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# A GUI Driven $\Sigma$ - $\Delta$ Modulator Design, Evaluation and Measurement Tool with a View to Practical Implementation

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**Abstract**—A user-friendly design tool created in the MATLAB/Simulink environment to speed up the design, analysis, evaluation and measurement of single-loop and multistage sigma-delta ( $\Sigma$ - $\Delta$ ) modulators at the system level is presented in this paper. The tool covers a variety of Simulink-based design topologies of low-pass, band-pass and high-pass  $\Sigma$ - $\Delta$  modulators.

**Keywords**-component; Sigma-delta A/D modulator, design tool

## I. INTRODUCTION

Sigma-delta modulators have been widely used to convert analogue signals to their digital equivalents because of their capability of providing better resolution as well as relative ease of implementation compared to conventional analogue-to-digital (A/D) converters. The wide range of applications offered by  $\Sigma$ - $\Delta$  modulators together with their numerous advantages is well detailed in the literature [1], [2]. On the other hand, the non-linear quantizer in combination with the feedback loop employed by the  $\Sigma$ - $\Delta$  modulators, cause significant stability and tonality problems. This is especially true for single-loop  $\Sigma$ - $\Delta$  modulators whose orders are higher than two [2].

To overcome instability, various single-loop or multistage  $\Sigma$ - $\Delta$  modulator topologies can be developed according to the desired specifications. Since there is not a unique methodology to guarantee the stability and proper operation of high-order  $\Sigma$ - $\Delta$  modulators, the design procedures can be optimized based on performance analyses. Thus, several design automation tools for Low-Pass (LP), and Band-Pass (BP)  $\Sigma$ - $\Delta$  modulators at the system level were reported in the literature [2]-[4]. The most common approach to automate the design procedure is based on iterative performance optimization via a performance evaluation loop using statistical methods or behavioral simulations for the desired modulator parameters [3],[5],[6]. The other common approach is the automation of the modulator coefficients by optimizing the Noise Transfer Functions (NTF) or Signal Transfer Functions (STF) for the desired design specifications. Inverse Chebyshev or Butterworth approximations are used for the NTF designs in order to obtain acceptable quantization noise reduction in the signal band [2], [4], [7].

In this paper, a user-friendly software tool to speed up the design, analysis and evaluation of LP and BP single-loop and multistage  $\Sigma$ - $\Delta$  modulators at the system-level is presented. In addition, this tool has been developed to design High-Pass (HP) based  $\Sigma$ - $\Delta$  modulators for the applications reported in [8]. The creation of a Graphical User Interface (GUI) driven design, evaluation and measurement tool for HP  $\Sigma$ - $\Delta$  modulators has not been previously reported in the literature to the best knowledge of the author. The proposed tool will allow researchers and practitioners to design and verify the feasibility of the  $\Sigma$ - $\Delta$  modulator at a much earlier stage of the design process thus saving considerable time, effort and cost. It can also be used as a valuable teaching tool for advanced courses on data converters.

This tool gives designers the ability to use easy-to-implement  $\Sigma$ - $\Delta$  modulator topologies and view their corresponding simulation results including the output spectrums, the NTF's and the power spectral densities (PSD) of the modulator outputs as well as their in-band signal-to-noise ratio (SNR) values. The MATLAB routines embedded in the GUI simulate the desired topologies according to the input parameters defined by the user. In contrast to many  $\Sigma$ - $\Delta$  design automation tools in the literature, which mostly employ specific design structures; this tool enables the user to design and analyze a variety of LP, HP and BP based  $\Sigma$ - $\Delta$  modulator topologies such as the single-loop, multi-stage (MASH), chain of accumulators with weighted feedforward summation and chain of accumulators with feedforward summation with local resonator feedbacks. Moreover, the tool allows the user to design the NTFs of Butterworth and Inverse Chebyshev approximations based on input specifications such as the cut-off frequency, bandwidth and modulator order. Butterworth and Inverse Chebyshev based  $\Sigma$ - $\Delta$  modulators have the advantage of offering improved stability and tonality performance for higher-order structures. Finally the tool is easily extendible to other  $\Sigma$ - $\Delta$  modulators structures and can also be used to design  $\Sigma$ - $\Delta$  modulators for digital-to-analogue converter (DAC) applications.

## II. SIGMA-DELTA MODULATOR DESIGN TOOL

### A. Modelling of Design Structure

The design topologies were developed from the general linearized form:

$$Y(z) = X(z)STF(z) + E(z)NTF(z) \quad (1)$$

Where,  $Y(z)$ ,  $X(z)$  and  $E(z)$  are the output signal, input signal and quantization error in the  $z$ -domain respectively. The output of the modulator is the superposition of the input signal and the quantization noise shaped by the NTF. The gain of the NTF is close to zero in the signal band and close to unity outside the signal band, which causes the noise components in the signal band to attenuate significantly by leaving the input signal unchanged. The  $\Sigma$ - $\Delta$  modulator output in an ADC scenario can then be digitally filtered in order to attenuate the out-of-band quantization noise.

1) *Low-Pass Modulators*: For a LP  $\Sigma$ - $\Delta$  data converter, the NTF should be high-pass in order to satisfy a large attenuation of quantization noise at lower frequencies and relative amplification at higher frequencies. For that reason, for a class of LP  $\Sigma$ - $\Delta$  modulators, the NTF has zeros at dc or its vicinity, i.e. at  $z = 1$ . Thus, the NTF can be expressed in the form of  $(1 - z^{-1})^N$  which behaves as a high-pass filter. This ensures large attenuation of the quantization noise at the lower frequencies [1].

2) *Band-Pass Modulators*: The NTFs realized by the band-pass  $\Sigma$ - $\Delta$  modulators have zeros in the signal band around the normalized centre frequency  $\nu$ , thus making the NTF a band-stop filter and the loop-filter to have infinite gain in the signal band. For this purpose, a low-pass-to-band-pass ( $LP \rightarrow BP$ ) transformation is applied at the points  $z = \pm e^{j2\pi\nu}$  to a suitable low-pass  $\Sigma$ - $\Delta$  modulator; so that from a low-pass  $\Sigma$ - $\Delta$  modulator of order  $n$ , a band-pass  $\Sigma$ - $\Delta$  modulator of order  $2n$  is obtained by maintaining the stability of the system as well as the noise properties of the original case [2]. By moving the zeros from  $z = 1$  to complex conjugates, the NTF yields  $1 - (2 \cos(2\pi\nu)) \cdot z^{-1} + z^{-2}$  where  $\nu$  refers to the normalized input frequency. A simplified version of this transformation:  $z^{-1} \rightarrow -z^{-2}$  is widely used in the literature for the case  $\nu = 0.25$  because it yields simpler design structures [2].

3) *High-Pass Modulators*: For HP  $\Sigma$ - $\Delta$  modulators, the NTFs have high gains at the lower frequencies in order to attenuate the noise components in the signal band, which is centered at high frequencies. Thus, an appropriate  $LP \rightarrow HP$  transformation in the  $z$ -domain is applied to the LP  $\Sigma$ - $\Delta$  modulators to shift the zero locations from  $z = 1$  to  $z = -1$ . For example, the NTF of a second-order HP  $\Sigma$ - $\Delta$  modulator becomes  $(1 + z^{-1})^2$  as a result of using the mapping  $z \rightarrow -z$  [9]. Alternatively, HP  $\Sigma$ - $\Delta$  modulators can be directly designed using LP based NTFs.

Using the relationship between the NTF and the transfer function of the loop-filter,  $H(z)$ , the desired loop filters for different design topologies can be developed from the NTFs.

$$NTF(z) = \frac{1}{1 + H(z)} \quad (2)$$

High-order  $\Sigma$ - $\Delta$  modulators with single-loop structures suffer from stability problems; therefore the cascaded (MASH) structure is widely used in many applications, which exploits the inherent stability of low-order  $\Sigma$ - $\Delta$  modulators. Alternatively, the single-loop  $\Sigma$ - $\Delta$  modulator topologies can be stabilized by adding feedforward or feedback paths with appropriately weighted coefficients. There are many possible design implementations such as the chain of accumulators in the feedforward and feedback loops. Butterworth and Inverse Chebyshev approximations can be used to compute the NTFs of higher-order  $\Sigma$ - $\Delta$  modulators. One approach to overcome the stability drawback is to introduce poles into the NTF to flatten the response at the higher frequencies. The poles of the NTF are directly related to the Butterworth NTF. In addition, the zeros can be moved to other frequencies from dc i.e. to the complex locations on the unit circle in the vicinity of dc (zero splitting). As a result of these complex alignments, compared to the Butterworth alignment, greater quantization noise attenuation and SNR improvements are achieved [2]. In order to have complex zeros, one could use inverse Chebyshev filters instead of Butterworth filters with the introduction of split zeros. The lower and upper frequencies of the NTF are selected so as to ensure that the input signal is positioned at the centre of the signal bandwidth.

### B. The Graphical User Interface

A GUI was created using MATLAB and the simulation models were designed in the Simulink environment. The models can be easily developed in Simulink using block formats which can be controlled by MATLAB routines so that it can be used in conjunction with the GUI. The models are navigated by the parameters defined by the user through the GUI and the simulation results are displayed on the GUI panes. The user has the option to open, modify and obtain the simulation results from a menu which contains a selection of  $\Sigma$ - $\Delta$  modulator structures.

The GUI consists of two parts. In the first part, the user can select low-pass, band-pass or high-pass  $\Sigma$ - $\Delta$  modulation.

#### 1) Input Parameters

In the second part of the GUI, for the chosen type of modulator, the desired specifications can be defined by the user such as the number of samples, the amplitude, frequency and nature of the input signal and amplitude of the dither signal, where it is appropriate. The GUI allows also the user to choose the order and the structure of the  $\Sigma$ - $\Delta$  modulator from a variety of design topologies such as the: single-loop, MASH and variants, chain of accumulators with weighted feedforward summation and chain of accumulators with feedforward summation with local resonator feedbacks. The MATLAB routines set the user-defined parameters to the Simulink models, run the simulation, analyze and display the simulation results and optionally the corresponding modulator model.

## 2) Measurement Setup

The plots of the  $\Sigma\text{-}\Delta$  modulator output spectrums, the corresponding NTFs, the PSDs and the histograms of the modulator outputs as well as the in-band SNR values are computed. The frequency spectrums are computed using the FFT of the corresponding signals, which are windowed with a Hanning window of the same length as the signal length. The FFT sample number is specified by the user in the parameter section, which is defined as ‘the number of samples’. The SNR values are calculated for oversampling ratios (OSR) of 32, 64, 128, and 256, as well as other user defined ones. Moreover, for the power spectrum densities, the Welch’s averaging method is used. The associated overlap number is chosen as 2048 as default but can be modified as needed.

## 3) Graphical User Interface in Operation

In this section, the tool is explained through an example for a 3<sup>rd</sup>-order high-pass  $\Sigma\text{-}\Delta$  modulator using a 1-1-1 MASH structure. The specified  $\Sigma\text{-}\Delta$  modulator is simulated for an input amplitude signal of 0.4, a normalized frequency of 0.499 and a dither amplitude signal of 0.3. All the simulations assume 10000 points for the transients. Thus, the total number of points chosen is 42768, which refers to  $2^{15}+10000$ . The mentioned parameters are directly set in the *Parameters* section. In the *Structure & Order Selection* Section, the third-order 1-1-1 MASH-option is chosen. The third-order high-pass  $\Sigma\text{-}\Delta$  modulator is called via the MATLAB routines and the user-defined parameters are assigned to the already created Simulink model blocks. The simulations of the histogram of the  $\Sigma\text{-}\Delta$  modulator output, the output spectrum and the corresponding SNR values are all shown in Fig. 1.

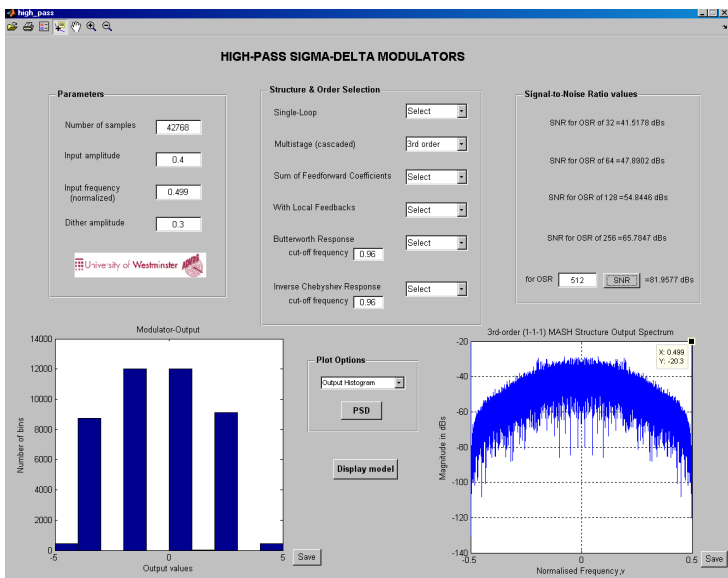


Figure 1. Screenshot of the GUI consisting of the input parameters and simulation results for a 3<sup>rd</sup> order HP  $\Sigma\text{-}\Delta$  Modulator

The output spectrum from Fig.1 shows the input signal at the correct frequency as well as spectral tones at lower frequencies.

The resulting SNR values for OSRs of 32 and 256 are 41 dB and 65dB respectively, which shows a good agreement with the reported results in the literature [8], [9].

The Simulink model for a 3<sup>rd</sup> order HP  $\Sigma\text{-}\Delta$  modulator using a 1-1-1 MASH structure is shown in Fig. 2.

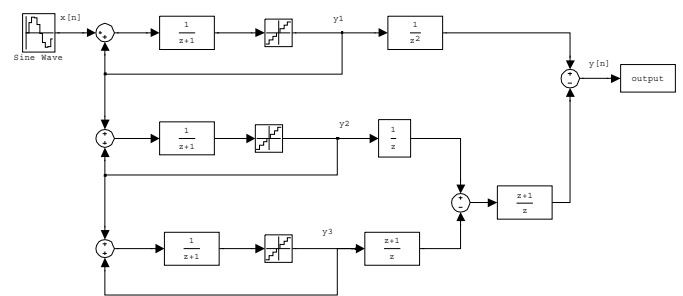


Figure 2. A 3<sup>rd</sup> order HP  $\Sigma\text{-}\Delta$  Modulator using 1-1-1 MASH structure

The power spectral density of the output signal is computed using the Welch’s averaged modified periodogram method of spectral estimation with 2048 overlaps. By averaging the output spectrum, the tones can be distinguished so that the  $\Sigma\text{-}\Delta$  modulators performance can be analyzed more effectively. The averaged PSD of the output of the corresponding HP  $\Sigma\text{-}\Delta$  modulator is plotted using the ‘PSD’ options in the GUI pane, as shown in Fig. 3.

The user can zoom in/out the plots and also save them to the desired file locations. The data cursor option provides the user effective simulation analyses and a comparison of the different output stages.

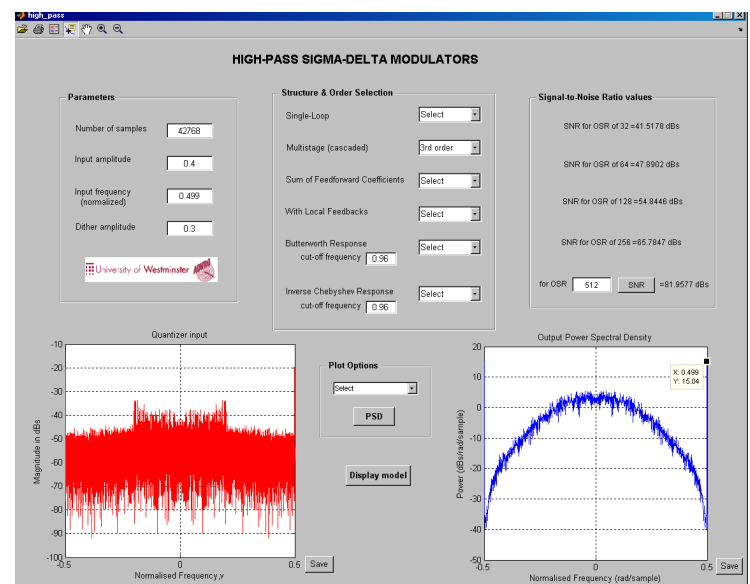


Figure 3. The quantizer input (left) and the output PSD (right) plots of a 3<sup>rd</sup>-order HP  $\Sigma\text{-}\Delta$  Modulator

When the user changes the input signal parameters or the modulator specifications according to the desired design structure, the simulation results are updated simultaneously.

As soon as the user selects the *band-pass modulation* option in the main window, the parameter-specification-window for BP  $\Sigma\text{-}\Delta$  modulators will be displayed. A 6<sup>th</sup> order mid-band band-pass  $\Sigma\text{-}\Delta$  modulator using the chain of accumulators structure with weighted feedforward summation was also simulated for an input amplitude of 0.55, a normalized input frequency of 0.25, and a dither amplitude signal of 0.2. The NTF and the output spectrum are plotted. The corresponding simulation results are shown in Fig. 4.

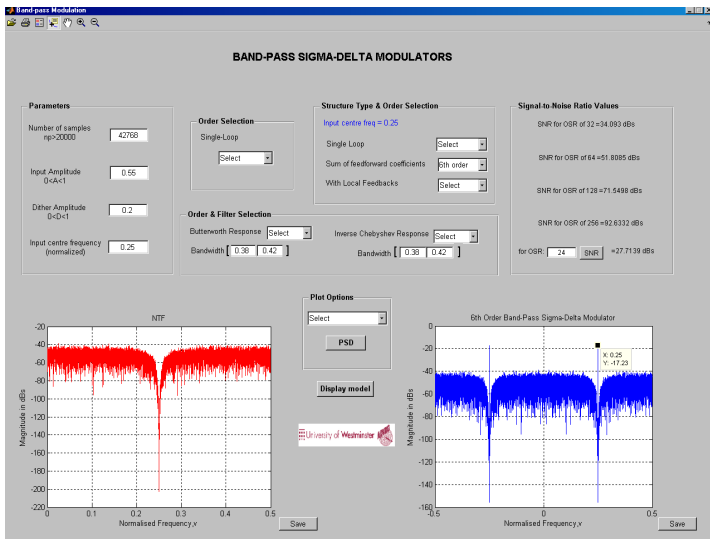


Figure 4. Screenshot of the GUI consisting of the input parameters and simulation results for 6<sup>th</sup> order BP  $\Sigma\text{-}\Delta$  Modulator

The corresponding structure of a 6<sup>th</sup> order BP  $\Sigma\text{-}\Delta$  modulator is given in Fig. 5. Using Butterworth approximation the coefficients  $a_1$ ,  $a_2$  and  $a_3$  are calculated as -0.2512, 0.0306 and -0.0018 respectively.

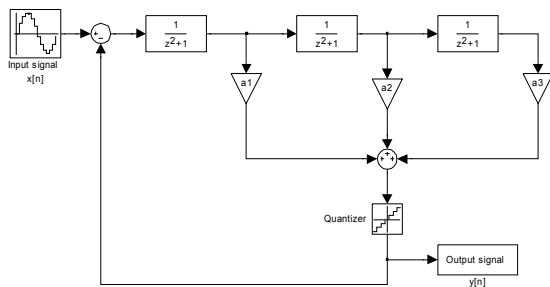


Figure 5. A 6<sup>th</sup> order BP  $\Sigma\text{-}\Delta$  Modulator using chain of accumulators with weighted feedforward summation

The GUI updates the frequency response of the output signal, the NTF and the input to the quantizer, as well as the in-band SNR values for each alteration that the user makes to the input parameters. Thus, the effects of small input changes can be easily observed in the output. As in the examples above, the user can define the specifications in order to design a variety of single-loop or multi-stage LP, BP and HP  $\Sigma\text{-}\Delta$  modulators.

### III. CONCLUSION

A software design, evaluation and measurement tool in a MATLAB environment in conjunction with SIMULINK is developed to speed up the design, analysis and evaluation at the system-level of various easy-to-implement LP, BP and HP  $\Sigma\text{-}\Delta$  modulators. This tool is user-friendly as it allows designers and practitioners to perform detailed simulations very easily. The MATLAB routines embedded in the GUI map the user-defined parameters into the building block specifications and display the simulation results in terms of SNR values, histograms and power spectral densities. The authors plan to develop the proposed GUI to incorporate the modeling of non-idealities at the system-level as well as stability and tonality indexes and other relevant metrics, and made make it available to the user community free on the ADVRG web-site in the future.

### REFERENCES

- [1] Pervez M. Aziz, Henrik V. Sorensen, Jan Van Der Spiegel, "An overview of sigma-delta converters," IEEE signal Proc. Mag., vol. 13, pp. 61-83, January 1996.
- [2] S. R. Norsworthy, R. Schreier, and G. C. Temes, "Delta-sigma data converters," IEEE Press, Newyork, 1997.
- [3] J. Ruiz-Amaya, J.M. de la Rosa, F. Medeiro, F. V. Fernandez, R. del Rio, B. Perez-Verdu and A. Rodriguez-Vazquez, "MATLAB/SIMULINK-based high-level synthesis of discrete-time and continuous-Ttme  $\Sigma\Delta$  modulators," IEEE Computer society, 2004.
- [4] T. Kuo, K. Chen, and J. Chen, "Automatic coefficients design for high-order sigma-delta modulators", IEEE Trans. on Circuits and Systems-II: Analog and Digital Signal Proc., Vol.46, No. 1, January 1999.
- [5] F. Medeiro, B. Perez-Verdu, and A. Rodriguez-Vazquez, "Top-down design of high-performance modulators," Kluwer Academic Publishers, Netherlands, 1999.
- [6] K. Francken, P. Vancorenland and G. Gielen, "DAISY: A simulation-based high-level synthesis tool for  $\Sigma\Delta$  modulators," Proc. IEEE Int. Conf. Computer-Aided Design, pp. 188-192, 2000.
- [7] S. Jantzi, C. Ouslis, and A. Sedra, "Transfer function design for  $\Sigma\Delta$  converters," IEEE Proc. ISCAS, pp. 433-436, May 1994.
- [8] S. Ralph, R. Farrell, "Using high pass sigma-delta modulation for class-S power amplifiers," Circuit Theory and Design ECCTD 18th European Conf, pp. 707 - 710, August 2007.
- [9] V. T. Nguyen, P. Loumeau, and J.F. Naviner, " High-pass  $\Sigma\Delta$  modulator: from System Analyses to Circuit Design", IEEE ISCAS, 2006