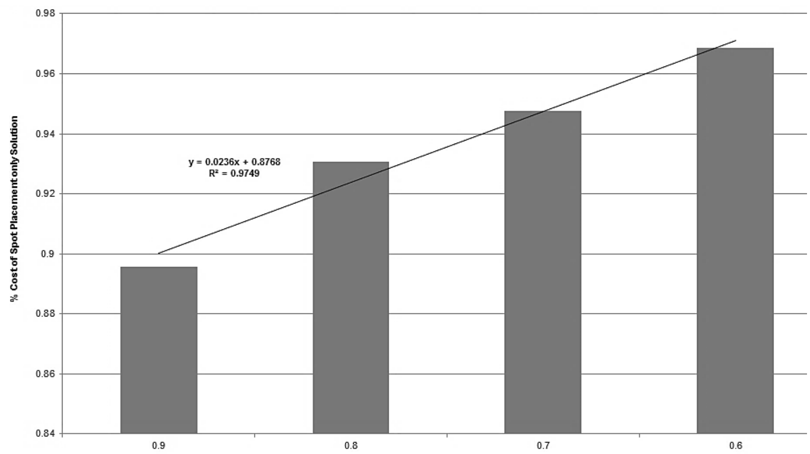

Figure 6.5–Percentage cost of spot contract only plan under different maximum market shares

In terms of run time, the increase in the number of care groups that leads to approximately double the number of variables in the model compared with case 1 has a dramatic effect on run times of the solver. Figure 6.6 shows how on average, run times for case 2 increase by a factor of 7.86 to 449 seconds compared with 57.07 seconds for case 1. Overall however, the run times are still within reasonable limits given that the average time to find a local optimum solution under case 2 is less than 8 minutes.

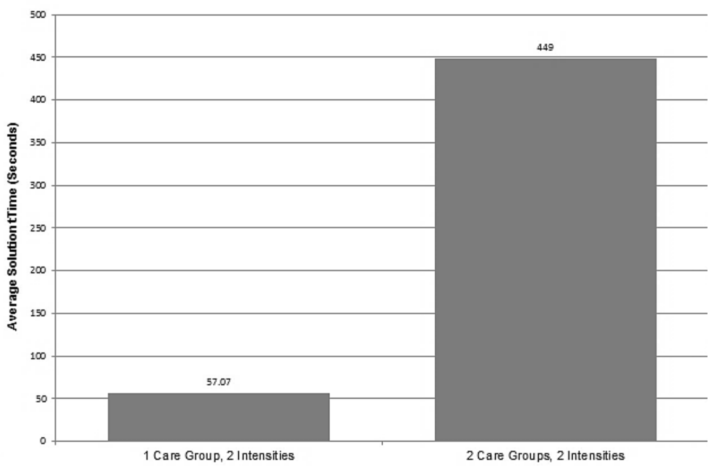

Figure 6.6–Average solution time for case 1 and case 2

6.6.3 Computational results

To ascertain the suitability of our formulation in combination with LINGO15 to solve contracting commitment problems of the type proposed we explore the impact on run times and total cost under different instance sizes. Here an instance size refers to the number of providers, time periods, care groups and intensity levels – thus an instance in which there were 2 providers, 1 period, 3 care groups and 2 intensity levels would be represented by the notation 2 x 1 x 3 x 2. Details of our initial computation results are shown in Table 6-15.

Table 6-15 – Computational results for difference instance sizes

Instance	**Instance Size**	**Total Cost**	**Run Time (hh:mm:ss)**	**% Error**
1	3 x 2 x 1 x 1	27,437	00:00:00	0
2	3 x 4 x 1 x 1	47,361	00:00:00	0
3	3 x 6 x 1 x 1	65,452	00:00:02	0
4	3 x 12 x 1 x 1	116,602	00:00:01	0
5	3 x 24 x 1 x 1	246,543	00:00:17	0
6	6 x 3 x 1 x 1	42,018	00:00:00	0
7	12 x 3 x 1 x 1	33,834	00:00:01	0
8	24 x 3 x 1 x 1	35,507	00:01:02	0
9	3 x 3 x 2 x 1	141,442	00:00:01	0
10	9 x 3 x 4 x 1	679,675	00:00:44	0
11	9 x 6 x 4 x 1	1334210	00:07:02	0
12	12 x 3 x 4 x 1	632,842	00:00:49	0

6.7 Discussion

We have considered a commitment problem in which LTC planners must choose the amount of commitment they want to make with respect to putting in place contracts with different providers; so as to satisfy demand over a planning horizon. Our proposed dynamic commitment model shows how volume and time based commitments can be incorporated into the planning decision and such commitments can be used to generate cost savings.

Compared with previous approaches that have investigated the supplier selection problem with discounting, we have used a price break approach that considers not only the volume or quantity of a commitment but the length of time for which the commitment is in place. Secondly, we have allowed the commitments themselves to be offset or staggered into the planning horizon. This is in contrast with previous studies that have assumed the commitment is in place throughout the planning horizon, if it is in place at all, and more appropriately reflects the real world problem in that contracts need not start or end at the same moment in time – especially when contracts span multiple providers.

A second feature of our model is that it allows for the purchaser to have some control over the market share awarded to different providers, such that the purchaser may set a hard constraint on volume of contracts awarded to any given provider. The way that we have modelled this maximum market share awarded is by considering the quantity-time volume of each all contracts held by a provider. This allows for situations in which a provider may be awarded a relatively large contract, providing it is not in place for long periods and similarly allows for the possibility of smaller contracts that are longer in duration. Compared with previous approaches that model market share based on quantity alone, we have adapted this market constraint to better reflect the service rather than product orientated nature of our problem. We have performed some sensitivity analysis to investigate what the trade of is with respect to the optimum cost plan under different maximum market share assumptions.

A third feature of our model is that it allows for the possibility of salvaging, that is to say that it allows for situations in which a larger commitment may be made than is strictly necessary to satisfy demand, with the remaining excess subleased or subcontracted to a third party. In this respect, the modelling approach more closely reflects how health care planners may share or subcontract provided care places to local authorities. Our model therefore considers plans in which it may be optimal to commit to more places than is necessary for some periods assuming that any excess can be salvaged at the salvage price. As the salvaging process may incur additional time, the time required to liaise with a local authority or third party to use the contracted place, we have add additional constraints on the maximum amount of salvage allowed in any given time horizon and set such values to reasonable limits.

Despite the many advantages of our approach it remains subject to a number of limitations. Firstly, we consider the case in which only 1 contract per provider, care group and intensity level may be formed. Furthermore, the contract size itself is fixed for the duration if it is in place. Thus we do not consider plans whereby it may for instance be optimal to have different sizes of contracts in different subsets of the planning horizon. In practice, this approach more closely reflects the contract formation process for the length of time we have considered. Had we considered a much longer time horizon, where it more realistic to allow for the possibility of having multiple contracts with the same provider, care group and intensity level, then perhaps it would have been necessary to incorporate this feature.

Secondly, we have only considered the impact of care home contracting and thus omitted the possibility of contracting with home care providers. In retrospect, we argue that such a feature could be incorporated by the addition of variables to represent demand for such services, the relative capacities of different home care providers and by modifying the intensity index such that it was extended by the number of possible home care intensities. We have not explicitly modelled home care in this version of the model due to uncertainty regarding the capacities of different home care suppliers.

Thirdly, we have assumed that the capacity of providers is known and despite allowing for changes in capacity to take place, we have not considered other purchasers of care. In practice, care homes may have less than their published capacity available due to the purchasing of care from neighbouring boroughs or indeed self-funding individuals that choose to liaise with the care home directly. We have purposely limited our approach to a known capacity model to simplify the formulation and because of the level of aggregation in our data; in that for instance we are considering the cumulative demand across London health authorities. With that said we recognise that a suitable extension of this model may therefore be to add some notion of uncertainty into the provider capacities. Alternatively, depending on how the model is applied the capacities could be parameterised by considering the procurement offers that are received through the early stages of a tendering process; in which providers specify different quantity discounts under different levels of commitment.

With regards to solution time we found that our formulation in combination with the LINGO15 solver was able to generate local optimum solutions to moderately sized problems within 2 hours. As the planning horizon is extended beyond 12 periods however or as the number of care groups under consideration increases, we observed a significant increase in the run time due to the presence of a non-linear term in the objective function. This non-linear term arises due to multiplication of the discount rate applied to a commitment together with the commitment cost; both of which being simultaneously determined by our model. Intuitively, as the value of a commitment increases (either in time or in quantity) it may be subject to a higher discount rate. When the commitment is such that it is subject to a different discount rate then the slope of the total cost of the commitment will change (flatten) in response to a new discount rate being enforced. As we have three price break thresholds in our application this would correspond with three distinct slopes of the commitment cost function.

An extension of this model may therefore consider how such parts of the formulation may be linearized or separated into terms with constant gradient that could be retrospectively added through a piecewise process. One way in which this could be achieved is through using a linear approximation of the contract discounting function to avoid the discontinuities in the commitment spend. An alternative suggestion would involve diving up the planning period into short periods, for example 6 months, and optimising each sub period problem individually. While this would help reduce the run time of the model it would come at the cost of increased formulation complexity. Secondly, some additional constraints could be added to the commitments so as to tighten the search space and reduce the number of possible arrangements i.e. by limiting the commitment contracts to a set of finite sizes and durations. Such constraints might for example specify that contracts are either 6 or 12 months in duration and that their size must be some multiple of the smallest contract size permitted. Lastly, there is the potential to explore application of one or more of the many heuristics and metaheuristics proposed[31] for larger instances of the CLSP.

6.8 Summary

In this chapter we have developed a methodology for modelling the contracting and commitment process for procurement-type problems. Using an example from LTC we have shown how a MIPP approach can be used to formulate the problem facing local planners and decide upon the number of fixed commitments to put in place over a time horizon; both with respect to time, size and their start and end date. An important aspect of our approach is that it requires information about the future pattern of demand for LTC services, something we have so far assumed to be known based on the calculation of observed demand from our pan-London LTC dataset.

[31] See Appendix A.4

Chapter 7

A hybrid grey-fuzzy model for LTC forecasting

7.1 Introduction

As we have seen in earlier chapters a variety of different approaches have been proposed for the purpose of modelling the demand for LTC, yet at the local level there are very few examples of how demand is modelled and indeed used to inform planning decisions. One explanation, which we have tried to address through our commitment model proposed in chapter 6, relates to the lack of examples as to how the results generated by such methodologies may aid local planning decisions. At the same time, the high data requirements combined with the degree of parameterisation necessary to populate and run existing published models may have also contributed to their lack of update by local level planners - organisations that may not have dedicated teams to carry out such analysis.

In this chapter we wish to explore the potential of a new hybrid approach (grey-fuzzy regression) to local level demand forecasting, whose benefits chiefly relate to its less burdensome data requirements and lack of concrete data assumptions needing to be satisfied. Our intention is thus to create reliable forecasts of local LTC activity that can be used as inputs to our dynamic commitment model.

7.2 Fundamentals of grey systems

Grey theory, as introduced by Deng (1988), is a multidisciplinary and generic theory that can be used to model systems in which there is poor, limited or incomplete information (Hsu and Chen 2003). In this context, any system can be described in terms of a colour. In the case of “black” systems inputs arrive and are transformed through some unknown process into outputs. Systems are “white” if the transformation from inputs to one or more outputs are known. In real world problems, it is argued that most systems can be represented as a mixture of both white and black models, where some input-output transformations are well defined whilst others can only be estimated with some level of uncertainty. It is in this case that we refer to the system as “grey” (Lin and Llu 2004).

To date, grey systems have successfully been applied in a number of problem domains (Kayacan, Ulutas and Kaynak 2010), including: social; economic; scientific and technological; military; agricultural and medical. Within the class of time series forecasting problems, grey inspired sequence prediction models have also been shown to deliver better model fitting and increased accuracy (Askari and Fetanat 2011). In the context of LTC grey systems appear particularly suitable to the problem of forecasting demand at the local level for four main reasons;

1. Low data requirements

Unlike alternative approaches[32] commonly used by local level LTC planners, grey systems theory requires very few data points in order to make a projection. In practice a grey model must have at least four observations and all observations must be in consecutive order (either backward or forward) with no gaps. This aspect of grey theory is particularly useful for local planners since often data on LTC is difficult to obtain due to changes in recording practices making older data obsolete or changes in policy which lead to sections of LTC activity being based on fundamentally different systems of care. In this case we can expect

[32] For example ARIMA, moving average models and exponential smoothing

at most 3-4 years of historic activity data being available, typically recorded at monthly intervals, leading to between 36 and 48 data points.

2. Few statistical assumptions

In contrast to other forecasting approaches, including ordinary least squares regression and autoregressive integrated moving average (ARIMA) which are often used for local level planning[33], grey theory makes very few assumptions surrounding the underlying data – including what distributional form they should take or the permitted relationships between sequential values.

3. Incomplete or vague information

Under grey theory the data to be analysed is assumed only to be reflective of the system under investigation rather than a true and highly accurate representation of it. As such it is suitable to LTC datasets where information about activity may contain missing data, recorded in a slightly different way or where there is very little information about an individual except that some form of care took place in a period.

4. Relative ease of calculation

Despite not being a unique feature of grey theory, the solution approaches proposed are however relatively easy to compute using standard office suites[34] due to their being only a small number of computational steps. The first step involves some pre-processing of the original data in order to create the grey variable. In the second step the grey model is defined in terms of the original data series and the grey variable. Thirdly, the parameters of the grey model are computed so as to provide the best fit to the underlying data using ordinary least squares. Finally, the predictions obtained using the grey model are then transformed so as to restore them to the original unprocessed form.

[33] This is based on our own experience of working with a pan-London LTC commissioning unit

[34] The version used in this thesis is Microsoft Office 2007 but it is also possible to use LibreOffice 5 (www.libreoffice.org) with the built-in non-linear solver.

7.3 The GM (1,1) model

While grey models can take a variety of forms the generic grey model is defined by the term GM(k,N) where k represents the number of differential terms and N the number of variables used to predict subsequent values in the sequence. In the case of LTC we are interested in the GM(1,1) model as, given the short planning period under investigation, we do not explicitly model the impact of factors other than time and at the same time we want to explore the grey model's ability using routinely collected activity data . Furthermore, choosing a value of k=1 implies that we are interested in mapping the behaviour of the demand process from one period to the next using only the information gathered in the previous period.

The formulation of the GM(1,1) begins by firstly creating a vector representing the grey variable X^1 from the original sequence of data which is contained within the vector X^0. Formally, the initial sequence of observations, in this case our LTC activity data per monthly period is represented by the vector X^0 and is constructed as shown in (7.1) (Kayacan, Ulutas and Kaynak 2010). Here n denotes the number of observations available and in our LTC corresponds with the value 48 as we use the recorded number of LTC packages taking place between the 1st of April 2005 and the 31st of March 2009.

$$X^0 = \{x^0(1), x^0(2), \dots, x^0(n)\} \tag{7.1}$$

The initial observations are then transformed by means of an Accumulated Generating Operation (AGO) to generate our grey variable X^1, a monotonically increasing sequence, subject to no non-negative observations, where X^1 is defined as the 1st AGO of X^0 as shown in (7.2). The requirements of the AGO are such that no observations can be negative but since LTC activity within a care group and intensity level must be greater than or equal to zero this assumption is satisfied. The intuition for the AGO is to provide sufficient pre-processing so as to be able to add additional regularity to the underlying data sequence and amplify hidden data patterns.

$$X^1 = \left\{\sum_{k=1}^{1} x^0(k), \sum_{k=1}^{2} x^0(k), \dots, \sum_{k=1}^{n} x^0(k)\right\} \quad (k = 1, \dots, N) \tag{7.2}$$

We define a new vector Z^1 which represents the average value of two adjacent neighbours in the AGO vector X^1 created previously (7.3). In the context of grey theory it is often referred to as the background vector. Intuitively, the vector Z^1 is created to transform our discrete AGO sequence into a smooth continuous one since at any incremental interval [t ,t+h] where 0 < h < 1 the value for $x^1(t+h)$ will lie somewhere between $x^1(t)$ and $x^1(t+1)$.

$$Z^1(k) = 0.5 \times [x^1(k) + x^1(k-1)] \quad (k = 2, \dots, N) \tag{7.3}$$

Formally, the derivative of the AGO with respect to time can be approximated as shown in (7.4) under the assumption that the interval between consecutive periods is 1 period (Bingyun and Malin 2009**.** The general convention is however to take the average of two successive periods in the AGO, as shown by the creation of Z^1 in (7.3), so as to have a more steady state value.

$$\frac{dX_t^1}{dt} \approx \frac{X_{t+1}^1 - X_t^1}{1} = X_{t+1}^1 - X_t^1 = X_t^0 \quad \forall\ (t \geq 1) \tag{7.4}$$

The GM(1,1) is defined as a difference equation (7.6) of the vectors Z^1, the steady state values of the AGO, and X^0, the original series (Deng 1988). The variables a and b are known as the development coefficient and the driving coefficient respectively. Their role is to control the mapping of the AGO sequence to observed data points. As a result, in order to use the grey model to make predictions both such variables need to be determined.

$$x^0(k) + az^1(k) = b \qquad (k = 1, \dots, N) \tag{7.5}$$

Equation (7.6) is the first-order differential equation based on the grey model in (7.5). In the context of grey theory it is known as the shadow equation for the GM(1,1) model.

$$\frac{\Delta X^1(t)}{\Delta t} + aX^1(t) = b \tag{7.6}$$

For values of k >= 2 we can rearrange and rewrite (7.5) in matrix form using the input data set X^0 and values from the background vector Z^1 to obtain (7.7):

$$\begin{bmatrix} x^0(2) \\ x^0(3) \\ \dots \\ x^0(n) \end{bmatrix} = \begin{bmatrix} -z^1(2), & 1 \\ -z^1(3), & 1 \\ \dots & \dots \\ -z^1(n), & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix} \tag{7.7}$$

By the least squares method the grey coefficients a and b can be estimated (7.8) :

$$\begin{bmatrix} a \\ b \end{bmatrix} = (B^T B)^{-1} B^T Y_n \tag{7.8}$$

$$where\ Y = \begin{bmatrix} x^0(2) \\ x^0(3) \\ \dots \\ x^0(n) \end{bmatrix}, B = \begin{bmatrix} -z^1(2), & 1 \\ -z^1(3), & 1 \\ \dots & \\ -z^1(n), & 1 \end{bmatrix}$$

In substituting coefficients a and b identified using least squares into (7.6), the approximate relationship between the next value in the AGO and the initial value in the original dataset can be found (7.9)

$$\hat{x}^1(t+1) = \left[x^0(1) - \frac{b}{a}\right] e^{-at} + \frac{b}{a} \tag{7.9}$$

While equation (7.9) represents the predicted value of the AGO sequence at time $(t+1)$, $\hat{x}^1(t+1)$, an inversed accumulated generating operation (IAGO) is required to remap the predicted AGO value back to the original input data. This can be achieved using equation (7.10) where $\hat{x}^0(t)$ is the predicted value in the original series at time t and $\hat{x}^1(t)$ is the predicted value in the AGO at time t.

$$\hat{x}^0(t+1) = \hat{x}^1(t+1) - \hat{x}^1(t) \tag{7.10}$$

Furthermore, the complete set of predicted values of the original sequence can be represented by the vector $\hat{X}^0$, namely:

$$\hat{X}^0 = \{\hat{x}^0(1), \hat{x}^0(2), \dots, \hat{x}^0(n)\} \tag{7.11}$$

7.3.1 Application to the London LTC dataset

Our objective is to investigate the suitability and accuracy of a grey inspired methodology to project LTC demand and cost at the local level. Specifically we want to evaluate the ability of a GM(1,1) model built solely on routinely available activity data to deliver reliable projections for the purposes of short to medium planning

Data

In order to develop a grey model for the London LTC data set we used data on recorded activity in London between the 1st of April 2005 and the 31st of March 2009[35]. Rather than model the number of individuals in LTC, we focus on the number of packages taking place by considering the number of days in care during each monthly period. For each monthly period, for which there are 48 in our dataset, we identify all care packages taking place by considering the start date and end dates of care for each individual. Once we have identified the individuals concerned we estimate the length of time in care during each period to calculate the number of care days. The number of care days is then summed over all individuals and divided by the number of days in a period to estimate the number of packages taking place.

The benefit of using the care days approach relates to its ease of calculation and how it can be more closely mapped to the total cost of care during a period. The weaknesses however it that it has a general tendency to understate the number of people in care since, for example, 10 individuals each receiving 3 days in care during a period would be reported as one care package taking place. Based on this metric, the number of care packages taking place in London during the data period across each of the six care groups and for both provision types (HC = home care, PL= institutional placement) is shown in Figure 7.1. In particular, we observe that while the total number of packages taking place has increased this is largely explained by the growth in the number of physically frail care packages and the number of organic mental health care packages taking place in institutions.

[35] Details of the data collection process can be found in §4.4

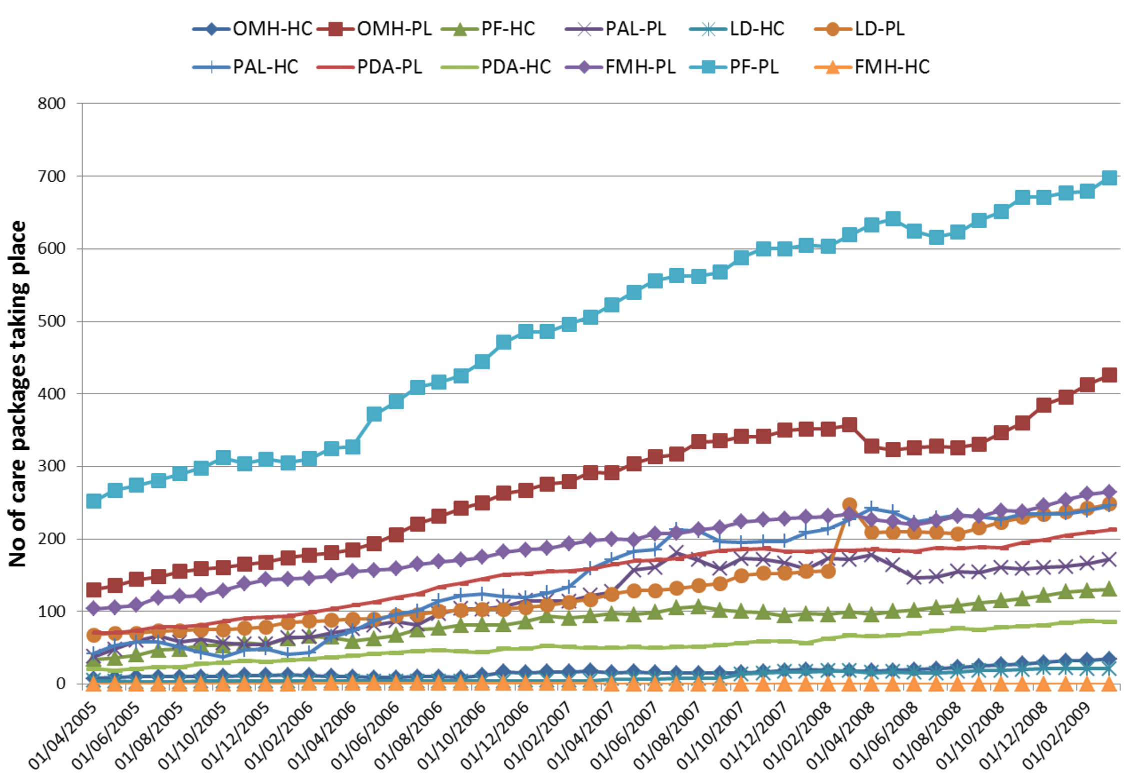

Figure 7.1– No. of care packages taking place between April 2005 and March 2009 in London.

Figure 7.2 presents the number of care packages taking place over time when the activity is group by those taking place in care homes and those taking place in institutions (placements). We observe that over time both activity types are increasing although over the period the proportion of activity that takes place in institutions has fallen by 6% from 86% in April 2005 to 80% in March 2009. In terms of the absolute numbers, the number of care packages taking place in the home has risen from just over 110 cases in the starting period to 496 by March 2009: an increase of approximately 450%.

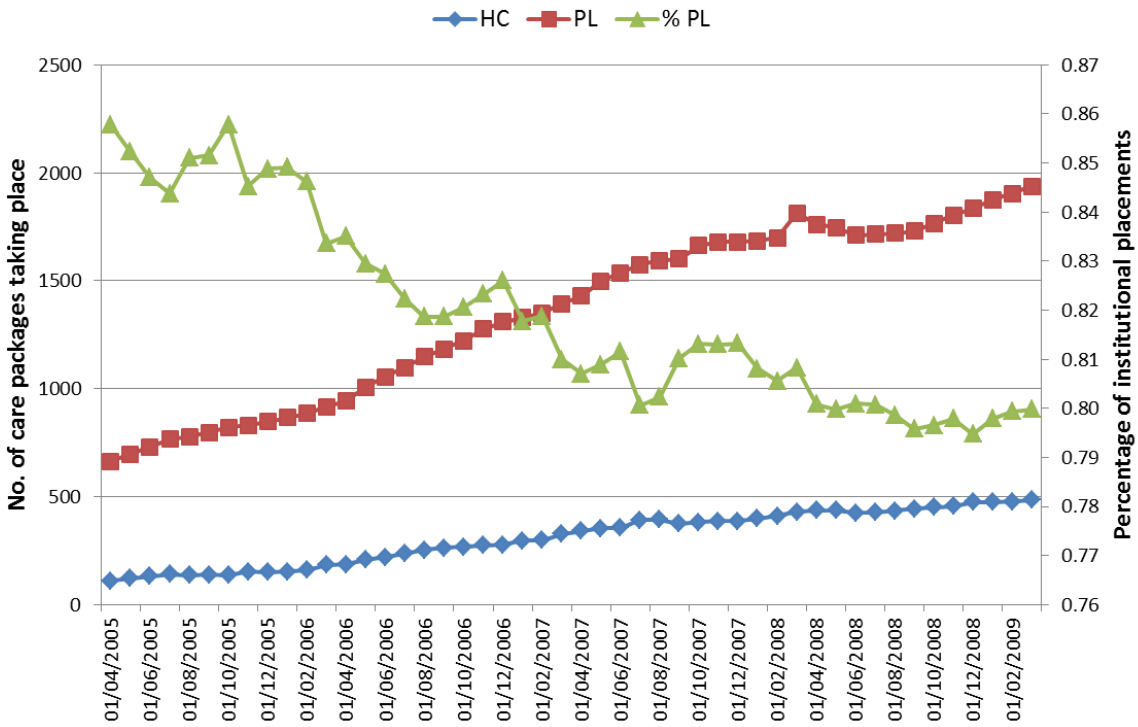

Figure 7.2– Proportion of home care and institutional placements taking place during April 2005 and March 2009 in London.

To test the time series for stationarity we performed[36] both the augmented dickey-fuller (ADF)[37] and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS)[38] tests. Under the ADF test that null hypothesis is there exists a unit root such that shocks to the time series have permanent effects, whilst the KPSS tests the null hypothesis is that an observable time series is stationary, in the sense that the joint probability distribution does not change when shifted in time, around a deterministic trend.

From Figure 7.3 do not find enough evidence to reject the null hypothesis of the ADF at the 5% level of significance and from Figure 7.4 we find sufficient evidence to reject the null hypothesis of trend stationarity. In this case, both tests would tend to support that the number of care packages taking place per time period is not stationary and hence any

[36] Both statistical tests are performed within the R statistical environment (www.r-project.org)
[37] See (Hatanaka 1996) for details
[38] See (Kwiatkowski, et al. 1992) for details

ordinary least squares (OLS) autoregressive model developed to make predictions of LTC demand could lead to unreliable parameter estimates.

```
> adf.test(x)

        Augmented Dickey-Fuller Test

data:  x
Dickey-Fuller = -1.4844, Lag order = 3, p-value = 0.78
alternative hypothesis: stationary

>
```

Figure 7.3– ADF test for total number of packages

taking place during April 2005 and March 2009 in London.

```
> kpss.test(x)

        KPSS Test for Level Stationarity

data:  x
KPSS Level = 2.4584, Truncation lag parameter = 1, p-value = 0.01

Warning message:
In kpss.test(x) : p-value smaller than printed p-value
>
```

Figure 7.4– KPSS test for total number of packages

taking place during April 2005 and March 2009 in London.

To test the amount of differencing required to induce stationarity in the number of packages taking place over time we performed one level of differencing using the R statistical environment and repeated the ADF and KPSS tests of stationarity. Figure 7.5 shows how with one level of differencing the upward trend in activity seen previously is almost completely removed and the mean and variance of the series appear more stable.

From Figure 7.6 the revised KPSS test shows that there is now significant evidence to support that the time series is now stationary at the 5% level, whilst the ADF test result has become less significant there remains sufficient evidence to support that the series continues to be non-stationary at the 5% level of significance. In particular, the ADF appears sensitive to the large fluctuations in activity during late 2007.

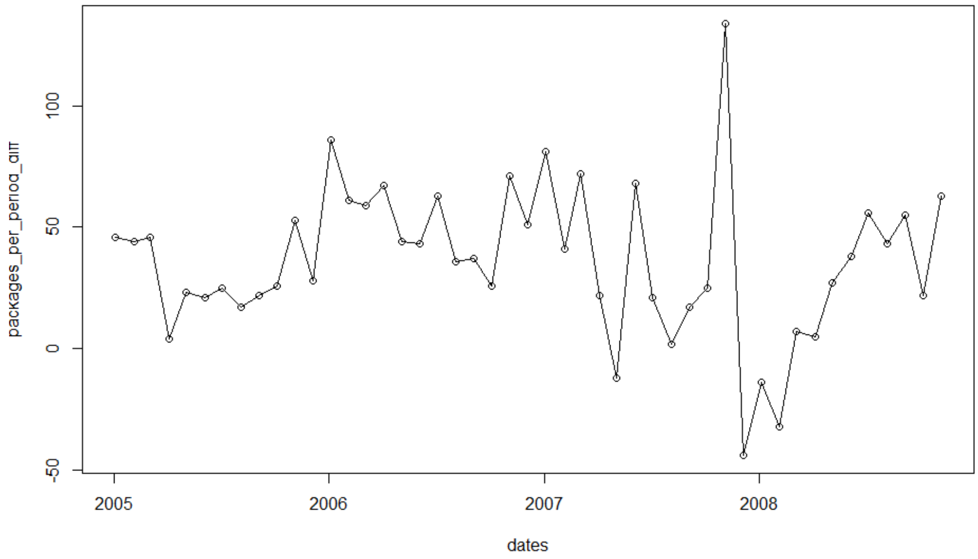

Figure 7.5– Plot of 1st difference in packages
taking place during April 2005 and March 2009 in London.

```
> adf.test(packages_per_period_diff)

        Augmented Dickey-Fuller Test

data:  packages_per_period_diff
Dickey-Fuller = -2.6129, Lag order = 3, p-value = 0.3298
alternative hypothesis: stationary

> kpss.test(packages_per_period_diff)

        KPSS Test for Level Stationarity

data:  packages_per_period_diff
KPSS Level = 0.1996, Truncation lag parameter = 1, p-value = 0.1

Warning message:
In kpss.test(packages_per_period_diff) :
  p-value greater than printed p-value
> |
```

Figure 7.6– ADF and KPSS test for 1st difference of packages
taking place during April 2005 and March 2009 in London.

To check for the presence of seasonality in the dataset we examined the autocorrelation and partial autocorrelation functions for the time series with one level of differencing for the total activity, home care activity and placement activity. The ACF function represents the tendency of lagged values of a series to be correlated with its current value, whilst the PACT function works in the same way it controls for the effect of any intervening lags. The resulting ACF and PACF plots for total activity are shown in Figure 7.7 and Figure 7.8. From the ACF and PACF we find that there are no significant lags in terms of moving average or autocorrelation terms at the 95% level of significance (represented by the dotted lines). Furthermore, the seasonal lags, at periods 12 are not significant and hence there is little evidence of monthly seasonality[39].

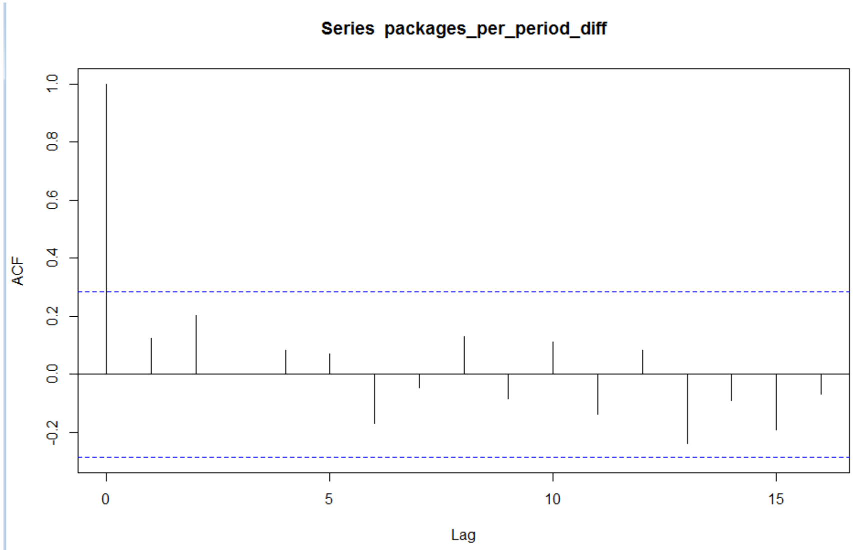

Figure 7.7– ACF for 1st difference of packages
taking place during April 2005 and March 2009 in London.

In addition to the total number of packages taking place we also considered the PACF and ACF for the number of packages taking place at home and in institutional settings. As was the case for the total number of packages we did not find any evidence of seasonality.

[39] Although not presented here the same results were found for the ACF and PACF for home care activity and placement activity when plotted individually.

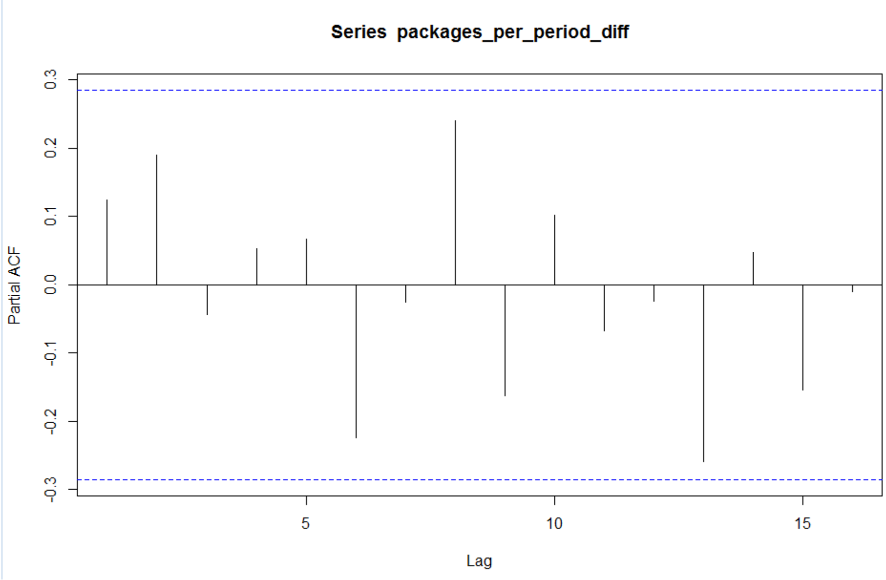

Figure 7.8– PACF for 1st difference of packages
taking place during April 2005 and March 2009 in London.

Formulation of the grey model in Microsoft Excel

To develop the grey model for LTC activity we used Microsoft Excel 2007 and adapted the equations (7.1) to (7.10) into the relevant Microsoft Excel formula. Table 7-1 provides an example of the resulting AGO and Z vector for the first 10 periods in our dataset. Recall that the AGO function can be calculated by summing the activity in a period k with the total sum of activity from 1 to k-1. Our background vector Z for a period k is the average of two advanced AGO values for k and k-1. The vector Z starts at k=2 since it is only defined for periods k>=2. Figure 7.9 provides a graphical overview of the three input vectors in our 10 period example.

Table 7-1 – Example of the initial grey data mapping functions

k	Period	Total Activity (X^0)	AGO (X^1)	Z^1
1	01/04/2005	773	773	
2	01/05/2005	819	1592	1182.5
3	01/06/2005	863	2455	2023.5
4	01/07/2005	909	3364	2909.5
5	01/08/2005	913	4277	3820.5
6	01/09/2005	936	5213	4745
7	01/10/2005	957	6170	5691.5
8	01/11/2005	982	7152	6661
9	01/12/2005	999	8151	7651.5
10	01/01/2006	1021	9172	8661.5

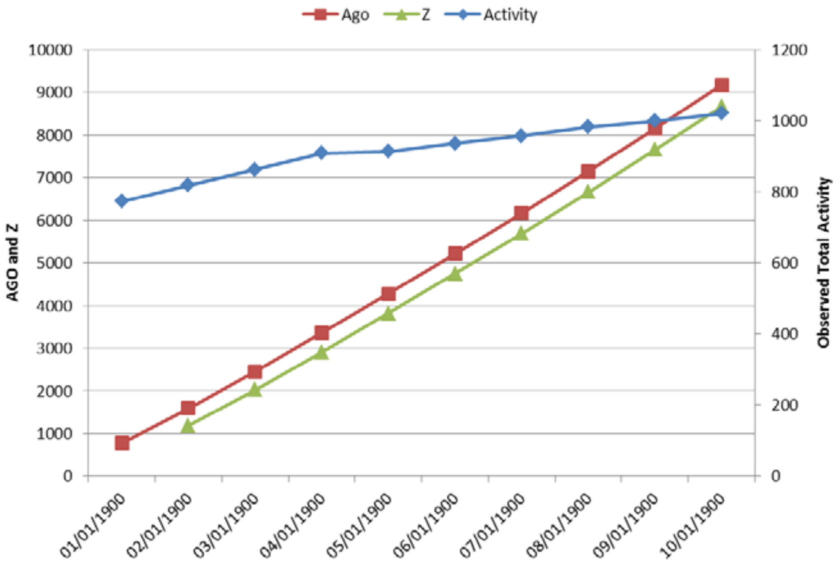

Figure 7.9– Graphical plot of the activity, AGO and Z values.

Minimising sum of squares

The grey model is solved for parameters a and b by the least squares method minimising the total squared errors in the AGO sequence as shown in (7.12). Depending on the assumption surrounding the relationship between the parameters in the grey model with the dependant variable the values a and b can be found using ordinary least squares (OLS), in the case that the model is assumed to be linear in the parameters, or a more general non-linear least squares approach in which the linear assumption is not necessary.

$$SSE = \sum_{t=2}^{N} \left(\hat{x}^1(t) - x^1(t)\right)^2 \tag{7.12}$$

7.3.2 Results

Table 7-2 provides an overview of our results after fitting the grey models to the different types of LTC activity and under two different solver approaches. The columns a and b represent the grey parameters estimates by the solver whereas the columns RMSE and MAPE represent the root mean squared error (RMSE) and mean absolute percentage error (MAPE) recorded for each resulting model.

Both the RMSE and MAPE are standard ways to record the diagnostic performance of forecasting models, with the RMSE being based on the square root of the total sum of squared errors and the MAPE being based on the average absolute percentage difference between each observed value and its corresponding predicted value. In both cases lower values of RMSE and MAPE indicate more favourable performance, with the RMSE penalising models that make even a small number of very large forecast errors.

Table 7-2 – Grey model results for different activity types and least square solvers

Model	Data	Solver	a	b	RMSE	MAPE
1	All activity	GRG Non-linear	-0.019	806.4601	28.02507	0.0839
2	All activity	OLS Linear	0.0195	826.7695	28.16823	0.0847
3	Home Care	GRG Non-linear	0.0266	152.5545	14.98825	0.1166
4	Home Care	OLS Linear	0.0266	159.0407	15.13362	0.1171
5	Placements	GRG Non-linear	0.0199	800.1466	27.27558	0.0806
6	Placements	OLS Linear	0.0195	826.7695	27.42248	0.0804
7	% Placement Activity	GRG Non-linear	0.0015	0.8514	0.217256	0.0068
8	% Placement Activity	OLS Linear	0.0015	0.8501	0.217256	0.0058

The two solver approaches used included the standard linear OLS regression solver and the GRG Non-linear solver available in Microsoft Excel, which is based on the generalised reduced gradient algorithm[40]. We set a maximum time limit of 30 seconds for each model solution attempt, using the solvers multi start feature and ran the algorithm on an Intel Core i7 system with 8GiB of RAM.

Figure 7.10 graphs the MAPE for models 1 through 8 based on the results obtained in Table 7-2. We find that of the models considered, all models except models 1 and 2 provide tolerable margins of MAPE (less than 10%). In general the differences in MAPE between the two solver types is very small (within 0.05%). In the case of models 7 and 8, both grey models provide a highly accurate fitted result by the MAPE of less than 1% forecast error per month. Recall that this was for a series that appeared to exhibit greater variability per period compared with the total number of packages taking place. The solution report for model 8 is shown in Figure 7.11– Solver solution output for grey model 8.Figure 7.11 and highlights the high significance of the parameter estimates for a and b in the grey model at the 1% level.

[40] https://support.microsoft.com/en-us/kb/82890

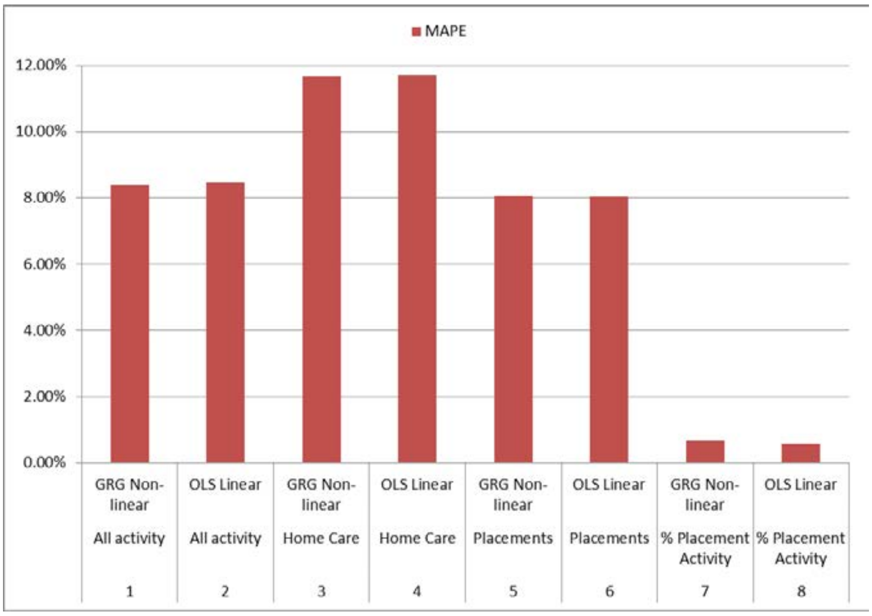

Figure 7.10– Graph of MAPE for different activity types and solver methods.

SUMMARY OUTPUT Model 8

Regression Statistics	
Multiple R	0.925205171
R Square	0.856004608
Adjusted R Square	0.849459363
Standard Error	0.007117688
Observations	47

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	0.013251288	0.006625644	130.7826671	3.04557E-19
Residual	44	0.002229105	5.06615E-05		
Total	46	0.015480393			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
b	0.850192631	0.002169488	391.8862504	1.35515E-79
a	0.001514324	9.36523E-05	16.16964487	3.95358E-20

Figure 7.11– Solver solution output for grey model 8.

For our commitment model proposed in the previous chapter we require the number of care packages per period per care group and intensity level. To apply the grey model by care group and provision type filtered our combined total activity dataset according to valid combinations of the care setting (e.g. home care and institutional placements) and the recorded care group (e.g. organic mental health, palliative etc.).

Table 7-3 shows the results of the fitting of the grey model to care group and provision type specific activity for 3 out of the 6 possible groups[41]. Compared with the grey models constructed for total activity we observed a general fall in MAPE although many of our sub-models were with a 10% range of tolerance. However, the palliative care group was an example of a care group where the grey model found not to accurately capture.

Table 7-3 – Grey model results for different activity types and least square solvers

Model	Care Type	Solver	a	b	RMSE	MAPE
9	OMH-HC	GRG Non-linear	-0.0272	7.5834	3.774109	0.1259
10	OMH-HC	OLS Linear	-0.0195	826.7690	4.081532	0.1479
11	OMH-PL	GRG Non-linear	-0.0202	164.1394	13.67691	0.1030
12	OMH-PL	OLS Linear	-0.0195	826.7690	13.64987	0.0999
13	PF-HC	GRG Non-linear	-0.0201	51.6369	7.944124	0.1043
14	PF-HC	OLS Linear	-0.0206	52.0642	7.487817	0.0912
15	PF-PL	GRG Non-linear	-0.0215	272.5379	16.96429	0.0745
16	PF- PL	OLS Linear	-0.0195	299.6341	16.15461	0.0778
17	PAL-HC	GRG Non-linear	-0.0307	66.3614	13.63452	0.2504
18	PAL-HC	OLS Linear	-0.0307	69.7438	14.30799	0.2513
19	PAL-PL	GRG Non-linear	-0.0220	68.3819	13.2204	0.1942
20	PAL-PL	OLS Linear	-0.0198	73.2856	12.63798	0.1826

[41] The same procedure was carried out for the remaining 3 care groups except we have selected a representative sample of results for conciseness.

Figure 7.12 plots the fitted grey models 17 and 18 where the MAPE was found to be 25% in the case of the GRG solver and 25.1% for the OLS linear solver. We observe that both the grey model tend to overstate the actual amount of PAL-HC activity in the first quarter of the period whilst understating it throughout the remainder – hence it appears not particularly well suited model to this type of series, where for instance we observe a high growth rate in activity throughout the period, rising from about 50 care packages taking place in April 2005 to more than 250 care packages by March 2009. We observe the non-linear nature of the fitted values for both grey models with the model estimated by OLS displaying a shallower gradient compared with the model estimated by GRG.

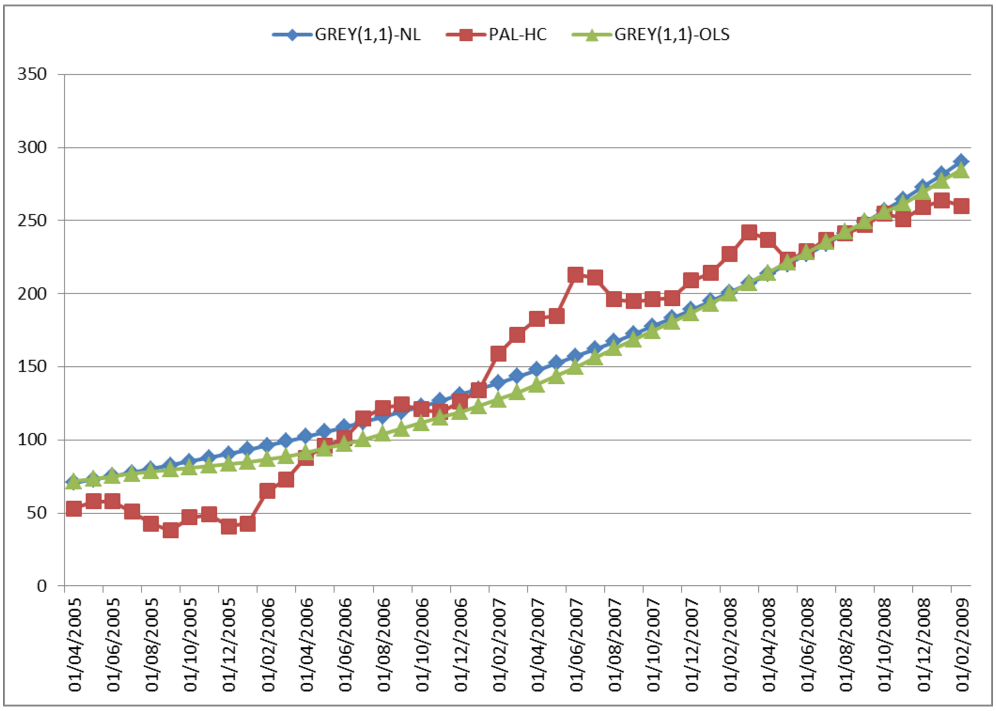

Figure 7.12– Models 17-18: Actual no. of PAL-HC packages vs. fitted values

Under close inspection of the PAL-HC series we find that as with the other series under investigation there is a clear upward trend taking place, as such 1 level of differencing is required to induce stationarity. However, from the resulting ACF plot in Figure 7.13, observe that unlike many of the other series investigated it future values of the series are found to be related to previous shocks to the system – hence in the context of ARIMA this could be modelled with the addition of moving average (MA) terms at lag 1.

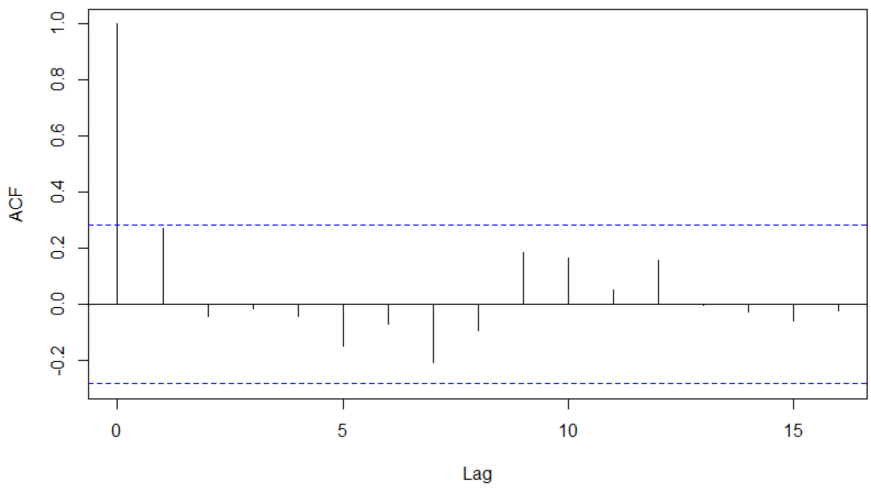

Figure 7.13– ACF plot for PAL-HC with 1 level of differencing

Following a study into the effectiveness of grey models for time series prediction it has been found that series exhibiting high growth rates can result in poorly fitting GM(1,1) grey models (Mao and Chirwa 2006). One suggestion has been to modify the underlying background generating function, as shown in equation (7.17), to modify the weights given to adjacent values in the AGO sequence prior to model fitting so as to increase the responsiveness of the GM(1,1) model under such situations.

$$Z^{1,m}(k) = \frac{1}{2m}\left[(m+1)x^1(k-1) + (m-1)x^1(k)\right]$$
$$where\ m = \left(\sum_{i=2}^{N} \frac{x^1(i)}{x^1(i-1)}\right)^{\frac{1}{n-1}} \tag{7.13}$$

Though experimentation with equation (7.13) we can see that when the number of values in our original time series is 2 (n=2) then the weights in the background value function are approximate the original background function with weights 0.509 and 0.490 given to the previous AGO value and the current AGO value respectively. However, as n $\rightarrow +\infty$ then m $\rightarrow$ 1, giving rise to the respective weights shown in (7.14) for the previous value in the AGO and the current value in the AGO.

$$\frac{m+1}{2m} \rightarrow 1 \text{ as } m \rightarrow 1\infty \qquad \frac{m-1}{2m} \rightarrow 0 \text{ as } m \rightarrow 1 \tag{7.14}$$

The inner summation of equation (7.13) measures the average growth rate of AGO as a whole, where individual periods in which growth is high can be cancelled out by other periods in which growth is negative. Positive rates of growth on average therefore tend to lead to increased weighting to the current period when estimating the background values whilst negative rates of growth on average have the effect of giving more weight to previous observations. To assess the impact of using the revised background function we plotted the background function Z(1,M) values as a percentage of those obtained using our original background function Z(1) for the PAL-HC series (Figure 7.14)

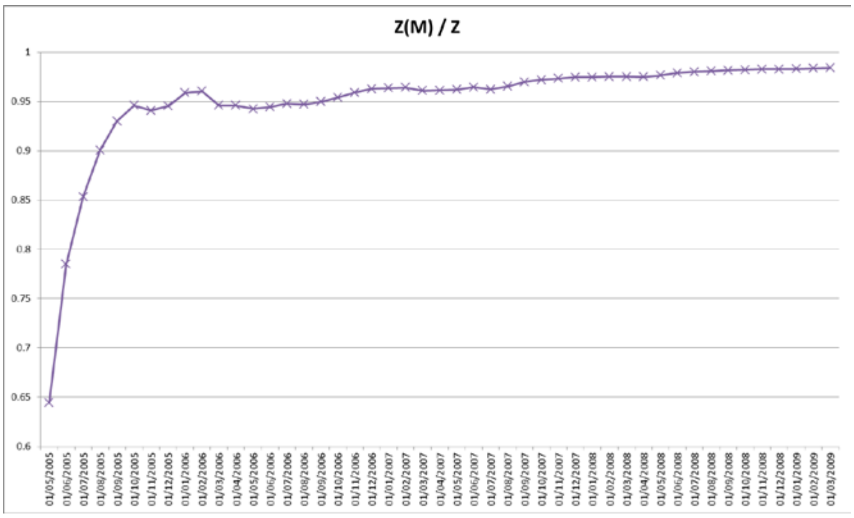

Figure 7.14– Plot of modified background function as proportion of original background function

We applied the revised GM(1,1) with the updated background function to models 9 through 20, the results of which are shown in Table 7-4 for the OLS Linear solver[42]. In general we observed a slight increase in the forecast accuracy across the various time series tested except for the OMH-HC series where MAPE rose slightly from 12.59% to 13.81%. Despite the use of the revised background function the results of the grey model for PAL-HC and PAL-PL while slightly improved did not fall within our tolerable forecast accuracy of +/- 10% MAPE.

Table 7-4 – Results from grey models with revised background function

Model	Care Type	Solver	a	b	RMSE	MAPE
21	OMH-HC	OLS Linear	-0.0292	7.6236	3.991529	0.1381
22	OMH-PL	OLS Linear	-0.0203	165.8413	13.65663	0.0981
23	PF-HC	OLS Linear	-0.0194	53.9402	7.457493	0.0919
24	PF-PL	OLS Linear	-0.0200	300.2185	15.74364	0.0448
25	PAL-HC	OLS Linear	-0.0195	826.7690	13.90086	0.2450
26	PAL-PL	OLS Linear	-0.0226	70.7932	12.30495	0.1703

For comparison with the GM(1,1) approach we also fitted a series of linear regression models and ARIMA models on the basis that such techniques are commonly used for short to medium term forecasting at commissioning organisations. To estimate the linear models we used ordinary least squares regression with time as our independent variable and the amount of care packages taking place as our dependant variable. For the ARIMA models we used the R package tseries to test a variety of functional forms, including ARIMA(0,1,0), ARIMA(1,1,0 and ARIMA(0,1,1) – the best fitting model was selected according to RMSE and MAPE performance. The resulting models and their respective performance is presented in Table 7-5.

[42] The results of the GRG solver have been suppressed so as to shown the most significant results. In general we found that the results of the GRG solver closely mimicked those of the OLS solver.

Table 7-5 – RMSE and MAPE for alternative model specifications

Model	Care Type	Model	RMSE	MAPE
27	OMH-HC	OLS Regression	3.9809	0.1464
28	OMH-PL	OLS Regression	11.7067	0.0592
29	PF-HC	OLS Regression	6.8298	0.0752
30	PF-PL	OLS Regression	13.3035	0.0463
31	PAL-HC	OLS Regression	11.1540	0.1627
32	PAL-PL	OLS Regression	12.1074	0.1486
33	All Activity	OLS Regression	24.1617	0.0444
34	Home Care	OLS Regression	10.8970	0.0552
35	Placement	OLS Regression	22.9125	0.0061
36	OMH-HC	ARIMA	1.3919	0.0650
37	OMH-PL	ARIMA	14.3943	0.0795
38	PF-HC	ARIMA	4.0285	0.0438
39	PF-PL	ARIMA	8.8952	0.0613
40	PAL-HC	ARIMA	9.7435	0.0685
41	PAL-PL	ARIMA	8.8952	0.0613
42	All Activity	ARIMA	47.1046	0.0257
43	Home Care	ARIMA	12.1526	0.0359
44	Placement	ARIMA	37.8856	0.0245

From Figure 7.15 we find that the GM(1,1) by MAPE is selected as the best model in only one of the 9 cases, followed by the OLS models in 4 out of the 9 cases and the ARIMA model in 5 out of the 9 cases. On the other hand, when RMSE performance is more desirable the GM(1,1) is shown to perform significantly better than the ARIMA models in 4 out of the 9 cases and better than the OLS regression models in 2 out of the 9 cases. In two other cases the OLS performs only marginally better compared with the GM(1,1) by RMSE.

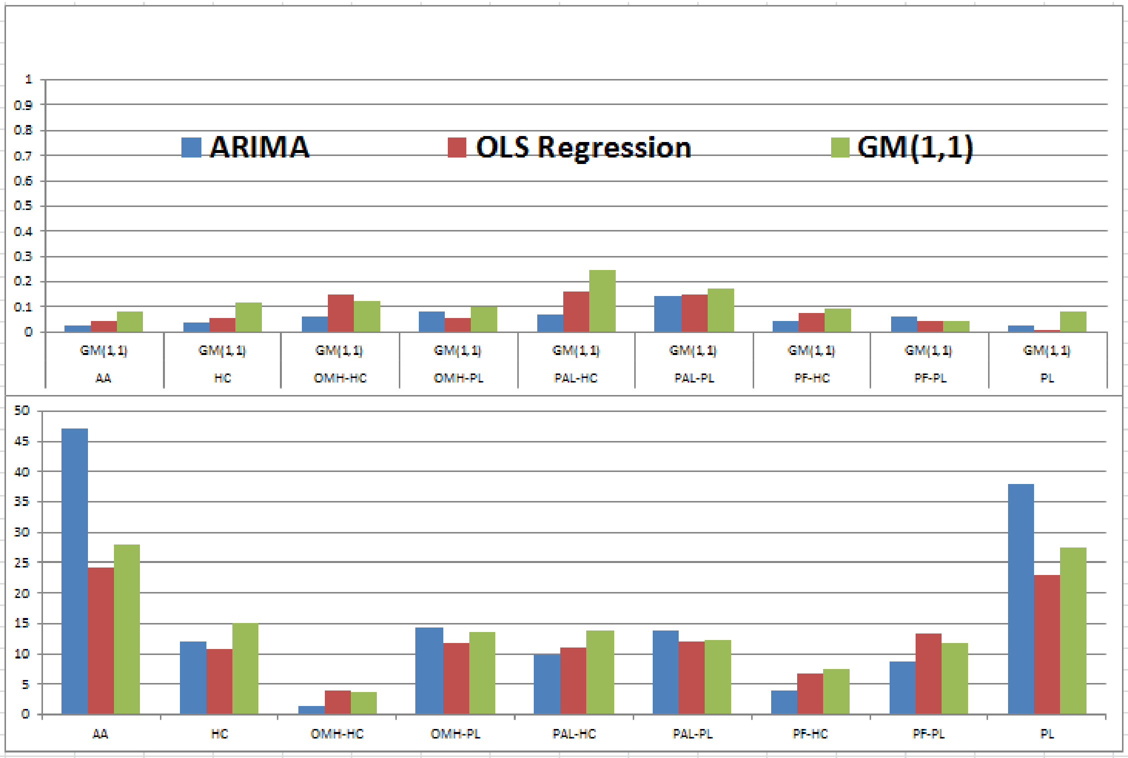

Figure 7.15– Comparison of model performance via MAPE(above) and RMSE(below)

Although GM(1,1) models require a minimum of 4 periods of input data, during our analysis we have used 48 periods since we have included activity over 4 full financial years. While 48 periods worth of data is not necessarily considered large in other domains, for the purpose of LTC activity this may in many ways represent a significant amount of historic data and indeed for the GM(1,1) many studies have used much smaller samples to highlight its performance on more restrictive datasets. Furthermore, it could be argued that within the 48 periods of data available, at least one policy change affecting the access to the LTC funding during the time horizon has taken place thus calling into question the suitability of using the earlier segment of the historic data to make future projections.

To test the performance of the grey model under a more restrictive data assumption we repeated our experiment except that in the case of models 45-50, shown in Table 7-6, we limited the test dataset to the last 18 periods.

Table 7-6 – Results from grey models with revised background function

Model	**Care Type**	**Model**	**Solver**	**RMSE**	**MAPE**
45	OMH-HC	GM(1,1)	OLS Linear	2.3147	0.0669
46	OMH-PL	GM(1,1)	OLS Linear	10.1624	0.0581
47	PF-HC	GM(1,1)	OLS Linear	3.8346	0.0506
48	PF-PL	GM(1,1)	OLS Linear	7.2985	0.0176
49	PAL-HC	GM(1,1)	OLS Linear	6.3681	0.0367
50	PAL-PL	GM(1,1)	OLS Linear	5.9903	0.0444

As shown in Figure 7.16, compared with the 48 period GM(1,1) models fitted the 18 period models perform significantly better by both MAPE. In particular, the PAL-HC series is forecast with a mean absolute percentage error of 3.57% compared with 24.5% when all 48 data points are used. Similarly, the MAPE in the OMH-PL series is falls from 5.81% to 9.81%. To some extent these reduction in MAPE can be partially explained by the removal of elements of the original series which proved difficult for the GM(1,1) to map, in particular the presence of an initial period in which the series was either flat or grew at a very high rate. However, in the case of time series like PAL-PL the time series represented by the 18 month period was observed to be far more variable in the later periods and indeed grew at a much faster rate on average compared to the situation when the entire data period was considered. As such, it is somewhat difficult to explain this decrease in MAPE based on the assumption that the trends in later periods were less variable and or more gradual.

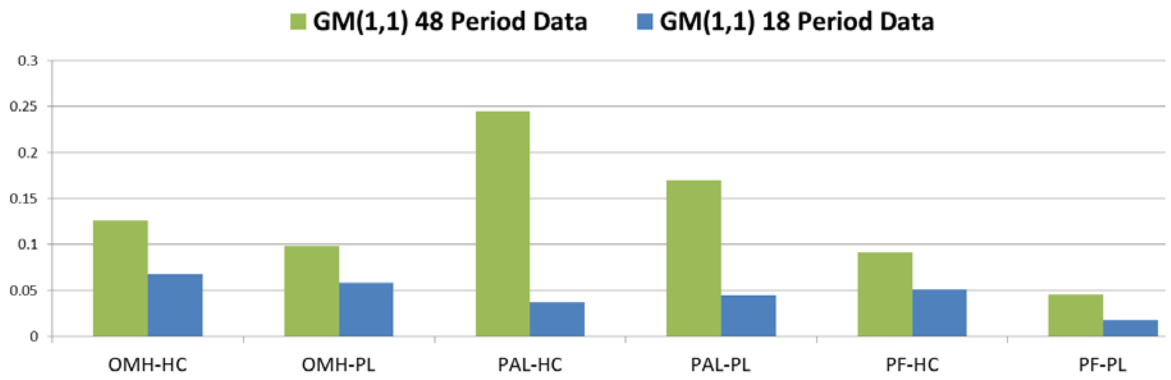

Figure 7.16– Comparison of MAPE for GM(1,1) when data is limited

7.4 Hybrid grey-fuzzy regression

Despite the benefits of grey regression for LTC planners already discussed, the grey regression approach is subject to several weaknesses. Three of the most important considerations for the purposes of forecasting LTC demand relate to (1) the reliability of the point estimates obtained from the grey model being sensitive to sampling (Tsaur 2008) , (2) that whilst coping in situations where data is limited the grey approach does not directly deal with the impreciseness of data, such as for example whereby we have approximated demand using the concept of no. of care packages taking place, and (3) the grey approach allows us to obtain point estimates. From a managerial perspective decision makers, we argue, are perhaps more interest in making interval extrapolations to understand the range of possible future values a variable might take.

7.4.1 Fuzzy regression

Fuzzy regression traces its origins back to fuzzy set theory (Zadeh 1965), a theory which recognises that it is often difficult to precisely categorise objects into predefined classes. For example, if we have a class of good products and bad products, the classification of each individual item could depend upon its mixture of certain qualities e.g. price, weight, reliability, etc. In this sense the extent to which any given item is good depends more on its degree of membership, or fuzzyness, to either the good or bad class of products defined by a membership function. Using the principles of fuzzy set theory, Tanaka et al. (1982) introduced the notion of fuzzy regression to show how the relationship between input X and output Y could be modelled depending on whether the relationship between X and Y was fuzzy or whether the inputs themselves were fuzzy. In our example, we consider only the case where the relationship between X and Y is fuzzy and X is a set of non-fuzzy observations.

Equation (7.15) represents the classical linear regression model, in which predictions of y are dependent on the intercept and slope of the estimated relationship between x and y. In this case each observation of x leads to a point estimate of y.

$$\hat{y}_t = a + bx_t \qquad (7.15)$$

$$\forall t = 1,2,..N$$

In contrast, fuzzy regression models take the general form shown in equation (7.16), whereby A = $\{A_0, A_1, A_M\}$ is a vector of membership functions and each $A_j = \{a_j, c_j\}$ represents the parameters that specify the triangular membership function with centre a and spread c for each column in X. The fuzzy output y is therefore estimated from the corresponding X observations adjusted by their respective membership functions. The membership function has the purpose of extending the range of permitted values of Y for any given value of X to accommodate the fact that the relationship is not crisp[43].

$$\hat{y}_i = A_0 x_{0,i} + A_1 x_{1,i} + \ \dots \ A_M x_{M,i}$$
$$\forall i = 1,2,..N \tag{7.16}$$

The membership function defined by each element of A is given by μ_{A_j} and shown in (7.17) . In our example, it represents the degree of truth (h) surrounding the slope of the relationship between all $x_{j,i}$ and y.

$$\mu_{A_j}(a_j) = \begin{cases} 1 - \dfrac{|\alpha_j - a_j|}{c_j}, & \alpha_j - c_j \le a_j \le \alpha_j + c_j \\ 0, & otherwise \end{cases} \tag{7.17}$$

$$where\ \alpha = [a_{0,} a_{1,} \dots a_M]$$

Figure 7.17 shows a potential membership function with parameters A = [0.66, 3.12]. The y-axis represents the degree of truth surrounding the slope of the relationship between x_j and y. If we set h =1, corresponding to crisp data we believe that the relationship is non-fuzzy. If however $0 \le h < 1$ then the fuzzy set of values for the slope coefficient includes those values bound by the two corresponding sides of the triangle along the range [-2.47, 3.78]. That is to say 0.66 ± 3.12.

[43] A crisp set is a set whereby we can evaluate a value's membership as either true or false.

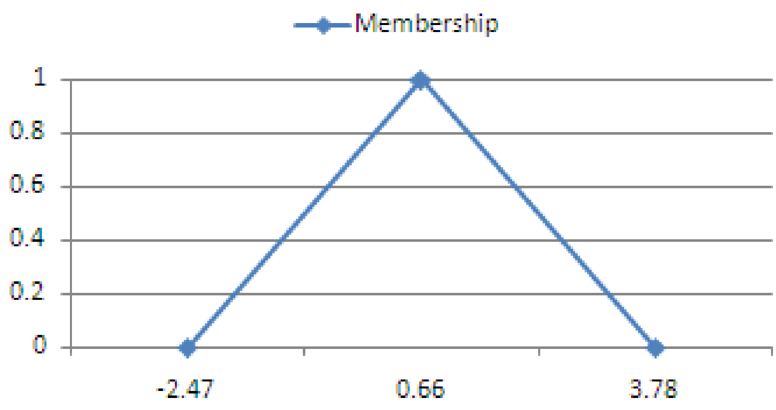

Figure 7.17– Example Triangular Membership Function

As in (Tsaur 2008), we can use the principle that the membership function of Y can be thought of as a combination of weighted membership functions corresponding to the fuzzy coefficients for X.

$$\pi(Y_i) = \left\{ \begin{array}{ll} 1 - \dfrac{|Y_i - X_i\alpha|}{C^T|X_i|}, & X_i \neq 0 \\ 0, & Y_i = 0, X_i = 0, \\ 1, & Y_i \neq 0,\ X_i = 0, \forall i = 1,2, \dots, M, \end{array} \right\} \tag{7.18}$$

To solve the regression equation (7.17) it remains to find values for A. One approach minimises the total fuzzyness in the model is minimized (7.19) subject to the membership function capturing the parameters of the model to at least a degree of truth h (7.20). Given that absolute spread is used it is possible to write the formulation in terms of a linear programming problem for solving.

$$Min\ \ W = \sum_{j=0}^{M} \left[c_j \sum_{i=0}^{N} |X_{j,i}| \right] \tag{7.19}$$

$$1 - \frac{\left|Y_i - X_i^T \alpha\right|}{C^T |X^i|} \geq h, \qquad \forall i = 1,2, \dots N \tag{7.20}$$

$$C \geq 0, \qquad \alpha \in R, \qquad 0 \leq h < 1$$

Once h has been chosen and A found, the fuzzy output Y_i can be estimated as a fuzzy number in the range (Y_i^L, Y_i^H) defined by the range of the respective spreads of the coefficients for each input variable. The equations for Y_i^L and Y_i^H are shown in equations (7.21) and (7.22).

$$Y_i^L = \sum\nolimits_{j=0}^{M} (a_j - c_j) X_{i,j} \tag{7.21}$$

$$Y_i^H = \sum\nolimits_{j=0}^{M} (a_j + c_j) X_{i,j} \tag{7.22}$$

Previous work

To date, hybrid grey-fuzzy regression has been used in a number of studies, including: (Tsaur, 2005), (Tsaur, 2010) and (Xia & Wong, 2014). (Tsaur, 2005) proposed a grey-fuzzy GM(1, 1) model by hybridising a fuzzy set into grey model GM(1,1) in order to obtain more valid forecast for extrapolative data which are of fuzzy type. (Tsaur, 2010) presents a fuzzy linear programming model to derive the interval grey regression model by necessity analysis. In this study, the developed grey-fuzzy regression model is applied to forecast LCD TV demand. (Xia & Wong, 2014) have extended the fuzzy grey regression model to consider seasonality based on the cycle truncation accumulation with amendable items to improve sale forecasting accuracy in the fashion retail industry, a case where sale data is not comprehensive and often scattered. Practicality and performance of the model are validated by applying the developed method on real sets of time series from three different types of fashion retailers and the experiment results show that the proposed model outperforms the state-of-art forecasting techniques.

7.4.2 Application to the London LTC dataset

To study the effect of using a hybrid grey-fuzzy methodology to predict demand for LTC activity we began by making use of the fitted values for LTC activity generated by the grey models proposed in §7.3.1. We then used equation (7.19) to extend the grey approach by passing the outputs of the grey models to a set of fuzzy regression models. Next we estimated the fuzzy relationship between the predicted values from the grey models and the observed historic cost.

An illustration of the hybrid model is shown in Figure 7.18 whilst the formulation of the estimated no. of packages taking place is represented in equation (7.23) as the fuzzy variable y_i. As in (7.19) our three models, one to represent each of total no of packages taking place, packages taking place at home and packages taking place in institutions, were solved for the parameters A by linear optimization using Microsoft Excel 2007 solver and assumed a symmetric triangular membership function. The fitted results of the hybrid and GM(1,1) models were evaluated in terms of their mean absolute percentage error (MAPE). In the grey-fuzzy models h was chosen to be 0.75 so as to simplify the working.

$$\hat{y}_i = A_0 + A_1 GM(1,1)_{1,i} \qquad \forall i = 1,2,..N \tag{7.23}$$

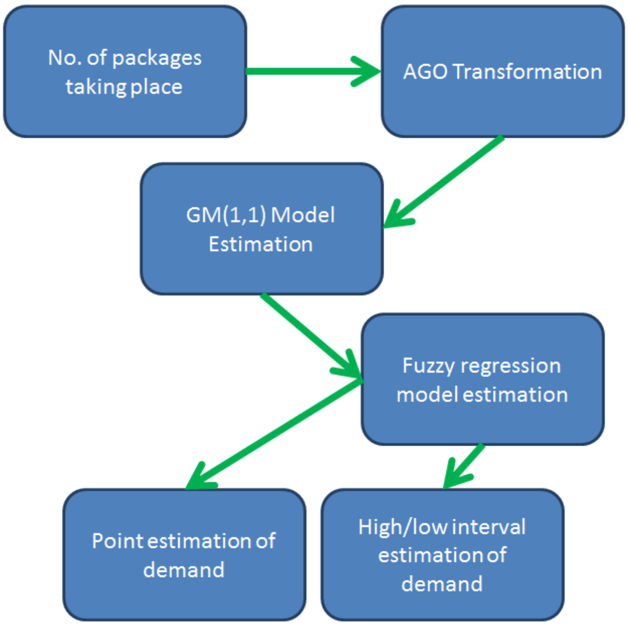

Figure 7.18– Proposed Hybrid Model

7.4.3 Results

Figure 7.19 and Figure 7.20 show a graph of the resulting hybrid grey-fuzzy model for all care packages taking place and fuzzy membership function respectively. Compared with the GM(1,1) model we now observe the upper and lower limits for the estimated amount of activity. The RMSE of 2.82% for the resulting model shows a low level of forecast error and is significantly lower than when the GM(1,1) alone is used.

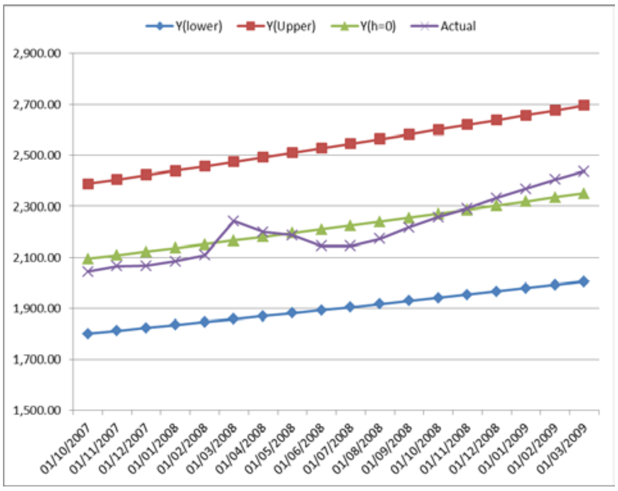

Figure 7.19– Grey-Fuzzy Regression All Activity (MAPE = 2.82% , RMSE=23.2126).

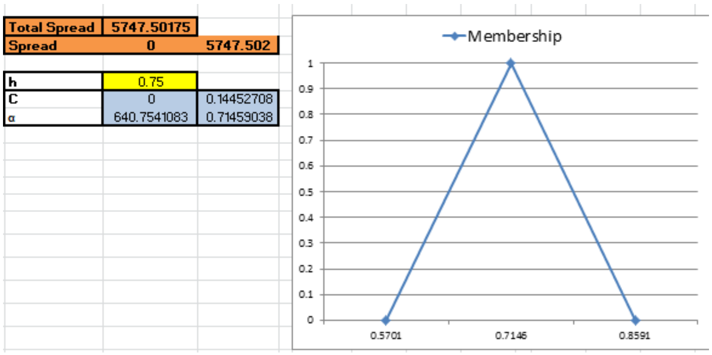

Figure 7.20– Membership function and fuzzy params (All Activity).

Figure 7.21 and Figure 7.22 show that the model fit for HC activity is slightly better than for all activity despite the tendency of the hybrid model to underestimate the observed activity in the second half of the dataset.

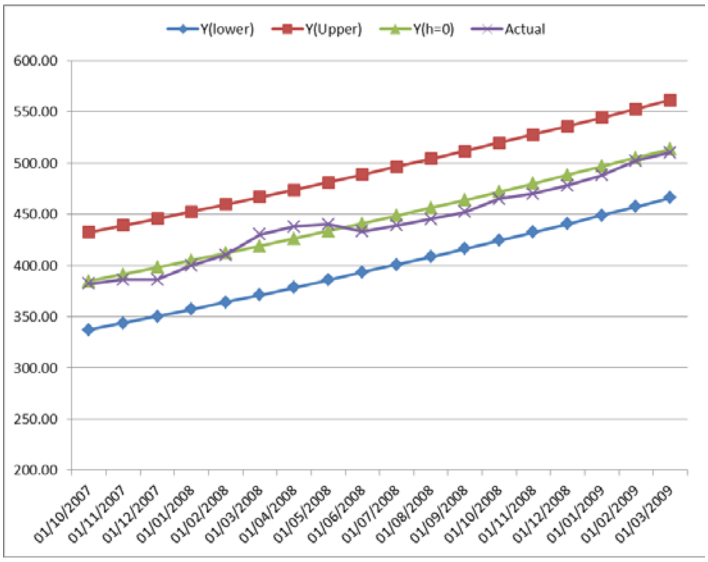

Figure 7.21– Grey-Fuzzy Regression HC Activity (MAPE = 2.17% , RMSE=11.4894).

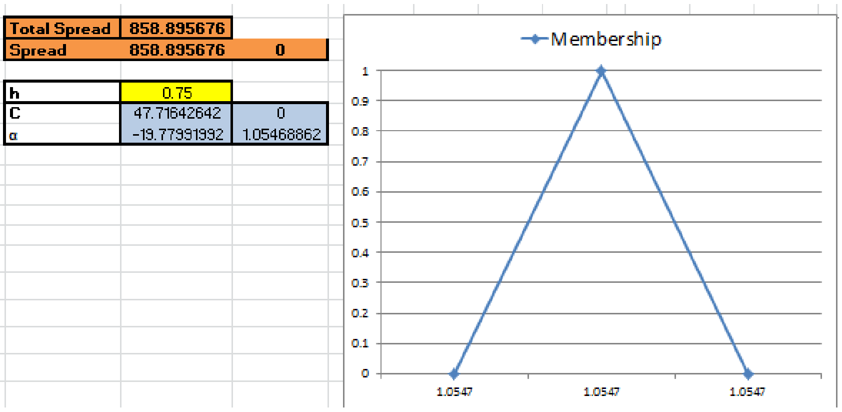

Figure 7.22– Membership function and fuzzy params (HC Activity).

Figure 7.23 and Figure 7.24 show that while the MAPE is lower for the grey-fuzzy model compared with the ARIMA, OLS and GM(1,1) models, the RMSE has not improved dramatically. The middle part of the data period appears to account for the largest amount of forecast inaccuracy, where the grey-fuzzy model makes a high amount of over prediction during June-Aug 2008.

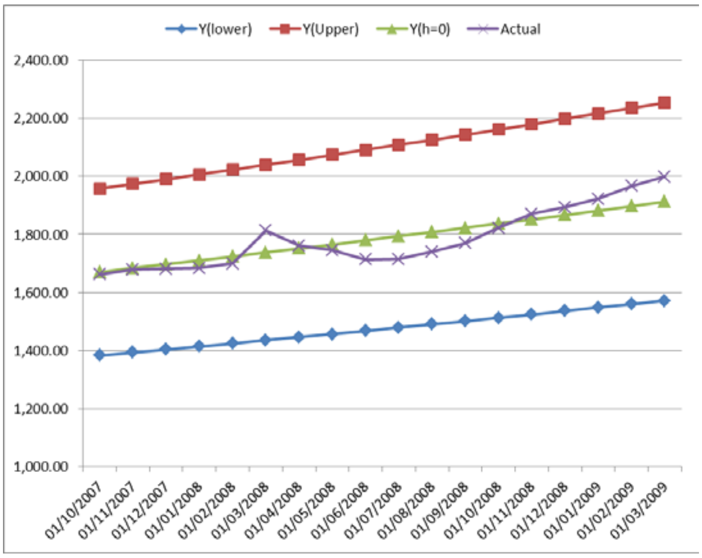

Figure 7.23– Grey-Fuzzy Regression PL Activity (MAPE = 2.47% , RMSE=22.5617).

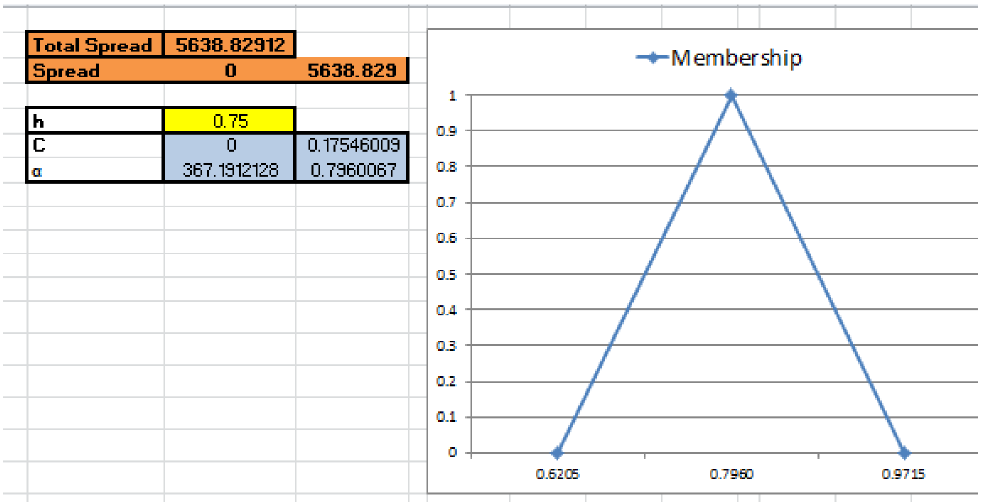

Figure 7.24– – Membership function and fuzzy params (PL Activity)

7.5 Discussion

Long-term care forecasting has historically been carried out at the national or indeed international level with the purpose of highlighting the potential impact of changing socio-economic factors on current models of funding and service delivery. Furthermore, the majority of such models have tended to focus on the longer term impact using forecasting horizons measured in decades. At the local level, where LTC is coordinated, there are far fewer studies investigating the impact at the local level over a more typical planning period of 1 to 2 years: where for instance it could be argued that such socio-economic play a more marginal role.

Compared with national planners and public health organisations, local LTC planners often have far fewer resources dedicated to forecasting and analysis in general yet we argue that there are several tangible benefits forecasting such activity can bring. Firstly, increased information about the future pattern of demand can held local LTC to budget for the next planning period by giving them increased information about the amount of activity taking place. Although not dealt with directly in this chapter, such information could then be linked to cost information to derive projections of expenditure. Second, through our dynamic commitment model presented in the previous chapter, we have shown how such demand information can be used to make cost savings through the use of provider commitments under a time and volume discounting regime. Thirdly, a greater understanding of the future pattern of activity can help local LTC plan how local services are used and designed so as to best meet the needs of the local population.

Despite such benefits we can find very little published evidence of LTC demand modelling at the local level. While it might be the case that such modelling work perhaps goes unpublished to commercial sensitivity or indeed the relative naivety of such models, that is not to say that it does not take place. We would tend to argue that, despite their comprehensiveness, the range of models published to date directed at national planners often require extensive parameterisation which, for local planners, may be beyond their capability and capacity to do. At the same time, national models often make strong assumptions about key variables and data sources that local LTC planners are unable to verify or adapt to their local circumstances. For these reasons we believe that in order to ease adaption by local planners, future models of LTC demand need to be more carefully tailored to the needs and capability of local planners. For these reasons we have proposed both a grey and hybrid grey-fuzzy forecasting approach.

In particular, in proposing a grey model of demand we wanted to verify the capability to use a methodology inspired by grey set theory to forecast LTC activity under the assumption that available input data is limited both in duration and richness. At the same time, our grey model, as a result of being based on grey set theory, makes no underlying statistical assumptions of the variables included except that they contain some information about the underlying demand generating process. Using the GM(1,1) model, which represents a grey model with 1 period differencing and 1 independent variable, we fitted a series of grey models to the number of LTC care packages taking place in London for financial years 2005/06 to 2008/09: yielding 48 monthly periods.

Our experimental results, using MAPE and RMSE as our measures of model performance, we found that the GM(1,1) was able to deliver fitted models with respectable levels of RMSE and MAPE within 10% per period. In the case of predicting the proportion of institutional to home care packages taking place we found that the GM(1,1) was able to deliver less than 1% MAPE. As a number of the time series representing LTC activity exhibited high rates of growth we applied a revised background function to the GM(1,1) to allow to model the pickup changes in trends and level shifts in the mean more quickly compared to the standard weighting scheme that gives equal weight to both current and previous observations. With the new background function in place we found that while reductions in MAPE and RMSE were observed, such improvements were rather modest. One of the most difficult series for the GM(1,1) model to estimate was the number of home care packages taking place, where for example the growth rate in the number of care packages rose sharply during the initial 6-12 months before stabilising.

In order to assess the relative performance of the GM(1,1) we partitioned LTC activity by care group and provision type and compared the fitted model results with more classical approaches – including ordinary least squares regression (OLS) and ARIMA. Whilst the results of the ARIMA and OLS models While the ARIMA and OLS models were generally more favourable compared with the GM(1,1) models on MAPE, except in a couple of cases where GM(1,1) outperformed either the ARIMA or OLS models, the GM(1,1) was able to deliver far more stable RMSE across a range of different time series. This is an important finding since RMSE as a measure of error gives more credibility to a model that makes a large number of smaller errors with one that makes even a small number of large errors.

Compared with ARIMA, the GM(1,1) has the ability to forecast several periods ahead without converging to the mean of the process after 1 step ahead, whilst the OLS assumptions were not always found to be satisfied in cases where the time series under investigation contained level shifts or changing rates of growth. When a more restrictive dataset was used, based on the last 18 periods, we found that there was a significant increase in the forecast accuracy of the GM(1,1) models which illustrated their relevance in restrictive data sets, such as LTC where changes in policy has taken place and thus call into question the suitability of using older sections of historic data to forecast ahead.

One issue relating to the use of the GM(1,1) model in practice is that, whilst it makes fewer assumptions, the results generated are point estimates of the series of interest. In practice, LTC planners may be more concerned with the future possible range of scenarios so as to be able to play for worse case and best case scenarios. Similar, whilst coping in situations with limited data it the grey model to some extent relies on a certain level of preciseness in the original series. For reasons relating to how LTC data is recorded, for example in terms of care days, absolute number of individual in care, there is a case to be made for making a certain level of approximation so as to simplify the analysis. In order therefore to be able to provide interval extrapolation with the GM(1,1) model, and handle situations in which several approximations may have to be made regarding the underlying data, we hybridise the GM(1,1) model with a fuzzy regression methodology.

When compared with the GM(1,1) model, the hybrid GM(1,1) fuzzy methodology provides lower RMSE and MAPE when predicting total activity, activity in care in the home and activity in institutions. More importantly though it's the ability of the methodology to create intervals of high and low scenarios so as to reduce the amount of uncertainty facing decision makers. From our experimental results we found that the GM(1,1)-fuzzy methodology generated prediction intervals in the range of 12-17% above the predicted value in the high case and 12-17% lower for the low case. With that in mind, we observe that the main weaknesses associated with the hybrid model is that it restricts the underlying model to one of linear type. Although many of the time series model we investigated could be approximately represented by a linear model, there were a number of sub series (for example for specific care groups) whereby a more appropriate choice would be non-linear due to the changing growth rate in activity witnessed over time. In this case the original GM(1,1) model might be more appropriate.

Despite the promise shown by the grey-fuzzy regression, our approach is subject to a number of limitations. Firstly, we only considered LTC activity whereas it could be argued that the cost of future activity is also of significant interest to planners. We have suggested that one approach would use our forecasted numbers and multiply by the distributional costs for each care group to obtain an approximate estimate of cost. Alternatively, the cost itself could be modelled as a time series in the same way we have focused on activity. Clearly the demand for LTC is dependent on factors other than time yet we have only used one predictor variable. Our justification is on the basis that we wanted to keep the data requirements of our approach low and further work is needed to assess what short term factors influence local demand. In principle, the grey-fuzzy and GM(1,1) models are not limited to one independent variable although the interpretation of the grey parameters do not have an intuitive explanation as is the case for the parameters of the OLS regression do.

7.6 Summary

In this chapter we have shown how the grey-fuzzy and GM(1,1) models can be used to forecast LTC activity under conditions of uncertain and limited historic data upon which to make projections. Compared with classical forecasting approaches our grey modelling approach does not require the practitioner to test and validate a large number of data assumptions and our experimental results show that GM(1,1) shows promise in providing forecasts with tolerable levels of MAPE. An important aspect of our methodology is the hybridisation of the original GM(1,1) to a combined grey-fuzzy regression approach. This second component of the approach allows facilitates powerful interval extrapolations based on the use of fuzzy set theory.

Chapter 8

Development of a local-level planning system for LTC

8.1 Introduction

Model-view-controller (MVC) is a design pattern used in the development of software applications. By separating data access, data manipulation and presentation of results, the MVC approach encourages clear separation of concerns which can lead to greater long-term maintainability of code. The layering of applications also helps to prevent segments of code being dependent on other sections, so that for instance the user interface can be altered without having to change how data is processed internally. In this chapter, we propose a web-based decision support tool incorporating aspects of our dynamic commitment and grey-fuzzy forecasting models developed using the using the MVC design pattern. Our goal is to investigate how a web-based planning system could help disseminate our research findings and provide LTC planners with relevant and insightful information about their local LTC system.

8.2 A demand planning tool for LTC

To date, a number of decision support systems have been proposed to support local LTC commissioners estimate future demand for LTC services. In particular, The “Institute of Actuaries and Urban Institute Studies” modelled the future number of patients using micro-simulation (Nuttall, et al. 1994). More recently in a follow up study by the Personal Social

Services Research Unit (PSSRU) at the University of Kent, future numbers of older people together with level of LTC services and cost were modelled by simulating changes in key drivers of patient demand (PSSRU 2005) (PSSRU 1998).

A tool developed by researchers at the University of Westminster in 2006 (Xie, et al. 2006) combined unit costs of care with an underlying survival model to provide forecasts of the cost of maintaining a cohort group of existing local authority funded patients over time. The model was developed in Microsoft Excel and used “R for Microsoft Excel” (RExcel n.d.) for data processing.

In 2008, the “NHS London Procurement Programme” (LPP) (NHS London Procurement Programme 2011) commissioned the development of a spreadsheet based tool to forecast the future resource requirements of LTC across London. Using information about both admissions and discharges from LTC in each area of London, the tool generated both cost and demand forecasts for each regional sector and London as a whole based on exponential smoothing of an adjusted local trend and length of stay estimate derived from the London-wide length of stay distribution.

At present we note that the existing tools are based around stand-alone software and as a result make collaboration between both the clinical and non-clinical planning teams problematic. Furthermore, except for the University of Westminster and the LPP tools, existing systems have tended to focus on catering to national planners in that outputs are geared towards the national picture, rather than for instance forecasting numbers of patients in institutions within a specific region.

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8.2.1 System objectives and requirements

The main aim of the proposed system is to provide analysis and reports on historic LTC activity and forecasts of future resource use for London LTC commissioners. In addition, the system must be able to generate outputs that are easily integrated into local planning documents and support data uploads from a range of LTC data recording formats. Furthermore the system design should take into account that it may be used by non-modelling experts and enable a number of different stakeholder teams to easily review and compare their analysis with the findings from planners in other regions.

Due to the large number of historic policy changes surrounding LTC and the NHS itself (Cheselden 2009) an important consideration is the flexibility of the system in terms of being able to accommodate new functionality in light of changes to the LTC system itself.

8.2.2 User requirements and needs analysis

The first step in identifying the user requirements of the proposed new system was to identify tasks and processes within the LTC planning process. To achieve this we held a series of interviews and meetings with LTC commissioners from a number of primary care trusts (PCTs) across London. Due to time and organisational constraints, we decided to limit the first round of interviews to just one representative from each London sector. In addition, we circulated questionnaires based on the issues with the previous spreadsheet tool to all LTC commissioners in London, so as to help ascertain its strengths and weaknesses.

From the insight gained in the interviews we drew cognitive maps[44] representing the scope of the tasks carried out on a day-to-day, quarterly and annual basis, and how the work related to longer term planning. By overlaying the cognitive maps of each commissioner we found common objectives, such as identifying high-cost patients with specific

[44] See Appendix A.10 and A.11

characteristics in the system, which could be included in the scope of the new decision support system.

While the principle of having the London-wide spreadsheet tool was strongly advocated by those interviewed, the questionnaire highlighted serious shortcomings in the previous tool in the areas of usability and data input. Some of the most notable problems included software compatibility and unofficial spreadsheet extensions, together with difficulty in inputting data into the system due to significant differences between the way the model expected data to be entered and the way in which it was in practice recorded by planners. As a result, the new web system would need to employ an efficient, consistent and easy to use user interface. With regards to data inputting, we planned to standardise the input format according to the mostly commonly used recording formats, so as to keep the input process efficient.

To more clearly understand data requirements, we employed an output driven design process in that we categorised the outputs of interest by commissioners into one of forecasting, benchmarking or analysis, and then determined which items of data we would need to collect so as to be able to deliver on these outputs. In the first prototype, we identified 10 outputs from all of the output categories, covering new admissions, high cost patients, discharges, care group category, length of stay and cost. Within the context of the MVC paradigm, each output below corresponds to a single controller. Details of these outputs are included in Table 8-1.

Table 8-1 – Outputs proposed for the planning system

Output No.	Description	Category	Type
1	Patient Pathway Analyzer	Analysis, Benchmark	Hierarchical Management Chart
2	Length of stay in Care	Analysis, Benchmark	Histogram
3	Care Group Distribution	Analysis, Benchmark	Histogram, Pie Chart
4	Age Distribution	Analysis, Benchmark	Histogram, Box Plot
5	Admissions and Discharges	Analysis, Benchmark, Forecasting	Time Series
6	Patients by Care Type over Time	Analysis, Benchmark, Forecasting	Time Series
7	Admissions and Discharges to external regions	Analysis, Benchmark, Forecasting	Time Series, Histogram
8	Costs by type of care over time	Analysis, Benchmark, Forecasting	Time Series, Histogram
9	No of patients fast-tracked into care	Analysis, Benchmark, Forecasting	Time Series
10	Types of patients admitted by age, gender and ethnicity	Analysis, Benchmark,	Pie Chart, Bar Chat

8.2.3 Data exchange

An analysis of existing data recording practices showed that by and large Microsoft spreadsheets were the most commonly used to record patient activity by LTC planners, followed by database management systems with support for outputting to a Microsoft spreadsheet file. In general, data files were typically around 3 to 5Mb in size and covered around 400-900 records, depending on the size of the region and the length of the data period.

8.2.4 Security considerations

In the past, we discovered that planners typically exchanged activity data using spreadsheet files over encrypted email connections, however for the new web-based tool we decided to streamline this process so that data could in fact be uploaded directly via the tools web interface, in much the same way as email attachments are added to emails on modern web-based email systems. To meet the needs for security, we required that data uploads would only be accepted over a HTTPS (Hypertext transfer protocol secure) connection.

To be able to generate outputs in the system we required patient level information. In particular, we needed a unique field to identify the movements of patients in the system. Due to security reasons and the law surrounding the safe collection and storage of such data, we decided to anonymise data prior to it being uploaded to the new system. Thus, we would apply a secure hashing function to potentially insecure items of data, such as NHS number, whose value was not needed implicitly but was required to be able to identify a unique patient. In addition, we decided not to require users to upload any otherwise personally identifiable patient information, such as date of birth and address, and instead would ask for year of birth and region respectively.

To protect each region's planning data, we decided to use a role-based access control (RBAC) list to protect unauthorized access to the system. This would enable the system to not only restrict access to the system as a whole, but would also prevent users from viewing the results and data associated with other regions. The role based policy also enabled individual regions to decide which of their planning team had access to particular categories of reports, namely analysis, benchmarking and forecasting, in addition to allowing them to restrict how existing data can be modified and which accounts would be permitted to upload new data. Users were also restricted to viewing their own region's data set and when benchmarking they were only able to observe results based on aggregated patient data from other regions.

8.2.5 System architecture

Prior to implementation of the system, an appraisal of various potential web architectures was considered, including: Microsoft ASP.NET MVC (Microsoft 2011), Ruby on Rails (Ruby on Rails 2011) and Java Spring (Spring Source 2011). Due the availability of skills, the support for Microsoft spreadsheet reading and manipulation and the crucial need for a development platform that enforced solid design foundations for both adding new features and long-term maintainable code, we elected to use ASP.NET MVC (Active Server Page Model View Controller) framework using Microsoft SQL server 2005 as the database engine. Despite Ruby on Rails coming a close second, due to its much simplified handling of database connectivity, lack of support for source code compilation made debugging more challenging compared with ASP.NET MVC.

The completed system consists of six key layers (Figure 8.1); the data access and model layer; the processing layer; the presentation layer; the charting and report reporting layer; the data importing layer; and the routing layer. These six components are each responsible for a limit subset of tasks undertaken in the complete decision support system, such that each layer has a clear and well defined responsibility, and is based upon the MVC design paradigm. The system is written in the C# programing language version 3.5 service pack 1.

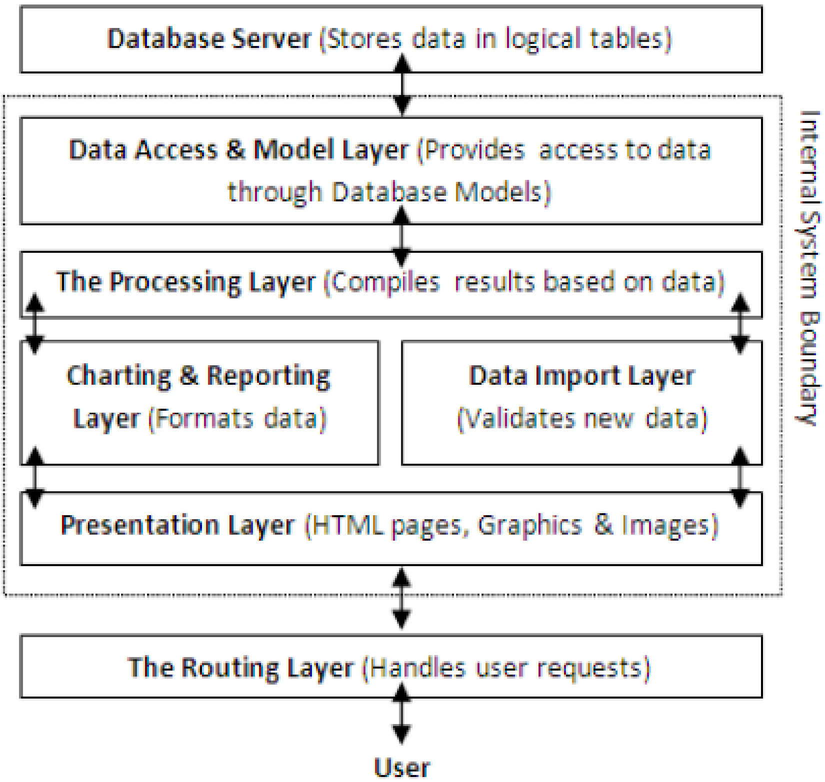

Figure 8.1–Key layers in the LTC MVC planning system

8.2.6 Model view controller design pattern

MVC is a design pattern that was developed in the 1960s for Smalltalk, an object orientated programming languages in which classes of programs communicate with one another via message passing (Smalltalk 2011) (Krasner and Pope 1988). In recent times, MVC has increased in popularity in the web development space due to the relative ease in which prototypes can quickly be deployed and the need for a framework which supports the developer in the management of large and complex web applications.

In the MVC approach, segments of code are separated into three distinct entities. Models represent classes of data within a relational data table, whereas controllers are responsible for processing and data manipulation. Views on the other hand are only responsible for laying out the results generated by controllers to users.

In our system, the patient model represents each patient's characteristics, whereas the episode model represents a precise period of care with an associated start and end date. A number of controllers are used, each containing the logic required to fetch, assemble and process the reports requested by end users. Views in the system correspond to HTML (Hypertext mark-up language) documents, bound to a particular model and controller. The tight separation of these entities also helps to ensure that changes to one or more components does not adversely impact upon the rest of the system and that parts can be added and removed without significant changes to existing code.

8.2.7 Routing with active server pages

Whilst the MVC design pattern enforces the logical layout of programming code, active server pages are used to route individual web requests to specific controllers when a user accesses a given URL (uniform resource location) associated with the tool. For example, by loading the page http://www.example/pathway/generate, the routing system firstly looks up the corresponding controller responsible for handling this request (Xaingjun, et al. 2009). In this case, it loads the pathway controller and instructs it to carry out the generate command. Internally, a single controller can contain several actions which may map to one or more URLs. When the controller is done reading and processing the input data, it passes the results to the associated view which displays the results using HTML to the user's internet browser.

8.2.8 Database access and data validation

Once LTC activity data is uploaded to the system it is saved into the underlying SQL server 2005 database. The data upload controller is responsible for ensuring that data is added to the relevant tables, so that for instance, each patient is recorded only once in the patients table but can be linked to several episodes in the episodes table using their unique patient identifier.

Data in the system is accessed and queried through models, where each model corresponds to a single table in the underlying database and is appropriately linked. Thus, when an action is performed on a specific model, the system automatically generates the necessary SQL (structured query language) statements to insert, edit, delete and select the data concerned within the database itself.

Data validation is carried out in both the database layer and in individual models. While the database is responsible for ensuring that both primary and foreign keys are respected, that is to say the same patient cannot appear more than once in the patient table, data annotations are used within models to enforce strict validation of data fields. For example, in the episodes model each start date of care is not allowed to be greater than the end date of care nor can the price per week or care be greater than £3000 per week. Although the latter is not based on any formal policy concerning maximum week cost, at the very least it prevent users from entering erroneous values.

8.2.9 Chart and report generation

All controllers in the system have access to a common charting framework, built on top of the Microsoft Charting Library. Charts available in the system include: line graphs, pie charts, histograms and box plots. To convert the results generated in a controller to a chart, the controller needs only to call the appropriate chart type in the charting framework and pass the relevant data. To display the chart to the user, the controller passes the resulting chart from the charting framework to the corresponding view for sending to the user's browser.

As the patient pathway diagram is not a chart which maps to an available chart type in the Microsoft Charting Library, the patient pathway controller instead uses the Google Charting Web Service to draw hierarchical management charts. To accomplish this, the controller first reads all episode models to find the stages of care for each patient in the system for a given time period. The controller then combines these linked stages to and links individual stages of care to the aggregated stages found when all patients are considered.

The next phase is to calculate the aggregate statistics for all the nodes considered in the aggregated pathway, that is to say it determines the number of patients in each stage of the pathway and their average length of stay. Finally, the controller then passes the results to the Google Charting Web Service and obtains the corresponding management map. An example of a patient flow map is illustrated in Figure 8.2, where we observe the movements of organic mental health patients who initially receive their care at home. The lines represent movement of a patient (from top to bottom) to different care types, each represented by a single node.

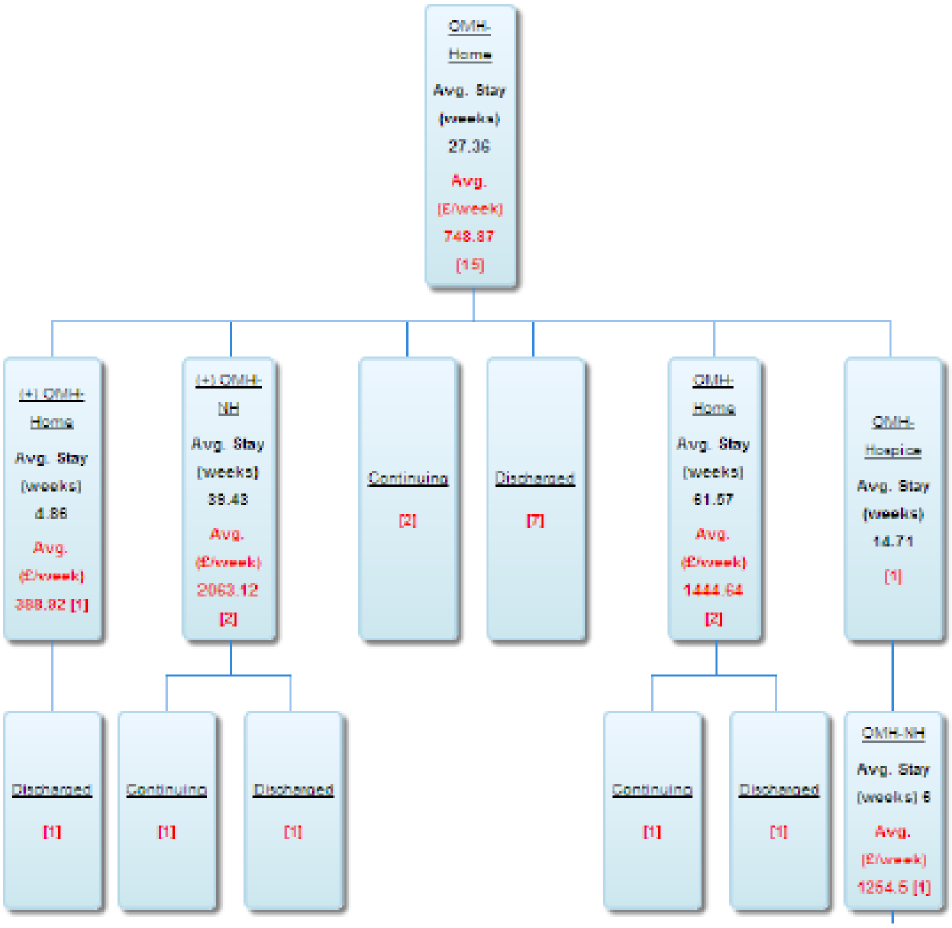

Figure 8.2–Patient flow map for OMH-Home

8.2.10 Analysis engine

Within the processing layer we have implemented a number of statistical techniques which users can utilize to analyse their LTC data, in addition to the outputs detailed in Table 8-1. For instance, we have included functions to evaluate mean, mode, medium variance, standard deviation, auto correlation, partial auto correlation and both Both Scotts Choice and Sturges formula are available for determining histogram bin width (Wand 1996).

In terms of forecasting, users can perform time series analysis through simple moving average, exponential smoothing and ARIMA modelling, with multiple options for analysing estimate errors including mean absolute percentage error (MAPE) and root mean squared error (RMSE). In addition to these classical techniques, which we found PCTs were typically most familiar with and more commonly used in practice, we also added an advanced option to conduct forecasts based on our GM(1,1) model and the hybrid grey-fuzzy regression model presented in §7.3 and §7.4 respectively.

As PCTs were much less familiar with grey and fuzzy set theory we added some additional documentation to the planning system to give guidance as to how to interpret the results and in what situations the alternative models might best be utilised: for example in situations where activity or cost was found to be more non-linear, where commissioners were more uncertain as to the underlying quality of the input dataset and or where commissioners wanted to forecast more than a couple of periods into the future. As the default C# programing framework does not come with the linear solver needed to identify the appropriate grey and fuzzy regression model parameters, we implemented a custom least squares solver based on the Math.Net numerics[45] .linear algebra solver : a software library that uses a free and open source licence permitting modification and redistribution on a royalty free basis[46].

8.3 Results and discussion

The web-based decision support system version 1.5 has now been released to London commissioners. To date, four PCTs have uploaded their LTC activity to the system and begin using it to evaluate future spend and compare historic reported spends with invoiced costs. In addition, one PCT has used it to retrospectively evaluate their recent purchase of a contract with a LTC provider. Although no formal evaluation of the system has been

[45] http://numerics.mathdotnet.com/
[46] Math.NET numerics is licenced under the MIT/X11 open source licence.

carried out, during demonstrations of the system several commissioners have commented on its ease of use and quick generation of reports[47][48].

One of the most challenging tasks during the development of the system was to enable seamless upload of LTC activity data. In part, this was due to a variety of different recording formats being used by providers. Many of these recording formats have been in place for some time and it was not reasonable to expect significant changes to them, although in some cases what we learned about how other PCTs recorded their data was passed in the form of best practices. Quality of LTC data varied significantly between PCTs, with some having much longer periods of historic data and less errors on average. This often resulted in the data having to be largely recoded manually before it could be uploaded.

A key concern of commissioners was the secure transfer and storage of data. We tried to meet with their requests by ammonising data and securing access to the system through a role-based access control (RBAC) policy. While in the most part this was sufficient for commissioners, there are clearly other security methods we could explore in later iterations.

We were very pleased with the time and developer productivity we obtained from using the ASP.NET MVC development framework. As we begin trailing the system, users we keep to point out interested new features that they would like to see and within one or two days we were able to develop a new prototype and present it for review. The separation of concerns also makes it easier to track down issues as they are discovered due to the precise location of the relevant programing logic being kept in a consistent location.

On the other hand, we had to implement and test much of the statistical functionality ourselves, which for particular modelling techniques, like ARIMA, took significant time. As a result, we have not included the breadth of functionality found in many common

[47] A screenshot of the decision support tool's homepage is shown in Appendix 290A.12
[48] A screenshot of the a sample output graph is shown in Appendix A.13

statistical packages, although given our intended user base, it is not clear whether all such functionality would unnecessary over complicate the user interface.

8.4 Summary

In this chapter we have presented a novel way in which models of demand and statistical insight into local LTC activity can be presented to LTC planners. Our approach uses the MVC(model view-controller) paradigm, which separates key aspects of our planning system into smaller logical units. This key benefit of our approach is that it allows wide-dissemination of our proposed mathematical models to health care planners whilst providing a platform in which different components can be updated and revised whilst lessening the impact on adjoining components. This allows for safe updating of our mathematical models in response to user feedback and the management of a wide variety of datasets from several London PCTs under a common generic codebase.

Chapter 9

Conclusion

9.1 Discussion

In this thesis, we have investigated several important issues concerning the system of LTC from the perspective of local health care planners. The main motivation of the research has been on using a quantitative modelling approach to help local health care organisations in their short-term planning of LTC service delivery and, perhaps more importantly, their efficient use of resources. As far as the running of the LTC system is concerned, local health authorities are particularly interested in how best they can meet the needs of LTC patients, the total cost of meeting such needs and how resources could more efficiently be used to deliver greater value for money. Indeed, in this thesis we have tried to address these three main issues in three conceptually linked stages. The first of which develops a model to illustrate the use of contractual commitments to generate cost savings related to the provision of LTC. The second stage provides a novel hybrid grey-fuzzy forecasting approach to model the short to medium demand for such services, demand forecasts which are then fed into our commitment model and presented in a web-based planning system for LTC.

Towards meeting our objective, we explored the inner workings of the LTC system in England including funding arrangements and organisation of the care system. More specifically, we conducted a limited cross-country analysis of the different forms of LTC systems around the world and their associated funding arrangements. We noted that internationally a key concern of those involved in the management of LTC related to the growth in both the nominal size and relative proportion of the elderly population – those 65 or over that are most likely to be in need of LTC. Furthermore, we highlighted several changes to the system of LTC in the UK and their potential implications. One of the most notable changes related to how funding for LTC had evolved from a devolved system whereby decisions were made on a case-by-case basis to a national framework for funding and access to care. In addition, we pointed out how much of the provision of LTC services had shifted towards private sector organisations, despite the organisation and coordination of such services remaining in the hands of local health and government authorities.

In chapter 3 we identified current and historical issues relating to the system of LTC, from both the perspective of its operation and previous modelling approaches. We found that a key research theme was future funding scenario for LTC, particularly given sharp rises it the cost of LTC in many developed economies as LTC had become increasing formalised since the term of the last century. Other notable issues related to the problem of staffing shortages within instructional care organisations, service disparities and the reliance on informal types of LTC. We also found that a large number of existing studies had focused on forecasting LTC activity at the national level. The reasons for this appear to stem from a lack of evidence that such forecasts may in fact prove more useful when carried out at the local level, where LTC is effectively planned, and perhaps reflects how existing LTC models had been used – mainly as a way to inform the policy debate surrounding future methods of funding.

In chapter 4 we outlined contractual elements of the LTC allocation decision facing commissioners and more general principles of contracting in the health care sector. In chapter 5 we illustrated how contracting and purchasing decisions relating to LTC bear relation to the more general lot-sizing problem, except that LTC concerned efficiently allocating demand for a service rather than for a manufactured good. In this chapter we also illustrated how, using a min cost model, commissioners could formulate the contracting decision in terms of a mathematical programme using data on LTC activity in London. In chapter 6, we proposed a dynamic commitment modelling framework for the contracting decision using a mathematical programming approach in which the decision is to select the amount of commitment in provider places to purchase at the beginning of a planning horizon subject to provider capacity constraints and under the assumption that either a time or volume based discount would be awarded to the commissioning organisation. Our approach differs from previous studies in that we model the demand for a service good, we allow for commitments to be offset into the planning period such that commitments need not all start or end at the same moment in time, maximum market shares for individual providers in contract time-quantity units can be set and we consider the ability of planners to salvage any excess commitment quantity by subcontracting with local authorities. We applied our formulation to reported LTC activity data in London together with a data from survey we carried out to determine estimates of care home capacity at individual providers in London, together with their care quality rating. Our results show that even in the case of a single LTC care group, over two intensity levels, involving 6 care providers and during 12 month planning horizon, approximately 10.5% cost savings could be generated. Whilst we used an example of LTC, we believe the formulation of this procurement problem can have more general applicability to procurement-type problems in involving price-breaks and for planning problems solved over short-to-medium term horizons.

Whilst extensive research into demand modelling of LTC at the national level has been carried out, few studies have examined the same LTC forecasting problem at the local level. In chapter 7 we proposed using a hybrid grey-fuzzy forecasting methodology to predict the demand for LTC in terms of the number of care packages taking place. We have shown how the grey-fuzzy approach can be used to deliver forecasts, through explanation of the theoretic considerations and together with an applied example using data surrounding LTC activity across London. The results of which can help long-term planners understand the possible future pattern of demand. Compared with the grey approach, the grey-fuzzy methodology was shown to improve upon the MAPE and provide commissioners with powerful interval extrapolation so as to be able to identify best and worst case cost scenarios. In contrast to using linear regression, commonly used at the local level in the short run, to build cost estimates of LTC demand the combined grey-fuzzy methodology fits well in situations, such as LTC cost prediction, where available data is limited and hence many of the statistical assumptions that form the basis of the OLS regression may not hold in practice.

Given the importance of having reliable estimates of LTC demand at the local level, in addition to how such demand estimates can be used to generate cost savings for local health planners, we believe this thesis and the models proposed therein will be of great interest to local health care planners, those involved in the procurement of service type goods and problems involving uncertain and a lack of rich data upon which to base demand projections.

Finally, in chapter 8 we present a web-based decision support tool that incorporates elements of our dynamic commitment model and forecasting models presented in chapters 6 and 7 in order to help disseminate our findings to LTC planners, managers of the health care system and other interested parties. Our web-based tool was designed using input from London LTC commissioners and is inspired by the MVC (model-view-controller) design paradigm. Unlike previous decision support tools currently used in LTC, our planning system is designed from the onset to be highly modular so as to provide a safe way for further adaptations with respect to changes in data recording formats used by different commissioning organisations, new government or health sector reporting requirements and the addition of new analytical reports to aid commissioner understanding of the local LTC population.

9.2 Limitations and future work

Whilst we have concentrated on the theoretical development of a contract commitment model for LTC and proposed a model to predict LTC demand at the local level under incomplete information, our approach is subject to a number of limitations. Here we outline those limitations and suggest possible directions for future researchers in this field.

- In our contract commitment model we consider the case in which only 1 contract per provider, care group and intensity level may be formed. Furthermore, the contract size itself is fixed for the duration if it is in place. Thus we do not consider plans whereby it may for instance be optimal to have different sizes of contracts in different subsets of the planning horizon. In practice, for long-term horizons it may be more realistic to presuppose that multiple contracts could be formed with a single provider. An extension of our approach may therefore consider revising the formulation so that it could for instance be used in situations where the planning horizon extends to multiple years.
- We have only considered the impact of care home contracting and thus omitted the possibility of contracting with home care providers. In retrospect, we argue that such a feature could be incorporated by the addition of variables to represent demand for such services, the relative capacities of different home care providers and by modifying the intensity index such that it was extended by the number of possible home care intensities. We have not explicitly modelled home care in this version of the model due to uncertainty regarding the capacities of different home care suppliers. A further work would therefore involve the sampling of care home providers to provide a detailed survey of home care provider capacity.
- We have assumed that the capacity of care home providers is known and despite allowing for changes in capacity to take place, we have not considered other purchasers of care. In practice, care homes may have less than their published capacity available due to the purchasing of care from neighbouring boroughs or indeed self-funding individuals that choose to liaise with the care home directly. We have purposely limited our approach to a known capacity model to simplify the formulation and because of the level of aggregation in our data; in that for instance we are considering the cumulative demand across London health authorities. With that said we recognise that a suitable extension of this model may therefore be to add some notion of uncertainty into the provider capacities. Alternatively, depending on how the model is applied the capacities could be parameterised by considering the procurement offers that are received through the early stages of a

tendering process; in which providers specify different quantity discounts under different levels of commitment.

- With regards to solution time we found that our formulation in combination with the LINGO15 solver was able to generate local optimum solutions to moderately sized problems within 2 hours. As the planning horizon is extended beyond 12 periods however or as the number of care groups under consideration increases, we observed a significant increase in the run time due to the presence of additional nonlinear variables. An extension of this model may therefore consider how parts of the formulation may be linearized or whether suitable heuristics could be developed to lower the run time of out model. One direction could be to investigate whether the heuristics proposed for the CLSP, a closely related problem to our contract commitment model, could be adapted to consider plans in which there is no stock transfer between adjacent time periods.
- In chapter 7 we proposed using a hybrid grey-fuzzy methodology to forecast LTC demand at the local level. A key reason for doing so related to the lack of a rich dataset upon which to base our forecasts, for example in having more detailed information surrounding the nature of each individual's care needs. In previous studies that have modelled demand we have seen how the use of specific diagnosis codes, the ability of an individual to perform common activities of daily living and their life style factors have been significant predictors in determining their consumption of LTC resources. In our model, we have used existing groupings of LTC patients by care group under the assumption that those individuals within the same care group would have similar levels of need. A future investigation could test these assumptions by collecting a smaller dataset than we have used yet one which is more comprehensive with regards to patient level characteristics.

Publications during research

Journal

Worrall, P. and Chaussalet, Thierry. (2014) A structured review of long-term care demand modelling, *Health Care Management Science*, In press (Published online).

Conference proceedings

Worrall, P. and Chaussalet, T, J. (2015) A mathematical modelling approach to determine time and volume based purchasing commitments in long-term care. In. Proceedings of the 27th European Conference on Operational Research, 12-15 July 2015, University of Strathclyde, Glasgow.

Worrall, P. and Chaussalet, T, J. (2012) Development of a hybrid grey-fuzzy methodology to forecast future demand for long-term care. In: *High Tech Human Touch: Proceedings of the 38th ORAHS conference*, 16-20 July 2012, University of Twente, The Netherlands.

Worrall, P and Chaussalet, T, J. (2012) Forecasting long-term care demand under incomplete information: a grey modelling approach. In: *2012 25th International symposium on computer-based medical systems* (CBMS), 20-22 June 2012, Rome, Italy. IEEE. ISBN 9781467320498.

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Worrall, P and Chaussalet, T, J. (2011) Towards an optimal purchasing policy for nursing home placements in long-term care. In: *Operational Research Information National Health Policy: proceedings of the 37th ORAHS conference*. School of Mathematics, Cardiff University. ISBN 9780956915801.

Poster presentations

Worrall, Philip and Chaussalet, Thierry J. (2010) Perspectives on modelling the demand for long-term care in the UK. *Sixth IMA International Conference on Quantitative Modelling in the Management of Health Care*: The Institute of Mathematics and its Applications, 29 – 31 March 2010, Mary Ward House, London.

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Appendix A

A.1 Table of literature review results

Author, year	**Category**	**Study Objective**	**Data Sources**	**Aspects of LTC System(s) Studied**	**Methodology**	**Time Horizon**	**Key Findings**
(Batljan, Lagergren, & Thorslund, 2009)		To investigate how changes in educational level of the older people may affect future prevalence of severe ill-health among old people in Sweden.	Population projections by age, gender and educational level under different trends in mortality. Swedish national survey of living conditions (SNSLC) carried out in the period 1975-99.	The educational composition of the older population during the next three decades.	Educational level classified into three categories based upon the years of education received. Logistic regression models used to estimate differences in the prevalence of severe ill health in different age, gender and educational level cohorts. Demographic extrapolation used, with constant morbidity, to project future no of those with ill health and in need of LTC. Additional scenarios added to include falling rates of morbidity and severe health needs using educational adjusted trends in mortality.	2000-2035	Population projections which take into account level of education within each age-gender subgroup can lead to higher expected numbers of elderly people. Including mortality differentials by education level has a strong impact on the size of the older population and a significant impact on the number of people with severe ill health. The number of people in Sweden suffering from severe health needs in old age will increase by 14% when the combined effects of age, education and gender are considered. This increase is small relative to the 75% projected increase over the same period, 2000-2035 when differentials in mortality among specific age groups are not considered. Projections on LTC need that consider changes in population composition by education result in less than half the increase in the number of elderly persons with severe ill-health compared with demographic extrapolation alone.
(Caley & Sidhu, 2011)		To estimate the future	Age specific health care	Future LTC health care	Three proposed models.	2006-	The rate of increase in health care cost

		healthcare costs facing healthcare organizations due population ageing.	costs published by the Department of Health 2005. Sub-national Population projections , death registrations and health expectations at birth from the Office for National Statistics 2009	costs using routinely available data. LTC costs in the years before death. Impact of changes in life expectancy with respect to LTC costs	Expected annual health care costs are derived by calculating the sum of the product of the current average health care costs for different age bands and the projected number of people in each age band until 2031. In the second model, age bands were adjusted to reflect an increase in life expectancy In the third model, age bands were adjusted by the increase in LE in good health by using the ONS projections of disability free life expectancy.	2031	differs substantially depending on how projections of future life expectancy are incorporated The projected future cost of care was highest in the model which made not account for changes in life expectancy or disability free life expectancy. The estimated annual health care expenditure due to ageing was almost double if expansions in life expectancy were not considered.
(Chahed, Demir, Chaussalet, Millard, & Toffa, 2011)		To predict length of stay in long-term care and the number of patients remaining in care at a specific future time horizon.	Dataset containing funded admissions to NHS long-term care supplied by 26 London primary care trusts.	Length of stay of patients with different characteristics, including which type of care they currently receive, age and gender. Movements between different LTC settings	A continuous time Markov model of the flow of elderly residents within and between residential and nursing care is used to model the flow of LTC patients between two conceptual states and a discharge state in which the patient leaves LTC. The transition probabilities were estimated by fitting survival curves to historic patient movements in care to establish further sub states corresponding to	2007-2008	There were significant variations in the proportions of discharge and transition between types of care as well as care groups. The proportions of discharge from home care are higher than from placement The proportions of discharge from short-stay and medium-stay states for Physically Frail patients are lower than those of from Palliative care.

					short, medium and long stay states. By running the model over 356 days the estimated number of individuals remaining in each of the six defined care categories was used to predict the demand for care at each point in time.		
(Chung, et al., 2009)		Derive quantitative estimates of future LTC expenditure in Hong Kong	Thematic Household Survey 2004 Hong Kong Annual Digest of Statistics Hong Kong population Projections 2007-2036 Hong Kong Domestic Health Accounts 1989-2002	The future number of elderly people and the number requiring LTC Expenditure on LTC given individual factors that drive need The future inflated costs of LTC and the disability benefits for older people.	Macro-simulation approach based on PSSR model. Probability of using each service estimated for each age-sex profile using logistic regression. Total utilization is estimated for each service in each year and multiplied by the inflated unit cost of care. Future projections obtained using population estimates	2004-2036	Demographic changes have a larger impact than changes in unit costs of care on overall expenditure Expenditure expected to increase by 1.5% of GDP in 200 4 to 3% by 2036. By service mix, the proportion allocated to institutional care would increase from 37% in 2004 to 46% by 2036. Spending on LTC could be contained within 2.3-2.5% of total GDP in 2036 if institutional care could be substituted by home and day care services.
(Wittenberg, Comas-Herrera, Pickard, & Hancock, 2004)		Project expenditure on long-term care services for older people in the UK to 2051	Government Actuary's Department (Population Projections)	Share of LTC expenditure between the public and private sector. Impact of providing free personal and nursing care. Impact of changes in patterns of care with respect to support for informal care givers.	Linkage of two micro-simulation models (PSSRU and NCCSU) PSSRU – demand for long-term care under different socio-economic assumptions NCCSU – models long-term care charges and the	2000-2051	Demand for LTC sensitive to projected numbers of older people, future dependency rates and real rises in the unit costs of care Much uncertainty surrounding how far expenditure on LTC as a proportion of GRP will need to rise to meet demographic pressures

					ability of groups of older people to contribute towards care home fees.		
(Comas-Herrera, et al., 2006)		To investigate which factors drive LTC in several EU countries and the sensitivity of the projections to alternative future scenarios	Eurostat 1999 population projections. (in addition to official national population projections from each country studied)	Expenditure on LTC in UK, Germany, Spain and Italy. Future numbers of dependent persons (65+), their respective probabilities of using different types of LTC services and volume of services required.	Distinct macro-simulation (cell-based) model for each country's LTC system, reflecting differences in entitlement, level of informal care and coverage of publicly available LTC. Incorporates assumptions surrounding the future changes in the macroeconomic environment, including real costs of care.	2000-2050	Proportion of GDP spent on LTC to double between 2000 and 2050 (assuming that the age-specific dependency rates remain constant). Future demand sensitive to assumptions about the future number of older people and future dependency rates. Future cost sensitive to real unit costs of care and the availability of informal care.
(Comas-Herrera, Northey, Wittenberg, Knapp, Bhattacharyya, & Burns, 2011)		To investigate how incorporating expert views on dementia would affect projections of future expenditure on dementia related care for older people.	19 responses to a question from experts in the field of Dementia care and Alzheimer's disease. (Carried out via a Delphi process) Survey from the Medical Research Council Cognitive Function and Ageing Society 1998	Future demand and expenditure on long-term care by older people with dementia in England.	Updated version of the PSSRU CI (Cognitive Impairment) macro-simulation model used to represent the LTC system in England The views of the Delphi panel were incorporated into the model as assumptions.	2002-2031	Expert option suggesting that there will be a reduction in age-specific prevalence rates of dementia will reduce the number of future suffers and the associated total expenditure on care by approximately 16% compared with no change in prevalence of dementia.. The expenditure effects of reduced institutionalization combined with increased care assistant wages will in effect cancel each other out.
(Comas-Herrera, Whittenberg, Pickard, & Knapp, 2007)		To project the future number of older people with cognitive impairment in England, the demand for LTC and associated cost. To investigate the	Government Actuary's Department 2005 projections on the number of older people. Future marital status and cohabitation projections	Sensitivity of the factors related to LTC on projections of future demand and cost. Use of services by those with cognitive impairment	Three part macro simulation model, built upon previous PSSRU model. First part projects future population into cells	2002-2031	Unless more effective treatments for cognitive impairment are development made widely available, expenditure on LTC for patients with CI will rise significantly over the next 30 years. Demand for LTC care depends on

		impact of specific assumptions surrounding future trends.	from the Office for National Statistics 2005 Prevalence of cognitive impairment from Cognitive Function and Ageing Studies study (1998) Resource implications for CI from Resource Implication Study (1999) General Household Survey for number of people in receipt of informal and non-residential care Number of people in care homes from Department of Health 2003 data Information about people in hospital for long –stays taken from 2001 Census data.	and or disability. Future household composition and implications for levels of informal LTC	which are defined by age, gender, cognitive impartment and disability. Second component assigns receipt of LTC services to each cell in the first stage based on the probability of receiving such services. Third stage projects unit cost of services for each composition of services in the second stage at constant 2002 prices. Projections for future years revise unit costs by labor related inflation to derive future projections of total expenditure.		availability of informal care from family and friends. Total expenditure on care sensitive to the supply of informal care, where expenditure on LTC could represent 1.11% of GDP compared with 0.96% if the supply of informal care fell significantly. Projected future LTC expenditure highly sensitive to assumed rate of growth in real unit costs of care.
(Costa-Font, et al., 2008)		To examine the sensitivity of estimates of future long term care demand under different official population projections.	Euro Stat 1999 based population projections	Variability in expenditure predictions across the UK, Germany, Italy and Spain. Effects of demographic uncertainty on both population and expenditure predictions. Future fertility rates and its influence on the numbers of informal care givers.	Country wide macro simulation model based on the PSSRU model Future population projections are partitioned by age, gender and level of dependency A second model classified services used by dependent older people according to type of care received and setting	2000-2050	The projected numbers of dependent elderly people were higher in Germany compared to the official national projections. Whilst in Spain and the UK there was a little deviation. Differences in relative expenditure between the highest and lowest population assumption varied from 35-50%, with Italy exhibiting the smallest difference and the UK the largest. For Germany and the UK, the difference in projected expenditure on LTC in 2050

					Expenditure projections are extrapolated by applying unit costs of the services in each group and multiplying by the respected population projection. A number of parameters for instance prevalence rates of dependency by age can be adjusted to accommodate different future demographic scenarios. Results were compared for both high and low population projections.		constituted 1% of GDP under the low and high population estimates. There is evidence of cross country convergence with respect to the cost of LTC as a percentage of GDP in Spain, UK, Italy and Germany. Growth in LTC expenditure over the period varied from 70-90% in the most optimistic scenario, to 150-180% in the most pessimistic.
(Fukawa, 2011)		To project long-term care expenditure in Japan between 2010-2050 by analysis of household transition	Population projects for Japan from 2006-2055, National institute of population and social security research, 2007. National Household survey Japan 2004.	Numbers of elderly people according to dependency and/or other living situations. Future cost of LTC relative to total healthcare expenditure The effect of the ageing of the "baby boomers" on LTC demand The household ratio or parents to children to asses potential future levels of informal care	A dynamic micro simulation model which transitioned individuals forward in time, subject to stochastic events taking place. An initial fixed population was simulated according to a sample taken from census data in 2005. Individuals were transitioned through the model according to estimated probabilities of life changing events in addition to changes in household circumstances. Transition probabilities	2010-2050	The proportion of those elderly who stay in institutions will steadily increase until 2050. The sum of health and LTC expenditure will increase from the preen 7.7% of GDP in 2010 to 11% of GDP by 2040 largely due to increased LTC expenditure. The future level of expenditure on LTC is sensitive to assumptions about the level of service use by different levels of dependency. Even if service use by level of dependency falls uniformly over the period by 20%, LTC expenditure in 2050 will be as a percentage of GDP will increase by 138% by 2050 when compared with 2005 levels.

					dependant on age, sex and level of disability for those aged 65 and over. Levels of dependency were classified into four groups and associated with the need for LTC. Movements from these levels and into an institution were dependant on each individual's personal circumstances. Future costs derived by applying future age specific population projections for each of the LTC insurance bands.		
(Hakkinen, Martikainen, Noro, Nihtila, & Peltola, 2008)		To investigate the claim that population ageing will not have a significant impact on healthcare expenditure	Finnish population registration system Finnish hospital discharge register. Finnish death register Registers from the Finnish Social Insurance Institution Finnish hospital benchmarking project	Impact of ageing on healthcare expenditure Impact of proximity to death on healthcare expenditure	Annual healthcare expenditure calculated for each individual aged 65 or over from 1998 until end of 2002 using 2000/01 deflated prices. Likelihood of using LTC service found using a logit/profit model based on patient characteristics. OLS regression model used to then estimate expenditure given patient predicted to require LTC using a general to specific selection of patient characteristics. Future LTC expenditure	2016-2036	LTC patients (excluding residential and home care) accounted for 55% of total healthcare expenditure despite the proportion aged 65 or over being 7%. Age has an important positive and increasing effect on the probability of being a LTC user. Females had a higher risk of needing LTC compared with males. Home care and home services excluded due to lack of national data. Projections based on the naïve age and gender specification showed an estimated annual LTC cost increase of 2.2% by 2036. Taking into account proximity to death, the expected annual increase in total LTC

					projects obtained by multiplying calculated age-gender specific expenditure according to survival status by future population estimates. In addition, an additional model where the probability of using LTC was delayed for three years was also used to consider falling rates of dependency with age.		cost was found to be lower at 1.9%. The model's projections were found to sensitive to the probability of individuals being in need of LTC. If LTC could be delayed by 3 years it was found that costs would decrease by 12% although part of this reduction would be met by a rise (2%) in other non-LTC healthcare costs.
(Hare, Alimandad, Dodd, Ferguson, & Rutherford, 2009)		To predict the future number of patients in different home and community care categories in British Columbia	Future population projections from "Population Extrapolation for Organization Planning with Less Error" (2007) provided by the British Columbia Ministry of Health Wealth demographics from Statistics Canada (2008) Quantity of non-publically funded home and community care estimated from telephone survey of all privately run facilities in British Columba (2007) Home and community care activity data from April 2001-March 2005 by client group provided by the British Columbia	Distribution of patients between different types of care, including assisted living environments and home care. Distribution of privately funded care to publically funded care.	Multi-state deterministic Markov model Home and community care groups divided into ten categories, 8 of which represent publicly funded care. Patients are not individually tracked through the system but rather the collective behavior of each care and age specific group is studied. Patients move between care categories and leave the model according to the age-independent transition rates. Movement between public and privately funded care according to	2002-2031	The model predicts that whilst patient counts will continue to rise over the next 20 years they will not reach their 2002 high levels until 2015. Without taking into account the privately funded care, the models prediction accuracy was poor as a number of clients are believed to use some mixture of both public and privately funded care. No attempt made to marry client counts with service loads for the prediction of budget requirements. The available of services has increased over the period and hence the six fold growth in HCC between 2002-2004. It is difficult to model the numbers of people who are seeking care but not receiving at the current time.

			Ministry of Health.		projected wealth distribution of the province. Movement between services based on historical usage of home care vs. assisted environments using fixed transition rates, and then dividing movers between public and non-public services. Transition probabilities estimated from historical data. Population projections used to estimate no of patients arriving to the system in each period.		
(Karlsson M. , Mayhew, Plumb, & Rickayzen, 2006)		To analyse the sustainability of the UK system for provision of long-term care in the light of the changes in demography and health status among older people that are expected in the future	OPCS survey of disability in Great Britain (1988) Health survey of England, Bajekal M. Care homes and their residents. London: The Stationery Office; 2002 for types of formal care by age and disability Costs of formal care Laing, Buisson. Calculating a fair price for care—a toolkit for residential and nursing care costs. London: Rowntree; 2001. and Netten A, Rees T,	Estimate of the future cost of LTC to the public purse as proportion of income tax The potential surplus or shortfall in the number of informal carers relative to the demand for informal care.	Multicomponent projection model based on Multistate disability model proposed by Rickayzen and Walsh (2002) The disability model generates an estimate of the number of individuals of each gender cohort split by age and severity of disease for each year of the projection period. People are transitioned over time into different levels of disability e.g. people becoming more disabled and people	2000-2050	Given our central assumptions, the demand for long-term care will start to increase considerably about 10 years from now, and reach a peak somewhere after 2040. The most important increase will be in informal care, since the number of older recipients is projected to increase from 2.2 million today to 3.0 million in 2050. In relative terms, the increase is similar in all care settings, amounting to between 30 and 50% compared to the levels today. The most noticeable increase is in formal

			Harrison G. Unit costs of health and social care. PSSRU; 2001.		dying. Trend data on healthy life expectancy used to update transition probability according to how rates of disability may improve. Different assumptions surrounding how these transition rates changes according to how mortality , speed of increased disability and level of disability may improve over time. Cohots of disability are then mapped to care settings. Estimates cost of LTC to the public purse as a percentage of income tax and the demand for informal care relative to no of care givers.		home care, however, which is projected to be almost 60% greater than the current level in 2040. Yet, since those services are relatively cheap, this item has a relatively small impact on total spending. The increasing demand for care will influence total costs. The total costs of formal long-term care defined in this paper amount to around £ 11 billion today and will, in constant prices, increase to around £ 15 billion around 2040. It transpires that our findings are relatively sensitive to the assumptions made concerning the trend in future disability rates in the older population. When we contrast our baseline scenario with a more pessimistic one—assuming no future health gains—we find that total costs keep on growing for longer and peak only in 2051 at a total of £ 20 billion (£ 80 billion when informal care is also considered). This translates into an implied tax rate of 1.8%, which is considerably higher than in the baseline scenario (1.3%). Regarding informal care, we find that under the baseline and optimistic scenarios, there is likely to be a sufficient supply of care to meet demand provided caregiving patterns remain as they are. However, if female care-giving patterns converge to those of males, then under the baseline health improvement scenario, there would be a shortage of between 10 and 20 million hours of care per week

(Ker-Tah & Tzung-Ming, 2008)		Predict values of the disability rate of the aged from 2006 to 2011 to estimate the future population in need of long-term care	Historical rates of disability in Taiwan from the Ministry of the Interior and the Department for Statistics over the period 1991-2006	The rates of disability in the Taiwanese elderly population that would require LTC services.	Gathered data on rates of disability in the elderly population and used a Grey forecasting model to forecast future rates of disability under different assumptions about the growth in the disability rate over time. Estimates of future rates of disability used to ascertain the size of the population in need of LTC in the future	2006-2011	The continual increase in the disability rate of the aged leads to a dramatic increase in the growth rate of the aged demanding LTC services over the period studied. A 1462% increase in the rate of aged related disability (from 1991-2011) far exceeds the expected growth rate in the aged population.
(Kinosian, Stallard, & Wieland, 2007)		Project long-term care service usage by enrolled veterans	Veterans Health Administration Survey National Long-Term Care Survey National Nursing home Survey National Health Interview Survey.	Demand and cost of nursing home care and community-based long-term care Services Persons who report receiving human or mechanical assistance to help with activities of daily living ADLs and instrumental activities of daily living.	Used a random sample of the Medicare-eligible VA population, to standardize the ADL and IADL disability levels from the 2002 VA Survey of Enrolees	2002-2012	The level of long-term-care use generally follows the distribution of disabilities in a population
(Lagergren M. , 2005)		Investigate the impact of changes in factors related to future LTC resource need	Official National Statistics on the Provision of Long-Term Care. Swedish National Survey on Living Conditions (ULF) ASIM Study in Solna municipality (1984-1994)	Consumption of different forms of LTC services by age, gender, marital status and disability. The future provision of LTC services in relation to care needs Balance of institutional	ASIM III-model subdivides the population into several cohorts by age group, gender, marital status and degree of ill health. For each group the number of persons in receipt of LTC for older	2000-2030	The population growth in the period 2000-2015 concerns mainly the younger old and thus does not have a large effect on the care service costs. Cost increases from 2020 onwards stem from 85+ year group, for the youngest old the costs diminish. Over period 2000-2030 35% increase in

			The Swedish National Survey on Ageing and Care at Kungsholmen, Stockholm (2001) Population projections from Statistics Sweden	and non-institutional care.	persons according to four different levels noted. Prevalence of ill health for each age, gender, civil status subgroup used to create a health index of four degrees (full, slight, moderate, and severe) Forecasts generated by multiplying population projections in each subgroup by respective proportion of persons in each group receiving services in 2000 levels. Different future scenarios surrounding ill health used to make projections. Two-step tend extrapolation of severe ill health from survey on living conditions.		less than 1hour of public services in the community setting per day. 27% more people in instructional care More intensive community care is less affected by projected increases in demand. By 2030 the oldest age group 85+ will account for 60% of all LTC expenditure from 50% in 2000. Proportion of married rise from 17% to 22% given mortality is expected to fall more rapidly for men than for women. Pessimistic future ill-health 69% increase in cost vs 25% increase in cost. At present 2.6% of GDP spent on care, could rise to 3.3-4.4% depending on future ill-health scenario.
(Macdonald & Cooper, 2007)		To estimate the future level of demand for care home placements from those suffering from dementia	Survey of 445 residents drawn randomly from 157 non-EMI nursing homes in South-East England. Commission for Social care and Inspection The Medical Research Council Cognitive Function and Ageing Society. UK Census 2001	The number of dementia cases in England and their associated care needs up to 2043.	Results from a local survey on the incidence of dementia are combined with age and sex specific prevalence ratios and extrapolated to estimate demand for dementia beds at the starting period. Future levels of demand are estimated by applying population projections under different assumptions surrounding	2003-2043	Assuming 50% of patients aged 60+ in care homes suffer from dementia, the number of dementia beds required would be around 740,000 by 2023 and over one million by 2043.

					the prevalence rate of dementia in care homes.		
(Malley, et al., 2011)		To examine the effect of different assumptions about future trends in LE on the sustainability and affordability of both the pensions and LTC system	2001 General Household Survey (GHS) 2002/3, 2003/4 and 2004/5 rounds of the Family Resources Survey (FRS) 2008 budget report (HM Treasury 2008).	Likely future cost to the public purse private expenditure on LTC LTC by source of expenditure Compare with GDP	To project expenditure on LTC, we use two models: the CARESIM micro-simulation model and the Personal Social Services Research Unit (PSSRU) aggregate LTC finance model. The PSSRU model is cell-based: it divides the current and projected future population into a large number of sub-groups or 'cells'. It simulates future demand for LTC and disability benefits for each of these groups, based on analysis of a sample of older people from the 2001 General Household Survey (GHS)4. Adjustments are made to the GHS analysis to include the residential care population and to reflect changes in the targeting of publicly-funded care provision since 2001 (Wittenberg et al., 2006). CARESIM simulates the incomes and assets of future cohorts of older people and their ability to contribute towards care home fees or the costs of home-based	2007-2032	expenditure on pensions and associated benefits is projected to rise in future years because of the increasing numbers of pensioners – more recent projections allowing for the further policy changes described above confirm this, and show even faster growth expenditure on LTC is projected to rise, although at a faster rate than pensions expenditure. The faster rate of growth in LTC expenditure is partly a consequence of the faster rate of growth of the oldest old group compared to the older population as a whole, as it is at the oldest ages where need for care is the greatest

					care, should such care be needed (Hancock et al., 2003). It is based on a pooled sample of older people from the 2002/3, 2003/4 and 2004/5 rounds of the Family Resources Survey (FRS) with money values updated to the base year (here 2007) 5. Together these two models can be used to project future expenditure on LTC by source of expenditure, under different funding reform options. The PSSRU model output on the characteristics of people requiring LTC is used as input to CARESIM to adjust the FRS sample to be representative of people receiving different LTC services in the projection year. CARESIM then simulates for each type of service the ability of older people to contribute to their care costs and the source of income used to pay for care. CARESIM output is used to break down expenditure in the PSSRU model into its constituent components and funding sources, i.e.		

					NHS, Personal Social Services, social security disability benefits and private money (Hancock et al., 2007). The projected levels of expenditure by each of these sources are compared with projected economic output, Gross Domestic Product (GDP).		
(Manton, Lamb, & Gu, 2007)		How trends in disability prevalence and in inflation-adjusted per capita, per annum Medicare costs affected total projected medicare costs	1982, 1984, 1989, 1994, and 1999 National Long Term Care Surveys (NLTCS) -roughly 20,000 persons sampled in each of the NLTCS, of those 65+	Implication of recent disability declines and their possible continuation for future Medicare costs	Applied a grade of membership analysis to 27 measures of disability from the 1982 to 199 9 National Long term care surveys,. This identified 7 disability profiles for which individual scores were obtained. These were used to extrapolate future Medicare spends by assuming different trends in the level of disability across the different groups.	2004-2009	At ages 85+ relatively more LTC and Medicaid expenditures are incurred for labor-intense maintenance and palliative care 16% savings
(Martini, Garrett, Lindquist, & Isham, 2007)		To project the impact of populating aging on total US health care cost per capita	1.2 million years of health care plan data from the HealthPartners database 2002-2003 US Census Bureau population projections 2000-2050 Medical Expenditure Panel Survey 2001	The monthly per capita costs of LTC covered by Medicare using insurance claims data. Per capita pharmacy costs associated with various conditions in LTC.	Medical and pharmacy claims data aggregated into individual episodes of care which are grouped by treatment group The total cost of each treatment group is added to their respective higher level illness or condition category.	2000-2050	Per capita costs a s result of ageing will increase by 18% from 2000 to 2035 as baby bombers and retirement and then level of as the age structure of the population stabilizes. 80% of the increase in per capita costs can be explained by 7 of the 22 illness categories, including: heart and vascular conditions, lung conditions and neurologic disorders.

					Monthly per capita costs estimated for each gender, age band and condition category and added together to estimate annual costs per capita. Future cost extrapolated by multiplying projections of population in each gender-age brand and multiplying by MEPS adjusted per capita costs.		Pharmacy costs were estimated to account for 1.5% of all care costs. The cost of care for males and females in the 85-89 year old group are 4.4 and 2.7 times as large as the per capita costs for the reference group of females aged 40-44.
(Peng, Ling, & Qun, 2010)		To project the future need of long-term care due to changes in demography and health status among the oldest Chinese	Chinese Longitudinal Healthy Longevity Survey, 1998, 2000, 2002 United Nations World Population Prospects of China in 2008 for population projections (2010-2050) assuming medium fertility and mortality		Calculated the observed self-rated health status transition probabilities for individuals with age I and gender j. Simulated this process using a non-homogeneous Markov process to obtain the simulation transition probabilities this was done separately for each initial health status k, using five-group discriminate analysis to estimate the probability of being in each of the five health status l 2 years later, as a function of a person's gender i and initial age j Health status transition probabilities were used to calculate the remaining	2010-2050	8066 thousand persons aged 80+ need long-term care in 2010, while in 2050 this number will increase to 42,581 thousand The care need person year number among males will increase from 23,159 in 2010 and to 115,460 in 2050, whereas the female person year number will increase from 40,401 to 208,210, and the total number for both genders will increase from 63,560 to 323,670, which implies a growth of more than 4 times during the 40 years. If we assume that the average care expenditure is 15 US dollars (about 100 Yuan RMB) per hour in 2010, then the total care expenditure rises from around 83.52 hundred million dollars in 2010 to around 425.30 hundred million dollars in 2050 (in 2010 prices). We have been able to show that, given our assumptions of average care cost is 15 US dol-R. Peng et al. / Health Policy 97 (2010) 259–266 265lars per hour, the care

					years of life and remaining years of healthy life in terms of age, gender and initial health. L Long-term care expenditures can be calculated by multiplying unhealthy person-years number by the annual average expenditure of care In order to define what is healthy, we made a split between good and fair because the two groups had great differences in mortality. We used Mantel–Haenszel statistic to test mortality relative risk (RR) between two health states. Results showed that the mortality of the elderly people who rated their health fair or poor significantly increased compared to those in the good category except for women aged 85–89 (RR > 1, P-value < 0.05). People who rated their health very good and good had		expenditure for long-term care will increase from 83.52 hundred million dollars to 425.30 hundred million dollars from 2010 to 2050. That means the total amount will grow more than 4 times over the next the 40 years, without considering inflation. The results also show that long-term care need is on the rise regardless of gender, and that the absolute number and increase rate of female care need are higher than those of male.

					no significant difference in mortality risk except for women aged 85–89 and 95–99, and men aged 80–84 (RR > 1, Pvalue > 0.05).		

A.2 Fields Collected as Part of the LTC Data Request across London

Variable Name	Definition
DOB	The date of birth of the patient
Care Group	The name of the care group assigned to the patient by the NHS
Payment Band	The funding arrangement in place, e.g. 100% NHS funded or jointly funded
Provision Type	The location where care will be provided, e.g. in the patient's own home or in a care home
Provision Start Date	The date upon which the patients care will start
Provision End Date	The date upon which the patients care ended due to death or cancellation
Discharge Reason	The reason for the cessation of the patient's care
Weekly Rate	The cost of the patient's care package in GBP
Ethnicity	The ethnic group to which the patient belongs
Gender	The sex of the patient
Host PCT	The name of the PCT corresponding to the location in which the patient's care will take place.
Commissioning PCT49	The name of the PCT that is responsible for funding and arranging the individual's care package.

[49] The commissioning PCT and Host PCT may refer to different organisation in the case that the commissioning PCT has placed an individual in care outside of their own catchment area.

A.3 Data Cleaning Phases for the London LTC Data Set

Phase No.	Phase	N Cases Removed	Remaining Cases
1	Initial import	0	13,700
2	No care group specified	363	13,337
3	No provision type specified	626	12,711
4	Weekly rate <= £5000 and weekly rate >= 0	2,990	9,721
5	Provision start date after 01.01.1999 and either provision end date blank or provision end date after 01.01.1999	399	9,322
6	Provision start date before 01.01.2010	1	9,321
7	Payment band not set to CHC funded	2,911	6,410
8	Provision end date after start date or blank	76	6,334
9	At least one recorded day in care	41	6,293
10	If provision end date set it must be on or before 1st April 2009	1	6,292
11	If weekly rate less than £112	744	5,548

A.4 Solution methods for the CLSP

Exact methods

Apart from the branch and bound method used to solve the relaxed version of the CLSP, two other exact methodologies have been proposed. The first by (Barany, Van Roy and Wolsey 1984) and later described by (Leung, Magnanti and Vachani 1989) is known as the cut-generation technique. In essence the cut-generation technique involves the addition of strong inequalities, as in (9.1) which says that the sum of demand in future periods must be less than or equal the maximum production of item j plus inventory carried over from the previous period, along the interval $[k, k+1, \ldots, t]$. As there are at most $O(n^2)$ such equalities of this type some or all can be added a priori to the formulation. Combinations of equalities like (9.1), a variable upper bound constraint, with (5.5) can be used to generate cuts in the solution space and allow improvement in the lower bound when the resulting reformulation is modelled using the branch and bound technique (Belvaux and Wolsey 2001).

$$I_{jk-1} + \frac{\sum_{t=k}^{T} C_t x_{jt}}{p_j} \leq \sum_{t=k}^{T} d_{jt} \quad (j = 1, \ldots, J; \quad t = 1, \ldots, T) \tag{9.1}$$

A second approach by (Eppen and Martin 1987) reformulates the original CLSP problem as a graph-based representation, adding additional constraints and variables but providing a much tighter formulation of the original CLSP linear programming relaxation. Their shortest path formulation used $O(n^3)$ variables and was solved by first considering the LP-relaxation, before applying the branch and bound method in the final stage. Despite the potential of both the cut-generation and graph-based methodologies in improving the quality of the solution obtained, versus the traditional MILP formulation, both require significant computation effort and neither have been shown to be able to solve real-world problems in reasonable amounts of time: other than those based on using small instances (Karimi, Fatemi Ghomi and Wilson 2003).

Among the general class of CLSP problems, (Van den Heuvel and Wagelmans 2006) have pointed out that a classification of CLSP problems has emerged to highlight the degree of complexity associated with solving CLSP problems under different structural assumptions. Under the notation $\alpha \,/\, \beta \,/\, \gamma \,/\, \delta$ for the CLSP where α represents setup costs, β holding costs, γ production costs and δ capacity, the abbreviations Z, C, NI, ND and G can be used to indicate how such features of the CLSP behave. Here the abbreviations stand for zero, constant, non-increasing over time, non-decreasing over time and no-prescribed pattern respectively. The authors noted that for specific sets of parameters the CLSP has been shown to be solvable in polynomial time. Specifically, (Florian and Klein 1971) presented an $O(T^4)$ algorithm for the G/G/G/C case, later improved by (Van Hoesel and Wagelmans 1996) to $O(T^3)$. Indeed, (Bitran and Yanasse 1982) showed in the original formulation of the CLSP that $O(T^4)$, $O(T^3)$, $O(TLogT)$ and $O(T)$ algorithms could be used to solve NI/G/NI/ND, NI/G/NI/C, C/Z/ND/NI and ND/Z/ND/NI formulations respectively. Furthermore, the special case of NI/G/NI/ND has been reduced in complexity to $O(T^2)$ by (Chung and Lin 1988).

Heuristic-based approaches

Aside from the MILP solution method to the CLSP, the cut-generation technique and reformulation of the CLSP, the other main class of solution methods that have been proposed involve the use of heuristics (Karimi, Fatemi Ghomi and Wilson 2003). Here a heuristic refers to a methodology that includes any strategy to find a solution to a problem that is not guaranteed to be optimal that trades some proportion of accuracy and precision for speed in computation. Heuristics are therefore best utilised in situations where finding an optimal solution, as in the case of the CLSP, may be infeasible due to the general formulation being NP-Hard. Here we identify some of the most common types of heuristics developed to solve the CLSP.

<u>Fix and relax heuristics</u>

Fix and relax (F&R) heuristics are those approaches that attempt to reduce the number of binary variables in the CLSP, stemming from presence of setup cost and modelled using the variables x_{jt} , by dividing the CLSP into a series of smaller sub-problems such that the number of binary variables considered simultaneously is reduced. Despite F&R heuristics presenting a computationally efficient way to solve the CLSP, setup decisions are only optimized on a small subset of the available periods. Within the literature, F&R heuristics are also referred to as period-by-period approaches.

The pioneering work within the F&R class of heuristics was by (Eisenhut 1975) in which a single pass of periods 1 through T is conducted to identify the necessary production to meet demand across all items at time t. Should any excess capacity at time t arise it is used to service demand in future periods according to an item-based priority index. When moving to the next period all previous period-based solutions to the CLSP are held fixed until production in the final period T is evaluated.

More recently, (Sürie and Stadtler 2003) perform a time-based decomposition of the CLSP problem in which a series of overlapping planning-windows is constructed. For each time-window the CLSP is solved by assuming all earlier periods have been planned and thus holding all decision variables in earlier time-windows constant. Here constraints concerning variables in periods after the current time-window are not enforced and thus capacity requirements in future periods are only approximated. A related heuristic by (Federgruen and Meissner 2007) uses an initial time-window that is repeatedly enlarged until it spans the entire time-horizon. Each iteration of the problem is solved optimally for decision variables related to the last τ periods, in contrast variables relating to t ... $t_{T-\tau}$ are held constant. The heuristic stops when the end of the planning horizon is reached. Other heuristics that are based on F&R include: (Absi and Kedad-Sidhoum 2007), (Sahling, et al. 2009) and (Wu, Shi and Duffie 2010).

Rounding heuristics

Rounding heuristics involve continuous relaxation of the MIP formulation of the CLSP. Once a continuous solution has been found the fractional binary variables are then rounded to obtain a feasible solution. Two key papers that have developed a rounding based heuristic include (Eppen and Martin 1987) and more recently (Alfieri, Brandimarte and D'Orazio 2002). In both papers the general approach concerns (1) determining thresholds for the binary setup variables, (2) rounding up or down the setup variables that meet these thresholds, and (3) solving the CLSP with those variables meeting the threshold held fixed whilst performing a branch and bound search with the remaining binary variables.

Improvement heuristics

Improvement heuristics are characterised by the generation of an initial infeasible solution to the CLSP, a solution that may be generated by ignoring capacity constraints. Once an initial solution is found, the solution is iteratively adjusted in an attempt to meet constraints previously ignored. In this step production is shifted between periods based on the additional cost that would be incurred. In the final step, an attempt is made to modify the solution so as to generate cost savings without breaching infeasibility. One of the earliest examples of an improvement heuristic was presented by (Dogramaci, Panayiotopoulos and Adam 1981) which shifted production by considering changes in costs across all items throughout the planning period. To limit the number of possible shifts that would be explored, (Karni and Roll 1982) defined conditions on the types of shifts that would be most effective and specified 10 different types of shifts that should be considered.

An approach which considers the change in cost by the reduction in capacity overuse was presented by (Trigeiro 1989) and named the *Simple Heuristic*. Under this approach the method works both backwards and forwards over the planning horizon in search of capacity violations. When a capacity violation is found production is shifted either forward or backward to a period in which there is excess capacity. The heuristic then moves on to the next period once all overtime in the incumbent period has been removed. The *Simple Heuristic* has since been modified by (Campbell and Mabert 1991) and (Hindi, Fleszar and Charalambous, An effective heuristic for the CLSP with set-up times 2003) to fix the length of time between periods in which production of an item takes place at a constant value.

Mathematical programming heuristics

Heuristics that attempt to solve the CLSP using optimum seeking mathematical programming have been a popular research theme within the literature. In part this may be explained by the advantage of their relative ease of application to a variety of CLSP problems and extensions, the availability of commercial solvers which allow some customisation and the ability to generate a lower bound on the optimal production plan to help assess the quality of a given solution.

Within the class of mathematical programming heuristics several sub-classes of approaches exist, including: those based on relaxation of the constraints so as to reduce the CLSP to a series of N single item uncapacitated lot-sizing problems (Thizy and Van Wassenhove 1985), (Millar and Yang 1994), (Chen and Thizy 1990); those based on using branch-and-bound integer optimisation with reformation and or variable redefinition (Hindi 1995), (Armentano, Franca and de Toledo 1999); and those based on set partitioning and column generation, (Cattrysse, Maes and Van Wassenhove 1990), (Dzielinski and Gomory 1965), (Salomon, Kuik and van Wassenhove 1993), whereby a master problem in which capacity constrains is revised with convex combinations of single item uncapactiated production plans whilst they do not exceed known capacity constraints.

<u>Metaheuristics</u>

A relatively new and niche area of CLSP research has investigated the use of metaheuristics for solving the CLSP. Metaheuristics can be thought of as more generalised heuristics that are both effective in finding solutions to complex optimisation problems and in their general applicability to broad classes of problems (Ólafsson 2006). Compared with heuristics, which require specialist knowledge of the problem domain and have been shown to suffer from the solution search getting stuck in local optima, metaheuristics require far less domain specific knowledge and can provide a more effective way to search across the entire solution space (Buschkühl, et al. 2010).

To date, several different metaheuristics have been used to solve variations of the CLSP, including but not limited to: simulated annealing (Özdamar and Barbarosoglu 2000), (Berretta, França and Armentano 2005); tabu search (Kuik, et al. 1993) (Gopalakrishnan, et al. 2001); and genetic algorithms (Hung and Chien 2000) (Xie and Dong 2002).

A.5 Model 1 Lingo Code

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MODEL:
SETS:

ITEMS / 1..2/;
INTENSITIES / 1..2/;
PROVIDER / 1..2/;
PERIODS / 1..2/;

CLINK(PROVIDER, ITEMS, PERIODS) : CAPACITY;
DLINK(ITEMS, INTENSITIES, PERIODS) : DEMAND;
PLINK(ITEMS, INTENSITIES, PROVIDER, PERIODS) : PRICE;

ALINK(PROVIDER) : ALPHA;

PDLINK(PROVIDER, ITEMS, INTENSITIES): USED_PROVIDER;
PDELINK(PROVIDER): PROVIDER_EXCESS;

!DECISION VARIABLES;
QLINK(ITEMS, INTENSITIES, PROVIDER, PERIODS) : QUANTITY;

ENDSETS

DATA:
                 !PERIOD 1 (ITEM1,ITEM2)     PERIOD 2 (ITEM1, ITEM2);

CAPACITY   =     25        50        50        50     !(PROV 1);
                 40        20        50        10;    !(PROV 2);

|
                 !INTENSITY 1;       !INTENSITY 2;
                 !T = 1;   !T=2;     !T = 1;   !T=2;
DEMAND     =     30        35        30        35     !(ITEM 1);
                 30        30        30        30;    !(ITEM 2);

                 !(PROV1); !(PROV1); !(PROV2); !(PROV2);
PRICE      =     500       500       400       400    !(INTENSITY 1); !(ITEM 1);
                 1000      1000      1100      1100   !(INTENSITY 2);
                 750       750       400       400    !(INTENSITY 1); !(ITEM 2);
                 1400      1400      600       600;   !(INTENSITY 2);


ALPHA      =     -0.10     -0.15;
ENDDATA

!OBJECTIVE FUNCTION;
MIN = @SUM(QLINK(I, L, K, P): (1 - ALPHA(K)) * PRICE(I, L, K, P) * QUANTITY(I, L, K, P));

@FOR(PDLINK(K, I, L):
     @SUM(QLINK(I, L, K,P) : QUANTITY(I, L, K, P)) = USED_PROVIDER(K, I, L));

! DEMAND CONSTRAINT;
@FOR(DLINK(I, L, P):
     @SUM(QLINK(I, L, K,P) : QUANTITY(I, L, K, P)) >= DEMAND(I, L, P));

! CAPACITY CONSTRAINT;
@FOR(CLINK(K, I, P):
     @SUM(QLINK(I, L, K,P) : QUANTITY(I, L, K, P)) <= CAPACITY(K, I, P));

! NON-NEGATIVITY CONDITION;
@FOR(QLINK(I, L, K, P) :
     QUANTITY(I, L, K, P) >= 0);

!INTEGER CONDITION;
@FOR(QLINK(I, L, K, P) :
     @GIN(QUANTITY(I, L, K, P)));
END
```

A.6 Model 1 Microsoft Excel Solution Report

Microsoft Excel 14.0 Answer Report
Worksheet: [Model 1 Example in Excel.xlsx]Sheet1
Report Created: 03/12/2015 21:53:39
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: Simplex LP
Solution Time: 0.016 Seconds.
Iterations: 18 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Min)

Cell	Name	Original Value	Final Value
C2	TC	249875	249875

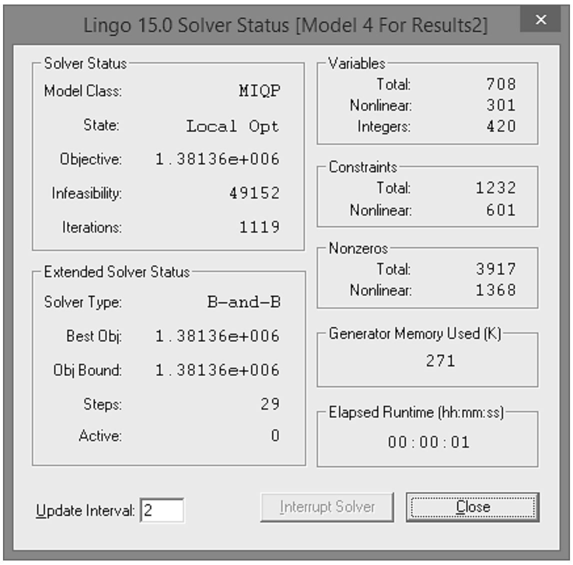
Variable Cells

Cell	Name	Original Value	Final Value	Integer
F7	A Assigned	0	0	Integer
F8	A Assigned	0	0	Integer
F9	B Assigned	30	30	Integer
F10	B Assigned	35	35	Integer
F11	A Assigned	20	20	Integer
F12	A Assigned	20	20	Integer
F13	B Assigned	10	10	Integer
F14	B Assigned	15	15	Integer
F15	A Assigned	10	10	Integer
F16	A Assigned	20	20	Integer
F17	B Assigned	20	20	Integer
F18	B Assigned	10	10	Integer
F19	A Assigned	30	30	Integer
F20	A Assigned	30	30	Integer
F21	B Assigned	0	0	Integer
F22	B Assigned	0	0	Integer

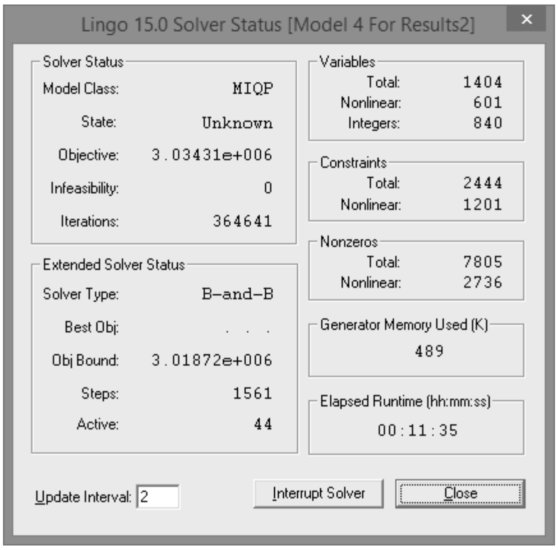
A.7 Table detailing care homes used in the application of model I

Nursing Home	Ownership	Postcod	User Rating	CQC	Avg. Scaled	α	Price	FM	LD	OM	PA	PD	PF	Total
Farm Lane	Care uk	SW6	0	3	0.375	-0.566	1.020	0	0	26	0	0	40	66
Red Court Nursing Home	Bupa	CR0	8.2	3	0.785	-0.092	0.980	0	0	5	15	10	5	35
Queens Court	Barchester	SW19	7.1	3	0.73	-0.156	0.970	0	0	0	0	0	37	37
Upminster Nursing Home	Havering Care Homes	RM14	7.5	3	0.75	-0.133	1.040	0	0	15	0	0	20	35
Broadlands Nursing Home Ltd	Prime Care Group	SM2	7.7	2	0.635	-0.266	1.000	10	0	10	0	5	0	25
Hadley Lawns Residential &	Bupa	EN5	7.9	3	0.77	-0.110	1.000	22	0	0	0	22	0	44
Candle Court	Tlcgroup	NW2	7.8	3	0.765	-0.116	0.960	30	0	20	10	0	28	88
Highbury New Park	Care uk	N5 2DS	7.6	3	0.755	-0.127	0.960	25	0	20	0	0	10	55
Stamford Nursing Centre	Bupa	N18	7	3	0.725	-0.162	1.050	20	20	20	10	20	0	90
Ashwood Care Centre	Lifestyle Care	UB4	6.8	3	0.715	-0.173	1.040	0	0	10	20	20	20	70
Greenhill Nursing Home	Misson Care	BR1 3SJ	7	3	0.725	-0.162	0.950	0	0	20	0	4	40	64
Albany Nursing Home	Lukka Care Homes	E10 7EL	7.1	2	0.605	-0.301	1.010	0	0	20	0	20	20	60
Groveland Park Care Home	Queensgate	DA7	9.8	3	0.865	0.000	0.960	0	0	20	20	13	0	53
Ebury Court Residential Home	Privately Owned	RM7	9.8	3	0.865	0.000	1.020	0	0	19	20	0	0	39
Barchester Park View Care Home	Barchester Healthcare	RM10	9.4	3	0.845	-0.023	0.990	9	20	20	20	20	20	109
Ash Court Care Centre	Forest Healthcare	NW5	9.1	3	0.83	-0.040	1.030	10	10	10	12	10	10	62
Dr French Memorial Home	Dr French Memorial	N12	9	3	0.825	-0.046	1.010	0	0	17	0	0	20	37
Barchester Lynde House Care	Barchester Healthcare	TW1	8.9	3	0.82	-0.052	0.970	0	30	16	30	0	0	76
Hawthorn Green Residential and	Sanctuary Care Ltd	E3 3DB	8.8	3	0.815	-0.058	1.010	0	0	30	30	0	30	90
Heathgrove Lodge Nursing Home	Bupa	NW11	8.7	3	0.81	-0.064	0.990	0	0	10	10	6	10	36
					0.865			126	80	308	197	150	310	1171

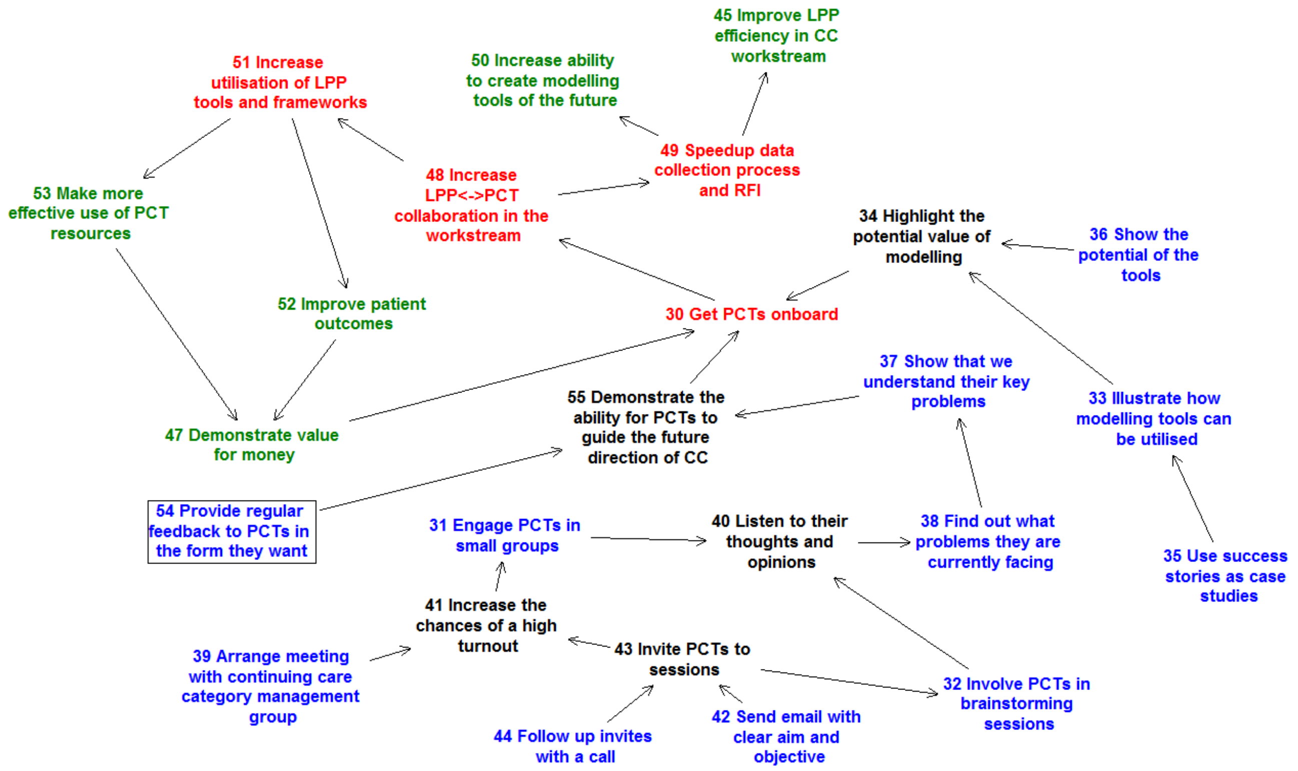
A.8 Screenshot of solver progress for the 12 period 1 care group instance



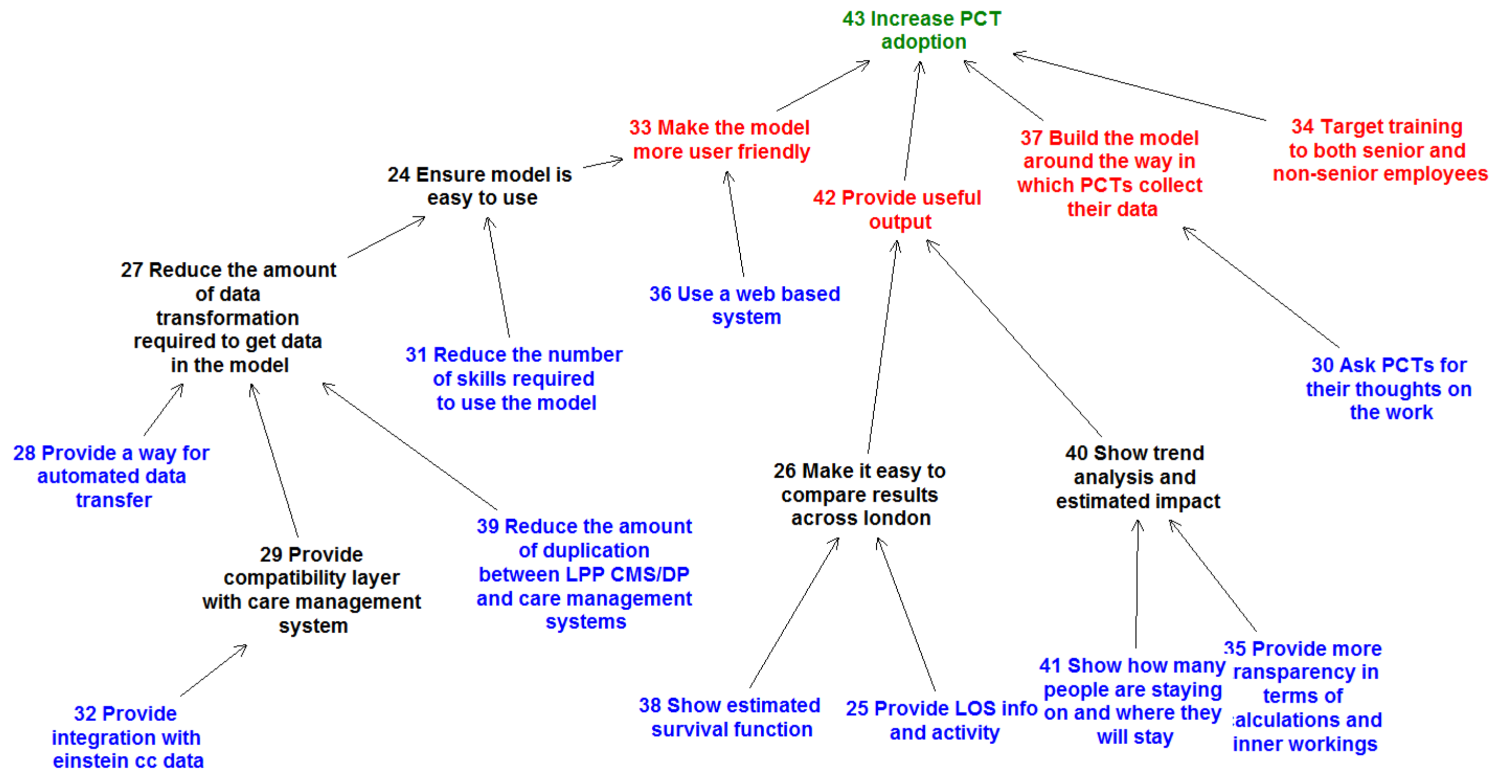
A.9 Screenshot of solver progress for the 12 period 2 care group instance



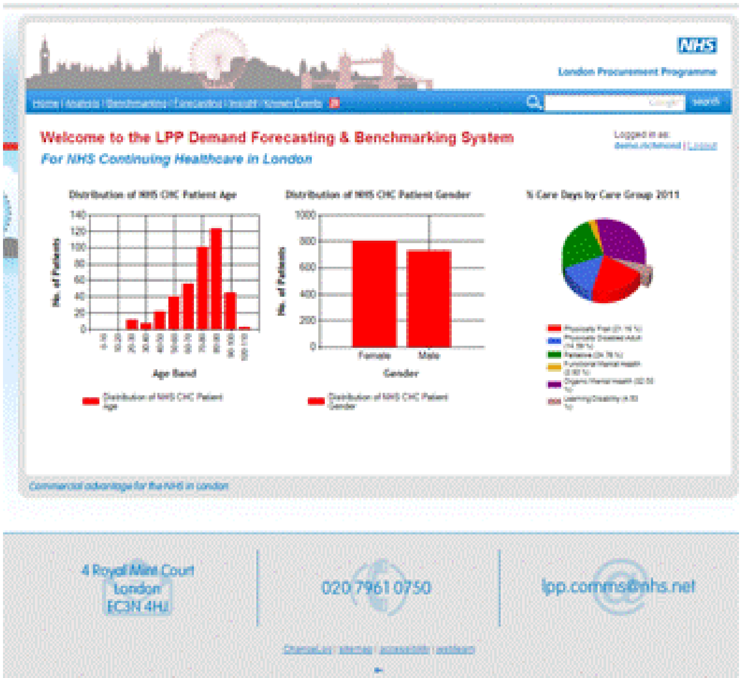
A.10 Cognitive map of issues relating to the pan-London LTC tool



A.11 Cognitive map based on interview held with a single LTC commissioner



A.12 Dashboard overview page



A.13 Dashboard forecast result page