Length of stay-based patient flow models: recent developments and future directions.

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Modelling Hospital Patient Flow: Recent Developments and Future Directions

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Abstract

Modelling patient flow in health care systems is considered vital in understanding the system's activity and may therefore prove to be useful in improving their functionality. A measure, extensively used, is the average length of stay which, although easy to calculate and quantify, assumes normally distributed data thus making the subsequent modelling of resources totally unsuitable. In fact, simple deterministic models are generally considered inadequate, hence the necessity for models to reflect the complex, variable, dynamic and multidimensional nature of the systems. This paper focuses on modelling length of stay and flow of patients. An overview of such modelling techniques is provided, with particular attention to their impact and suitability in managing a hospital service.

Running Title: Modelling Patient Flow through Hospitals

Keywords: Health, Stochastic models, Simulation

Introduction

The provision and planning of hospital resources has always been a matter of great importance. To this extent, it is critical for scientifically sound and valid methods to be employed when it comes to developing models that can capture the complexity of health care systems. Modelling patient flow in health care systems is considered to be vital in understanding the system's activity and may therefore prove to be useful in improving the functionality of the health care system.

Hospital length of stay (LoS) of in-patients has been employed as a proxy for measuring the consumption of hospital resources. A measure frequently used is the average LoS, which although easy to quantify and calculate it is often not representative of the underlying distribution as in many cases the data are skewed.[1] It is also commonly used in more generic but rather simplistic models for planning and managing hospital resources and capacities. These models usually take the form of some deterministic, spreadsheet-based calculations.[2] However, since a hospital is a complex stochastic system, simple deterministic approaches for planning and managing the system are considered to be inadequate.[3, 4]

A better way of assessing the system's activity is to consider the measurement of flow of patients through hospitals and other health care facilities. Patient flow is an important aspect in the systemic approach of health care services as it brings out the temporal dimension of the system as well as the structural. An accurate and reliable model of patient flow would enable hospital managers to predict future activity on the wards. Such predictions would be extremely useful in assessing future bed usage and forthcoming demands on various hospital resources such as the number of beds required, the length of time for which the beds are required, the case-mix of each ward – the type of beds required and the various associated staffing levels needed.

Patient flow can be seen from two perspectives, the clinical and the operational.[5] From the operational perspective, patient flow represents the movement of patients through a set of locations in a health care facility. Operational models of patient flow are very detailed and complex, usually taking the form of simulated queuing systems (for example, see [6, 7]). Although capable of providing very accurate predictions for various future system activities, these models are very costly and time consuming to build. This is due to the sheer complexity of the models and to the fact that most of the required input data are not readily available. The latter necessitates expensive and time consuming on-site observations. Furthermore, such models are usually tailor made to the needs of specific health care settings and as a result, cannot be easily generalised.

In contrast, from the clinical perspective patient flow represents the progression of a patient's health status.[5] Models developed under this perspective are considered to be less costly and time consuming than the operational models since routinely collected data can be used for estimating the input parameters. These models are of particular value to epidemiological studies where the behaviour of certain patient populations is modelled over a long period of time and the cost effectiveness and efficiency of different interventions or screening programmes is evaluated.[8, 9] The problem is that most hospital departments deal with a variety of patient populations and diseases thus, models developed under this perspective are not suitable for micro-level resource allocation and capacity planning managerial activities.

Alternatively, patient flow can be seen from a behavioural perspective where staff, patients, and facilities interact with each other to form specialty specific, locally determined streams of flow [10]. The assumption behind such models is succinctly described by Snowden [11]:

> "Humans, acting consciously, or unconsciously are capable of a collective imposition of order in their interactions that enables cause to be separated from effect and predictive and prescriptive models to be built".

In general, the goal of most behavioural research is to infer the underlying process that generated the observed data [12]. The following observation lends support to the introduction of an appropriate technique for modelling LoS:

> "… despite the immense complexity of a hospital system, there is a simplicity: patients occupy beds for a measurable amount of time" [13]

Observing a health system from the behavioural perspective offers, in certain cases, the correct level of simplification and abstraction for the models. Concepts such as acute care, assessment, fast stream, continuing care and long-term, imply dimensions of time as well as actual facilities. Routinely collected administrative data can be used and thus, expensive and time consuming on-site observations can be avoided [14]. Additionally, models can be scaled to accommodate departmental, hospital, regional and national levels of analysis. A complicating factor however, is that establishing such models is not a trivial task. Advanced and sound statistical techniques need to be employed or developed before being linked to decision models from the broad OR/MS portfolio.

The scope of this paper is to bring together some recent developments that are related to this particular domain of patient flow modelling. In the next section different probabilistic solutions for modelling LoS, namely Markov models, phase-type distributions, and conditional phase-type distributions are presented and discussed in terms of their impact and suitability in assisting with the management of a hospital service. We then discuss how a mixed-exponential model can be used as the basis for a compartment model of patient flow, which in turn can be converted to a discrete-event simulation model. Finally, we provide a discussion on the general merits of these models before concluding with possible future directions in the field.

Probabilistic Modelling of Patient Flow

Markov Models

Markov models are often used to represent stochastic processes in statistical theory [15]. The stochastic process is formalised by a set of states to which the system may belong and probabilistic laws that govern movement between the states. Such a model assumes a probabilistic behaviour of patients moving around the system and therefore gives a realistic representation of the actual system.

Irvine et al. [16] describes the development of a continuous time stochastic model of patient flow. Essentially, it is a two-stage continuous-time Markov model that describes the movement of patients through geriatric hospitals. The compartments in the model can be regarded as states and the probabilities of patients moving within those states can be calculated. Patients are initially admitted to the acute state from which they transfer to the long-stay state or leave the hospital completely through discharge or death state. Such an approach takes into account different types of patients thus enabling the extraction of variances from the model by considering patient variability.

McClean et al. [17] extends the previous stochastic Markov model to a three stage one and attaches different costs to each of the three stages thus providing a model that can facilitate planning of health and social services for the elderly while taking cost into account. Taylor et al. [18] uses the above approach of a continuous time Markov model and applies it to the case of a four compartmental model [19], where the four stages are acute, long-stay, community, and dead. The model estimates the expected number of patients at any time t in each stage for a cohort of patients, all admitted on the same day and enables the estimation of the variances of the number of patients in each stage at time t. Taylor et al. [20] extends these models to contain six stages, which can be used to determine the interactions between hospital geriatric medical services and community care. The methodology allows the number of compartments in the model to be governed by the data in order to obtain a model that gives the best representation of the system.

Markov models are based on well established statistical methodologies and provide a viable approach to measuring and modelling flow. The models follow the patient journey and give insights into the hazard rates and probabilities involved. However, such models give no knowledge of the internal processes of care and therefore need to be combined with other techniques such as the compartmental models of patient flow, described later in this paper. Alternatively, probabilistic networks based on phase-type distributions (the Conditional phase-type models described below) can incorporate prior knowledge of the internal process along with other contributing covariates.

Phase-Type Distributions

Another statistical model that can be employed to represent the variable nature of LoS is phase-type distributions (Ph). Such distributions describe the time to absorption of a finite Markov chain in continuous time, where there is a single absorbing state and the stochastic process starts in a transient state [21]. The assumptions of the distributions state that the $1,\ldots,n$ states are all transient, so absorption into the state $n+1$, from any initial state is certain. These models describe duration until an event occurs in terms of a process consisting of a sequence of latent phases - the states of a latent Markov model. For example, patient LoS can be thought of as a series of transitions through phases such as acute illness, intervention, recovery, discharge. In most instances, phase-type distributions can be generalised to include almost all continuous distributions [22] such as the exponential, which will only have one phase, the Erlang, and mixed exponential distributions, a feature that makes them appealing to use.

Phase-type models were originally introduced as a natural probabilistic generalisation of Erlang distributions. The key difference is that movement between all the transient stages and the absorbing phase can occur in the phase-type distribution. Conversely, in the case of the Erlang transitions, movement can only occur between sequential phases. In other words, the phase-type distributions allow a patient to leave the system completely at any stage and move directly into the absorbing state. However, the generality of the phase-type distributions makes it difficult to estimate all the parameters of the model. To overcome this problem Coxian Phase-type distributions were introduced.

Coxian phase-type distributions [23] are a special sub-class employed to describe the probability P(t) that the process is still active at time t [22]. They differ from general phase-type distributions in that the transient states (or phases) of the model are ordered. The process begins in the first phase and may either progress through the phases sequentially or enter into the absorbing state. Such phases may then be used to describe stages of a process which terminates at some stage (see figure 1). For example, in the case of a patient spell, transitions through the ordered transient states could correspond to various stages of patient stay in hospital such as diagnosis, assessment, rehabilitation and long-stay care.

[Figure 1]

Faddy and McClean [24] uses this model to find a suitable distribution for the LoS of a group of geriatric patients in hospital. They conclude that phase-type distributions are suitable for measuring the LoS of patients in hospital and show how it is also possible to consider other variables that may influence it such as age on admission and year of admission. The three-term mixed exponential model that will be described later (equation 2.3) can be regarded as a three phase distribution where the patients are split into three groups, short, medium, and long-stay according to their LoS.

Coxian phase-type distributions are based on cohort data and can identify the presence of different compartments. They give better insight into the reality of the two extremes of flow modelling, namely the log-normal start and the long-term patients. In addition to LoS being represented in a sound mathematical sense, the model provides a useful representation for interpretation by non mathematicians where LoS consists of a sequence of phases mimicking a type of behaviour or property such as acute or long stay.

Conditional Phase-Type Models (C-Ph)

The conditional phase-type (C-Ph) distribution is a novel approach which uses Coxian phase-type distributions conditioned on a Bayesian Network (BN). This approach allows the incorporation of discrete and continuous variables [25]. Unlike previously developed models the C-Ph model can represent a continuous distribution which is highly skewed while also incorporating causal information from interrelationships between explanatory variables.

The incorporation of a BN into the model allows the inclusion of statistical graphical models which provide a framework for describing and evaluating probabilities when there is a network of inter-related variables representing causality.[26] Figure 2 illustrates the model as consisting of these two components where the Coxian phase-type distribution is the process model and the BN the causal network.

[Figure 2]

The conditional phase-type (C-Ph) model is defined as consisting of causal nodes $\mathbf{C} = \{C_1, \ldots C_m\}$ belonging to the causal network, and process nodes $\mathbf{Ph} = \{Ph_1, \ldots Ph_n\}$ representing the phase-type distribution. The Coxian phase-type distribution can be fitted to the patient LoS using a sequential procedure, as described in Faddy [22].

Marshall et al. [25, 27] uses the C-Ph model to model the LoS of elderly patients in hospital. The approach is illustrated using data on hospital spells (the process) for a number of geriatric patients along with personal details, admissions reasons, dependency levels and destination (the causal network). The final model represents patient LoS in terms of five of the most significant patient variables in the data set, namely patient age, gender, admission method into hospital, Barthel grade (dependency score) and destination on departure from hospital. The approximations of the parameters of the phase-type distributions are recorded for the optimal number of phases in the distribution along with the appropriate joint probability distribution of the causal network. The BN may therefore be used to select the most likely situation for a patient based on the other variables in the network. The patient LoS may then be modelled using the estimates of the phase-type distribution for that particular cohort of patients.

In summary, the C-Ph model provides a better understanding of resource use. It integrates probabilistic networks with Coxian phase-type distributions to prototype a forecasting tool based on historic activity. Such models can be used to give valuable insights into the interaction between important variables (e.g. dependency, previous illness, local social circumstances, etc.) on the probability that a patient will be short, medium or long stay. For all these reasons, it has the potential to be a useful explanatory management tool. However, the construction methodology is more complex than using traditional methods hence further testing is required to ensure it is more clinically and managerially meaningful.

Compartmental and simulation modelling

Mixed Exponential Distributions

Millard [27, 28] observed a distinct distribution to bed occupancy that was initially assumed to be staff skill related. It was later proposed that the observed change in the distribution represented the interaction of the movement of two or three types of patients through the system. The hypothesis that the time pattern of bed occupancy in departments of geriatric medicine is expressed by a mixed exponential equation was later tested on data collected from thirteen health districts in the South West Thames Region, England [29]. McClean and Millard [30] used hazard functions to consider various models of survival of geriatric patients in hospital and observed that the pattern of bed occupancy in departments of geriatric medicine could be expressed using mixed exponential distributions. This two-term mixed exponential model with probability density function (p.d.f.) of the form

$$f(t) = p\lambda_1 e^{-\lambda_1 t} + (1-p)\lambda_2\, e^{-\lambda_2 t} \qquad 0 < p < 1; \lambda_1, \lambda_2 \geq 0 \tag{4}$$

has shown to give a good fit for durations of occupancy of geriatric beds [31, 32]. In general, the model also gives a reasonable approximation to the numbers of patients departing each week from hospital. The development of such a model led to a new method of estimating the usage of hospital beds. McClean and Millard [30] use the fits of the mixed exponential models to provide a method for predicting future behaviour of patients and identifying where there is a change in patterns. The mixed term exponential model of survival may help explain why a small proportion of the people who enter long-term care stay for a very long time. This also provides reasoning as to why the majority of beds in a long-term care unit are occupied by very long-stay patients.

A variation of the mixed exponential model is a more sophisticated lognormal and exponential mixture which could provide a better description of the early peak in departures to death/discharge. However, the lognormal model requires the estimation of an additional parameter and is hence more complicated to apply than the exponential.[30]

Compartmental modelling

Godfrey [33] defines compartmental systems as those consisting of a finite number of homogeneous, well mixed, lumped subsystems, called compartments. These exchange with each other and with the environment so that the quantity or concentration of material within each compartment may be described by a first-order differential equation. Compartmental models can be linear, non-linear, deterministic or stochastic depending on the process they represent. In recent years compartmental models have been applied to the movement of patients throughout hospital systems.

Harrison and Millard [32] suggested that the movement of geriatric patients around departments of geriatric medicine is best described by a model consisting of two compartments. The model, illustrated in figure 3, describes patients who are initially admitted to a short-stay or acute state, from which they either die or are discharged at a rate r, or are transferred to a second long-stay state at a rate v from which they either die or are discharged from the hospital at a rate d. The model is deterministic with discrete time. It provides a means of estimating the numbers of acute and long-stay patients and their expected lengths of stay.

[Figure 3]

The system may be described by two linear difference equations, the solution of which is a two-term expression which can easily be written as the two-term mixed exponential model discussed earlier in the paper. The two-compartment deterministic model enables hospital planners to optimise the use of geriatric beds and gives a realistic and intuitive insight into the movement of geriatric patients around the geriatric department.

Further work into the pattern of bed occupancy in acute hospitals showed that some patterns of bed occupancy were best represented by an equation with three exponents. Harrison [34] extends the two compartment deterministic model to incorporate a third compartment thus representing patient behaviour as acute stay, rehabilitative stay and long-stay care. Taylor et al. [35] describes a four compartmental model which takes into account an extra compartment for patients in the community.

A specially designed analysis program, called Bed Occupancy, Management and Planning System (BOMPS), implements the above mathematical model. The e-fit module of this software program generates performance statistics based upon the fit between a mixed exponential curve and bed census data by using a nonlinear least squares algorithm [36, 37]. Dependent on whether the best-fit mixed exponential equation has one, two or three components; one, two or three compartment statistics are generated respectively. The number of compartments used in a model is determined on the basis of the number of exponents required to obtain the best mathematical fit to the actual data.

Given patient admission and discharge dates and a census date, BOMPS estimates individual bed occupancy times, displays the cumulative pattern of occupancy, determines and displays the best fit curve, and generates resource utilisation statistics such as the number of patients in a stream, the estimated average LoS of patients within a stream, the rates of admission and release to and from a stream, and the conversion rates from one stream to another. This also facilitates scenario modelling as it is possible to change the conversion and discharge parameters in each compartment. Several examples of using the software in different areas of health services have been published collectively [38, 39], while further applications have been reported separately [40-45]. They all report successful application of the methodology to different hospital departments.

In summary, compartmental modelling is a well-established methodology that has been validated mathematically and clinically. Its disadvantage is that it is based on a one-day bed census and thus, it is highly dependent on the day of the census. It ignores seasonality and the cyclical effects of admissions and discharges.

Simulation modelling

Another logical extension to the compartmental modelling framework is to consider the problem as a queuing system. Queuing systems, whether developed as analytic approximations or simulations can be used to extend the capabilities of the compartmental models described above. Queuing performance measures such as time in the system and time spent waiting in queues can help planners to test different scenarios and avoid bottlenecks in the flow of patients.

Given the complexity of the system and the required flexibility in modelling terms, discrete event simulation (DES) is usually preferred to analytic approximations. DES concerns the modelling of a system as it evolves over time by a representation in which the state variables change instantaneously at separate points in time, called events [46]. Events are defined as instantaneous occurrences that may change the state of the system. DES is widely used in modelling health care systems as recently discussed by Jun et al. [47]. The basic components of a patient flow simulation model can be summarised as:

a) entities, the simulated elements of the system e.g. patients,
b) activities, the operations and tasks that transform the state of the entities e.g. compartments and queues, and
c) the state of the system, a collection of the variables that describe the system at a certain point in time e.g. the number of available beds, the number of patients in a queue etc.

El-Darzi et al. [48] first described the development of such a queuing system, using DES to perform the numerical evaluations. Initially, a steady-state simulation model with three compartments (short-, medium-, long-stay or acute, rehabilitation and long-stay) without capacity constraints was developed and tested against the results of the established compartmental models. The duration of service of each compartment was exponentially distributed and the random nature of the patients' progression through the system was simulated by a probabilistic node between the compartments. The very long warming up period that was observed and was needed for reaching stable state, apart from having technical implications to the model (the batch means method was preferred to the replication/deletion method), highlighted the real-life problems associated with very long LoS. Any changes that will affect the flow of the long-stay patients, will take time before their consequences stabilise and become apparent. A constrained (capacitated) model was then used to evaluate the effect of changes in various model parameters on refusal rates, blockage between the compartments and generally, the flow of patients. In line with findings of previous studies, it concluded that the key for the smooth flow of patients in the system is the emptiness in the long-stay compartment.

The main advantages of these DES models arise from the fact that by adapting the conceptual model of the compartment model to that of a queuing system, the incorporation of capacity constraints and bed blockage in the evaluation of patient flow is made possible. By employing DES to evaluate the resulting queuing system numerically, the stochastic nature of the modelled systems is taken into account along with the required flexibility and adaptability for the conceptual modelling capabilities of the model. This has been demonstrated by adding external compartments (independent home and support home) to the basic configuration [49] and by modifying the basic model to cater for a possible hypothesis on the causes of the winter bed crisis in English hospitals [50].

The main disadvantage of DES models is the long execution time and the amount of output data they generate. Each scenario and thus model set-up needs, even under the more efficient batch means method of output analysis, one very long run of hundreds of thousands of simulated days. It follows naturally that the modeller has to be very careful when it comes to selecting the different scenarios to be evaluated.

However, these problems have been partially resolved by employing data warehousing and On-Line Analytical Processing (OLAP) techniques to handle the data generated by the DES model. [51-53] More specifically, the proposed data warehouse environment provides the means for automating the necessary algorithms and procedures for estimating different parameters of the simulation. They include initial transient in steady-state simulations and point and confidence interval estimations. This data warehouse environment can substantially reduce the computational complexities in analyzing and interpreting simulation output data and it constitutes a significant step toward rendering simulation engines "black boxes" for the end-users.

Discussion

This paper discusses some of the methodologies that have been recently developed in the health care modelling domain that seek to discern in-patient populations in two or three separate streams of flow. The first probabilistic approach describes a special type of Markov model known as the Coxian phase-type distribution and its further development into the Conditional phase-type distribution. The Coxian phase-type distribution allows the representation of the continuous duration of stay of patients in hospital as a series of sequential phases which the patients progress though until they leave the hospital (the system) completely. Such identification of the presence of different patient states is not only mathematically sound but also provides an additional means of interpretation for the clinician or health care manager. The phase-type representation of patient length of stay can be interpreted as different types of patient behaviour, for example in the case of a three phase model these were identified by the clinician as being equivalent to three groups of patient activity namely; acute, rehabilitation and long-stay. This gives a better insight into the reality of the system.

It is possible to expand the theory of Coxian phase-type distributions to include a network of additional interrelated variables that may interact to influence patient LoS. This is facilitated by the recent development of the Conditional phase-type distribution which conveniently allows the representation of additional variables (such as patient characteristics) to be considered and taken into account using a Bayesian network. Although this representation is mathematically complex, its graphical nature provides a visual representation that is easy for clinicians to interpret. However the technique does require expert assistance in its development due to the complex fitting of the Coxian phase-type model to the conditional probabilities attached to the network variables. Nonetheless such complexity in model fitting makes the resulting conditional phase-type distribution a much more powerful model than some of its counterparts. In fact the resulting model has the potential of becoming a valuable tool for clinicians and health care managers by more realistically representing the health care systems under consideration.

In the second general approach described in this paper, compartmental model of patient flow provided the conceptual basis and the input parameters to a DES model for evaluating the interaction of the different streams of patient flow through health care systems. By adapting the conceptual model of the compartment model to that of a queuing system, the incorporation of capacity constraints and bed blockage in the evaluation of patient flow was made possible. By employing DES to evaluate the resulting queuing system numerically, the stochastic nature of the modelled systems was taken into account along with the required flexibility and adaptability for the conceptual modelling capabilities of the model.

Although the DES models of patient flow inherit the assumptions of the compartmental models of patient flow (discharge independent of LoS, compartments operating at full capacity, system in stable state), they are not bound by them insofar as their parameters can be estimated by different statistical and mathematical models. These include survival analysis, phase-type distributions, and data mining algorithms. However, as it has been demonstrated by several case studies reported in the literature, the compartmental models of patient flow give an accurate picture of the ongoing process in health and social care services. Thus, it is safe to assume that the simulation models reported here hold the same favourable characteristics. Furthermore, the latest developments in information technology and data processing have enabled the development of a decision support framework that incorporates the compartmental and DES models.

Conclusions

The management of hospital resources is a critical issue in today’s society. In order to assist with such an issue, health care systems are continually being developed to try to capture the activity that takes place in hospital wards and how the management of such can be modelled and improved for future allocation of resources. It is hoped that modelling patient flow in health care systems can assist in the overall understanding of the system's activity and may therefore prove to be useful in improving the functionality of the health care system. With this in mind, it is vital to have modelling methods which are scientifically sound and valid but also acceptable and easily understood by clinicians and health care managers.

Previously developed bed usage measures do not adequately represent the true activity or situation in the hospital ward. Therefore it is necessary to consider new models that for example, do not focus on the average measure such as the average LoS, but use other modelling techniques to represent LoS and patient flow in a hospital ward. Such models would be considered extremely beneficial to the hospital manager for instance an accurate and reliable model of patient flow would enable hospital managers to predict future activity on the wards. Such predictions would be extremely useful in assessing future bed usage and forthcoming demands on various hospital resources such as the number of beds required, the length of time for which the beds are required and the case-mix of each ward.

Current trends in health care modelling have expanded the portfolio of methods and techniques being employed to include recent developments in scientific fields such as artificial intelligence, data mining and information technology. Consider the framework described by Harper [3] where there are various components or stages of analyses combining a preliminary statistical analysis with a further data investigation using methods such as classification and regression tree analysis (CART) and modelling techniques on patient length of stay. This is complemented by a final stage of modelling using simulation techniques. In this case, various data mining, statistical and operational research methods come together to provide operational modelling for hospital resources. Another example is Walczak et al.'s [54] use of the artificial intelligence method of neural networks to facilitate the modelling and prediction of resource utilization associated with patient LoS. There are some drawbacks in using such an approach where the models are considered to be 'black box' in nature, however the combination of such a technique with other forms of analysis could overcome such problems thus making the method applicable to health resource allocation.

The authors believe that the future of modelling patient activity in health care systems, can be built on the successes of current models and evolving hybrid approaches to form a *toolbox* consisting of data mining, data analysis, operational research and artificial intelligence methods. This *toolbox* would facilitate preliminary data preparation and initial statistical analysis with advanced methods for modelling health care resources and inference techniques for providing further predictions. The reported complexities of health care systems coupled with the availability of vast amounts of health related data necessitate the inter-disciplinary collaboration described in this paper.

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Figure 1 An illustration of the Coxian phase-type distributions

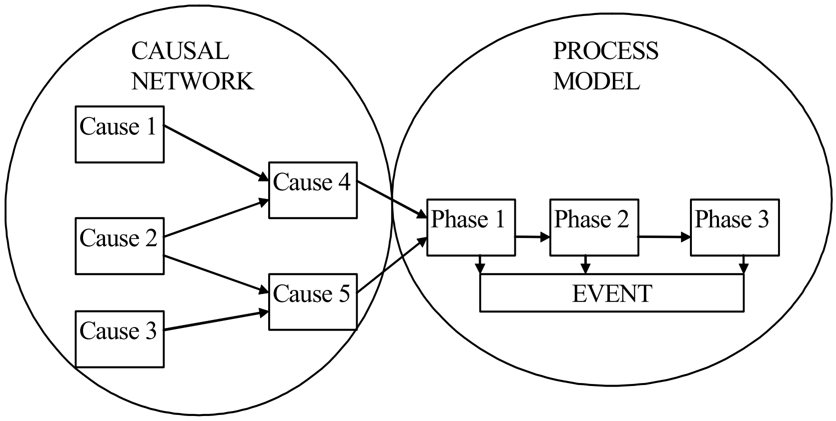

Figure 2 The Conditional Phase-Type Distribution (C-Ph)

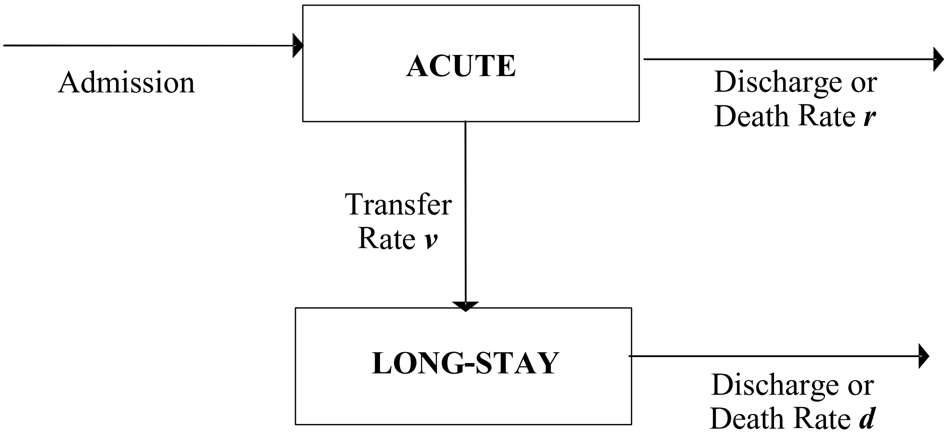

Figure 3 The Two-Compartment Model