On the performance of a blind source separation based I/Q-corrector.

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ON THE PERFORMANCE OF A BLIND SOURCE SEPARATION BASED I/Q-CORRECTOR

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ABSTRACT

I and Q Channel phase and gain mismatches are of great concern in communications receiver design. In this paper we carry out a detailed performance analysis of the Blind-Source Separation (BSS) based imbalance compensation structure. The results indicate that the BSS structure can offer adequate performance for most communication systems. Since the compensation is carried out before any modulation specific processing, the proposed compensation method works with all standard modulation formats.

I. INTRODUCTION

Gain and phase imbalances in quadrature transceivers are known to degrade the overall communication link performance [1]-[7]. Receivers with good balance in the in-phase and quadrature channels must rely on stringent specifications on mixers and RF/IF filters that are difficult to achieve as the frequency of operation increases to microwave or mm-wave region.

Several techniques have been proposed to estimate and compensate the quadrature receiver errors [1]-[7]. In [4] and [5] the Gram-Schmidt orthogonalisation procedure is proposed for correcting the I/Q errors by using test/pilot signals. In [1] and [2] an interference cancellation based adaptive I/Q corrector is proposed. This paper explores the structure and the performance capability of a non-data/pilot aided adaptive DSP technique developed for the quadrature receivers in [7]. In this study, we assume that the gain and phase imbalances are introduced at the receiver and we carry out a detailed performance analysis of the BSS based imbalance compensation structure.

The paper is organized as follows: Section II defines the model of the gain and phase imbalance compensator. Section III describes the performance analysis and the simulation results, while concluding remarks are given in Section IV.

II. BSS-BASED I/Q CORRECTOR

A. APPLICATION OF BSS TO I/Q CORRECTION

In the theoretical derivation of the algorithm the following notations will be used:

Transmitted I/Q Signals:	$\mathbf{s}(z)=[s_I(z)\ \ s_Q(z)]^T$
Received I/Q Signals:	$\mathbf{r}(z)=[r_I(z)\ \ r_Q(z)]^T$
Corrected I/Q Signals:	$\mathbf{c}(z)=[c_I(z)\ \ c_Q(z)]^T$
Mixing Vector:	$\mathbf{H}_i^{[k]}(z)=[h_i^{[k]}(0)\ldots h_i^{[k]}(L_i)]^T$
Coefficient Vector:	$\mathbf{W}_i^{[k]}(z)=[w_i^{[k]}(0)\ldots w_i^{[k]}(L_i)]^T$

where L_i (i=1,2) is the filter length. While deriving the structures for the solution of the I/Q phase and gain mismatch problem, the only assumption we make is that the ideal transmitted signals, $s_I[k]$ and $s_Q[k]$ are uncorrelated. Hence, this assumption implies that

$$E[s_I[k]s_Q[k-n]]=0 \qquad \forall n \tag{1}$$

In the presence of I/Q impairments however, this relationship no longer holds and there is a correlation between the I and Q channels [7]. In the presence of I and Q phase and gain mismatches the received signal $\mathbf{r}$[k] can be expressed as [7]:

$$\mathbf{r}[k]=\mathbf{H}\mathbf{s}[k] \tag{2}$$

where $\mathbf{H}$ is the unknown nonsingular *mixing matrix* which is determined by the phase and gain errors [7] and $\mathbf{s}[k]$ is the transmitted signal. Given the received vector $\mathbf{r}$[k], the source separation problem comprises the recovery of the original signals in an unsupervised way by finding a *de-mixing* matrix $\mathbf{W}$ hence recovering the sources:

$$\begin{aligned}\mathbf{c}[k]&=\mathbf{W}\mathbf{r}[k]\\&=\mathbf{W}\mathbf{H}\mathbf{s}[k]\end{aligned} \tag{3}$$

Application of the BSS to the I/Q problem is depicted in Figure 1.

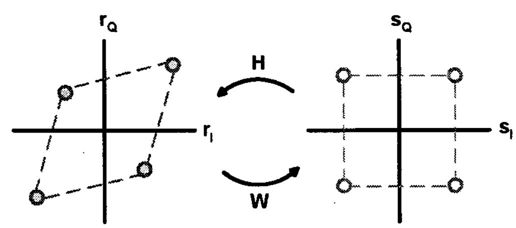

Figure 1 Application of BSS to I/Q correction

As it can be seen from Figure 1, the mixing matrix $\mathbf{H}$ transforms the transmitted s_I[k] and s_Q[k] signals to the new received r_I[k] and r_Q[k] signals. Therefore, the constellation points get closer to the edge of their decision regions. As a result, it takes less noise power to perturb the constellation points and move them into the wrong decision regions. The result is a higher probability of Bit-Error-Rate (BER) than would otherwise be expected. Figure 2 depicts the BER for

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QPSK modulation with I/Q channel phase and gain errors.

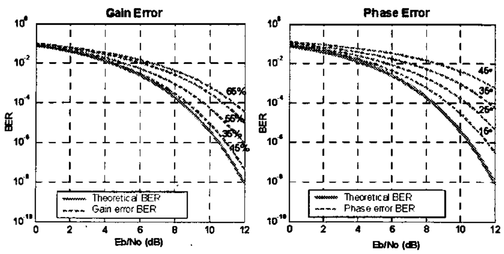


Figure 2 BER as a function of phase and gain errors

Referring back to Figure 1 again, the de-mixing matrix **W** on the other hand, transforms the received $r_I[k]$ and $r_Q[k]$ signals back to the original transmitted ones, eliminating the effects of analog front-end phase and gain errors and restoring the BER to be close to the ideal case. In the next section the structure for the I/Q channel phase and gain error compensation will be looked at.

B. STRUCTURE FOR THE SOLUTION

A block diagram of the BSS-based I/Q corrector is given in Figure 3 [7].

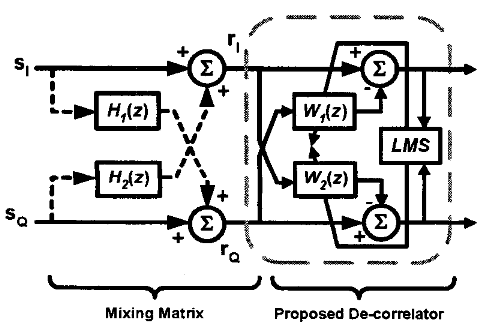


Figure 3 BSS-based I/Q corrector [7]

Due to its simplicity and ease of implantation the LMS algorithm [8] is used to update the filter coefficients of $W_1(z)$ and $W_2(z)$. The update equations are given as:

$$\begin{aligned} w_1^{[k+1]}[m] &= w_1^{[k]}[m] + 2\mu_1 c_I[k] r_Q[k-m] \qquad m = 0..L_1 \\ w_2^{[k+1]}[n] &= w_2^{[k]}[n] + 2\mu_2 c_Q[k] r_I[k-n] \qquad n = 0..L_2 \end{aligned} \tag{4}$$

where L_1 and L_2 are the filter lengths. The source estimates, $c_I(z)$ and $c_Q(z)$, can be expressed as:

$$\begin{aligned} c_I(z) &= (1 - W_1(z)H_2(z))s_I(z) + (H_1(z) - W_1(z))s_Q(z) \\ c_Q(z) &= (H_2(z) - W_2(z))s_I(z) + (1 - W_2(z)H_1(z))s_Q(z) \end{aligned} \tag{5}$$

When the filters converge, i.e. $W_1(z) = H_1(z)$ and $W_2(z) = H_2(z)$ then the source estimates become:

$$\begin{aligned} c_I(z) &= (1 - H_1(z)H_2(z))s_I(z) \\ c_Q(z) &= (1 - H_1(z)H_2(z))s_Q(z) \end{aligned} \tag{6}$$

As it can be seen from (6) the I and Q channels have the same gain and are orthogonal again. Also, $(1 - H_1(z)H_2(z)) \approx 1$ and can be safely ignored.

An interesting property of the LMS algorithm is its decorrelation property [8]. Consider the update equations (4) and assume that the algorithm has converged then the signals $c_I[k]$ and $r_Q[k]$ as well as $c_Q[k]$ and $r_I[k]$ are decorrelated over the length of the adaptive filter. This gives:

$$\begin{aligned} E[c_I[k] r_Q[k-m]] &= C_{c_I r_Q}[m] = 0 \qquad m = 0,\ldots,L_1 \\ E[c_Q[k] r_I[k-n]] &= C_{c_Q r_I}[n] = 0 \qquad n = 0,\ldots,L_2 \end{aligned} \tag{7}$$

Hence resulting in $c_I[k]$ and $c_Q[k]$ being decorrelated:

$$\begin{aligned} C_{c_I c_Q}[m] &= E[c_I[k] c_Q[k-m]] = 0 \qquad m = 0,\ldots,L_1 \\ C_{c_I c_Q}[n] &= E[c_Q[k] c_I[k-n]] = 0 \qquad n = 0,\ldots,L_2 \end{aligned} \tag{8}$$

Therefore we can conclude that decorrelation is indeed a necessary condition for the correction of the phase and gain errors of quadrature receivers.

III. PERFORMANCE EVALUATION

To analyse the performance of the proposed structure, we consider linearly modulated communications signals, namely M-PSK and M-QAM with ideal symbol rate sampling. We assume an AWGN channel and phase and gain errors of 30° and 6 dBs respectively.

The performance of the adaptive algorithm is characterised by the Modelling-ERRor (MERR). This gives a global figure for the quality of the identification of the coupling filters $H_1(z)$ and $H_2(z)$ by $W_1(z)$ and $W_2(z)$. The curve of the MERR versus time (or frames) shows the modelling performance of the proposed algorithm. What is more, the MERR can be used to observe the convergence rate and/or the steady state performance of the proposed adaptive system. The MERR is defined as the squared norm of the difference of the transfer functions between the original filters used in the mixture and the estimated filters, relative to the squared norm of the mixture filter. It is given as:

$$MERR_{w_i} = \frac{|H_i(z) - W_i(z)|^2}{|H_i(z)|^2} \tag{9}$$

In the time domain it is defined as the expected value of the sum of squares of the difference between the original and the estimated filters. It is expressed as follows:

$$\varepsilon_i[k] = \frac{E\left[\sum_{l=0}^{Li-1} (h_i[l] - w_i^k[l])^2\right]}{\sum_{l=0}^{Li-1} h_i^2[l]} \tag{10}$$

where L_i is the filter length. First we will investigate the influence of the filter length (L_i) on the performance of the proposed solution. The filter length cannot be chosen arbitrarily small. There must be sufficient degrees of freedom to model the unknown impulse responses $h_1[k]$ and $h_2[k]$. Figure 4 depicts the modelling error for different filter orders using 16-PSK modulated signals.

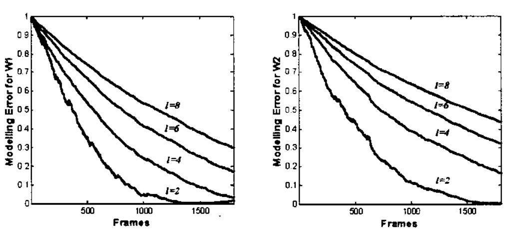


Figure 4 Modelling Error for different filter length L_i

From Figure 4 we can see that, longer filters converge slower. What is more, increase of the filter length leads to larger misadjustment as expected [8]. Hence, filter length of 2 (i.e. 2-taps) is chosen for the proposed algorithm. Modelling error for different step-sizes (μ) using 16-PSK modulated signals is shown in Figure 5.

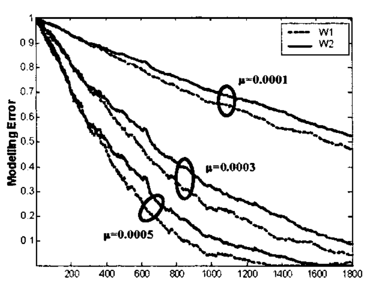
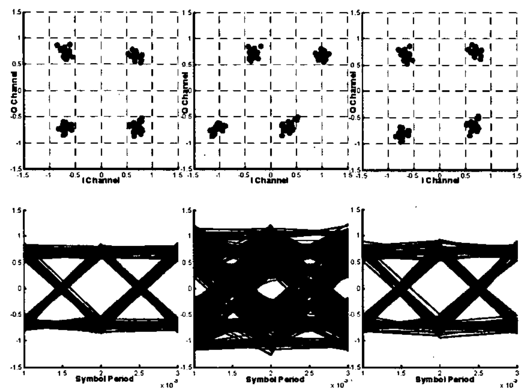


Figure 5 Modelling Error for different step-sizes (μ)

Step-size values, smaller than 0.0005 and larger than 0.001, that made the system unstable were discarded from the Figure 5. As it can be seen from Figure 5 step-size, μ=0.0005, gives the best performance.

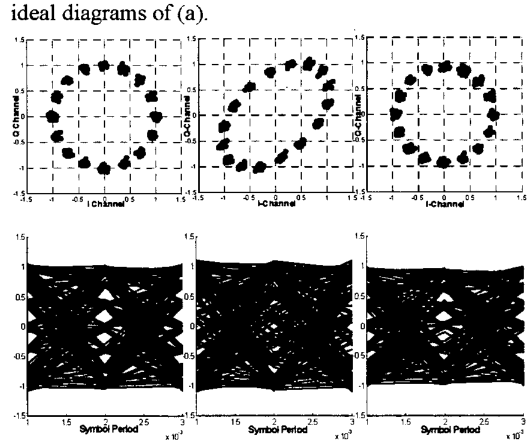
We now concentrate on the application of the proposed algorithm to different modulation formats. We consider three cases: (i) Q-PSK with an SNR of 20 dB, (ii) 16-PSK with an SNR of 20 dB and (iii) 32-QAM with an SNR of 10 dB. Figures 6, 7 and 8 depict the eye and constellation diagrams for the application of the BSS-based corrector to QPSK and 16-PSK for an SNR=20 dB and 32-QAM for an SNR=10dB, modulation formats.



(a) (b) (c)

Figure 6 Scatter and Eye Diagrams (a) Original, (b) With phase and gain error and (c) Corrected, for QPSK modulated signals.

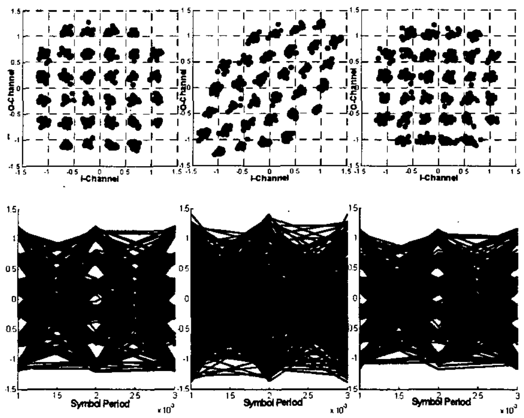
As it can be seen, the erroneous constellation and eye diagrams (b) are transformed (c) almost matching the ideal diagrams of (a).



(a) (b) (c)

Figure 7 Scatter and Eye Diagrams (a) Original, (b) With phase and gain error and (c) Corrected, for 16-PSK modulated signals.

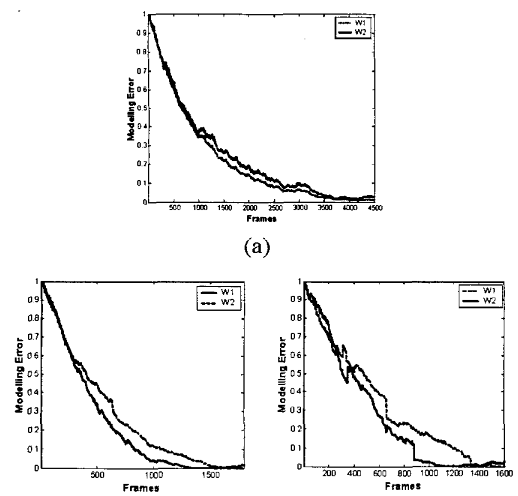
As before, the erroneous constellation and eye diagrams (b) are transformed (c) almost matching the ideal diagrams of (a).



(a) (b) (c)

Figure 8 Scatter and Eye Diagrams (a) Original, (b) With phase and gain error and (c) Corrected, for 32-QAM modulated signals

Once again, the erroneous constellation and eye diagrams of (b) are transformed to those of (c) almost matching the ideal diagrams of (a). Figure 7 depicts the modelling error for both of the above cases.



(a)

(b) (c)

Figure 7 Modelling Error for (a) QPSK, (b) 16-PSK and (b) 32-QAM

As it can be seen, de-mixing filters W_1 and W_2 almost match the mixing filters H_1 and H_2; hence the modelling errors are almost zeroed starting from frames 4000 for (a) and from 1400 for (b) and (c).

IV. CONCLUSIONS

In this paper we have presented the simulation and analytical results for the performance of the BSS based I/Q compensator using the LMS algorithm. The robustness of the algorithm in terms of step-size and filter-tap lengths was demonstrated through simulations. The robustness against different modulation formats; M-QAM and M-PSK were also demonstrated through simulations. Simulation results demonstrate substantial improvements with the effects of the phase and gain errors almost removed. The algorithm is extremely simple to implement consisting of two, 2-tap adaptive FIR filters and lends itself to efficient real-time realisation. What is more, since the algorithm is implemented before any modulation specific processing, it should work with all standard modulation formats such as PAM, QAM, PSK, GMSK and OFDM.

V. REFERENCES

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