Multi-Factor Dynamic Modelling and Forecasting of Interest Rates and Equity Markets

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ABSTRACT

In this thesis, several theoretical specifications and estimation techniques are employed towards the dynamic modelling and forecasting of the term structure of interest rates, both independently and in conjunction with equity markets.

The first empirical investigation is motivated by the recent call for richer specifications following the global financial crisis of 2007-2009. In that regard, several existing multi-factor continuous-time models are extended to four and five factors to assess the benefit of richer models. The Gaussian estimation methods for dynamic Continuous-Time models yield insightful comparative results concerning the two different segments of the yield curve. The dynamics of the more volatile short-end of the yield curve are best explained by the most flexible models which consistently outperform all the other less complex models in terms of both in-sample and out-of-sample performance. For the long-end flatter segment, the benchmark discrete-time parsimonious models seem hard to beat, while the addition of extra factors has a minimal benefit in terms of forecasting performance.

In a second empirical study, the term structures of three Scandinavian countries are modelled using multi-latent-factor models. The empirical results produced by Kalman filter estimation method indicate that the three-factor specification captures most of the changes over time in the shape of the yield curve for Denmark and Norway, while for Sweden the statistical tests do not reject the two-factor model against the three-factor formulation.

Finally, the third investigation brings new empirical evidence of the impact of the 2007-2009 financial crisis on the return and volatility linkages between the U.S. - the country where the shock originated and other major economies using a multivariate methodology for the simultaneous modelling of interest rates and equity markets. During the global financial crisis of 2007-2009 the financial markets around the world have communicated through a more complex network of information transmission routes. The channels with most intensity of information transmission were the indirect international ones, bringing new evidence of the importance of this type of routes that has previously been investigated very little in the spillovers literature

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Last but not the least, I would like to thank my friends and family, especially my daughters for putting up with a student mother and shorter holidays for such a long time.

Declaration

I hereby declare that except where specific references are made to the work of others, the contents of this doctoral thesis are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or other university.

Elena Tunaru

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List of Abbreviations

ACF – Autocorrelation Function

ADF – Augmented Dickey–Fuller test

AIDS – Almost Ideal Demand System

ANN Artificial Neural Networks

AR(1)– Auto-Regressive of order one

ARFIMA – Autoregressive Fractionally Integrated Moving Average

ARIMA - Autoregressive Moving Average

BBA – British Bankers' Association

BDFS – Balduzzi, Das, Forezi and Sundaram 1996 model

BDT – Black Derman Toy 1990 model

BDNS – Block Dynamic Nelso Siegel Model

BEKK – Baba, Engle, Kraft and Kroner 1995 model

BIC – Bayesian Information Criterion

BK – Black and Karasinski 1991 model

BN – Babbs and Nowman 1999 model

BoE – Bank of England

BS – Brennan and Schwartz 1980 model

CDCP – Correct Direction Percentage Change Error

CEV – Constant Elasticity of Variance 1975 model

CIR – Cox, Ingersoll and Ross 1985 model

CKLS – Chan, Karolyi, Longstaff and Saunders 1992 model

DNS – Dynamic Nelson Siegel

EDM – Exact Discrete-time Model

EMM – Efficient Method of Moments

EMU – European Monetary Union

EMS – European Monetary System

FH – Flesaker and Hughston 1996 model

GARCH – Generalized Autoregressive Conditional Heteroskedasticity model

GBM – Geometric Brownian Motion

GC- General Collateral

GDP – Gross Domestic Product

GFC – Global Financial Crisis

GMM – Generalised Method of Moments

HJM – Heath, Jarrow and Morton 1992 model

I(1) – Integrated of order one

IV – Instrumental Variables

KF – Kalman Filter

LL – Local Linear regression

LRSQ – Linear Rational Square Root

LTCM – Long-Term Capital Management

MAE – Mean Absolute Error

MAPE – Mean Absolute Percentage Error

ML – Maximum Likelihood

MMC – Mesh-based Monte Carlo

PCA – Principal Component Analysis

PDE – Partial Differential Equation

QE – Quantitative Easing

QMLE – Quasi-Maximum Likelihood

SDE – Stochastic Differential Equation

SONIA - Sterling Overnight Interbank Average rate

TSIR – Terms Structure of Interest Rates

USV - Unspanned Stochastic Volatility

VAR - Vector Autoregressive

VARMA – Vector Autoregressive Moving Average

VARMAX - Vector Autoregressive Moving Average with explanatory variables

VRP – Variable Roughness Penalty model

ZCYC - Zero Coupon Yield Curve

ZLB – Zero Lower Bond

Chapter 1

Introduction

Over the last three decades, interest rate modelling has become both theoretically and empirically, one of the most important and challenging areas of modern finance. Factors such as the expansion of international financial markets, the increased trading of new derivative products and the progress attained in the computational field, have all contributed substantially to this evolutionary process, whose starting point can be traced back in time to Vasicek's (1977) dynamic interest rate model.

1.1 The Importance of Interest Rate Modelling

Interest rates are one of the most important economic and financial variables at both, macroeconomic and microeconomic levels. At the macroeconomic level, the level of economic activity can be influenced through conventional monetary policy tools. One of the most common tool is the decision taken by central banks and the Monetary Policy Committee (MPC) to change the official (base) interest rates with the aim to deliver price stability (low inflation targeting) which is crucial to the ultimate objective of economic stability (Hamilton and Wu, 2012). For example, a decrease in the official interest rates will be reflected in other borrowing and lending rates, encouraging borrowers (spending) and discouraging lenders (savers) which feeds further into increasing output and employment levels. Therefore, the decision of altering interest rates is made conditionally upon current and predicted levels of other economic variables such as money supply, inflation and gross domestic product (GDP). Recently, academics and economists consider complex interest rate models in macro-finance in the attempt to better understand and explain interest rate movements and their relation to these economic variables (e.g., Ang et al., 2006; Rudebusch and Wu, 2008). At the microeconomic level, understanding the behaviour of interest rates is crucial to areas such as derivatives pricing, portfolio allocation, risk management and forecasting (Diebold and Rudebush, 2013). The financial trading of derivative products in general and of fixed-income products in particular (e.g. bond options, interest rate swaps, swaptions, caps, floors and mortgage-backed securities), constitute a valuable component of the world economy as most of the financial institutions, investment banks and government treasuries rely on this kind of activity, with billions of dollars traded on a daily basis (Dempster et al., 2014). As the largest user group of interest rate models, investment banks focus on current valuation of various financial products and hedging strategies. Risk managers also rely on interest rate models in order to simulate market behaviour, so that they can dynamically assess the return on their holding portfolios and calculate their risk exposure to fluctuations in the level of interest rates. Thus, a model that successfully explains the dynamics of the yield curve is an absolute requirement for any successful financial strategy followed by banks, insurance companies and other financial institutions.

1.2 The Gaps in the Literature and Motivation

Compared to other financial concepts or variables, the term structure of interest rates has two dimensions, the static cross-dimension given by the maturity (tenor) of the interest rates and a dynamic time-dimension that refers to the change in the shape of the yield curve over time. That is, at one point in time there is information about multiple points on the yield curve.

Given the importance and the duality of the yield curve concept, the literature on the term structure of interest rates models is vast and complex. According to Diebold and Rudebusch (2013, pp 1) it is looking more like “a tangled web” with numerous overlapping categories and lacking a unified platform that would allow the selection of a reference model such as the Black and Scholes (1973) model for the pricing and hedging of stock derivatives. Thus, the choice for the theoretical model it is a difficult one and it has to fit the purpose at hand.

Nevertheless, it is widely recognised by academics and practitioners that continuous-time models are more appropriate than discrete-time models (Bergstrom and Nowman, 2007), and that multi-factor models are superior to single-factor models (Dai and Singleton, 2000). However, there is no definite answer to how many factors should a model include. Until recently, most of the term structure of interest rates (TSIR) studies relied on the early findings of Litterman and Scheinkman (1991) who concluded that three unobservable factors (described as level, slope and curvature) can explain over 95% of the fluctuations in yield curves. Consequently, only few empirical studies[1] have considered beyond three sources of uncertainty despite the early existence of several generalized n-factor models[2]. The severity of the last global financial crisis of 2007-2009 (GFC) has been partially blamed on the reduced number of risk sources in of the financial models used by financial market participants (Shiller, 2012). This point of view has prompted the recommendation by financial regulators (Basel II Committee on Banking Supervision, 2010) that banks should increase the number of risk factors when modelling the yield curve. One category of dynamic term structure models that intuitively accommodate for such extensions is the multi-factor yields-only models. Following Nowman (2001, 2003), the general framework developed by Chan et al. (1992) (hereafter CKLS) will be extended to four and five factors in the first comparative empirical investigation of this thesis.

Turning to the empirical literature, the picture is again unclear. Even from early stages the empirical evidence did not converge, as the estimation results seem to be very sensitive to aspects such as the frequency and source of the data, the data period and the estimation method employed (see Treepongkaruna and Gray, 2003; Lo, 2005). A well-known problem in the estimation of a continuous-time model is the choice of a discrete-time model based on a particular discretisation method, since the data available is only discrete. Recently Duffee and Stanton (2012, pp 2) have suggested that "due to our limited understanding of the properties of the estimation techniques available when applied to sophisticated term structure models" the empirical testing of such models is rather immature compared to the corresponding theoretical literature. Looking for an optimal estimation technique they concluded that the Kalman filter technique combined with the maximum likelihood (ML) estimator is superior to other alternative such as moment-based and simulation methods. In the second empirical investigation, the Kalman filter technique will be employed in this thesis to estimate the Babbs and Nowman (1999) multi-factor model for three Scandinavian countries.

A large part of the interest rate modelling literature analyses TSIR models based on their implications for pricing interest rate sensitive securities like bonds and bond options, with less focus on the dynamic aspect of forecasting that is equally important as recognised by Dempster et al. (2014, pp. 251) who assert that the key to the accuracy of

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[1] Egorov et al. (2011) and Steeley (2014a) used four factors, while Christensen et al. (2009) considered a six-factor model.

[2] A list of such generalised models includes Langetieg (1980), Jamshidian (1996) and Babbs and Nowman (1999).

any global macroeconomic model “are the yield curve models that forecast interest rates and upon which the determination of all other variables depends”. The application of the yield curve modelling to predict future movement in the level of interest rates has recently become an important focus in the relevant literature. However, these studies employ frequently the semi-parametric affine Nelson-Siegel model augmented with macroeconomic factors (see Steeley, 2014b and Ullah, 2016). Therefore, would be of great interest to analyse also the forecasting performance of newly extended the four- and five-factor CKLS models.

The three aspects discussed above (richer models, better estimation method and forecasting performance) have been depicted from the term structure of interest rates literature to constitute the main motivation behind the empirical research presented in the first two empirical studies of this thesis.

The latest phase of the crisis, more precisely the European sovereign debt crisis in 2009 had increased the interest on examining the inter-linkages among international bond markets. In a third investigation, the multivariate BEKK (1,1) model that is widely used in measuring the return and volatility spillovers between different types of markets is employed to analyse the flow of this type of information between the shock-source country (the U.S.) and one of the following major economies: the U.K., Eurozone, Japan and Canada. Most of the spillovers studies keep the domestic and the international transmission channels in isolation with only few studies (Christiansen, 2010; Ehrmann et al., 2011) combining simultaneously equity and bond returns in a discrete-time econometric framework. The new data provided by the last financial crisis will be subject to pre- and post-crisis analysis in order to measure the impact of the crisis on the information transmission process within a more complex network of channels (domestic/international and equity/interest rates markets).

1.3 The Structure of the Thesis

The chapters in this thesis proceed as follows. Following this introduction chapter, the second chapter presents an extensive literature review of continuous-time term structure of interest rate (TSIR) models. Given their practical implications for econometric and forecasting analysis, factor models are critically discussed in larger detail emphasising their main contributions and limitations in terms of their mathematical specifications and of their statistical properties. The main research of the thesis comprises three empirical investigations. The first two empirical studies (Chapter 3 and 4) consider different approaches of dynamic modelling and forecasting of various bond markets, while the third study (Chapter 5) investigates the simultaneous modelling of bond and equity markets during the 2007-2009 financial crisis. The final conclusions and lines of future research are presented in Chapter 6.

1.4 The Aims and the Contributions to Knowledge

The main purpose of the first empirical study is to assess if there is any benefit in enriching the models by adding extra factors as required by the financial regulators in the aftermaths of the GFC of 2007-2009. This aim is achieved by extending the CKLS multivariate framework for the first time to four and then to five factors and by comparing the performances of the two extensions in terms of goodness of fit and prediction power. In addition, a range other classic multi-factor interest rates models nested in the CKLS model will be estimated using the Gaussian estimation methods of continuous-time dynamic systems developed by Rex Bergstrom (1983, 1985, 1986, 1989, 1990). In contrast with the classic multi-factor models such as Chen (1996) and Balduzzi et al. (1996), the CKLS framework can be interpreted as an *intrinsic* yield curve model, as all the factors are interest rates of different maturities bringing in as much information as possible. Hence, it accounts for both (time and cross-sectional) dimensions of the yield curve and also for the theoretical element that interest rates move together in a very complex fashion by modelling their correlation matrix over time.

The short end of the yield curve is estimated in an international comparative context involving five currencies of countries chosen as most important and diverse within the G10 group: the U.K., the U.S., the Eurozone, Japan and Canada. In addition, the long end of the yield curve is estimated using U.K. Government nominal interest rates. The empirical results from the dynamic estimation of a total of forty-eight models provide the in-the-sample estimates that are used to comparatively conduct an extensive forecasting analysis.

The empirical results of the dynamic estimation favour the five-factor models over the four-factor models in terms of goodness-of-fit. The addition of the fifth factor increased substantially the goodness of fit of the more complex models to the data, with some nested models being very close to equal the performance of the general CKLS model. The forecasting analysis suggests that for shorter maturity (up to six months) interest rates, the continuous-time models nested in the CKLS framework outperform consistently the discrete-time models. This provides empirical evidence to advocate the use of more complex and richer models to better explain the more volatile segment of the yield curve.

Hence, using more sources of risk than previously used in the literature improves the predictive performance of the models on the shorter maturity spectrum of the yield curve. For longer maturities, it is concluded that the addition of the fifth factor brings minimal improvement to the forecast-reliability of the models.

The second empirical study (Chapter 4) explores another TSIR modelling framework, namely the general multi-factor linear Gaussian model of Babbs and Nowman (1999) (BN hereafter). On the strong grounds of its multiple advantages (generality, tractability, and correct treatment of the state variables) this type of model is a real candidate in the race for “the best” TSIR model. One of the aims of this study is to compare the performance of one-, two- and three-factor versions of the BN model in terms of both goodness of fit and predictive power. For the first time in the literature this model will be applied to a panel data of government nominal interest rates from three Scandinavian countries: Denmark, Norway and Sweden. The models are estimated by using the Kalman filter technique combined with the ML estimator based on daily zero-coupon yields with a cross-section of eight maturities over the January 2000 - September 2014 period. The Kalman filter technique allows explicitly for measurement errors in the data, avoiding therefore the common approach (e.g. Chan et al., 1992; Nowman, 1997) of using short-term rates as a proxy for the instantaneous interest rate. Once the models are estimated the time series of the measurement errors implied by the Kalman filter are analysed to test for misspecification bias. Moreover, the Kalman filter facilitates the extraction of the latent factor time series and the identification of the factor loadings as function of term to maturity. This is of great importance to risk management where consistent revaluation is possible because the factor simulations play the role of the parameters used in the valuation process (Geyer and Pichler, 1999). A factor loading analysis will determine the impact that each factor has on the yield change and therefore the nature of the factors in terms of level, slope and curvature. The model comparison continues with the forecasting analysis as the best in-sample performing model is not necessarily best in predicting future interest rate values. It is of interest to see if the models perform in a similar way across the analysed Scandinavian countries and if there are any differences between the number of factors that explain best their TSIR.

Based on formal statistical tests and residual analysis, the empirical results indicate that the three-factor specification explains best the changes over time in the shape of the yield curve at least for Denmark and Norway, while for Sweden the two- and three-factor specification perform in a similar way. There is evidence of a structural break as a result of the last global financial crisis, as the estimation results for the pre-crisis data-sample differ considerably from those from the post-crisis period. In terms of factor analysis, the latent factors can be interpreted as the level, slope and curvature only for Denmark and Norway. The empirical results are in general different for Sweden, suggesting that the term structure of Swedish interest rates has simpler dynamics for which two factors are sufficient.

The third empirical investigation (Chapter 5) will bring new evidence to the existing spillovers literature by investigating the impact of the GFC on the return and volatility spillovers between the U.S. and four major markets, namely U.K., Germany, Japan and Canada. Despite the high degree of integration among these major economies, their relationship with the U.S. can still be country-specific due to the differences in the structure of their financial systems, in the state of their economies and in the monetary policies implemented during the GFC.

The most recent global financial crisis of 2007-2009 has prompted a new wave of research on information spillovers, with numerous studies exploring new transmission channels and developing new methods to model the dynamics of a crisis (Longstaff, 2010). The modelling framework employed is the discrete-time multivariate generalised autoregressive conditional heteroscedasticity (MGARCH) framework. More precisely, the four-dimensional full BEKK(1, 1) is used to model simultaneously the linkages between bond and equity markets as in Christiansen (2010) and Ehrmann et al. (2011). The four-factor model allows for a more complex network of channels (internal and external) is examined simultaneously. The short- and long-term segments of government bond markets are considered separately, in conjunction with the respective equity markets. By considering the short- and long-term bond markets separately one could determine if the information is transmitted in a different and specific way between the stock markets and each maturity segment of the fixed income markets.

In addition, the models employed are estimated over two periods - before the crisis and during the crisis – to observe any significant changes in the structural parameters and to assess the impact of the last financial crisis on the return and volatility spillover effects between the markets considered.

The comparative analysis of the summary results concludes that out of the three types of routes of information transmission, the most active routes are the indirect external route followed by the domestic one. These results are valid for both return and volatility channels and it emphasises the importance of considering this type of routes, ignored previously in the spillovers literature. Along these routes, the information flows unidirectionally from the interest rate markets to the equity markets and not vice-versa, implying that the interest rate markets dominate the equity markets in transmission of information. When comparing the results of the two segments of the yield curve it is found that the return and volatility spillover effects are much stronger when the equity markets are modelled in combination with the long-term markets than with the money markets. Among the countries considered, the results for Canada are rather different as the Canadian markets seem to influence indirectly the U.S. markets, reflecting the relative stability that Canadian markets sustained during the crisis.

Chapter 2

Interest Rate Modelling - Literature Review

2.1 Introduction

The material presented in this chapter intends to provide a comprehensive literature review of continuous-time models of the term structure of interest rates, with a focus on factor models, since this kind of specifications will be used in the subsequent chapters towards several empirical applications. The current modern financial literature offers a profusion of theoretical interest rate models and as a result numerous models have been selected for discussion. Highlighting the similarities and differences among the models and critically presenting their main contributions and limitations will help other researchers with the selection of the appropriate interest rate model for their investigations. This will contribute towards a clear common platform on which the models will be relatively compared in terms of their mathematical specifications, their statistical properties and of their implications in pricing interest rate sensitive securities like bonds and bond options.

This chapter is structured in two main sections, dedicated to the theoretical and empirical streams of interest rate modelling, respectively. The first section tries to illustrate the theoretical development of the modelling of interest rates by presenting first certain fundamental concepts related to the construction of the term structure and the challenges posed by the process of selecting the "best" model. The description of term structure models starts with single-factor interest models, followed by multi-factor interest rate models. The theoretical literature review continues with the brief description of other types of important interest models such as Heath, Jarrow and Morton (1992), market and macro-finance models. The second main section provides a comprehensive literature review of the empirical evidence on interest rate models, emphasizing the difficulties encountered in the estimation of complex continuous-time interest rate models.

2.2 Literature Review of the Theoretical Interest Rate Models

2.2.1 The Term Structure of Interest Rates (TSIR)

All financial assets can be valued by applying the appropriate discount rate function to their expected future cash flows. Different valuation techniques involve distinct types of interest rates such as spot rates, short rates, forward rates and yields-to-maturity. The pattern observed in a type of interest rate from instruments with different terms to maturity but similar credit risk at a fixed point t in time, is called the *yield curve* or the *term structure* of that particular type of interest rate. With the taxonomy of interest rates there is a range of different term structures/yield curves such as spot curve, forward curve, yield-to-maturity curve and swap-rate curve, all considered to be smooth functions of time to maturity T, $T-t$.

A close relationship exists between the discount curve (the curve of zero-coupon bond prices) $T \mapsto P(t,T)$, the implied spot curve $T \mapsto R(t,T)$ and the forward curve $T \mapsto f(t;T,S)$ where $f(t;T,S)$ is the forward rate over the future period (T,S) calculated at time t ($t < T < S$). Given one of the curves the other two can be uniquely determined through the following pricing equalities in a continuous compounding setting (Cairns, 2004):

$$R(t,T) = -\frac{1}{T-t}\ln P(t,T) \qquad (2.1)$$

$$f(t;T,S) = -\frac{1}{S-T}\ln\frac{P(t,S)}{P(t,T)} \qquad (2.2)$$

where the forward rate prevails over the time interval $[T,S]$ and there is a boundary constraint $P(T,T) = 1$.

However, the yield curve fitting exercise is facilitated in the context of spot yields because in this case the curve is reasonably flatter than the exponentially decaying curve of the bond prices. Importantly, the spot rates $R(t,T)$ being derived from market prices of government zero coupon bonds, can be regarded as the risk-free interest rates over fixed periods of time. For these reasons, in most of the financial literature the *term structure* of (risk-free) interest rates is represented by the zero-coupon yield curve (ZCYC).

In essence, the role of interest rate models is to describe and explain the dynamics of the price-curve of zero coupon bonds under the assumption of a known initial position. Although financial markets are unable to provide sufficient observations on prices of discount bonds, there are certain liquid securities that offer a straightforward derivation of these prices. Even then, the continuity of the curve is not accomplished as only a finite number of observations are available. Hence, the term structure of interest rates is not directly observable and it needs to be statistically inferred from market prices. Yield curve construction forms the basis of interest rate modelling and involves various techniques from simple interpolation methods to more advanced *continuous-time models*.

Among the first attempts, McCulloch (1971) estimated a discount yield curve using cubic splines, while studies such as Merton (1973) and Vasicek (1977) were among the first continuous-time approaches to yield curve construction.

The concept of the *short* rate $r(t)$ plays a fundamental role in modelling the term structure of interest rates. In a continuous-time setting the *short* rate is the instantaneous spot rate, defined as the yield of a bond with an infinitesimal maturity, that is $r(t) = \lim_{T \to t} R(t,T)$, which is also equal to $f(t;T,T)$, the instantaneous forward rate (see James and Webber, 2000).

The practical issue of the best proxy for the short rate is still debatable, with considerable implications on the empirical results and their interpretation. While the stability of the Fed Funds rate provides a reason for considering the overnight rate as a fair proxy for the short rate, this choice has been avoided because of its minimal correlation with other spot rates and the different nature of the forces driving the overnight market from those existent in the longer-term money market. Any good model for the short rate should have the ability to replicate most of the stylized facts withdrawn from historical data regarding the dynamics of the yield curves. For example, a main feature of the interest rates time series is the mean reversion property, which means that the interest rates are pulled systematically towards a long-run average level. The average shape of the yield curve is concave and increasing, evolving in time and passing through various shapes - from upward sloping to downward sloping, humped, or inverted humped shape. The short-term segment of the yield curve is characterised by higher volatility than the long-term segment; the dynamics of the yield curve are in general persistent with a higher degree of persistence being observed at the long end of the yield curve. (see Diebold and Li, 2006).

Over the years, various empirical studies have presented substantial evidence of the diversity in the interest rate dynamics from periods of stability to high volatility, from persistence to surprising jumps, and from movements between the levels to even the possibility of cyclical patterns[1]. Models that accommodate most of these features of the yield curve are practically important, especially for the dynamic forecasting of future interest rates and for correct/fair pricing of interest rate sensitive financial instruments. Hence, the dynamics of the term structure of interest rates have been the subject of intensive and sophisticated mathematical modelling. The randomness of interest rates has been examined following different approaches. While the discrete-time approach has represented a major step towards understanding and explaining interest rate dynamics as in Cox, Ross and Rubinstein (1979) and Ho and Lee (1986), the continuous-time modelling framework is currently unanimously adopted in recognition of the continuous evolution of the modern financial markets.

Despite all the effort put into developing "better" interest models it is still difficult to produce a model that would entirely capture the randomness observed in the behaviour of the interest rates. According to James and Webber (2000), the main features for a model to be "good" include the ability to accurately value simple and novel financial products, easy calibration and robustness. However, the literature often expands on the list of criteria acknowledging at the same time the difficulty for a model to simultaneously satisfy all of them. For example, Rogers (1995) considers also theoretical and computational arguments and advocates the following list of criteria that an interest rate model should satisfy:

- to be flexible and be capable of generating a variety of yield curves
- to be based on inputs that are either directly observable or easy to estimate
- to be consistent with market prices
- to be computationally fast
- to exclude negative interest rates and other impossible situations
- to be free of arbitrage

However, following the GFC of 2007-2009 the financial markets around the world have been subject to prolonged periods of near-zero and even slightly negative interest rates for some countries (Denmark and Switzerland). As a result, the requirement for a model to exclude negative interest rates has to be reconsidered.

[1] Most of these patterns could be observed for example, in the behaviour of the interest rate in the daily time series of three-month LIBOR rate from 1988 to 1995 (James and Webber, 2000).

2.2.2 The Taxonomy of Continuous-time Interest Rate Models

Depending on specific criteria term structure models could be classified in many different ways. For example, there are discrete-time models competing against continuous-time models, single factor models versus multi-factor models, linear drift models against nonlinear drift interest rate models. In terms of calibration we can differentiate between no-arbitrage models and equilibrium models. The no-arbitrage models fit exactly the observed market data, providing a snapshot in time of the yield curve but losing the time homogeneity of the parameters, while the equilibrium models consider the current market prices as an output that only approximates the current term structure.

The multitude of theoretical TSIR models can also be categorised depending on the specific form taken by the short rate: as a state variable, as an affine combination of state variables, as a sum of the squares of the state variables, as an exponential of state variables or just as a point on the forward curve. An intuitive classification of interest rate models, depending on the state variable used and on its specific dynamics is presented in James and Webber (2000, p.60). Accordingly, there are six main categories of interest rate models: affine yield models such as Duffie and Khan (1994, 1996); whole yield curve models such as Heath, Jarrow and Morton (1992) (hereafter HJM); market models as Jamshidian (1997); price kernel models like Constantinides (1992) and Rogers (1997); positive models (log-r models) like Black and Karasinski (1991) and consol models such as Brennan and Schwartz (1979). More types of models can still be added to this impressive list of interest rate models. For example, most diffusion models can be jump-augmented where the resulting models recognise jump existence in the dynamics of interest rates. A list of jump-diffusion models includes those developed by Ahn and Thompson (1988), Das and Foresi (1996), Das (1997, 2002), Attari (1999), Duffie, Pan and Singleton (1998), Heston (2007) and Sorwar (2011). Another class of interest rate models are the regime-switching (RS) models that attempt to capture the non-linearity empirically observed in both, conditional drift and volatility of the short rate and the near unit-root persistence in interest rate data. Building on the work of Hamilton (1989, 1994) a large number of RS models have been constructed and empirically tested with important implications for the macro-economic context as shown in Gray (1996), Ang and Bekaert (2002), Bekaert, Hodrick and Marshall (2001) and Naik and Lee (1993).

A specific and popular class of short rate models is the class of *affine models* that possess a high degree of analytical tractability and flexibility. First introduced by Brown and Schaefer (1994) and later extended by Duffie and Kan (1994)[2], affine models are very useful for derivatives pricing and in particular for econometric analysis. A short rate model is affine if both, the drift and also the square of its diffusion component[3] are affine (linear) functions in the level of the interest rate (Andersen and Piterbarg, 2010). An important result demonstrated by Duffie and Kan (1996), is that the bond prices and zero-bond yields are affine functions of the short interest rate if and only if the short rate follows an affine process, hence affine specifications fully describe the term structure of interest rates with a "cross-section" of interest rates computed at any time (Lemke, 2006). Affine type models can be classified further in three categories: Gaussian affine models for which all the state variables follow the Vasicek model, CIR affine models where all state variables have a square-root-volatility and finally, affine models that allow for a combination of both types: Gaussian and CIR-type state variables. A more technical classification based on two dimensions was elaborated by Dai and Singleton (2000), who defined nine classes of equivalence organised in three categories depending on the number of factors. Another distinct group of affine models is represented by the quadratic affine type (see Andersen & Piterbarg, 2010) where the short rate is a quadratic function of a Gaussian stochastic process.

With so many, sometimes overlapping classes of interest rate models, it is simpler and more relevant for the empirical research carried out in this thesis to broadly distinguish (see Gibson et al., 2010) between two main types of interest rate models: factor-models and whole yield curve models based on the forward rate such as HJM and market models. It is important to recognize that the two groups have different practical implications. On one side, being dynamic, factor interest rate models bring essential information based on historical data about the pattern of future rates, hence they are more suitable for econometric and forecasting analysis. The market models on the other side are static, in the sense that they describe the position of the yield curve at one particular point in time and they involve frequent recalibration that ultimately will change the model. Nevertheless, due to their facile calibration to observed market prices they are preferred by trading desks and other practitioners, making them extremely popular in the last decade in comparison with multi-factor models. However, due to a higher tractability the classical short-rate models are still used in conjunction with more advanced models for risk-management purposes where valuation of derivative products is frequently

[2] Cox, Ingersoll and Ross (1985b) have only mentioned a similar class of models; however, Brown and Schaefer (1994) considered the affine property for the first time only for a single factor; later Duffie and Kan (1994) studied a detailed extension to a finite multidimensional space of state variables.
[3] Affine models are also called exponentially affine models.

required. As the empirical investigations carried out in the following chapters of this thesis consider estimation and forecasting analysis of several multi-factor models, the literature review of interest rate models presented in this chapter focuses more on factor models while briefly describing the market models.

2.2.3 Factor Models

2.2.3.1 Single-Factor Interest Rate Models

Although multi-factor models perform better and are more realistic, single factor models are not obsolete, providing the necessary foundation for the development of more complex term structure models. Given their historical importance and the necessity to understand the basic principles behind interest rate modelling, certain factor models will be presented in more detail than others, including their mechanics and the implied closed formulae for the term structure of interest rates and zero-coupon bond prices.

The continuous-time "classical" approach assumes that interest rates follow a stochastic process (Ito process[4]), expressed mathematically as a stochastic differential equation (SDE), for which the main state variable is the *short* rate[5], $r(t)$. During the 1970's and 1980's the models proposed for capturing the dynamics of interest rates involved only one factor – the short rate, whose randomness is driven by a standard Brownian motion[6]. A generic specification for a diffusion model of interest rates is given by the following SDE:

$$dr(t) = a(r(t),t)dt + b(r(t),t)dW(t) \tag{2.3}$$

where $W(t)$ is a standard Brownian motion (BM) under the real-world measure P, $a(r(t),t)$ and $b(r(t),t)$ are deterministic processes potentially dependent on both the time t and the level of the short rate $r(t)$. The equation above assumes that the infinitesimal change in the level of the interest rates is the sum of a drift $a(r(t),t)dt$ and a normally distributed fluctuation $b(r(t),t)dW(t)$. The part $a(r(t),t)$ is called the drift, while the diffusion part $b(r(t),t)$ represents the local instantaneous volatility[7] of the short rate process.

[4] The development of Ito calculus played a major role in modelling random processes, just like the underlying factors that drive the movements in the interest rates.
[5] Other approaches (for example HJM) use the instantaneous forward rate as the state variable.
[6] The Brownian motion is interchangeably represented by a Wiener process.
[7] The drift and the volatility are conditional on the history of the Brownian motion up to time t.

Any arbitrarily chosen form for drift and volatility functions will create a specific short rate model as long as the SDE can be solved. There are particular characteristics about the drift that distinguish between *mean reversion* models and non-linear drift models (e.g. Ait-Sahalia, 1996), while specific forms of the diffusion classify models as *Gaussian* models (e.g., Vasicek, 1977), *square-root* models (e.g., Cox et.al.,1985a and b, hereafter CIR), *power-type* models (e.g., Chan et al., 1992).

An important feature of the dynamics of interest rates is the historically observed mean reversion property. This means that while temporally persistent at a high level, the interest rates are most likely to fall towards an equilibrium level (and vice versa when interest rates are low). The most common way to model the mean reversion property is by considering the drift as a linear function of the short rate:

$$a(t,r(t)) = \alpha + \beta r(t) = -\beta(\frac{\alpha}{\beta} - r(t)) = k(\mu - r(t)) \quad (2.4)$$

where $k = -\beta > 0$ is the constant speed of reversion towards, and μ is the constant long-term equilibrium level of the interest rates.

Another important property observed in the behaviour of interest rates and empirically tested in the literature (see Chan et al., 1992; Tse, 1995; Episcopos, 2000) is the so called "level-effect" that describes the relationship between the volatility of the interest rates and their level. This dependence is incorporated in the diffusion function as a nonlinear expression in the level of the instantaneous interest rate as follows:

$$b(t,r(t)) = \sigma r(t)^{\gamma} \quad (2.5)$$

where $\sigma > 0$ is a volatility scale factor and $\gamma > 0$ is the level-effect parameter.

Early on, the model specifications were relatively simple with the majority of them including a mean reversion drift and assuming particular restrictions for the level-effect parameter γ. Some of the classical theoretical single-factor models include models such as Merton (1973), Cox (1975), Vasicek (1977), Dothan (1978), Brennan-Schwartz (1980), Rendleman and Bartter (1980), Cox et al. (1985a and b).

The Merton Model (1973)

One of the first continuous-time formulations of interest rate behaviour was presented in Merton (1973), as a standard Brownian motion with a constant drift:

$$dr(t) = \alpha dt + \sigma dW(t) \quad (2.6)$$

where both coefficients/parameters α - the instantaneous drift and σ - the conditional instantaneous volatility, are real constants and $W(t)$ is a standard Brownian motion. As a

Gaussian affine model, Merton's model is tractable, providing relatively simple closed formulae for the term structure of interest rate, of pure discount bond prices and of bond option prices, as presented below.

Given an initial value $r(0)$, the short-rate in Merton's model satisfies the following stochastic integral equation (see James and Webber, 2000):

$$r(t) = r(0) + \alpha t + \sigma \int_0^t dW(s) \quad (2.7)$$

As a result, the short-rate is conditionally normally distributed with the conditional mean and variance of $E[r(t) \mid r(0)] = r(0) + \alpha t$ and $Var[r(t) \mid r(0)] = \sigma^2 t$, respectively.

Under the expectations hypothesis of the term structure, Merton (1973) assumed that all the pure discount bonds for all the maturities will return on average over the next period a yield equal to the short-rate, i.e. $E\left(\frac{dP}{P}\right) = rdt$. Cootner (1964) pointed out that since at maturity the bond price equals its face value the total returns on the bond must be correlated over the life of the bond. Furthermore, Merton (1973) concluded that the variance of the unanticipated returns should be a function of the time-to-maturity $\tau = T - t$. Consequently, the analytical bond price proposed by Merton (1973) was particularly specified in order to satisfy these two properties:

$$P(t,T) = \exp\left(-r(t)(T-t) - \frac{\alpha}{2}(T-t)^2 + \frac{\sigma^2 (T-t)^3}{6}\right) \quad (2.8)$$

Hence, the zero-coupon bond price at time t with maturity date T, $P(t,T)$ is a function of the short rate and the time-to-maturity. The term structure is determined using equation (2.1) that relates the bond prices to the spot rates:

$$R(t,T) = -\frac{\ln P(t,T)}{T-t} = r(0) + \frac{\alpha}{2}(T-t) - \frac{\alpha^2}{6}(T-t)^2 \quad (2.9)$$

Merton (1973) himself acknowledged that the model is unrealistic and unstable because the Gaussian distribution of the short rate implies the possibility of negative values of interest rates with a positive probability, which is in contradiction with economic theory[8]. Additionally, the model is not flexible enough as only two shapes of the yield curve are possible. From equation (2.9) one can derive the slope of the term structure and observe the two shapes are a humped and a decreasing shape when $\alpha > 0$ and when $\alpha \leq 0$, respectively. While each spot rate is normally distributed, equation (2.9) also implies that

[8] Allowing for negative interest rates invalidates the no-arbitrage condition when there is cash in the economy (Gibson et al. 2010). However, during 2012 and 2013 short-term interest rates such as one-week, one-month and two-month CHF-LIBOR rates were negative.

the volatility of the spot rate is constant, i.e. independent of the maturity which is incompatible with the observational fact that in general short-term rates are more volatile than other maturity interest rates (Gibson et al., 2010). Another considerable criticism of Merton's model is the simplicity of the drift function which ignores the historically observed mean reversion feature.

The Constant Elasticity of Variance (CEV) Model (1975)

The CEV model was developed by Cox (1975) and by Cox and Ross (1976), in response to a possible inverse relationship observed in the equity market between stock price and the stock price volatility. The central feature of the model is the elasticity of variance parameter γ that measures the so-called *level effect* or the degree of dependence of the local volatility on the level of the state variable. In the context of interest rates, the SDE of the CEV model is written as a non-linear process:

$$dr(t) = \beta r(t)dt + \sigma r^{\gamma}(t)dW(t) \tag{2.10}$$

where the parameters β and σ are real constants and $0 \le \gamma < 1$[9].

The simplicity of the power-type volatility specification in the CEV allows for non-flat volatility surfaces (smiles or skews) and at the same time facilitates the determination of an explicit transition density function, and therefore a closed-form formula for the price of options and other simple claims (caplets). Cox (1975) provided the option pricing formulae implied by the CEV model in (2.10) that are rather complex, involving a standard complementary Gamma distribution function. Both the conditional mean and conditional variance of the instantaneous changes in the interest rate depend on the level of the instantaneous rate: $E[dr(t)\,|\,r(t)] = \beta r(t)$ and $Var[dr(t)\,|\,r(t)] = \sigma^2 r^{2\gamma}(t)$. It can also be shown that as the name of the model affirms the elasticity of the volatility is constant, that is $\frac{\partial \ln \sigma^2(r(t),t)}{\partial \ln r(t)} = 2(\gamma - 1)$ (Epps, 2002). Hence, if $\gamma = 1$ the elasticity is zero which is consistent with the constant volatility in the GBM model; and if $\gamma = 0.5$ the elasticity is -1 as in the model proposed by Cox and Ross (1976) which is a square-root process. It is known that for $0 \le \gamma < 0.5$ the CEV equation (2.10) admits multiple solutions, explaining why applications are mostly confined to the rest of the interval $[0.5,1]$. For $\gamma > 1$ the CEV specification was studied by Emanuel and MacBeth (1982)

[9] Having values under unity for the elasticity parameter, corresponds to equity markets where there is an indication of a possible inverse leverage effect, i.e. as the share price increases the volatility of the price changes decreases. Elasticities larger than unity are observed in commodity markets.

and Chen and Lee (1993). They found that the non-central chi-square distribution of the process will converge to a lognormal distribution as γ approaches one. Despite its tractability, the CEV model can permit negative interest rates and may display exit boundaries (Brigo and Mercurio, 2006); moreover, when strict positivity constraints are imposed on the CEV process, Delbaen and Shirakawa (2002) demonstrate that there always exist arbitrage opportunities.

The Vasicek Model (1977)

A reference classical model of term structure is that of Vasicek (1977) for which the dynamics of risk-free rate of interest were assumed to follow an Ornstein-Uhlenbeck[10] process (sometimes called the elastic random walk), mathematically described by the following SDE:

$$\mathrm{d}r(t) = k(\mu - r(t))\,\mathrm{d}t + \sigma \mathrm{d}W(t) \tag{2.11}$$

All the parameters k, μ and σ are strictly positive constants and $W(t)$ is a standard Brownian motion. The Vasicek model is the first model that incorporates the mean reversion feature of interest rate behaviour, which is modelled using a linear expression of the current level of the process. The drift involves two parameters: μ - the long-term risk-neutral mean, called the *mean reversion level* and k - the adjustment rate at which the risk-free rate is expected to revert to its long run mean, known as the *mean reversion speed*. The diffusion component/coefficient σ is time and process invariant and represents the *homoscedastic conditional volatility* of the short rate process.

In other words, the Vasicek model assumes that the short rate follows a deterministic mean reverting path subject to a continuous normally distributed random shock. In time the model becomes statistically stationary with a long run finite variance and it admits a Gaussian invariant probability distribution[11]. By applying the Ito lemma to the function $\exp[(kt)r(t)]$ the SDE (1.6) can be solved and its solution is given by the following expression:

$$r(t) = r(0)e^{-kt} + \mu(1 - e^{-kt}) + \sigma e^{-kt} \int_0^t e^{ks} dW(s) \tag{2.12}$$

[10] In contrast with a Wiener process where the drift is constant, an Ornstein-Uhlenbeck process is a Gaussian process that has mean-reversion drift. In addition, an OU process can be regarded as the continuous-time analogue of the discrete-time AR(1) process.

[11] Through the joint parametrization of μ and σ^2, the Vasicek model generates both marginal and transitional normal densities (Ait-Sahalia, 1996b); sometimes the model is referred to as the affine Gaussian short rate (GSR) model (Andersen and Piterbarg, 2010).

It follows that for a fixed initial value $r(0)$ the implied marginal expected value and variance are: $E[r(t)\,|\,r(0)] = r(0)e^{-kt} + \mu(1-e^{-kt})$ and $\mathrm{Var}[r(t)\,|\,r(0)] = \frac{\sigma^2}{2k}(1-e^{-2kt})$

Asymptotically, the parameters of the implied Normal distribution under the risk-neutral measure are given by: $\lim_{t\to\infty} E[r(t)] = \mu$ and $\lim_{t\to\infty} \mathrm{var}[r(t)] = \frac{\sigma^2}{2k}$

Hence, the faster the rate of mean reversion the smaller are the deviations from that mean. Under the assumption of market efficiency Vasicek (1977) provides an explicit characterization of the term structure invoking the no-arbitrage principle used by Black and Scholes (1973). In order to determine the term structure of interest rates implied by the OU process (2.11), Vasicek (1977) follows the PDE approach for the price of zero-coupon bonds and derives the analytical solution for the term structure in the special case of a constant market price of risk λ. However, a more modern approach, which has become a standard procedure in term-structure modelling, is the martingale approach where the probability measure is the risk-neutral measure Q. It can be shown (Cairns, 2004) that under this equivalent martingale measure (EMM) the term structure of interest rates implied by the Vasicek model has the following form:

$$R(t,T) = R(t,\infty) + \frac{1-e^{-(T-t)k}}{k(T-t)}(r(t) - R(t,\infty)) + \frac{\sigma^2}{4k^3(T-t)}(1-e^{-(T-t)k})^2 \qquad (2.13)$$

where $R(t,\infty) = \lim_{T\to\infty} R(t,T) = \mu - \frac{\sigma^2}{2k^2}$. This means that infinite maturity interest rate (the yield on a consol bond) is constant.

From (2.13) one can reconstitute the prices of the discount bonds for all the maturities, which are found to have a special exponentially affine form as follows:

$$P(t,T) = \exp[A(t,T) - B(t,T)r(t)] \qquad (2.14)$$

where

$$B(t,T) = \frac{1-e^{-k(T-t)}}{k}$$

$$A(t,T) = (\mu - \frac{\sigma^2}{2k^2})[B(t,T) - (T-t)] - \frac{\sigma^2}{4k}B(t,T)^2$$

Despite some desirable properties like tractability, time homogeneity and the economic autoregressive feature of mean reversion, the Vasicek model also was considered to have several limitations. The most considerable one is its positivity problem as the model allows for negative values of spot and forward rates, however, with an arguably very small probability as suggested by Rogers (1995). Additionally, it lacks great flexibility in the way that it generates only three (upward, downward and slightly humped) term structure shapes. The assumption of homoscedasticity seems to be unrealistic as historical records of interest rates clearly indicate at least a non-constant variance, as pointed out by James and Webber (2000).

The Dothan Model (1978)

Another single factor model is the Dothan (1978) model that can be characterised under the objective historical measure P as a geometric Brownian motion without drift, i.e.

$$dr(t) = \sigma r(t) dW(t) \tag{2.15}$$

With the assumption of a constant market price of risk and an equivalent transformation of the probability measure P into the risk-neutral measure Q, the new specification of the Dothan model includes a drift, more precisely it becomes a geometric Brownian motion (GBM), see Filipovic (2009):

$$\mathrm{d}r(t) = \beta r(t) dt + \sigma r(t) dW_Q(t) \tag{2.16}$$

Integrating the SDE (2.16) one could find the expression of the short rate for a known $r(0)$, as:

$$r(t) = r(0) \exp\left((\beta - \frac{\sigma^2}{2}) t + \sigma \int_0^t \mathrm{d}W_Q(s) \right) \tag{2.17}$$

The model can be also represented as a lognormal model with the $\log r(t)$ following a standard Brownian motion. The short rate has a lognormal conditional distribution with the two parameters, the conditional mean and the conditional variance given by $E_Q[r(t) \mid r(0)] = r(0) e^{\beta t}$ and $Var_Q[r(t) \mid r(0)] = r^2(0) e^{2\beta} (e^{\sigma^2 t} - 1)$, respectively.

As with all lognormal short-rate models, the Dothan model yields positive interest rates, but it does not admit analytical formulae for bond and bond option prices. However, the Dothan model is an exception in the sense that a “semi-explicit” expression for bond prices could still be obtained (for more details see Brigo and Mercurio, 2006).

The Rendleman and Bartter (GBM) Model (1980)

In the Rendleman and Bartter (1980) model the interest rate follows a Geometric Brownian Motion (GBM), the same process assumed for the share prices in the derivation of the Black and Scholes option pricing formula:

$$dr(t) = \beta r(t)dt + \sigma r(t)dW(t) \tag{2.18}$$

For an arbitrary initial short rate $r(0)$ the analytical solution to SDE (2.18) is:

$$r(t) = r(0)e^{(\beta - \frac{\sigma^2}{2})t + \sigma W(t)} \tag{2.19}$$

It is well known that the process defined by (2.19) is log-normally distributed with the conditional mean and variance having the following expressions: $E[r(t) \mid r(0)] = r(0)\mathrm{e}^{\beta t}$ and $Var[r(t) \mid r(0)] = r^2(0)\mathrm{e}^{2\beta}(e^{\sigma^2 t} - 1)$, respectively. A main disadvantage of GBM is that it does not incorporate mean reversion – a specific feature of interest rates behaviour that share prices do not possess.

The Brennan and Schwartz (BS) Model (1980)

This is a model that fully captures the mean reversion and nests the Merton, Dothan and GBM models. The short rate dynamics are described using the following SDE:

$$\mathrm{d}r(t) = k(\mu - r(t))\mathrm{d}t + \sigma r(t)\mathrm{d}W(t) \tag{2.20}$$

where the parameters k, μ and σ are real positive constants, and $W(t)$ is a standard Brownian motion under the risk-neutral measure.

Originally, Brennan and Schwartz (1980) proposed a model for valuing convertible bonds where the prices of such securities depend on two random variables - the value of the issuer firm and the interest rate; the process assumed for the latter factor was specified as in equation (2.20) above. Unfortunately, the distribution of the short rate cannot be explicitly found and as a result the prices of interest rate contingent claims have to be derived using numerical methods as in Courtadon (1982) who employs the Brennan and Schwartz (1980) model to value pure discount bonds.

The Cox, Ingersoll and Ross (CIR) Model (1985)

Another classic short rate model was proposed by Cox, Ingersoll and Ross (1985a) as an alternative to the Vasicek model in the attempt to rectify the problem of possible negative interest rates. Derived from an equilibrium asset pricing model, the CIR model assumes the following SDE with the same linear drift function as in the Vasicek model, but with a nonlinear (square root) still affine diffusion coefficient:

$$dr(t) = k(\mu - r(t))dt + \sigma\sqrt{r(t)}dW(t) \tag{2.21}$$

where the drift parameters are strictly positive and $\sigma^2 \leq 2k\mu$. The conditional standard deviation of the changes in the interest rate is positively related to the level of interest rate, more specifically it is proportional to $\sqrt{r(t)}$. The structure in SDE (2.21) has important empirical implications for the behaviour of the interest rate, based on some important regularity aspects regarding the parameters involved. While negative interest rates are avoided, in the case of reaching the zero-lower-bound, when $\sigma^2 > 2k\mu$, the interest rate will be subsequently elastically pulled upward to a positive level. However, if $\sigma^2 \leq 2k\mu$ the magnitude of the drift is sufficiently large to preclude the absolute zero level. It is very important to note that the CIR model is not Gaussian because its joint parameterization leads to a non-central chi-square transitional (conditional) distribution with the following parameters $\chi^2\left(\frac{4k}{\sigma^2(1-e^{-kt})};\frac{4k\mu}{\sigma^2},\frac{4kr(0)}{\sigma^2(e^{kt}-1)}\right)$(Cairns, 2004).

Given the initial value $r(0)$, the conditional expected value and variance of the instantaneous rate $r(t)$ are calculated as:

$$\mathrm{E}[r(t)\,|\,r(0)] = r(0)e^{-kt} + \mu(1-e^{kt}) \tag{2.22}$$

$$\mathrm{Var}[r(t)\,|\,r(0)] = r(0)\frac{\sigma^2}{k}(e^{-kt}-e^{-2kt}) + \mu\frac{\sigma^2}{2k}(1-e^{-kt})^2 \tag{2.23}$$

Asymptotically, as t increases the distribution of the interest rate approaches a gamma distribution, with the steady-state mean and variance given by:

$$\lim_{t\to\infty}\mathrm{E}[r(t)\,|\,r(0)] = \lim_{t\to\infty}\left(r(0)e^{-kt} + \mu(1-e^{kt})\right) = \mu$$

$$\lim_{t\to\infty}\left(\mathrm{Var}[r(t)\,|\,r(0)]\right) = \lim_{t\to\infty}\left(r(0)\frac{\sigma^2}{k}(e^{-kt}-e^{-2kt}) + \mu\frac{\sigma^2}{2k}(1-e^{-kt})^2\right) = \frac{\sigma^2\mu}{2k}$$

The model is still tractable with the implied bond prices having the same general form as in the Vasicek case, but mathematically rather more complex. Cox et al. (1985) determine the term structure of interest rate by specialising their equilibrium model for preference structures with constant relative risk aversion utility functions. As their original economic framework is rather complex and extensive (and beyond the scope of this work), the martingale approach is once again invoked for the presentation of the analytical formulae for the discount bond prices. It can be shown (Cairns, 2004) that under the risk-neutral measure Q the term structure of interest rates implied by the CIR model has the following analytical form:

$$\begin{aligned} P(r,t,T) &= \exp\left(A(T-t) - B(T-t)r(t)\right) \\ &= \exp\,[A(\tau) - B(\tau)r(t)] \end{aligned} \tag{2.24}$$

where

$$A(\tau)=\frac{2k\mu}{\sigma^2}\log\left(\frac{2\gamma e^{(k+\gamma)\tau/2}}{(k+\gamma)(e^{\gamma\tau}-1)+2\gamma}\right)$$

$$\gamma=\sqrt{k^2+2\sigma^2}$$

$$B(\tau)=\frac{2(e^{\gamma\tau}-1)}{(k+\gamma)(e^{\gamma\tau}-1)+2\gamma}$$

Cox et al. (1985b) also derived explicit analytical formulae for prices of European call options on discount bonds, making the CIR model extremely popular for some time among practitioners.

The Longstaff Model (1989)

Motivated by the findings in Fama (1984) and McCulloch (1971) that the term premiums have a humped pattern, Longstaff (1989) presents another rational expectation equilibrium model within the CIR framework, by allowing technological change to affect production returns in a nonlinear way. The new “double square-root” model was more flexible relative to the original CIR model as it allows for those patterns in the term premiums unlike the CIR model:

$$dr(t)=(\mu-\sqrt{r(t)})dt+\sigma\sqrt{r(t)}dW(t) \tag{2.25}$$

Additionally, Longstaff (1989) derives the analytical formulae implied by the model (2.25) for both the yield to maturity and the price of discount bonds and finds an uncommon non-linear dependency of the term structure on the level of the short rate. However, Longstaff’s bond pricing equation fails some boundary condition leading to infinite expected rates of return on the bond as highlighted by Beaglehole and Tenney (1991).

The Chan, Karolyi, Longstaff and Sanders (CKLS) Model (1992)

In a seminal paper Chan et al. (1992) measure the sensitivity of the volatility with respect to the level of instantaneous rate by considering a general flexible nonlinear function for the diffusion function. Their general model provided a common theoretical framework that nests eight classical models[12] and therefore it allowed for a consistent performance comparison between those models as part of an important empirical exercise. The power-type CKLS model is represented by the following SDE:

[12] The CKLS representation nests the following models: Merton, Vasicek, CIR-SR, Dothan, GBM, Brennan and Schwartz, CIR-VR and CEV models.

$$dr(t) = (\alpha + \beta r(t))dt + \sigma r^{\gamma}(t)dW(t) \quad (2.26)$$

where $\alpha, \sigma > 0$, $\beta < 0$ and γ is a real constant.

This general univariate framework can be extended to the multivariate case as in Nowman (2003, 2006) where the state variables are correlated yields of different maturities.

The Ait-Sahalia Model (1996)

In an influential study, Ait-Sahalia (1996b) developed a test statistic to detect any misspecification in various classical models. By comparing the density functions implied by the parameterised single factor short rate models with that of a non-parametric estimator (Ait-Sahalia, (1996a)) the test resulted in the rejection of all the eight CKLS nested models. According to Ait-Sahalia (1996b) the main source of misspecification arises from the assumption of a linear drift and to challenge that assumption he suggested a more general model that accommodates a non-linear drift and a more flexible diffusion function:

$$\mathrm{d}r(t) = \left(a + br(t) + cr^2(t) + \frac{d}{r(t)} \right)\mathrm{d}t + \sigma\sqrt{r(t)}\mathrm{d}W(t) \quad (2.27)$$

where the drift and diffusion parameters are subject to certain conditional constraints.

The Ahn and Gao Model (1999)

Ahn and Gao (1999) proposed another important single-factor short rate model with a quadratic drift and a constant elasticity of volatility equal to 1.5, that is tractable and appears to empirically outperform all the standard models nested in the CKLS framework:

$$dr(t) = k(\mu - r(t))r(t)dt + \sigma r(t)^{1.5} dW(t) \quad (2.28)$$

Arbitrage -Free Interest Rate Models

All the single factor short rate models discussed so far are time-homogeneous as all model parameters are time-invariant and hence with a small number of free parameters they provide only an approximation to the currently observed term structure. This has a considerable impact on the valuation of interest rate derivatives as a 1% error in pricing the bond could eventually lead to a much higher error in the value of a bond option (Hull, 2003). However, it is possible for some of these models to be converted into no-arbitrage models, i.e. to eliminate the discrepancies between the actual and the modelled yield curve. More specifically, they can be extended by allowing the parameters to vary deterministically over time. Consequently "calibrated" models like the extended Vasicek model and the extended CIR model were proposed by Hull and White (1990). Their work follows the approach introduced by Ho and Lee (1986) where interest rate models are innovatively designed to be automatically consistent with a given initial yield curve. The same idea has been embraced and extended in other studies including Black, Derman and Toy (1990) and Black and Karasinsky (1991).

The Ho and Lee Model (1986)

The Ho and Lee (1986) model is the first no-arbitrage model originally presented in a discrete-time setting in the form of a binomial tree model for bond valuation. Later studies including Dybvig (1988) and Jamshidian (1988) have derived its continuous-time equivalent as:

$$dr(t) = \alpha(t)dt + \sigma dW(t) \tag{2.29}$$

This is an extension to the Merton random-walk model, with the same constant volatility but a more general, deterministic function of the time drift component. Solving the SDE (2.29) for the instantaneous short rate one obtains the following expression:

$$r(t) = r(0) + \int_0^t \alpha(s)ds + \sigma W(t) \tag{2.30}$$

The new element brought by Ho and Lee's framework is the use of the forward curve in the derivation of the discount bond and bond options prices. Being an affine term structure (ATS) model, the bond prices will be exponentially affine and from (2.2) the forward curve is obtained as follows (see Filipovic, 2009):

$$f(t,T) = f_0(T) - f_0(t) + \sigma^2 t(T-t) + r(t) \tag{2.31}$$

where $f_0(T)$ and $f_0(t)$ represent instantaneous forward rates observed at time zero for maturities t and T, with the initial forward rate curve $f_0(T) = -\dfrac{\partial \ln P(0,T)}{\partial T}$.

Integrating the second expression in (2.31) and reverting equation (2.2) the bond prices are:

$$P(t,T) = \exp\left(-\int_t^T f_0(s)ds + f_0(t)(T-t) - \frac{\sigma^2}{2}t(T-t)^2 - (T-t)r(t) \right) \tag{2.32}$$

For this model it is shown that the short rate "fluctuates along the modified initial forward curve" (Filipovic, 2009; pp.89):

$$r(t) = f_0(t) + \frac{\sigma^2 t^2}{2} + \sigma W(t) \qquad (2.33)$$

The model is tractable with the possibility of reconstructing prices for discount bonds and bond options as illustrated in James and Webber (2000, p. 184).

The Hull and White Models (1990)

According to Hull and White (1990, p. 576) the time-dependence of the parameters "can arise from the cyclical nature of the economy, expectations concerning the future impact of the monetary policies and expected trends in other macroeconomic variables". Hull and White (1990) investigate extensions to the Vasicek (1977) and Cox et.al. (1985) models that fit exactly the initial term structure. The general extensions that they explore admit all the parameters as functions of time:

$$dr(t) = k(t)(\mu(t) - r(t))dt + \sigma(t)dW(t) \qquad (2.34)$$

$$dr(t) = k(t)(\mu(t) - r(t))dt + \sigma(t)\sqrt{r(t)}dW(t) \qquad (2.35)$$

Of the two extended single-factor model the extended Vasicek model is particularly attractive, because of its analytical tractability. The price of a zero coupon bond at a future time t as implied by the Hull and White model, depends on the short rate $r(t)$ and the bond prices of two bonds of different maturities observed today, $P(0,t)$ and $P(0,T)$, respectively:

$$P(t,T) = \exp(A(t,T) - B(t,T)r(t)) \qquad (2.36)$$

where

$$B(t,T) = \frac{1 - e^{-k(T-t)}}{k}$$

$$A(t,T) = \ln\frac{P(0,T)}{P(0,t)} + B(t,T) f_0(t) - \frac{1}{4k^3}\sigma^2 t(\mathrm{e}^{-kT} - e^{-kt})^2(e^{2kt} - 1)$$

Other single-factor short rate models proposed in the literature consider the short rate to be lognormally distributed, a natural approach to ensure interest rate positivity.

The Black-Derman-Toy Model (BDT) (1990)

The Black-Derman-Toy (1990) arbitrage-free model (hereafter BDT) was initially presented in discrete-time as a one-factor binomial model, and later several authors including Rebonato (1998), Wilmott (1998) and Bali (1999) derived its continuous-time limit as a familiar SDE in terms of $\log r(t)$:

$$d\log r(t) = [\alpha(t) + \beta \log r(t)]dt + \sigma(t)dW(t) \tag{2.37}$$

where β is a real constant and $\alpha = \alpha(t)$, $\sigma = \sigma(t)$ are deterministic functions of time. Another formulation of the BDT model is:

$$\mathrm{d}\log r(t) = [\alpha(t) + \frac{\sigma'(t)}{\sigma(t)}\log r_t]\mathrm{d}t + \sigma(t)\mathrm{d}W(t) \tag{2.38}$$

Unfortunately, the BDT model inherits some shortcomings that consequently make it impractical. Intractability is one undesirable feature, but more significantly the model suffers from "mean-fleeing", which means that there is some possibility for the mean reversion level to become negative[13]. Also, the dynamics of the BDT model are path independent, i.e. the short rate is an outright function of the Brownian motion $W(t)$. Initially the BDT model considered only the mean reversion parameters as time variants, and later in Black et al. (1990) the model was extended to allow also for a time-dependent volatility.

Black and Karasinski Model (1991)

Trying to improve the dynamics and to rectify some of the drawbacks of the BDT model, Black and Karasinski (1991) (BK hereafter) assumed that $\log(r(t))$ follows an exogeneous standard Gaussian process (Andersen and Piterbarg, 2010). Black and Karasinski (1991) proposed a lognormal short rate model with all three parameters - the target rate, the mean reversion speed and the local volatility - as deterministic functions of time:

$$d(\log r(t)) = k(t)[\mu(t) - \log r(t)]dt + \sigma(t)dW(t) \tag{2.39}$$

To price more complex interest rate contingent claims like swaptions, Peterson et al. (2003) developed a multifactor extension of the log-normal model of Black and Karansinski (1991) using a chain of stochastic means from one factor to another.

The Sandmann and Sondermann Model (1993)

Another short rate model in the lognormal framework was proposed by Sandmann and Sondermann (1993), who differentiate between the instantaneous and compounding periods in order to avoid the explosion of interest rates present in the other lognormal models. Instead of the short rate their setting models a *simple* lognormal rate $r^*(t)$ that is

[13] As it can be observed from the SDE (2.38) of the BDT model the speed of the mean reversion cannot be controlled as it depends on the local volatility. When the local volatility is an increasing function of time, the BDT model implies a negative target level.

compounded over a fixed finite period while the short rate is a nonlinear function of $r^*(t)$:

$$r(t) = \log(1 + r^*(t))$$
$$dr^*(t) = \mu(t) r^*(t) + \sigma(t) r^*(t) dW(t) \tag{2.40}$$

The dynamics of the short rate implied by this model are neither normal nor lognormal (Gibson et al., 2010) as they satisfy the following related SDE:

$$dr(t) = \left[\mu(t) - \frac{\sigma^2(t) b^2(r(t))}{2} \right] dt + \sigma(t) b(r(t)) dW(t) \tag{2.41}$$

where $\mu(t)$ and $\sigma(t)$ are some deterministic functions of time and $b(r(t)) = 1 - e^{-r(t)}$.

The Duffie and Kan Model (1994)

Almost all of the single-factor diffusion interest rate models presented so far in the academic literature could be entered under the umbrella of a very general parametric specification of a short rate model presented in Duffie and Kan (1994), a model that allows for both linear and non-linear drift and is given by the following SDE:

$$dr(t) = [\alpha_1(t) + \alpha_2(t) r(t) + \alpha_3(t) r(t) \log(r(t))] dt + [\beta_1(t) + \beta_2(t) r(t)]^\gamma dW(t) \tag{2.42}$$

The Goard Model (2000)

Goard (2000) generalised the Ahn and Gao time-homogeneous model by considering a time-dependent moving target for the drift, and derived an explicit solution for the price of a zero-coupon bond. His model is given by the following SDE:

$$dr(t) = [c^2 r(t)(a(t) - qr(t))] dt + cr^{3/2}(t) dW(t) \tag{2.43}$$

where c and q are time invariant independent parameters and $a(t)$ is an arbitrary function of time.

The Das Model (2002)

More realistic specifications of interest rate dynamics that explain some of the discontinuities historically observed in the evolution of interest rates would include jump processes. In this regard, Das (2002) developed an analytical framework represented by a class of Poisson-Gaussian models in the attempt to capture the effect of surprise information (e.g. supply and demand shocks, economic news or exogeneous interventions from central banks) on the level of interest rates. The short rate dynamics are described by a mean reverting drift in conjunction with two independent processes, a diffusion and a Poisson process, respectively:

$$dr(t) = k(\mu - r)dt + \sigma dW + Jd\pi(h) \tag{2.44}$$

where the Poisson process π with the arrival frequency parameter h is scaled by the random jump J. An extension to the above Gaussian-Poisson process may incorporate regime switches as in Naik and Lee (1993), Gray (1996) and Piazzesi (1998) with the choice of the jump process being conditioned by the type of the regime as for higher interest rate regimes the jumps are more pronounced.

The Mahdavi Model (2008)

Under the minimum restriction of no arbitrage, Mahdavi (2008) derived a very general one-factor model for short-term interest rates, claiming that the expected change in short-term rate can be partially observable. It was shown that the expected change in the short-term rate it is equal to the slope of the forward curve, which is observable, plus a term involving the market price of interest rate risk. The model parameterization is very general:

$$dr(t) = [\, f_T(t,t) - \lambda(t)\sigma(t)]dt + \sigma(t)dW(t) \tag{2.45}$$

where $f_T(t,t)$ represents the slope of the forward curve at the origin. In this form the model allows an accurate estimation of market price of risk parameter. In his paper, Mahdavi (2008) also presented a more detailed parameterization of the model that nests most classic single factor models:

$$dr(t) = (\, f_T(t,t) + \alpha_1 + \alpha_2 r(t) + \alpha_3 r(t)^2)dt + \sqrt{|\,\alpha_4 + \alpha_5 r(t) + \alpha_6 r(t)^2 + \alpha_7 r(t)^3\,|}dW(t) \tag{2.46}$$

While the single-factor specifications represent the initial phase of the theoretical development of interest rates modelling, it is widely recognised that only one factor is quite restrictive and richer dynamics are needed to give rise to various yield curve shapes is needed. Also, they unrealistically assume that the bond returns are perfectly correlated, making single factor models of the term structure unsuitable for pricing more complex interest derivatives like caps and swaptions.

2.2.3.2 Two-Factor Models

A natural step in the theoretical development of interest rate models was to consider a more realistic approach by increasing the number of sources of randomness. The volatility of the interest rates observed over long periods of times indicated a possible stochastic nature that could be modelled by a process involving a separate Brownian motion. Also, the single-factor short rate models could be generalised to a stochastic mean or/and volatility, evolving this way into two- or three-factor models. These developments have coincided with the important Principal Component Analysis (PCA) of Litterman and Scheinkman (1991) who claimed that over 95% of the variability in the interest rates changes could be explained by three common factors - the level, the slope and the curvature, with 88% attributed solely to the first factor. As a result, many researchers have explored this idea and have considered various candidates for the second factor: Brennan and Schwartz (1979) chose the long-term rate, Schaefer and Schwartz (1984) preferred the spread between the short- and long-term rates; Heston (1986), Pearson and Sun (1994), Sun (1992), Cox et al. (1985) and Pennacchi (1991) considered the inflation, Balduzzi et al. (1997), Naik and Lee (1993) selected the mean level of the short-term interest rate, Schaefer and Schwartz (1987), Fong and Vasicek (1991) and Longstaff and Schwartz (1992) considered the volatility of the interest rate changes.

The Brennan and Schwartz Two-Factor Model (1979, 1982)

Derived in a partial equilibrium framework, the two-factor model proposed by Brennan and Schwartz (1979, 1982) is defined by two sources of uncertainty: the short rate $r(t)$ and a long-term *consol* rate $L(t)$[14]. Initially, the logarithms of these variables constitute the two factors that follow an Ito joint diffusion process, with a linear and a quadratic transformation of an OU process, respectively.

$$\begin{aligned} d\ln r(t) &= k[\ln L(t) - \ln r(t)]dt + \sigma_1 dW_1(t) \\ d\ln L(t) &= \alpha[\beta - \ln L(t)]dt + \sigma_2 dW_2(t) \end{aligned} \tag{2.47}$$

The model can be rewritten in terms of more complex processes for the short rate and the consol rate themselves, providing a useful financial interpretation with the two factors interpreted as the level and the steepness of the yield curve, respectively. Under no-arbitrage conditions Brennan and Shwartz (1979) derived the pricing equation for default-free pure-discount bonds which is also satisfied by any contingent claims that depend on

[14] Roughly speaking the *consol* rate is the return on a claim that pays perpetually a constant dividend, providing a "synthesis of the whole term structure up to infinity" (Brigo and Mercurio, 2001).

r and L, such as bond and bond futures options. Thus, in their framework the yield curve is entirely specified by the joint stochastic evolution of its short and long-term extremities. However, this joint specification of the state variables has been questioned by Hogan (1993) and Duffie et al. (1995) who proved that there is no real-valued solution to their diffusion equations.

The Richard Model (1978)

A rather different affine two-factor model of the term structure of interest rates was developed by Richard (1978) who employed two independent stochastic factors: the expected real short-term rate $q(t)$ and the expected instantaneous inflation rate $\pi(t)$, respectively:

$$dq(t) = k_1(\mu_1 - q(t))dt + \gamma_1\sqrt{q(t)}dW_1(t)$$

$$d\pi(t) = k_2(\mu_2 - \pi(t))dt + \gamma_2\sqrt{\pi(t)}dW_2(t) \tag{2.48}$$

The model is additive, in the sense that the short rate is modelled as a linear combination of the two factors, hence allowing for the decomposition of both bond prices and yields, into their real and inflationary components.

The Schaefer and Schwartz (1984)

Motivated by empirical evidence of orthogonality between the long-term rate and the spread, Schaefer and Schwartz (1984) proposed another affine two-factor model where the two uncorrelated state variables are the long-term rate $l(t)$ and the spread $z(t) = r(t) - l(t)$. While the spread follows a standard OU process, the long rate process is more complex with a non-arbitrary drift and CIR type diffusion function:

$$dz(t) = k_1(\mu_1 - z(t))dt + \gamma_1 dW_1(t)$$

$$dl(t) = (\gamma_2^2 - l(t)z(t))dt + \gamma_2\sqrt{l(t)}dW_2(t) \tag{2.49}$$

The Fong and Vasicek Model (1991, 1992)

Fong and Vasicek (1991, 1992) considered two sources of uncertainty for explaining the term structure of the interest rate: the short rate and the instantaneous variance of the changes in the short rate. The behaviour of these stochastic variables is described by the following diffusion processes:

$$dr(t) = k_1(\mu_1 - r(t))dt + \sigma(t)dW_1(t)$$

$$d\sigma^2(t) = k_2(\mu_2 - \sigma^2(t))dt + \eta\sqrt{\sigma^2(t)}dW_2(t) \tag{2.50}$$

Both processes incorporate mean reversion, the instantaneous volatility of the short rate has itself a volatility proportional to the current level of the short rate volatility and the two driving Brownian motions are assumed correlated. Under the condition of no-arbitrage Fong and Vasicek derived the closed formula for computing the price of pure discount bonds that involves complex algebra calculations.

The Longstaff and Schwartz Two-Factor (LS) Model (1992)

From the category of two-factor models, the Longstaff and Schwartz (1992) (LS) model evolves from a general equilibrium model of the economy and leads to a term structure model with a stochastic volatility. The model is both tractable and flexible, with closed formulae for the prices of pure discount bonds. Starting with two underlying state variables x_t and y_t that follow individual CIR standard processes the short rate and the volatility are additive functions of the two underlying economic state variables:

$$\begin{cases} r(t) = \alpha x(t) + \beta y(t) \\ v(t) = \alpha^2 x(t) + \beta^2 y(t) \end{cases} \quad \text{where} \quad \begin{cases} dx(t) = (a - bx(t))dt + c\sqrt{x(t)}dW_x(t) \\ dy(t) = (d - ey(t))dt + f\sqrt{y(t)}dW_y(t) \end{cases} \tag{2.51}$$

In the LS model the two factors are the short-term interest rate and interest rate volatility. An alternative interpretation is one in which the two factors are the short-term rate and a long-term rate, which is similar in spirit to the work of Brennan and Schwartz (1979).

The Hull and White Two-Factor Model (HW) (1994)

Following Brennan and Schwartz (1979), Hull and White (1994) propose a two-factor model where the additional state variable is a random long-term equilibrium rate. The model is made arbitrage-free by including a time variant shift in the drift, allowing therefore for consistency with the currently observed term structure.

$$\begin{cases} dr(t) = [\theta(t) - \mu(t) - ar(t))dt + \sigma_1 dW_1(t) \\ \\ d\mu(t) = -b\mu(t)dt + \sigma_2 dW_2(t) \end{cases} \tag{2.52}$$

where the parameters a, b are real constants and $\sigma_1, \sigma_2 > 0$ are real constants and the two separated Brownian motions W_1 and W_2 are correlated.

The Andersen and Lund Model (1997)

In line with the Dybvig (1988) and Longstaff and Schwartz (1992) theoretical specifications, Andersen and Lund (1997) developed a two-factor model that incorporates the main behavioural features observed in the evolution of interest rates: mean reversion and volatility heteroscedasticity. Their model can be seen as an extension of the CKLS model with the addition of a stochastic log-volatility factor:

$$\begin{cases} dr(t) = k_1(\mu_1 - r(t))r(t)dt + \sigma(t)r(t)^{\gamma} dW_1(t) \\ d\log\sigma^2(t) = k_2(\mu_2 - \log\sigma^2(t))dt + \zeta dW_2(t) \end{cases} \tag{2.53}$$

where $W_1(t)$ and $W_2(t)$ are independent standard Brownian motions.

The Bali Model (2003)

The lognormal BDT single factor model has been extended by Bali (2003) to a two-factor formulation, with the second factor represented by a stochastic variance or standard deviation that is modelled within a diffusion-GARCH framework. In the original paper two alternatives are considered for the discrete-time GARCH effect - a linear symmetric GARCH model (Bollerslev (1986)) and a TS-GARCH model (Taylor (1986) and Schwert (1989)). The continuous-time model implies mean reversion for both the log-interest rate level and the instantaneous standard deviation of the log-interest rate changes:

$$\begin{cases} d\ln r(t) = \phi_1(\mu_1 - \ln r(t))dt + \sigma(t)dW_1(t) \\ d\sigma(t) = \phi_2(\mu_2 - \sigma(t))dt + \eta_2\sigma(t)dW_2(t) \end{cases} \tag{2.54}$$

where $W_1(t)$ and $W_2(t)$ are independent Brownian motions and the diffusion process is parameterised as a function of the interest rate level $r(t)$ and the stochastic volatility factor $h(t)$, i.e. $\sigma^2(t) = h(t)r^{2\gamma}(t)$. By applying the Ito lemma to the first equation an equivalent model is obtained with the level of the short rate as the first state variable:

$$
\begin{cases}
dr(t) = r(t)[\phi_1(\mu_1 - \ln r(t)) + \frac{1}{2}\sigma^2(t)]dt + \sigma(t)r(t)dW_1(t) \\
d\sigma(t) = \phi_2(\mu_2 - \sigma(t))dt + \eta_2\sigma(t)dW_2(t)
\end{cases}
\tag{2.55}
$$

2.2.3.3 Three-Factor Interest Rate Models

The results of the PCA analysis conducted by Litterman and Scheinkman (1991) led to the acceptance of the three-factor formulations as sufficient to capture most of the dynamics of the interest rates. The inclusion of extra factors brings more complexity to the mathematical formulae of reconstruction of bond and derivative prices, with the effect of reducing the tractability of the model. However, some three-factor models such as Fong and Vasicek (1991), Sorensen (1994) and Chen (1996) still possess explicit solutions. The most common choice for the three state variables is a natural one with the short rate, the long-term mean and the volatility of the changes in the interest rates being driven by separate Brownian motions that are assumed to be either independent or correlated.

The Chen Model (1996)

In the three-factor model proposed by Chen (1996) the short rate dynamics evolve as follows:

$$
\begin{cases}
dr(t) = k(\mu(t) - r(t))dt + \sqrt{v(t)}\sqrt{r(t)}dW_r(t) \\
d\mu(t) = \alpha(\beta - \mu(t))dt + \eta\sqrt{\mu(t)}dW_\mu(t) \\
dv(t) = a(b - v(t))dt + \omega\sqrt{v(t)}dW_v(t)
\end{cases}
\tag{2.56}
$$

All the state variables are modelled as CIR processes, with the third factor as the instantaneous conditional variance of the first factor - the short rate. In terms of tractability, only in specific cases there exist analytical solutions for discount bonds and certain interest rate derivatives (see Chen, 1996).

The Balduzi, Das, Forezi and Sundaram Model (BDFS) (1996)

A popular model in the group of three-factor models was proposed by Balduzzi et al. (1996) (thereafter BDFS). In the BDFS model the mean μ_t and the volatility σ_t of the short rate are also stochastic, following a Vasicek and a CIR process, respectively. Therefore, the model is defined by three processes:

$$\begin{cases} dr(t) = k(\mu(t) - r(t))dt + \sqrt{\sigma(t)}dW_r(t) \\ d\mu(t) = \alpha(\beta - \mu(t))dt + \eta dW_\mu(t) \\ d\sigma(t) = a(b - \sigma(t))dt + \omega\sqrt{\sigma(t)}dW_\sigma(t) \end{cases} \tag{2.57}$$

where only two factors are correlated: the short rate and the volatility. Despite the fact that the model is intractable with solutions for the term structure obtained by numerical methods, it offers greater flexibility than two-factor models, giving rise to less common shapes of the yield curve like humped and spoon-curved (James and Webber, 2000).

The Andersen and Lund Three-Factor Model (1997)

Another important three-factor model with the same three factors as in the BDFS model but following different stochastic processes was proposed by Andersen and Lund (1997). For both three-factor models, the factors – the short rate, the mean and the volatility, are identified as exactly the three components from the PCA approach, i.e. the level, slope and curvature, respectively.

$$\begin{cases} dr(t) = k(\mu(t) - r(t))dt + \sigma(t)r^\gamma(t)dW_1(t) \\ d\mu(t) = \alpha(\beta - \mu(t))dt + \nu\sqrt{\mu(t)}dW_2(t) \\ d\ln\sigma(t) = a(b - \ln\sigma(t))dt + \omega\sqrt{v_t}dW_3(t) \end{cases} \tag{2.58}$$

The Diebold and Li Model (2006)

Swapping the role of the three parameters as initially set-up by Nelson and Siegel into time-dependent variables, Diebold and Li (2006) achieved a three-factor dynamic version of the deterministic Nelson and Siegel (1987) model.

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\left(\frac{1 - e^{-\gamma\tau}}{\gamma\tau}\right) + \beta_{3t}\left(\frac{1 - e^{-\gamma\tau}}{\gamma\tau} - e^{-\gamma\tau}\right)$$

More recently, Diebold and Rudebusch (2013) revealed an insightful interpretation of this model, suggesting a combination of three dynamics: 1) the latent factors $\beta_{1t}, \beta_{2t}, \beta_{3t}$, 2) the dynamic of y_t for a fixed time to maturity τ and 3) the coefficients as factor loadings $(1, (1-e^{-\gamma\tau})/\gamma\tau, ((1-e^{-\gamma\tau})/\gamma\tau - e^{-\gamma\tau})$ that are responsible for movements of the cross section of yields for any *t*.

2.2.3.4 General Multi-Factor Models

Other researchers have proposed more general theoretical specifications with n-factors, arguing that the term structure of interest rates is "embedded in a large macroeconomic system" (Langetieg, 1980, p. 71). Some of the most known general representations include Langetieg (1980), Beaglehole and Tenney (1991), Babbs (1993), Nunes (1998) and Babbs and Nowman (1999). In general, n-factor models follow an additive structure with the short rate as a linear combination of an arbitrary number of stochastic factors. Given the limited space, only two most important models will be presented in the rest of this chapter.

The Langetieg Model (1980)

The general multi-factor model suggested by Langetieg assumes that the short rate is represented by a linear combination of n independent stochastic factors that follow a joint elastic random walk process:

$$r(t) = w_0 + \sum_{i=1}^{n} w_i X_i(t)$$
$$dX_i(t) = k_i(\mu_i - X_i(t))dt + \sigma_i dW_i \qquad (2.59)$$

where the coefficients k_i, μ_i and w_i can be time variant. The model is tractable only in the case of three alternative distributions corresponding to Vasicek, CIR and Dothan diffusion processes, with the price of the zero-coupon bond keeping the same form as in the univariate setting as an exponentially affine expression of the underlying factors. For example, in the case of an extended Vasicek model where the short rate is the sum of the factors involved, the price of a discount bond with maturity $(T-t)$ is given by:

$$P(t,T) = \exp\left(\sum_{i=1}^{n} A_i(t,T) - \sum_{i=1}^{n} B_i(t,T) X_i(t) \right) \qquad (2.60)$$

where $B_i(t,T) = \frac{1}{k_i}(1 - e^{-k_i(T-t)})$ and

$$A_i(t,T) = -\frac{\sigma_i^2}{4k_i^3}(1 - e^{-k_i(T-t)})^2 + \frac{1}{k_i}(\mu_i - \frac{\sigma_i^2}{2k_i^2})(1 - e^{-k_i(T-t)}) - (\mu_i - \frac{\sigma_i^2}{2k_i^2})(T - t)$$

The Babbs and Nowman Model (1999)

The general model of n factors proposed by Babbs and Nowman (1999) assumes that the short rate is a particular linear combination of unobservable factors that are interpreted as n streams of economic and financial *news* concerning interest rate decisions taken by central banks through monetary policies or concerning regular economic statistics news. Mathematically, the model is given by the following specifications:

$$\begin{cases} r(t) = \mu(t) - \sum_{i=1}^{n} X_i(t) \\ dX_i = -m_i X_i dt + p_i dW_i \end{cases} \qquad (2.61)$$

where μ represents the long-run average level of the short interest rate, $\{X_i\}_{i=1,n}$ are the news-factors as diffusion processes with m_i and p_i as the mean reversion and diffusion coefficients; and the $\{W_i\}_{i=1,n}$ are correlated Brownian motions that model news arrivals.

2.2.4 The Heath, Jarrow and Morton (HJM) Framework (1992)

Despite the tractability of the affine-type short rate models and of the richer dynamics provided by the multifactor interest models, the empirical results obtained within the short rate framework were somehow disappointing, one reason being the inability of the short rate models to provide sufficient information about the covariance structure of the forward rates. Historically, the first attempt to develop an alternative to short-rate models was made by Ho and Lee (1986) who considered modelling the evolution of the whole yield curve in a discrete-time setting of a binomial tree. This intuitive idea was continued and adapted in continuous-time by Heath, Jarrow and Morton (1992) (HJM hereafter) who developed a rather different general theoretical framework of interest rate modelling. Unlike the one-factor short rate models where the diffusion coefficient partially characterises the dynamics of the state variable, the arbitrage-free HJM framework explains the evolution of the instantaneous forward rates only through their volatility structures, as the drift of the instantaneous forward rate is a transformation of its own diffusion. However, this represents a restriction on the choice of the drift form in comparison with short rate models. The generality of the HJM framework is theoretically extremely important, but this generality level cannot be maintained in practice as only a limited range of volatility structures will lead to a Markovian short-rate process (see James and Webber, 2000).

For example, Carverhill (1994) decomposed the volatility structure into the product of strictly positive and deterministic functions of time and proved that this formulation represents the HJM specification that is equivalent to the Hull and White one-factor short rate model with time variant coefficients. If the volatility structures of the instantaneous forward rate are not subject to constraints, the pricing of derivatives becomes more difficult as the discretization of the general short rate model (that is not markovian anymore) will encounter computational difficulties. In order to address this shortcoming, Ritchken and Sankarasubrramanian (1995) increased the flexibility of the markovian condition by requiring a multidimensional two-state Markov process (with one component being the short rate process) which allowed a complete computation of the interest rate derivatives. Another model within the HJM framework was proposed by Mercurio and Moradela (2000), which is a one-factor Gaussian model that accommodates a specific humped volatility structure that implies normally distributed instantaneous forward rates. Among other advantages, this model allows closed form formulae for prices of discount bond options and outperforms empirically the Hull and White (1994) extended Vasicek model.

2.2.5 Market Models

Market models constitute another building block in the literature of interest rate modelling. With a desirable practical feature of easy calibration, they have become increasingly popular among market practitioners. Market models are rather straightforward as they employ quoted market rates instead of the instantaneous interest rates by following two main approaches – direct and indirect. In the direct approach a numeraire and a measure are identified such that the market interest rates become martingales. This allows for a log-normal imposition which naturally leads to the derivation of Black-like formulae for option pricing. The indirect approach uses an underlying model that when subject to specific restrictions provides log-normal market rates which are ultimately fed into the market model.

A reference model in this category is the Brace, Gatarek and Musiela (1997) (hereafter BGM) model that follows an indirect approach, deriving the market forward rates processes from the HJM framework. A landmark result within the BGM model is the straightforward valuation of caps and swaptions by using Black-like formulae and by calibrating the volatility of the forward rate process to Black's implied volatilities. For the valuation of swaptions the BGM model uses an approximation formula; however, the authors also proposed a modified swaption price formula that takes into account the different market tenors for caps and swaptions (see James and Webber, 2000).

Building on the work of Miltersen, Sandmann and Sonderman (1997), market models following the direct approach have been developed by other researchers such as Jamshidian (1997) and Musiela and Rutkowski (1997).

The general framework of the above market models is mainly defined by an *a priori* fixed tenor structure, a set of assumptions about how markets operate[15] and a reference pricing measure usually chosen as the terminal forward measure. Within this framework, market models are able to price two dominant instruments in the interest-rate-option markets - caps and swaptions - that have as underlyings forward LIBOR rates and forward-swap rates, respectively. Accordingly, the lognormal forward-LIBOR model (LFM) values caps using the standard Black's cap formula, while the lognormal forward-swap model (LSM) prices swaptions with Black's formula for swaptions (Brigo and Mercurio, 2001). Despite their compatibility with the market practiced models, the two types of market models, LFM and LSM are not compatible with each other. Hence, this lack of generality requires a different market model to be employed for each specific derivative instrument.

2.2.6 Pricing Kernel Models

A totally distinct approach to interest rate modelling involves the pricing operator also called the *pricing kernel*. Early studies employing this line of research include Constantinides (1992), Flesaker and Hughston (1996) and Rogers (1997). Trying to address some of the limitations of the CIR model, Constantinides (1992) developed a general model of the nominal term structure of interest rates using a positive price kernel based on independent OU diffusion processes. His model is still tractable and more flexible than the CIR model as it allows inverted-humped yield curves as well. An important feature of the model is the possibility of a sign-changing term premium as a function of the state variables and the term to maturity, unlike most classical univariate models where it is considered constant. In the Flesaker and Hughston (1996) (FH) framework an important model is the *rational lognormal* model that retrieves bond prices as a rational function of a lognormal variable, and provides closed Black-like analytical formulas for the prices of caps and swaptions. Another price kernel approach stemming

[15] Some of these assumptions are: caps and swaptions payoffs take place and cashflows in the markets occur only on specified reset dates that determine the tenor structure.

more from the theory of Markov processes has been developed by Rogers (1997) who also identified that the FH general framework does not always lead to an interest rate model. Important to note is that these approaches are naturally suitable for modelling interest curves of different currencies.

The dynamics of most of the multi-factor interest rate models evolve within the exponential-affine framework developed by Duffie and Kan (1996) and Dai and Singleton (2000). This type of model could not simultaneously accommodate the positiveness and the unspanned stochastic volatility (USV) observed in the behaviour of interest rates. Filipovic et al. (2014) bring together the multi-factor interest rate models and the kernel approach into a new general framework called the *linear-rational* framework. In this, additional to the factor processes (that constitute the *term structure* component), a state price density is considered following Constantinides (1992) as a second component of the model. Two linearity assumptions are imposed on the two components, namely: the dynamics of the multivariate term structure component have a linear drift, while the state price density is a linear function of the first state component. Using the kernel of the term structure the uncertainty in the dynamics of the term structure component can be separated into two sources, corresponding to m intrinsic term structure factors and n unspanned factors, respectively. In this linear-rational framework the model specifications respect the zero lower bound (ZLB), are highly tractable and also allow for analytical solution to the pricing of more advanced derivative products like swaptions.

2.3 Literature Review of the Empirical Evaluation of Interest Rate Models

2.3.1 Empirical Evidence on Single-Factor Interest Rate Models

The vast empirical literature on the estimation of interest rate continuous-time models portrays an inconclusive and rather complex picture in which various aspects are identified as affecting the empirical findings. Once the theoretical models have been considered to be conceptually suitable, there are multiple estimation routes to translate them into practice.

Early empirical studies tested the single-factor theoretical models, with a focus on certain aspects such as the selection of the best model in capturing the dynamics of interest rates, the evidence for reversion to the mean and the determination of the sensitivity level of the volatility to the interest rate level. Initially, lacking a common framework for comparison, empirical testing was conducted on individual models only.

For example, the CIR model was empirically tested by Brown and Dybvig (1986) using monthly quoted prices of US Treasury bills, bonds and notes between 1952 and 1983, while Edsparr (1992) estimated the same model based on Swedish data. In a seminal paper Chan et al. (1992) presented a general framework, facilitating a multidirectional comparison among different classical models. Eight[16] short-term rate single-factor models could be nested in the unrestricted CKLS model, so their relative performance in terms of explanatory power could be consistently evaluated.

Following the proposal of the more general CKLS framework, numerous subsequent empirical studies provided early evidence of discrepancies. In an extensive international study Episcopos (2000) emphasized this sensitivity of the empirical results, concluding that the choice of the estimation techniques, sample period, data frequency, and country should be taken into account. Several comparative empirical studies (Treepongkaruna and Gray, 2003; Ioannides (2003); Lo 2005) have confirmed and demonstrated that different estimation routes lead to different results in terms of parameter estimates and implications for pricing interest rate contingent claims. Therefore, assessing the relative empirical performance of such an impressive number of theoretical models becomes extremely complex, and the following survey of key empirical evidence on testing interest rate models tries to illustrate just that.

For the estimation of the parameters Chan et al. (1992) employed the Generalized Method of Moments (GMM) of Hansen (1982). Based on one-month monthly US Treasury bills rates between June 1964 and November 1989, in the case of the unrestricted model the estimates of the drift parameters did not support the mean reversion feature. The level effect parameter was estimated at around 1.5 implying a high degree of dependence of the local volatility on the level of interest rates. The relative performance of the nine models was examined using two statistical tests and a metric that measured the ability of the models to capture the volatility of the changes in the risk-free rate. Based on the goodness-of-fit measure provided by the GMM objective function which is χ^2 distributed, the models with $\gamma \geq 1$ (Brennan and Schwartz (1980) and CEV (1975)) performed best, whereas Merton (1973), Vasicek (1977), Cox et.al. (1985a) and Cox et.al. (1985b) models were rejected against the unrestricted model. Employing the Newey and West (1987) hypothesis-testing method, the restrictions imposed by the nested models on the unrestricted CKLS model were evaluated and pair-wise comparison was conducted with no rejection being observed between models with similar diffusion

[16] Another general framework was proposed previously by Marsh and Rosenfeld (1983), but it nested only three restricted models (Chan et al. 1992).

coefficient. Additionally, the hypothesis of a structural break at the point October 1979 marking the Federal Reserve experiment (1979-1982) was rejected. Following an experiment to value a 2-year call bond option Chan et al. (1992) found that the option values varied substantially from one model to another, a result with great implications for valuing interest contingent claims and hedging interest rate risk.

Adopting the CKLS framework, Tse (1995) conducted an extensive international comparative empirical exercise based on data from eleven countries. The estimation results by the GMM method are rather mixed with countries grouped in three categories according to the magnitude (high, medium and low) of the estimated level-effect parameter. The most sensitive volatility of interest rate changes occurred in the U.S ($\gamma_{Tse}(US) = 1.73$), France $(\gamma_{Tse}(France) = 1.63)$ and Holland ($\gamma_{Tse}(Holland) = 1.60$), while the lowest elasticity of volatility estimate was observed in the UK ($\gamma_{Tse}(UK) = 0.11$) and Canada ($\gamma_{Tse}(Canada) = -0.36$). In contrast with another Chan et al. (1992b) study on the Japanese market where $\gamma_{CKLS}(Japan) = 2.44$, Tse (1995) found $\gamma_{Tse}(Japan) = 0.62$.

In another important empirical study, Dalhquist (1996) looked at six alternative interest rate processes (CKLS, Vasicek, CIR SR, GBM, Brennan-Schwartz and CEV) for Denmark, Germany, Sweden and the UK over similar time periods. Employing GMM methodology and monthly one-month maturity data sets (Euro-currency and US Treasury bills rates), Dalhquist (1996) found evidence of a positive relationship between interest rate level and volatility as indicated by Chan et.al. (1992). Moreover, the estimates of the level-effect parameter vary with higher values for Sweden (1.154) and Denmark (0.970) and are less pronounced for Germany (0.387) and especially for the UK (0.156). However, in contrast with the CKLS results for the U.S. significant mean reversion was found for Denmark and Sweden. In terms of relative explanatory power, the best models that could not be rejected at the 5% level of significance against the unrestricted CKLS model, were the CIR and Brennan and Schwartz models for Denmark and Sweden, and the Vasicek (1977) and the CIR SR (1985) for the U.K. and Germany, respectively. Moreover, in the case of Denmark, Dalhquist (1996) discovered parameter instability during August 1985 when arguably the Danish central bank had adjusted its monetary policy.

Applying for the first time in finance the Gaussian estimation method developed by Bergstrom (1983, 1985, 1986, 1990), Nowman (1997) estimated the eight single-factor continuous-time short rate models within the CKLS setting. Based on the U.S. Treasury (1964-1989) and the U.K. interbank rates (1975-1995), the final quasi-maximum likelihood estimates for the true parameters of the initial model were rather different between the two markets. Regarding the mean reversion parameters, the empirical results for the US contradicted those in CKLS indicating a weak presence of mean reversion, while the level effect parameter was found to be insignificant, with an estimate of $\gamma_{Nowman}(US) = 1.3610$. For the U.K., the evidence for mean reversion was still weak, but the estimate for the level effect was inferred as highly significant at $\gamma_{Nowman}(UK) = 0.2898$. A larger but similar study was conducted by Nowman (1998) for US, Japan, France and Italy, covering the period 1981-1995. The mean-reversion effect was in general weak with some significant evidence only in the case of US. For France and Italy, the level-effect parameter was in excess of two, whereas for Japan and the US was close to one.

In another comparative empirical study, Shoji and Ozaki (1998) developed a statistical method of model selection that was applied to data on Japan, US and Germany. An advantage of their method was that it could involve models that are not nested in a best unrestricted model as in the CKLS framework. Several continuous-time models for the term structure were estimated, including models with a nonlinear drift. According to their method, for Germany, the model with nonlinear drift outperformed the best models with linear drift.

In Episcopos (2000) various classical one-factor short rate models across a sample of ten countries[17] based on one month interbank rates. Some of the results are surprising with the CEV model outperforming the other competing models when other studies such as Tse (1995) and CKLS reject it, while the level-effect parameter varies across the ten countries from 0.20 to 1.56. For seven out of ten countries the level effect is under unity suggesting a less sensitive volatility than that in the CKLS findings. Also, the data sets used provided significant evidence for structural breaks in the case of six countries.

Yu and Phillips (2001, 2011) proposed a new estimation approach to a non-linear CKLS type diffusion model of interest rates, which is related to that of Nowman (1997). Their time changing technique has great empirical appeal as it allows for non-equidistant observations and it converts the continuous-time model into a Gaussian one. In an extensive study, Treepongkaruna and Gray (2003) tested the robustness of various one-factor short-term interest rate models (Vasicek, CIR and CKLS) over different estimation techniques (GMM and QMLE) and data sets (eight countries) covering different sub-periods, with different frequencies (daily and weekly). The stability of the parameters

[17] The group of ten countries is: Australia, Belgium, Germany, Japan, Netherlands, New Zeeland, Singapore, Switzerland, the UK and the US.

across sub-periods is uniformly rejected, suggesting that more complex models permitting for parameters to change over time could be more appropriate. The mean reversion parameters are insignificantly different from zero for all the countries, models, frequencies and estimation methods other than for the Italian Lira where there is some evidence of mean reversion in the Vasicek model estimated by QMLE. However, this evidence can be eliminated if four observations during the European currency crisis (September 1992) were excluded. The dependence of the volatility on the level of the interest rate differs from country to country as many previous studies have shown. The results were sensitive when the estimation technique and sampling frequency changed, requiring therefore some robustness checks.

To shed some light in this direction, Lo (2005) investigated the estimation of the Cox et al. (1985) and Chan et al. (1992) models in a comparative analysis of three Gaussian exact and approximate estimation methods implemented by Nowman (1997), Shoji and Ozaki (1998), and Yu and Phillips (2001). The conclusions from both a simulation study and empirical analysis of short-term interest rates for Canada and the UK, indicate that the Nowman (1997) and Shoji and Ozaki (1998) methods perform in a similar fashion, while the performance of the Yu and Phillips (2001) method was crucially impacted by the window width parameter used in the approximation. In terms of the best fit the Shoji and Ozaki method gave the best performance for both data sets, one-month Canadian Treasury bills (January 1980 to June 2002) and the one-month sterling interbank middle rate (March 1975 to March 1995), respectively. With regard to Yu and Phillips (2001) method it was observed that a large window width leads to a disappointing model fit and the estimation bias of the drift parameters could be significant as it depends on the choice of the window width. Lo (2005) concluded that all these aspects are important as they significantly affect the empirical results and emphasise the relative nature of any empirical work involving estimation of short-term interest rates models.

In a more recent comparative analysis of alternative single-factor continuous-time short rate models, Sanford and Martin (2006) employ a Bayesian inferential approach to estimate four models nested in the CKLS framework and to determine the magnitude of the level effect parameter that supports empirically the Australian interest rates over the 1990-2000 period. Their findings suggest that for this particular data set the CIR model is the most appropriate as indicated by the highest posterior probabilities provided by the Bayes factors, relative to the other models considered; therefore, the pricing equations implied by the CIR model are reasonable enough in the Australian context.

Modelling the Drift Component

The issue regarding the existence or non-existence of mean reversion in the dynamics of interest rates remains controversial. Looking at the overall evidence in terms of mean reversion most of the relevant empirical studies do not support statistically such a phenomenon. Following Ball and Torous (1996) who indicated a considerable estimation bias of the mean reversion estimates for the CIR model under popular estimation methods such as GMM and MLE, Faff and Gray (2006) investigated this problem further and demonstrate that the GMM estimates for the drift parameters in the single-factor models were severely overestimated, and therefore unreliable. At the same time, they asserted that the diffusion parameters are estimated with high precision under both GMM and MLE estimation methods, respectively.

Recently, Barros et al., (2012) investigated the mean reversion property of short-term rates for ten new EU countries. Using long memory fractionally integrated models and daily data covering the period January 2000 to December 2008, they concluded that interest rates are non-stationary (or stationary of order one, I(1)) and non-mean-reverting, except for Hungary. Testing for structural breaks, Lithuania is the only country for which in 2007 a structural break was statistically detected, while for all the other countries there is evidence of a structural break around 2001/2003. Once the structural breaks were considered, the mean reversion appeared more evident in some countries in the first sub-period, while after the break point the interest rates were clearly non-stationary with a higher degree of integration in all instances.

The parameterization of the short rate processes by restrictively assuming specific forms for the drift and diffusion functions could be another reason for such diversity of results. The natural alternative was to consider the most general SDE where the drift or/and the diffusion were not subject to parameterization.

In a famous article Ait-Sahalia (1996a) tested the validity of various classic parametric specifications in comparison with a non-parametric estimation technique. Based on 7-day Eurodollar deposit rates from 1 June 1973 to 25 February 1995 the empirical results indicated a certain degree of non-linearity in the drift component with values of the drift close to zero in the region of 4% - 17% and substantially higher outside this region. As a result, Ait-Sahalia (1996b) proposed a richer parametric model that involves a non-linear drift and nests four well-known short-rate models (Vasicek, CIR, BS, CKLS). The specification test instrumented by Ait-Sahalia (1996b) failed to reject only the model with a non-linear drift, suggesting that the main source of rejection of the classical parameterizations is the linearity of the drift. These important findings were explored further by various authors including Stanton (1997), Duffee (1999) and Chapman and Pearson (2000). However, their research led to mixed results about what is the appropriate drift specification.

Stanton (1997), for example, presented a general non-parametric procedure that allowed the estimation of both components (drift and the diffusion) by deriving a family of approximations to the true parameters. The procedure also permitted the market price of risk to be estimated by examining the daily excess returns between three month and six-month Treasury Bills rates from January 1965 to July 1995. The findings suggested that the drift exhibits a similar nonlinear pattern as in Ait-Sahalia (1996a) with a rapid increase in mean reversion when the interest rates level is high. Using Monte Carlo simulation Stanton (1997) examined the economic significance of the price of risk and finds that the assumption of a specific functional form for the price of risk has important implications for the evaluation of interest rate contingent claims especially as the maturity increases.

The linearity of the drift is also examined by Chapman and Pearson (2000) who applied the techniques developed by Ait-Sahalia (1996b) and Stanton (1997) to data generated through a linear drift by Monte Carlo simulation. The unexpected non-linear pattern measured in the drift was explained as the possibility of a source of bias stemming from the estimation approach. However, opposite results were obtained by Connolly et al. (1997) and Durham (2003), who pointed out that the stationarity of the short rate may be induced by the dynamics of the volatility while the drift is fairly stable.

More recently, Goard and Hansen (2004) employed the GMM method to conduct an empirical comparison within a general non-linear drift framework that nested three important models: the CKLS (1992) model, the Ahn and Gao (1999) model and Goard and Hansen (2004) model. Empirically Goard and Hansen's model seemed to outperform the other models even for smaller sampling periods, which indicates that the particular form of the drift as second order Fourier was able to capture very well the time dependence of the long-term equilibrium mean and also to explain the periodicity of the yield curve. Using an arbitrage-free framework Mahdavi (2008) estimates using the GMM approach the short-term interest rates of seven industrialised countries and the Euro zone. With no single model performing consistently across all countries, the empirical results strongly reflect once again the particularities of each market. While for the US, UK, Sweden and Canada there is evidence of mean reversion and non-linear volatility, the drift for Australia is non-linear, whereas for Japan it is constant indicating a log-normal process. For Denmark the volatility structure is close to that reported by Chan et al. (1992) for the U.S., in the case of the Euro-zone the volatility is an increasing function of the level of the interest rate.

Modelling the Volatility Component

Using several jump models Das (2002) examined seven different empirical features in the Fed Funds data and finds that the models captured well the effects of new information with evidence that the volatility of interest rate changes is substantially higher following the arrival of news. The inclusion of the jump process as an intrinsic feature of financial markets seems to render a linear drift. Otherwise the drift is nonlinear due to information effects. In summary, the nature of the drift is not exactly known, as the two scenarios are arguably equally supported by empirical evidence.

As emphasised by Chan et al. (1992), volatility is a crucial component in the dynamics of interest rates and its modelling has important implications for the pricing of interest rate sensitive products and for the hedging of interest rate risk- the better the model captures the volatility, the more efficient the hedge implied by the model. Most of the theoretical models assume a simple parameterization of the volatility as a function of the interest rate level. Simultaneously, while the literature lacked consensus on the degree of this relationship, there was clear evidence of another feature of the volatility that emerged from serial correlation based (GARCH) modelling in discrete-time.

Volatility clustering and high level of volatility persistence should be also taken in consideration when modelling the volatility of the interest rates. The two features, the level effect and the conditional heteroskedastic (GARCH) effect, were combined in a new class of models by Brenner et al. (1996), who extended the CKLS allowing for the volatility to be affected by information shocks. They concluded that the sensitivity of the volatility on the levels has been overestimated in the literature implying that modelling the volatility solely on the levels it is an important source of model misspecification. A similar study by Koedijk et al. (1997) reconfirmed that the inclusion of the GARCH effect renders a weaker level effect. The new models developed by Koedijk et al. (1997) (KNSW hereafter) in a discrete-time setting, were estimated using the QML method and the consistent estimators based on weekly and monthly one-month Treasury Bills rates (January 1968-July 1996) provided a superior fit relative to both, pure GARCH and pure CKLS type models. Additionally, the more flexible KNSW specifications were found to have important implications for bond option prices that differ from the prices implied by CKLS models.

In another comparative study, Vetzal (1997) examined two classes of continuous-time interest rate models, the standard univariate short rate models and their variants of stochastic volatility models with an E-GARCH effect. The iterative GMM estimation method provided lower estimates of the volatility from stochastic volatility models relative to those implied by the classical one-factor models nested in the CKLS model. Consequently, this led to lower prices for bond options under the stochastic volatility process for the short rate. Vetzal also emphasised the advantage of the tractability possessed by Longstaff and Schwartz (1992) model when it comes to pricing interest rate contingent claims, and that this advantage should be taken into account against the easier estimation of E-GARCH models.

Moreover, the effect combining models suffer from some limitations as pointed out by Andersen and Lund (1997); they lack practical appeal due to the presence of discretisation bias and the erratic behaviour of the internal dynamics of the discrete-time models. An earlier continuous-time alternative was suggested by Longstaff and Schwartz (1992) who developed a multivariate CIR general equilibrium model with the volatility as the second stochastic factor that in the discrete form follows a standard GARCH (1,1) model enhanced with the effect of the one period lagged interest rate. The econometric specification of the model served to testing of the equilibrium restrictions implied by the model, and less to the investigation of the form of both, drift and volatility. Collectively these findings suggest that a possible reason for the rejection of the standard models by the nonparametric procedures is the choice for the volatility function and not that much the non-linearity of the drift. Modelling volatility as an additional factor was also supported by an important shortcoming of single-factor models – the implied perfect correlation among the bond returns across all maturities, which contrasts the empirical evidence. Andersen and Lund (1997) proposed the first direct consistent estimator for a two-factor short rate model involving both the level effect and a stochastic –log-volatility factor. Employing the EMM approach of Gallant and Tauchen (1996), the consistent estimates based on weekly three-month Treasury Bills rates between January 1954 and April 1995, indicate evidence of mean reversion and a level effect close to 0.5. The EMM facilitates a comparative analysis of various discrete and continuous-time models that led to the following results: inside the CKLS framework the CIR model extended with a stochastic log-volatility performs best in terms of explanatory power, while the Level-GARCH models are rejected based on serious instability. Alternatively, Andersen and Lund (1997) incorporate an asymmetric volatility effect using Level-EGARCH discrete specifications that seem to perform reasonably well.

Collectively, these findings suggest that a possible reason for the rejection of the standard models by the nonparametric procedures is the choice of the volatility function rather than the non-linearity of the drift. Modelling volatility as an additional factor was also supported by an important shortcoming of single-factor models – the implied perfect correlation among the bond returns across all maturities, which contrasts with the empirical evidence.

2.3.2 Empirical Evidence of Multi-Factor Interest Rate Models

The affine framework illustrated in Duffie and Kan (1994) provided an important platform for numerous empirical investigations of multifactor models along two approaches. The first approach considers an additive structure of latent factors for the short rate (e.g., Chen and Scott (1993), Pearson and Sun (1994), Duffie and Kan (1996) and Babbs and Nowman (1999)), while the second approach presents the model in terms of the lagged short rate and other state variables, see Chen (1996), Balduzzi et al. (1996), Backus et al. (2001).

Bergstrom and Nowman (1999) considered a particular case (two factors) of the Babbs and Nowman (1999) general model, assuming the instantaneous interest rate as a specific[18] linear combination of two unobservable state variables that can be interpreted as short-term and long-term streams of economic news modelled as possibly correlated Gaussian processes. The two-factor model was estimated using Gaussian estimation methods for seven currencies based on one-month euro-currency rates, and despite the fact that some additional restrictions were necessary for identifying the diffusion and correlation parameters[19], the model provided good empirical results.

Pearson and Sun (1994) proposed a more flexible version of a two-factor CIR model by allowing the two state variables - the real interest rate and the expected inflation rate to become negative unlike the original two-factor CIR model. A comparative analysis between the two models was conducted based on three data sets that combined monthly Treasury bills, notes and bonds. In all instances the original CIR model was rejected based on the likelihood ratio test. When only bills were used the estimation results for the extended model were misleading, with problems of parameter identification and substantial pricing errors for securities of longer maturity.

[18] The coefficients of the two factors are minus unity.

[19] The correlation parameters had to be constrained to zero which means that the two news factors are uncorrelated and the feedback matrix is zero in off-diagonal positions.

The empirical testing of the three-factor model developed by Balduzzi et al. (1996) proved to be quite challenging as the mean reversion level and the volatility were unobservable. While for the short rate the proxy was one-month US Treasury bills rates, the mean was extracted using data on bond prices and the volatility was modelled by a GARCH process. The system to be estimated was a quasi-GARCH-M formulation that proved to fit data very well. The results also confirmed some theoretical features of interest rate behaviour: volatility is a main factor for short-term and medium-term interest rates while the mean has a larger influence on the long-term yields. Dai and Singleton (2000) examine several three-factor affine models from their framework and based on a simulated method of moments they found that for the U.S. money market data the best results are achieved by a specific class that BDFS belongs to as a particular case.

Focusing on the BDT term structure model, a comparative analysis of one-factor diffusion and two-factor stochastic volatility models was conducted by Bali (2003). Based on a Monte-Carlo simulation exercise the two-factor BDT model outperformed the original one-factor BDT model, with a better performance in forecasting the volatility of interest rate changes sampled from daily, one-, three- and six-month Eurodollar deposit (LIBOR) rates between 1971 and 1999. The results concerning the sensitivity parameter gama (the level effect) and the significance of the stochastic volatility factor (the GARCH effect) were assessed for robustness in a context of various functional forms of the drift, that can be nested in a third-order polynomial drift. While there is evidence of a nonlinear drift, across different periods, different maturities and different drift specifications the empirical results converge, confirming previous findings that the level and GARCH effects play a complementary role in the description of the dynamics of the volatility of interest rate changes.

A more recent two-factor interest rate model has been developed by Koutmos and Philippatos (2007) to test for asymmetric mean reversion in European interest rates. By combining two previous theoretical models, the Longstaff and Schwartz (1992) two-factor CIR model and the Bali (2000) model, respectively, the authors found evidence of asymmetric mean reversion and also asymmetric volatility. The MLE estimates based on weekly three-month interest rates for France, Germany and the U.K. indicated that the mean reversion parameter is significantly negative and stronger following a decrease in interest rates while a non-stationary feature is present after an increase in interest rates. However, the mean reversion appeared to dominate the non-stationarity pattern, hence implying that the mean reversion phenomenon exists and its misspecification is rather an empirical than a conceptual issue.

Over a series of articles Nowman (2001, 2003, 2006) estimated several multi-factor (two- and three-factor) models such as the CKLS, Vasicek and CIR models, for the UK and Japan. Initially no feedbacks were considered, and the two factors were the short-term and the long-term interest rates for the two factor models; in Nowman (2003) feedback effects in the conditional mean component were introduced in the model for Japan. The results selected Vasicek as a better model compared to CIR based on the likelihood ratio test against the unrestricted CKLS model.

Ait-Sahalia and Kimmel (2010) estimated all nine of Dai and Singleton's (2000) canonical affine multi-factor interest rate models with US treasury data using a new estimation technique for a closed form approximation of the ML function. Based on simulated and real data they demonstrated that the new techniques produce highly accurate estimates with an insignificant approximation bias, also with less computation due to the analytical closed forms obtained.

After the recent financial crisis, the interest rates have decreased and kept stable at near zero level. This observation can be translated into the *collapse* of the two first factors - level and slope - in a single factor, with the former factor disappearing. Kim and Priebsch (2013) investigated if in this environment the affine Gaussian multi-factor models of interest rates are any longer suitable as these models do not respect the zero lower bound. They empirically tested the performance of a three-factor affine Gaussian model against its equivalent shadow-rate model where the short rate was constrained to respect the zero lower bound. Using the Kalman filter method Kim and Priebsch (2013) estimated the two models and found that the three-factor shadow-rate model outperformed the three-factor affine Gaussian model, which produced larger estimated fitting errors and unrealistic long-horizon forecasts of the short rate.

Most recently, Filipovic et al. (2014) empirically analysed a particular specification – the LRSQ (linear rational square root) model, inside their new general framework. Their findings, based on a combined estimation approach of the QMLE and Kalman filter (KF) methods, indicated that a minimum of five factors, more specifically, three term structure factors and two unspanned factors seem to capture very well the dynamics of both term structure and the volatility of interest rate changes over the period that followed the 2007-2009 last financial crisis. A valuable but controversial finding based on the one-year spot rates, was that the level-effect was more pronounced as the interest rates get closer to the near zero bound.

Given the lower accessibility of data on prices of derivative products relatively fewer studies employ such data. In practice the calibration is also extended to the volatility term structure implied by prices observed in the market of option-related derivatives like caps and floors. According to Longstaff et al. (2000) it is necessary to use both, data on interest rates and data on prices of contingent claims, in order to explain the economic argument behind the rejection of theoretical models. Following these suggestions, Jagannathan et al. (2003) examine the classical one-, two- and three-factor CIR model using data on LIBOR and swap rates, where the short rate is the sum of a constant and the factors. The MLE estimates are then used for pricing derivative products like caps and swaptions. Increasing the number of factors seems to create larger pricing errors especially around the LTCM (Long-Term Capital Management) collapse and in the regions where the slope of the yield curve is negative, while the one-factor specification achieves a better fit to the prices of short-term derivatives.

Another popular strand in TSIR literature has focused on various dynamic versions of the parametric Nelson-Siegel (NS) (1987) model. First, Diebold and Li (2006) developed a dynamic three latent factor model by making the parameters of the NS model time-variant and modelling them as vector autoregressive processes. Following this reformulation of the NS model, numerous recent studies have employed various extensions of the dynamic Nelson-Siegel (DNS) term structure model. For example, a block dynamic Nelson-Siegel model (BDNS) was developed by Philip (2010) who used a hierarchical clustering algorithm to disentangle the term structure into two maturity clusters that are found to have a time-varying dependence and also separate dynamics. Based on US zero coupon yields and Libor-swap rates, the forecasting results produced by the block dynamic BDNS model are superior when compared to DNS model that does not account for the clustering feature. Further extensions of the DNS model with promising forecasting performance have been suggested by Koopman et al. (2010) who considered the single loading parameter as a fourth factor. The authors employed an extended Kalman filter to estimate their model and found evidence of considerable improvement especially in the in-sample performance of the new model. This new dynamic Nelson-Siegel (DNS) framework takes into account the dual feature of the yield curve just like the multi-factor CKLS model empirically examined by Nowman (2001, 2003, 2006). However, given the latent nature of its factors, increasing the number of the factors inside DNS framework is limited by the lack of their interpretation in terms of level, slope and curvature.

2.3.3 Macro-Finance Interest Rate Modelling

Over the last decade the class of multivariate Gaussian models has been extensively used in macroeconomics and finance. The standard way of modelling the term-structure of interest rates has been using unobservable state variables within a no-arbitrage framework. However, the recent literature especially in the aftermath of the last global financial crisis tries to find a proper economic explanation of the yield curve dynamic movements. Various studies, combining both interest rates and macroeconomic variables, document a new direction for term structure modelling by employing a macro-finance structure. The relationship between interest rates, monetary policy and macroeconomic fundamentals has been empirically examined by Ang and Piazzesi (2003) and Piazzesi (2005) who incorporated macroeconomic variables into the Duffie and Kan (1996) affine models and assumed that bond yields span macroeconomic risks. Similar approaches include Rudebusch and Wu (2003) and Hordal et al. (2006). Continuing on this earlier research, Rudebush and Wu (2008) developed a macro-finance framework which jointly estimated an arbitrage-free term structure model with a New Keynesian rational expectations macroeconomic model. This combination enabled the interpretation of the latent factors - the level and the slope - of the yield curve as the perceived inflation target and the cyclical monetary policy response, respectively.

Despite the inclusion of several macroeconomic factors there is evidence that while the short end of the yield curve is clearly affected there is still a substantial misfit of the longer maturity interest rates, suggesting a missing but necessary additional factor. Extending the work of Kozicki and Tinsley (2001), Dewatcher and Lyrio (2006) proposed a macro model combining the TSIR with the inflation rate and the output gap. Their empirical results on the US economy reconfirm that macro variables such as the real interest rate and the inflation rate play a crucial role in the evolution of short-term maturity rates. Based on evidence of a reasonable fit to long-term maturity rates, they also conclude that long-run inflation expectations should be included in the model.

Other strands of macro-finance research explore different financial and economic environments where policy makers had to intervene and investigate the effectiveness of their monetary policy instruments. Christensen et al. (2009) employed a six-factor arbitrage-free Nelson-Siegel (NS) model to demonstrate that the provision of bank liquidity by central banks in December 2007 has substantially lowered the LIBOR rates during the crisis. Most recently, Ullah (2016) extended the arbitrage-free NS model by including five macroeconomic variables and demonstrated that there is a relationship between the yields and these variables that should be accounted for in order to improve the forecasts of the yield curve movements.

During late 2012 and 2013, interest rates reached record low levels with the short-term rate at the zero lower bound for many consecutive days, culminating with negative values in some markets[20]. In a recent study Steeley (2014) examined the dynamics of the yield curve in the current context of near zero interest rates and the impact of the quantitative easing (QE) policy on the shape of the yield curve. Following a PCA analysis, he concludes that four factor are necessary to explain the data; a fourth factor - the change in volatility - is found responsible for undulations observed in the shape of the yield curves (especially the forward curve) on the day QE policy was announced.

The zero lower bound of interest rates presents researchers with some challenging grounds, as an arbitrage-free model that keeps nominal rates positive is still to be developed. Diebold and Rudebusch (2013) suggested some future potential routes involving non-affine structures as in Kim and Singleton (2012) where a Quadratic-Gaussian model is considered. Their key result is that the non-diagonal feedback matrix of the model ensures a better fit to the zero bound than affine structures, and hence correlation among state variables is significantly important. According to Diebold and Rudebush (2013), a different possibility is to use still affine but non-Gaussian models that accommodate nonnegative interest rates such as the CIR model.

Another emerging and controversial aspect in the macro-finance literature of yield curve modelling is the relationship between the bond supply and the risk premium. The empirical facts of the recent financial crisis seem to contradict the conventional theory of "no supply effects" in the sense that changes in the long-term bond supply do not affect bond prices.

In that regard, some studies find that the unconventional policies implemented by the BoE and the Fed of buying long-term bonds have been effective in lowering long-term yields and therefore encouraging economic growth. However, the real mechanism that underlies the dynamics of the long end of the yield curve as a result of these actions still remains unclear.

[20] The one-week and one-month CHF-LIBOR rates were negative in November 2012 and early 2013.

2.4 Conclusions

The aim of this extensive literature review of continuous-time interest rate models is to offer to any new researcher a true picture of the complexity of the task of modelling interest rates, given the highly-technical mathematical apparatus behind the theoretical models and the challenging transition process from theory to practice via sophisticated econometric techniques and advanced computational software. Over the last four decades a taxonomy of theoretical interest rate models has been developed trying to capture most of the features observed in the dynamics of interest rates. However, when it comes to empirical evidence, no clear direction is drawn, due to the multifaceted (different data sets, different discretization methods and/or different econometric estimation methods) aspects of translating continuous-time processes into numbers using discrete-time data. While there is sufficient evidence to support multi-factor specifications over single-factor models, there are several questions within the short rate approach that are still unanswered, such as which estimation technique is most appropriate, how many factors should a model consider and how these models can forecast future interest rates over turbulent periods such as the last financial crisis of 2007-2009. The following three empirical investigations try to bring new evidence that will contribute to a better answer to these questions.

Chapter 3

Gaussian Estimation and Forecasting of Extended Multi-Factor Term Structure Models

3.1 Introduction

Over the last thirty years, interest rate modelling has been developing at a remarkable speed helped by the computational and technological progress during the same period. Most financial market variables, among them the short-term interest rates, are considered to evolve randomly in a continuous dynamic fashion. However, their continuous recording is not available yet[1]. While the continuous-time specification for short-term interest rate models is well established in the literature (as reviewed in Chapter 1), results from new empirical studies employing continuous-time models could always bring valuable insights towards a robust comparative framework in terms of both model specification and method of econometric estimation.

Given the implications of the findings by Litterman and Scheinkman (1991) that three factors (described as level, slope and curvature) can explain over 95% of the fluctuations in yield curves, the empirical literature on TSIR is mostly limited to three factors such as in the Chen (1996) and the Balduzzi et al. (1996) models. However, in the aftermath of the last global financial crisis of 2007-2009, recent studies[2] and financial regulators[3] have suggested that the models used by market participants should account for more information by increasing the number of factors included in the model. The purpose of this chapter is to assess if the recommendations by the financial regulators of using richer

[1] In recent years the technological advances (bandwidth, computing power and storage) have considerably increased the availability of high-frequency data such as tick-by-tick data or trade and quote data especially in finance. These data are intraday transactions and quotes of stocks, bonds, options, currencies and other financial products.

[2] Following a PCA analysis Steeley (2014a) identified the change in the volatility as an important fourth factor, responsible for some changes in the shape of the yield curve, while Christensen et al. (2009) considered a six-factor model.

[3]Basel II Committee on Banking Supervision (2010) recommended that banks should model the yield curve using more risk factors.

yield curve model is of any benefit. Most central banks are still widely using one- or two-factor models and this has been showed to be inappropriate. The Federal Reserve employed a three-factor interest rate model in their Comprehensive Capital Analysis and Review program only in 2014 (van Deventer, 2014). For central banks to implement models beyond three factors, there is a great need for more empirical evidence on the benefits of adding extra factors. The next natural extensions are dimensions such as four-, five-factors. To determine if there is any advantage in doing so, in this chapter we compare the performance of four- and five-factor models within various types of markets (money market and bond market).

One could continue to add extra factors as long as this brings improvement in the model accuracy, in terms of both, goodness of fit and prediction power. While this could be very time-consuming given the increased computational complexity of the estimation techniques, the number of estimated parameters will increase considerably and as a result, the model could suffer of the econometric overparameterization problem. How many factors should be included in a model could be determined by employing formal tests such as principal components analysis (PCA), however the answer to this question is relative to the degree of accuracy which is required for the model to reach: if 95% (as in Litterman and Scheinkman, 1991) is considered good enough three factors are sufficient, if 98% accuracy is required then more factors will be necessary. It was shown (see van Deventer, 2014) that for a 99% accuracy target up to 10 factors are necessary and sufficient. However, for our specific purpose mentioned above the transition from four- to five-factors will provide valuable insights about the implementation of such more complex models.

The extensive theoretical literature offers several modelling frameworks that can be extended beyond three factors such as the general models of CKLS (1992), Babbs and Nowman (1999) and HJM (1992). The modelling framework adopted in this study is rather different and is motivated by several aspects. From an intuitive and practical point of view, the factors - interest rates of different maturities - are important points across the yield curve, whose historical values can be observed in the most liquid markets. In addition, the framework allows for the theoretical element that interest rates of different maturities move together in a very complex fashion by allowing feedbacks in the drift component and by modelling their correlation matrix over time. Moreover, the multi-factor CKLS model has a similar structure to that of the parsimonious vector autoregressive VAR(1) model and hence, a consistent comparison between the two models can be conducted.

Following Nowman's (2003, 2006) approach, the CKLS multivariate framework will be extended to four and then to five factors, by employing the Gaussian estimation methods of continuous-time dynamic systems developed by Rex Bergstrom (1983, 1985, 1986, 1989, 1990, 1997). In the context of interest rates, this method yields quasi maximum likelihood (QML) estimates and its empirical application is justified by the considerable gain in the predictive power of continuous-time models compared with less efficient methods (for example standard methods like 2SLS and 3SLS) or less sophisticated models such as discrete simultaneous equation systems and VAR models (see Nowman, 1997). In addition, a range of short-term interest rates models nested in the multi-factor versions of the CKLS model will be estimated over the period 2000-2013 that includes the recent global financial crisis of 2007-2009. The short end of the yield curve is estimated in an international comparative context involving the some of the most important and diverse countries within the G10 group: the UK, the US, the Eurozone, Japan and Canada. Using the UK Government nominal interest rates, the remaining part of the yield curve is also estimated, hence by connecting the two estimated segments at the point of one-year maturity, the whole UK TSIR is obtained. The empirical results from the dynamic estimation of a total of forty-eight models will provide the in-the-sample estimates that will be used for the out-of-sample model performance.

Another important aim is to conduct an extensive comparative forecasting analysis using the dynamic optimal forecasts to construct a range of statistical and economic loss functions as measures of forecasting accuracy. Three elements of forecasting analysis are brought together to construct a robust forecasting comparison framework: across six different forecasting methods (the four continuous-time models are compared with two discrete time econometric methodologies such as AR (autoregressive) and VAR (vector autoregressive)), across three different horizon-lengths of the holdout samples and between the two model-extensions (four- and five-factors). Moreover, the out-of-sample performance of the competing models is formally tested using the Clark-West (2007) and Diebold-Mariano (1995) for nested and non-nested specifications, respectively.

The structure of this chapter is as follows: In Section 2 a literature review on the development of continuous-time econometric methods is presented with a focus on the Gaussian estimation method developed by Bergstrom. Section 3 outlines the methodology based on a gradual extension of the CKLS model with feedback effects to four and five factors. Section 4 presents the data sets from the interbank and bond markets. Section 5 presents the empirical results from the estimation of the continuous-time models and their interpretation. Section 6 presents the forecasting analysis and comparisons with other methodologies. Finally, the concluding remarks are drawn in Section 7.

3.2 Literature Review

3.2.1 Early Developments in Continuous-time Econometrics

In the late 1940s, econometricians were increasingly aware of the problem caused by measuring the variables at discrete times, whereas their observed values represented the outcome of multiple interactions inside a complex economic system. Most of the macroeconomic variables at that time (national income (GDP), unemployment and inflation rates) were observed annually, although over such a long period, they were obviously influenced by other variables like national debt, money supply and exchange rates. The natural form of such causal system was provided by a system of stochastic differential equations.

During the first half of the 20th century, the mathematical theory of continuous-time stochastic models had been well developed mainly around the concept of Brownian motion. Most important contributions were made by leading mathematicians like Einstein (1906), Wiener (1923) and Kolmogorov (1931). But the area of the estimation of the structural parameters of continuous-time models from discrete data was still short of producing convincing results. The first significant contribution to this area was made by Bartlett (1946) who recognised that the assumption of innovations in the form of Brownian motion was not quite appropriate for modelling economic phenomena. In his article, Bartlett obtained estimates for the parameters of single first and second order differential equations from discrete observations. However, they were considerably biased, emphasizing the technical difficulties of obtaining asymptotically efficient/consistent estimates from a discrete sampling scheme. Koopmans (1950) distinguished for the first time between stock and flow data and recognized a series of advantages from using econometric continuous-time specifications over the recently introduced discrete time simultaneous equations models. Stock variables such as the money supply, stock of capital and interest rates are measured at discrete points in time, whereas flow variables such as output and consumption are discretely averaged as integrals over the observation period.

A general advantage of considering continuous specifications is that they allow for the variables to interact. As a result, the continuous-time models can be interpreted as causal systems unlike the simultaneous equation models. Wold (1952) developed a recursive simultaneous equation system that could be interpreted as a casual chain, but he also (see Wold (1956)) recognised that formulating the causal chains as systems of differential equations could have an even greater importance. Adding to the list of advantages, the continuous-time models naturally allow for the different treatment of stock and flow variables. Also, the continuous-time specifications are independent of the unit observation period, unlike discrete time models. Moreover, governments and policy makers can benefit from the use of continuous-time models as they can produce forecasts of variables of interest (for example GDP) in continuous-time, even though the data are observed discretely.

A rigorous analysis of the arguments put forward by Bartlett (1946) was conducted by Edwards and Moyal (1955) under more relaxed and appropriate assumptions for economic phenomena. The disturbances could be generated by more general processes (for example a mixture of Brownian motion and Poisson process). Some other early studies (Quenouille 1957, Phillips 1959, Durbin 1961) have explored the econometric specification in terms of stochastic differential equations with unsatisfactory but useful outcomes. Phillips (1959) developed the first rigorous algorithm for the estimation of a complete system of differential equations using Fourier transforms; his methodology, however, produced asymptotically biased estimators. These issues seem to have been ignored at least temporarily, one possible reason being the fact that the dominant estimation procedure in the literature of that time was Haavelmo's (1943) discrete simultaneous equations methodology.

3.2.2 Bergstrom's Early Work on Continuous-Time Modelling

The arguments for continuous-time modelling discussed in the literature of the late 1950s and the previous work done by Phillips (1959) convinced Rex Bergstrom of the benefits brought by the continuous-time specifications of econometric models. Phillips' model failed to take certain *a priori* restrictions into account that Bergstrom considered to be critical, given the limitations econometricians face in terms of sample sizes and availability. Some a priori restrictions implied by economic theory were necessary for obtaining reliable estimates for the structural parameters of more complex continuous-time models. The implementation of such models was difficult due to high computational costs at the time.

Following two research directions at the same time, Bergstrom (1966a) proposed an estimation algorithm for closed systems of first order differential equations, while in

Bergstrom (1967) he developed a prototype disequilibrium growth model for the UK macro-economy. The continuous specification used by Bergstrom (1966a) was given by the following equation:

$$dx(t) = A(\theta)x(t)dt + \zeta(dt) \tag{3.1}$$

where $\{x(t), -\infty < t < \infty\}$ is an n-dimensional continuous random process, A is a matrix whose elements are specific functions of the unknown vector of p structural parameters $(p < n^2)$ and $\zeta(t)$ is an n-dimensional vector of white noise innovations. The equally spaced discrete observations $x(0), x(1), x(2), \ldots.$ extracted from the continuous-time model (3.1) satisfy an *exact* discrete model given by

$$x(t) = F(\theta)x(t-1) + \varepsilon_t \tag{3.2}$$

where F is an exponential matrix valued function defined as

$$F = e^A = I + A + \frac{1}{2!}A^2 + \frac{1}{3!}A^3 + \ldots..$$

and $E(\varepsilon_s \cdot \varepsilon_t') = 0$ when $s \neq t$.

Simple *a priori* conditions (some elements of matrix A are zero or linear functions of the parameter θ) translate into complicated transcendental functions in the structural parameter θ. To avoid this difficulty Bergstrom (1966a) suggested a non-recursive model as a discrete approximation[4] to the continuous model (3.1) given by the following equation:

$$x(t) - x(t-1) = A\{0.5[x(t) + x(t-1)]\} + u_t \tag{3.3}$$

together with the assumption that $E\{u(t)u'(t-h)\} = 0$ for $h \neq 0$.

Bergstrom (1966a) analysed the importance of the specification bias introduced by the approximate discrete model (3.3). To find out which discrete model should be used, the exact or the approximate one, Bergstrom suggested that one should determine how important are the differences in the precision of the estimates from the two discrete models. The main advantage of Bergstrom's approach was that the underlying parameters of the differential equations system could be deduced from the estimates of the approximate non-recursive model using standard simultaneous equation methods, for example Three Stage Least Squares (3SLS) and Full Information Maximum Likelihood (FIML) methods. In an important Monte Carlo study Phillips (1972) applied the Minimum-Distance Estimation (MDE) method to the equivalent exact discrete model of a simple three equation trade cycle differential closed system. By comparison with the

[4] In later studies, Wymer (1973) and Sargan (1976) used this type of discrete trapezoidal approximation for more general continuous specifications that included exogeneous variables as well as stock and flow variables.

standard 3SLS estimator from the approximate discrete model the minimum-distance estimator had improved asymptotic properties (consistent and asymptotically efficient), with a reduction of more than 50% in the standard errors of the estimates of the parameters of the exact discrete model over the approximate discrete model. However, when exogeneous variables are included and higher order stochastic differential equations are considered, Wymer (1973) points out the difficulty of estimating the new derived exact discrete model given the complexity[5] implied by the a priori restrictions. If an unrestricted exact model would be estimated, the variances of these consistent estimates are larger than of those estimates obtained by using the approximate discrete model, especially when dealing with small samples.

Phillips (1974, 1976) considered first order open continuous-time models by including smooth non-random exogeneous variables as polynomials in time of degree no more than two. With a more complicated exact discrete model the estimation procedure involved some approximations concerning the exogeneous component, hence the presence of some approximation bias that is shown to disappear as the observation period converges to zero. Using the new "exact" discrete model Phillips (1974) derived the exact Gaussian (quasi-maximum likelihood) estimator with a biased mean and covariance matrix.

An alternative approach to estimating continuous-time models was suggested by Robinson (1976a, 1976b) who proposed a range of estimation methods applied to more general linear differential equations systems that accommodate certain particularities. When constraints on the parameters are non-linear or when the models are formulated as differential-difference equations, Robinson (1976a) applied numerical methods such as nonlinear least squares and maximum likelihood that produced consistent and asymptotically efficient estimators. As a complementary method to these two estimation procedures, the Instrumental Variables (IV) method presented in Robinson (1976b) involves two steps in order to obtain first consistency and then efficiency of the estimates. Using a discrete approximate Fourier transformed system, the IV method led to closed expressions for the estimates, avoiding numerical optimization therefore surpassing the other methods of estimation on computational grounds especially when the parameter space is very large.

[5] It is shown in Bergstrom (1983) that the vector of the disturbances of the EDM equivalent to a continuous time higher –order stochastic differential equation system is generated by an MA process.

3.2.3 Early Empirical Studies

Despite involving a single-equation model, the first empirical application of a continuous-time model was Houthakker and Taylor (1970). They modelled the demand for consumer durable goods in the US using a continuous formulation and derived the approximate discrete model, in the same way as Bergstrom (1966a). Some early multiple equation continuous models include the disequilibrium adjustment model of the United Kingdom financial markets, developed by Wymer (1973) and the US business cycle model by Hillinger et al. (1973).

A major reference for the subsequent empirical studies related to continuous modelling in econometrics was the completion by Bergstrom and Wymer (1976) of the first continuous-time macroeconometric model a Neoclassical-Keynesian cyclical growth model of the United Kingdom. Their model was extended to a larger financial sector by Knight and Wymer (1978) for the UK at the International Monetary Fund. The Bergstrom and Wymer model (hereafter BW model) model was employed as a prototype for the development of macroeconometric models for various countries such as Australia (Jonson et al., 1977) and Italy (Gandolfo and Padoan 1982, 1984, 1987, 1990). Meanwhile, other economy-wide continuous-time models were proposed for the United Kingdom (Jonson (1976)), Canada (Knight and Mathieson, 1979), Germany (Kirkpatrick, 1987), Italy (Tullio, 1981; Gandolfo and Padoan, 1990), and the United States (Armington and Wolford 1983; Donaghy, 1993).

During the 1980s, continuous-time macromodels were increasingly used for policy analysis (Jonson and Trevor, 1981; Bergstrom, 1984b; Gandolfo and Padoan, 1982, 1984). Econometricians were trying to measure the impact of various types of policy feedbacks on the continuous-time model's asymptotic stability. Stefansson (1981) applied a controlling procedure to a small continuous-time econometric model for the Icelandic economy and was unable to obtain exact optimal feedbacks. Other econometric studies considered continuous-time models at the microeconomic level. For example, Richard (1978) investigated a commodity continuous-time model for the world copper industry, Brennan and Schwartz (1979) studied an asset pricing model and Levich (1983) used the Armington and Wolford (1984) model with impressive results in forecasting exchange rates.

3.2.4 The Gaussian Estimation Method of Continuous-Time Models with Discrete Data

Most of the continuous-time models developed during the 1960s and 1970s were either of first order or their estimation involved the use of an approximate discrete model. More realistic and flexible continuous-time models were largely acknowledged as most appropriate in order to capture the dynamics observed in many economic phenomena. However, using more sophisticated models has led to increasingly complex estimation procedures; hence, the importance of developing a rigorous theoretical framework of statistical inference that could be applied for open and closed linear higher order continuous-time systems with both stock and flow data.

Continuous-time formulations that intensively use economic theory in the attempt to model the relationship between economic variables were at the centre of Rex Bergstrom's research agenda. In a seminal paper, Bergstrom (1983) presented the first theoretical elements of the Gaussian econometric methodology applied to linear stochastic differential equations systems with discrete data. This article represented the foundational study that led, through a systematic approach, to major subsequent developments in all areas of research related to the Gaussian estimation methodology.

Following Bergstrom (1983), the general formulation of a closed linear continuous model of order k, is:

$$d\left[D^{k-1}x(t)\right]=\left[A_1(\theta)D^{k-1}x(t)+...+A_{k-1}(\theta)Dx(t)+A_k(\theta)x(t)+b(\theta)\right]dt+\zeta(dt) \qquad (3.4)$$

where $x(t)$ is a real continuous n - dimensional random process, θ is a p - dimensional vector of structural parameters, $A_1, A_2,...A_k$ are $n\times n$ coefficient matrices whose elements are known functions of θ, b is an $n\times 1$ vector whose elements are also known functions of θ, and D represents the mean square differential operator. It is assumed that the vector of disturbances $\zeta(dt)$ satisfies the following Assumption 1 in Bergstrom (1983):

ASSUMPTION 1: $\zeta=[\zeta_1,...,\zeta_n]'$ is a vector of random measures with a finite Lebesgue measure on all subsets of the real line, such that $E[\zeta(dt)]=0,\quad E[\zeta(dt)\zeta'(dt)]=(dt)\Sigma$ where $\Sigma=\Sigma(\mu)$ is a positive definite matrix whose elements are known functions of μ another vector of parameters and $E[(\zeta_i(\Delta_1)\zeta_j(\Delta_2)]=0$ for all $i,j=1,...,n$ and any disjoint sets Δ_1 and Δ_2. (See Bergstrom (1984a) for a discussion of random measures).

In order to avoid the restriction of stationarity, Bergstrom (1983) considered some boundary conditions that $x(t)$ should satisfy:

$$x(0) = y_1,\ Dx(0) = y_2,\ \ldots\ ,\ D^{k-1}x(0) = y_k\ , \qquad (3.5)$$

where $y_1, \ldots, y_k$ are n - dimensional random vectors verifying the following assumption:

ASSUMPTION 2: $E[y^i \zeta'(\Delta)] = 0,\ (i = 1,\ldots,k)$ for any set Δ in the half line $0 < t < \infty$ with finite Lebesgue measure.

Therefore, the continuous-time model comprises the differential equation system (3.4), the boundary conditions (3.5) and the Assumptions 1-2 and it will be hereafter referred to as the *basic* continuous model. The complete vector parameter to be estimated will comprise θ, μ and y, where y includes the unobservable elements of the initial state vector corresponding to the flow data and $y_2, \ldots, y_k$ defined above. The generality of the model has expectedly increased the level of complexity in the estimation procedure. The higher order feature brought an element of complication as the quantities $Dx(t), \ldots, D^{k-1}x(t)$ are unobservable quantities, while new sources of autocorrelation were created in the residual vector by considering mixed data. As a consequence, the estimates of the parameters would be affected by a *temporal aggregation* bias.

The central theoretical development in Bergstrom (1983) was a fundamental theoretical theorem that proved the existence and uniqueness of a discrete solution for the basic continuous model. Despite the additional complexities, the exact discrete model that is satisfied by the discrete data generated from the continuous model with mixed data was shown to be a vector autoregressive moving average model. In the case when only stock variables are considered the solution is given by a $VARMA(k, k-1)$. For continuous models involving only flow data, where the variables are measured as integrals over the observation period ($x(t) = \int_{t-1}^{t} x(r)dr$), the solution becomes a $VARMA(k, k)$ model. When the data is mixed the exact discrete model implied by the basic continuous-time model still maintains the $VARMA(k, k)$ form. The exact solution of the basic continuous model on the domain $[0, T]$ could be expressed mathematically as:

$$x(t) - F_1(\theta)x(t-1) - \ldots. - F_k(\theta)x(t-k) - g(\theta) = \varepsilon_t + G_1(\theta,\mu)\varepsilon_{t-1} + \ldots + G_k(\theta,\mu)\varepsilon_{t-k} \qquad (3.6)$$

where the coefficient matrices $F_1, \ldots, F_k$ are highly nonlinear transcendental (matrix exponential) functions involving the parameter vectors θ and μ. The innovations $\{\varepsilon_t\}_t$ have the following properties: $E(\varepsilon_t) = 0,\ E(\varepsilon_t \varepsilon_t') = K(\theta,\mu),\ \ E(\varepsilon_s \varepsilon_t') = 0,$ for $s \neq t$.

The form of the exact discrete model given by (3.6) is used for analysing the asymptotic sampling properties of the maximum likelihood estimator, however for the purpose of computing the Gaussian estimate and likelihood function an intermediary version (with only the autoregressive coefficients) was considered. The moving average side was embedded in the wide sense stationary process of the disturbances η_t. According to Bergstrom (1983, Theorem 2), the form of the exact discrete solution implied by a closed continuous-time model of order k is:

$$x(t) = F_1(\theta)x(t-1) + ... + F_k(\theta)x(t-k) + g(\theta) + \eta_t, \quad t = k+1,...,T \tag{3.7}$$

where $\{\eta_t\}_t$ is a wide sense stationary first-order vector moving average disturbance with the following properties:

$$E(\eta_t) = 0,$$

$$E(\eta_t \eta'_{t-r}) = \Omega_r(\theta, \mu) \quad \text{for } r = 0,..., k-1$$

$$E(\eta_t \eta'_{t-r}) = 0 \qquad \text{for } r > k-1$$

The complexity of the explicit form of the matrix functions coefficients of the autoregressive part $F_1(\theta), \ldots, F_k(\theta)$ and $g(\theta)$, $\Omega_0(\theta,\mu), \ldots, \Omega_{k-1}(\theta,\mu)$ increases as the order of the system gets higher (Bergstrom, 1983 and 1984a,b).

For the estimation of the vector parameter (θ, μ), Bergstrom suggested two alternatives that implied additional assumptions. For computational reasons, the procedure followed assumed the wide sense stationarity property for the n- dimensional process $\{x(t), -\infty < t > \infty\}$. The exact Gaussian estimates were obtained by minimizing $\hat{L}$ with respect to (θ, μ), where $\hat{L}$ equals minus twice the maximum likelihood function and is computed as:

$$\hat{L} = \sum_{i=1}^{n(T+1)} \left\{ z_i^2 + 2\log m_{ii} \right\} \tag{3.8}$$

This simplified expression for $\hat{L}$ was derived using a common procedure in Bergstrom's algorithms for computing the maximum likelihood function: the Cholesky factorization of the correlation matrix valued function $V(\theta, \mu)$. Mathematically, there is a lower triangular real matrix M with positive elements m_{ii} on the first diagonal, such that $V = MM'$. The vector $z = [z_1, ... z_{nT}]'$ can be recursively determined from the $Mz = \eta$. The algorithm for computing $\hat{L}$ was rigorously outlaid in Bergstrom (1983, 1985, 1986) and it will be reapplied in the current study and presented at a later stage in the estimation section. Once $\hat{L}$ has been computed, the Gaussian estimates could be obtained by various optimization procedures which involve repetitive evaluation of $\hat{L}$ for a set of parameter values. Bergstrom (1983) suggested two such numerical procedures: the maximum gradient method and approximation method for larger continuous models using the spectral density function. While the former method is simpler it does not necessarily lead to the minimum of $\hat{L}$ in case there are multiple local minima. Therefore, the procedure should be repeated using different inputs for the initial values of the vector parameter (θ, μ) . The latter method concerned finding an approximation to $\hat{L}$ in order to avoid the computation of high dimension ($nT \times nT$) matrices. Bergstrom (1983) computed these approximations using the spectral density function of the stationary process $\{x(t), -\infty < \mathrm{t} < \infty\}$, that after a specific decomposition can be represented as an autoregressive process.

3.2.5 The Development of Computational Algorithms, Hypothesis Testing, Forecasting and Control

Bergstrom (1985) derived a new efficient algorithm for computing the exact Gaussian likelihood for the parameters of a higher order closed continuous-time dynamic model with flow data. A new set of parameters was added for estimation, as the initial state vector y is unobservable[6]. An approximate estimate (extrapolation) for y is accepted instead of an exact one. With y fixed in this way then the estimation procedure will provide asymptotically efficient estimates for (θ, μ). Bergstrom (1985) conjectured that the new efficient estimation based on the VARMA type discrete models, should considerably increase the precision of estimates and reduce the prediction errors of future observations.

Applying Phillips' (1974, 1976) exogenous variable methods, Bergstrom (1986) extended the efficient algorithm for the closed model to an open model of a higher order continuous-time system with both stock and flow types of data. The exogenous variables introduced into the model are assumed to be generated by polynomials in time of degree not exceeding two. The model can be extended even more: for higher order systems,

[6]Only the first vectorial component of y , made of the initial states $y_1 = \{x_i(0)\}_{i=1,\ldots,n}$ is considered known or observable; the rest of the components $y_j(0) = \{D^{j-1} \mathrm{x}_i(0)\}_{i=1,\ldots,n}$ for $j = 2, \ldots k-1$ are unobservable, hence they will be endogeneously estimated.

instead of using quadratic interpolation, polynomial interpolation can be used, of a degree dependent on the order of the system. The exact discrete model specification was shown to be a generalized VARMAX model, a convenient form in terms of estimation, testing and forecasting.

According to Phillips (1974, 1976) the biases introduced by these assumptions are smaller (of third order) than those (of second order) obtained by employing Fourier methods developed by Robinson (1976a, b, c). For a model with Gaussian innovations and exogenous variables with such continuous paths it was shown that the method yields exact quasi-maximum likelihood estimates of the structural parameters. Bergstrom provided the exact formulas for the implementation of the Gaussian methodology for the most general second order continuous-time model in which both the endogenous and exogenous variables are a mixture of stock and flow variables. As in Bergstrom (1986), the most general continuous-time linear model allowing for greater dynamics (higher order, considering both types of data, including exogenous variables) would have the following equation:

$$d\left[D^{k-1}x(t)\right]=\left[A_1(\theta)D^{k-1}x(t)+A_2(\theta)D^{k-2}x(t)+....+A_{k-1}(\theta)Dx(t)+A_k(\theta)x(t)+Bz(t)\right]\cdot dt+ +\zeta(t),\quad t\geq 0 \tag{3.9}$$

After the development of a complete theoretical framework of the Gaussian estimation of continuous dynamic systems and robust computational algorithms for its implementation, Bergstrom further looked into various other problems using this type of model, concerning optimal control methods for policy makers in Bergstrom (1987), a forecasting algorithm in Bergstrom (1989), and statistical hypothesis testing in Bergstrom (1990, Chapter 7).

Bergstrom (1987) considered the approximate discrete continuous-time model as the true model and provided a rigorous mathematical solution to the problem of controlling a continuous-time linear stochastic model with the control variables as exogeneous. His approach was extended for control and non-control exogeneous variables. The feedbacks are shown to be optimal in the class of linear feedbacks. The optimal level of control was estimated by minimizing the infinite horizon quadratic cost function for a second order dynamic model. If the estimated optimal feedback is applied from time zero onward, then the costs are minimised. The main advantage of Bergstrom’s method was that it could be applied to higher order systems with more realistic specifications of cost functions and the estimates were still consistent. Bergstrom (1989) presented an optimal forecasting algorithm of discrete mixed data together with a theorem that demonstrates the optimality of the forecasts. They are exact Gaussian estimates of the post-sample observations conditioned by the information contained in the sample.

A final econometric aspect that Bergstrom was determined to address was the statistical testing and model evaluation of the higher order continuous-time models. Bergstrom (1990) developed some practical procedures for testing hypotheses concerning a specific continuous-time model with a mixed data sample. A more detailed analysis of the VARMA type models satisfied by the exact discrete time models was conducted. The findings, with important practical implications, were proved in a theorem about the behaviour of the moving average coefficient matrices. Bergstrom observed that they are time variant, and he demonstrated that they converge rapidly to a limit set of three matrices that is asymptotically stable stationary. Nowman (1991) suggested for practical applications that after twelve steps the limit matrix is found to seven decimal places. Following this result, a three-stage testing strategy was presented and its extension to an open and higher order system case was discussed. The strategy was tested for both general and restrictive hypotheses in a VARMA framework. The exact discrete models represented by the VARMA specification provided the basis for exact asymptotic tests of the specification of a continuous-time model and tests of hypothesis of a set of restrictions on the parameters.

3.2.6 Related Other Work on Gaussian Estimation and Continuous-Time Models

Over the last two decades a series of studies have expanded the range of alternative differential models, providing closed forms for such models, ready for estimation and forecasting analysis. This was driven by exploring more flexible and complex continuous-time models that would better fit time series data with particular dynamic features. The exact discrete model approach presented in Bergstrom (1983) represents one of the major methods applied in the estimation of linear stochastic differential equation systems besides approaches based on Kalman filtering of state space forms developed by Harvey and Stock (1985, 1988) and spectral representations considered by Robinson (1976a, b).

Inspired by a model developed by Bailey et al. (1987), Chambers (1991) extended Bergstrom's econometric framework and derived the exact discrete model (EDM) equivalent to a more general continuous-time system, more specifically a second order differential equations system that included the first and second derivatives of the exogeneous variables in addition to their levels. Another alternative that would accommodate for the new dynamics while still using the framework in Bergstrom (1986), was the adjustment algorithm applied to the exogeneous variables prior to the estimation suggested by Nowman (1991). In a theoretical paper Chambers (1998) presented a detailed estimation technique that involved the derivation of a frequency domain Gaussian estimator of the parameters of a joint differential-difference equation system[7]. It was shown that this estimator is strongly consistent and asymptotically normally distributed without requiring the Gaussianity of the data. A more flexible specification of continuous-time models incorporated unobservable stochastic trends instead of deterministic trends. Studies exploring this feature include Phillips (1991), Simos (1996) and Harvey and Stock (1988, 1989, 1993). Extending the estimation algorithm of Bergstrom (1986), a new exact Gaussian estimation procedure was developed in Bergstrom (1997) with unobservable stochastic trends in a continuous-time model combining first and second order differential equations with white noise innovations and mixed data. Chambers (1999) derived the formulae for an EDM corresponding to a continuous system of higher order stochastic differential equations that can be applied to stationary, non-stationary and even explosive systems. The differential-difference type equations were employed by Chambers and McGarry (2002) in modelling cyclical behaviour in an unobserved components framework[8]. Using the discrete form of the Whittle likelihood the authors have proposed a flexible estimation technique for the derivation of a frequency domain Gaussian estimator of the parameters of a more dynamic model than those models previously considered in the literature. On the same line of research, Ercolani and Chambers (2006) and Ercolani (2009, 2011) conducted rigorous econometric analyses of various continuous-time specifications with unknown lag-parameters or driven by fractional noise.

Overcoming the complications brought by the inclusion of exogeneous variables constituted the central objective of many studies. Following suggestions made by Robinson (1992) regarding a pragmatic approach to estimating the exogeneous component in the EDM, McCrorie (2001) provided an order-selection criterion for

[7] Among relatively few previous attempts there are Robinson (1976a) and Robinson (1977b).

[8] The authors considered a univariate first-order three-component (trend, seasonal and cyclical) continuous time model and provided conditions for the parameters of the differential-difference equation concerning the cyclical component (containing lags), so that the initial process becomes stationary and allows for a business cycle.

choosing the optimal interpolant[9] that would close the model. He also showed in a Monte Carlo experiment that the choice of a wrong degree polynomial could lead to seriously biased estimates of the variables of interest. Some early studies, including Telser (1967), had mentioned the aliasing problem in a continuous-time differential equation system. However, it was Phillips (1973) who looked first at possible ways of minimizing the identification problem of the structural coefficient matrix in a first order linear differential equation system. Assuming that *a priori* restrictions on the system are simple linear functions of the elements of the coefficient matrix, Phillips (1973) showed that structural parameters are in some cases identifiable. In McCrorie (2003) a sharper characterization of the identification problem is presented, allowing for the joint treatment of the coefficient and the covariance matrices.

McGarry (2003) derived the EDM equivalent to a novel continuous-time formulation that included seasonal dummies, avoiding in this way the widely practiced seasonal data adjustments. The SDE system was of forth order allowing for a mixture of stock and flow inside all the vector processes. When open systems were considered exogeneous variables assumed a higher degree of smoothness which, according to Phillips (1974), should reduce the asymptotic bias induced in the estimation procedure. Another EDM was obtained by Simos and Taylor (2009) from a third order differential underlying equation system with fixed initial condition driven by I(1) observable stochastic and white noise disturbances.

Cointegrated continuous-time models form another class of models studied in the continuous-time modelling literature. An early approach to estimating the parameters of cointegrated systems was proposed by Phillips (1991) with two different procedures: a frequency domain regression method for the cointegrating parameters and a non-parametric treatment for the dynamic parameters. Chambers (2009) derived the EDM analogue to a first order cointegrated continuous-time system in a triangular error correction format with mixed stock and flow variables and observable stochastic trends. Following the recursive computation algorithms presented in Bergstrom (1985, 1990), he also provided a time domain full Gaussian estimation procedure applied to both sets of parameters. The statistical properties of the Gaussian estimators are revealed by Chambers and McCrorie (2007) where frequency domain Gaussian estimators had been derived in a more general continuous-time context.

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[9] In most of the empirical work a quadratic interpolation is used; Bergstrom et al. (1992) and Bergstrom and Nowman (1999) have used this type of interpolation with successful results.

More recently econometricians explored different ways of estimating continuous-time models driven by moving average innovations, a complex feature that is retained in the discrete time representation. Following the exact discrete time approach in Bergstrom (1983), Chambers and Thornton (2012) derived the exact discrete models for a general $ARMA(p,q)$ specification of the continuous-time model with stock or flow variables. In another recent study, Park and Jeong (2010) developed an asymptotic theory for maximum likelihood estimators of the parameters of continuous dynamic processes that possess a zero root.

3.2.7 Empirical Applications of Gaussian Estimation Methods

The next phase that would make Bergstrom's research programme of continuous modelling complete was empirical applications of econometric Gaussian methods in continuous-time. During the 1980s, the estimation of demand models based on consumer behaviour theory failed to produce good results despite the increasing complexity in the model specifications. Bergstrom's econometric general framework was employed for the first time in Bergstrom and Chambers (1990) to model the dynamic responses of consumer demand for goods to variations in disposable income. The dynamic formulation of the continuous models from Bergstrom's econometric methodology accommodated previous shortcomings like ignoring the influence of the lagged dependent variables and the fact that the consumers' stocks were unobservable. Exact maximum likelihood estimates were obtained by maximizing a pseudo-Gaussian likelihood function and the dynamic quarterly forecasts of two-year post-sample period were superior to those resulting from simpler models. Later Chambers (1992) estimated the first multivariate continuous-time model represented by a complete demand system, for durable and non-durable goods. His results re-confirmed previous findings in the demand theory literature, with interest rates and the price of the durable goods as main determinants. Another empirical application of the Gaussian estimation in the area of consumer behaviour was Chambers and Nowman (1997), where an alternative dynamic specification for the almost ideal demand system (AIDS) of Deaton and Muellbauer (1980) was considered for estimation and dynamic multi-step ahead forecasting.

Following the earlier Bergstrom-Wymer (1976) model of the UK economy, the first major empirical exercise is represented by the development of a 14-equation second order continuous-time macro-model of the UK economy (see Bergstrom, Nowman and Wymer (1992)). The study produced promising results with small forecasting errors for all the main variables. The performance of the model was later improved by replacing the deterministic trends with segmented trends in Nowman (1998).

The Bergstrom's (1997) method that incorporated unobservable stochastic trends and comprised differential equations of first and second order was first applied in a major empirical study of the UK economy in Bergstrom and Nowman (2007). It was shown that the model performed satisfactorily over an eight-quarter forecast period. Continuous ARMA dynamics seem to provide encouraging empirical results. An empirical exercise in Chambers and Thornton (2010) highlights the impact of considering a more dynamic continuous specification by allowing for the disturbances to be a moving average process. They demonstrate the superiority of ARMA (2,1) continuous specifications for stationary processes with stock variables (sunspot data and short-term interest rates) and non-stationary processes with flow variables (US non-durable consumer expenditure) over purely autoregressive formulations. In a comparative study, Gough et al. (2014) modelled the interest rate spread for Germany, Japan, UK and the USA during the recent global financial market crisis of 2007–2009. Based on monthly and weekly data they found that the Merton continuous-time model produced the best predictions when compared to discrete time benchmark model such as ARMA and ARFIMA models.

3.2.8 Gaussian Estimation of Multi-Factor Interest Rate Models

For the first time in the finance literature, Nowman (1997) estimated a range of single factor continuous-time models of the short-term interest rate nested in the famous CKLS framework applying the Gaussian methods developed by Bergstrom (1983, 1985, 1986, 1990). This application opened a new field of research in modelling the term structure of interest rates. Various studies (Brennan and Schwartz, 1979; Longstaff and Schwartz, 1992; Chen and Scott, 1995; Babbs and Nowman, 1999) have indicated that considering the multi-factor specification of term structure models will increase the ability of the model to better capture the dynamics of the interest rates. A particular subclass[10] of Langetieg's (1980) linear Gaussian models was considered and estimated by Babbs and Nowman (1999) on U.S. zero coupon yields using Kalman filtering methods. Their no-arbitrage specification is in fact the multi-factor Generalized Vasicek model studied by Babbs (1993). Bergstrom and Nowman (1999) successfully applied the Gaussian estimation method to the Generalized Vasicek multi-factor model of Babbs and

[10] The multi-factor models in this subclass assume that the short rate can be written as a specific linear combination of state variables whose dynamics are simplified by not including any feedback from the other state variable in the drift component.

Nowman (1999) with unobserved state variables of seven currencies. Two-factor versions of the general CKLS, Vasicek and CIR models were estimated by Nowman (2003) with empirical results suggesting evidence of feedback from the long-term to the short-term interest rate. Similar models were employed in Nowman and Saltoglu (2003) and Saltoglu (2003) in an extensive forecasting comparison between the parametric Gaussian estimation method and a series of non-parametric estimation methods, namely, Artificial Neural Networks (ANN), k-Nearest Neighbour (k-NN) and Local Linear regression (LL). Following Bergstrom's (1966a) approach, Nowman (2003) introduced feedbacks in the conditional mean of the CKLS and the CIR model was compared to the non-feedback case in Nowman (2001). Evidence of feedback effects was provided for Japan. In Nowman (2006) the CKLS model was extended even further to a three-factor version and the Gaussian estimation method was applied to Japanese interest rates. This way of increasing model flexibility will be followed in the methodology of this chapter, with two extensions of four- and five-factors being empirically tested in an international context.

3.3 Methodology

3.3.1 The Theoretical Modelling Framework

In line with Nowman (2001, 2003, 2006) the modelling of the yield curve will be extended to four and five factors. If for the two-factor extended models nested in the CKLS formulation the factors were the short-term and long-term interest rate respectively, when more than three factors are included they represent interest rates of a range of maturities along the yield curve. The term-structure models that will be empirically tested are the multi-factor general Chan et al. (1992), Vasicek (1977), Cox et al. (1985) and Brennan and Schwartz (1980) models. The last three traditional models are nested in the CKLS model, corresponding to certain restrictions on the parameter of elasticity of volatility, namely $\gamma = 0, \gamma = 0.5$ and $\gamma = 1$. Each specification will be extended, for all the five data sets, to four and five factors leading to a total of twelve continuous-time models of the term-structure to be estimated. These models will be applied to five distinct interbank markets (UK, US, Eurozone, Japan and Canada) using shot-term data and to the UK Gilts market using both, short and long-term interest rates.

The theoretical modelling framework is presented in terms of the CKLS specification as it nests all the other analysed models as the level effect parameter takes particular numerical values. It is important to emphasize that the analysis involves three distinct theoretical models: the true underlying continuous-time model (also called the *basic* model), the approximate/modified continuous-time model introduced by Nowman (1997, 2003) and the exact discrete model (see Phillips, 1972; and Bergstrom, 1983) that will be estimated.

The True Continuous-Time Multi-Factor Model

The general single factor CKLS short-rate model is given by the following stochastic differential equation:

$$dr(t) = [\alpha + \beta r(t)]dt + \sigma r^{\gamma}(t)dZ(t), \text{ for any } t > 0 \tag{3.10}$$

The multi-factor version of the general CKLS continuous-time model can be written as a system of stochastic differential equations:

$$\begin{cases} dr_1(t) = [\alpha_1 + \beta_{11}r_1(t) + \beta_{12}r_2(t) + \ldots + \beta_{1n}r_n(t)]dt + \zeta_1(dt) \\ dr_2(t) = [\alpha_2 + \beta_{21}r_1(t) + \beta_{22}r_2(t) + \ldots + \beta_{2n}r_n(t)]dt + \zeta_2(dt) \\ \vdots \\ dr_n(t) = [\alpha_n + \beta_{n1}r_1(t) + \beta_{n2}r_2(t) + \ldots \beta_{n,n}r_n(t)]dt + \zeta_n(dt) \end{cases} \tag{3.11}$$

or in vector-form as:

$$dr(t) = [\alpha + \beta r(t)]dt + \zeta(dt), \quad \text{for any } t > 0 \tag{3.12}$$

where

- $r(t) = [r_1(t), r_2(t), ..., r_n(t)]'$ is the vector of the observable interest rate variables,
- $\alpha = [\alpha_1, \alpha_2, ..., \alpha_n]'$ is the vector of the *drift* parameters,
- $\beta = \{\beta_{ij}\}_{1 \le i, j \le n}$ is the *feedback* matrix whose elements are assumed to be non-zero, as implied by the close relationship between interest rates of different maturities, and
- $\zeta = [\zeta_1, \zeta_{2,} ..., \zeta_n]'$ is a vector of random measures under certain conditions defined by the following assumption:

Assumption 1 Nowman (2001) (generalised): $\{\zeta_i\}_{i=1,...,n}$ are correlated random measures defined on all subsets of the half line $0 < t < \infty$ with finite Lebesgue measure, such that $E[\zeta_i(dt)] = 0$ for all $i = 1,...,n$ and $E[\zeta(dt) \cdot \zeta'(dt)] = (dt)\Sigma(r,t)$, where $\Sigma(r,t) = \{\sigma_{ij}\}_{1 \le i, j \le n}$ is a positive definite matrix, with $\sigma_{ii} = \sigma_i^2 r_i^{2\gamma_i}(t)$ and

$\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j r_i^{\gamma_i}(t) r_j^{\gamma_j}(t)$ for any $i \neq j$, $i, j = 1,\ldots,n$. The parameter γ_i measures the dependence of the volatility of the interest rate $r_i(t)$ on its level, ρ_{ij} represents the correlation coefficient between any two distinct factors $r_i(t)$ and $r_j(t)$, and σ_i is the proportional volatility factor for the conditional volatility of the interest rate $r_i(t)$.

The Approximate Continuous-Time Multi-Factor Model

Bergstrom's (1983, 1984a) estimation methodology assumed constant volatility of the state variables which is considerably restrictive in the case of interest rates. Nowman (1997) relaxed this assumption by allowing a special type of heteroskedasticity. The volatility was considered to be a step function, changing value at the beginning of each unit observation period and then remaining constant over that unit time interval. Mathematically, for any $t > 1$ the unit period is denoted by the interval $[t'-1, t']$ where $t'-1$ is the largest integer less than t. For the single factor case Nowman (1997) adjusted only the conditional volatility component, leaving the drift component unchanged. Therefore, the new continuous-time model was a better approximation to the original continuous-time model with the potential benefit of reducing the temporal aggregation bias. With a temporarily constant volatility the new model can be estimated over each observation interval by implementing the Gaussian methods developed by Bergstrom for higher order linear Gaussian continuous-time models discussed in Section 3.2.5.

An important advantage of generalizing the CKLS framework to yields-only multi-factor formulations (see Nowman, 2001, 2003 and 2006) is that the assumption of constant volatility during the unit period allows to explicitly compute the variance-covariance matrix of the innovations as follows:

$$\Sigma^*(r,t) = \{\sigma_{ij}^*\}_{1\le i,j\le n}, \text{ where } \sigma_{ii}^* = \sigma_i^2 r_i^{2\gamma_i}(t'-1) \text{ and } \sigma_{ij}^* = \rho_{ij}\sigma_i\sigma_j r_i^{\gamma_i}(t'-1) r_j^{\gamma_j}(t'-1).$$

Possessing this feature, the multi-factor CKLS framework takes into account the close relationship that exists among yields of different maturities. These dynamics are intrinsic to the term structure of interest rates and this *interaction* across the yield curve is measured by the multi-factor CKLS framework via two components: the feedback matrix and the covariance matric.

3.3.2 The Continuous-time Multi-Factor Interest Rate Models with Feedbacks

The Four-Factors Continuous-Time Term Structure Models

The continuous-time systems of the stochastic differential equations for the four-factor CKLS, Vasicek, CIR and BS models (hereafter CKLS4F, Vasicek4F, CIR4F and BS4F respectively) have the same mathematical form:

$$\begin{cases} dr_1(t) = [\alpha_1 + \beta_{11}r_1(t) + \beta_{12}r_2(t) + \beta_{13}r_3(t) + \beta_{14}r_4(t)]dt + \zeta_1(dt) \\ dr_2(t) = [\alpha_2 + \beta_{21}r_1(t) + \beta_{22}r_2(t) + \beta_{23}r_3(t) + \beta_{24}r_4(t)]dt + \zeta_2(dt) \\ dr_3(t) = [\alpha_3 + \beta_{31}r_1(t) + \beta_{32}r_2(t) + \beta_{33}r_3(t) + \beta_{34}r_4(t)]dt + \zeta_3(dt) \\ dr_4(t) = [\alpha_4 + \beta_{41}r_1(t) + \beta_{42}r_2(t) + \beta_{43}r_3(t) + \beta_{44}r_4(t)]dt + \zeta_4(dt) \end{cases} \tag{3.13}$$

The four multivariate specifications are different only in the way their random measures are correlated, as their specific covariance matrix depends on the level effect parameter. By imposing specific restrictions on γ ($\gamma = 0$, $\gamma = 0.5$ and $\gamma = 1$ for the Vasicek, CIR and BS model, respectively) each model will assume a specific adjusted matrix $\Sigma^*(r,t) = \{\sigma_{ij}^*\}_{1 \le i,j \le 4}$ for measuring the autocorrelation in the innovations series $\zeta = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4\}$ Consequently, the maximum likelihood function $L(\theta)$ will have another different expression for each of the four models. According to Nowman (2001, 2003 and 2006) the adjusted covariance matrix $\Sigma^*(r,t)$ can be computed as follows:

The four-factor general CKLS model (hereafter CKLS4F) for $\gamma = [\gamma_1, \gamma_2, \gamma_3, \gamma_4]'$ unrestricted has the following adjusted covariance matrix:

$$\Sigma^*_{CKLS4F}(r,t) = \begin{pmatrix} \sigma_1^2 r_1^{2\gamma_1}(t'-1) & \rho_{12}\sigma_1\sigma_2 r_1^{\gamma_1}(t'-1) r_2^{\gamma_2}(t'-1) & \rho_{13}\sigma_1\sigma_3 r_1^{\gamma_1}(t'-1) r_3^{\gamma_3}(t'-1) & \rho_{14}\sigma_1\sigma_4 r_1^{\gamma_1}(t'-1) r_4^{\gamma_4}(t'-1) \\ \rho_{21}\sigma_2\sigma_1 r_2^{\gamma_2}(t'-1) r_1^{\gamma_1}(t'-1) & \sigma_2^2 r_2^{2\gamma_2}(t'-1) & \rho_{23}\sigma_2\sigma_3 r_2^{\gamma_2}(t'-1) r_3^{\gamma_3}(t'-1) & \rho_{24}\sigma_2\sigma_4 r_2^{\gamma_2}(t'-1) r_4^{\gamma_4}(t'-1) \\ \rho_{31}\sigma_3\sigma_1 r_3^{\gamma_3}(t'-1) r_1^{\gamma_1}(t'-1) & \rho_{32}\sigma_3\sigma_2 r_3^{\gamma_3}(t'-1) r_2^{\gamma_2}(t'-1) & \sigma_3^2 r_3^{2\gamma_3}(t'-1) & \rho_{34}\sigma_3\sigma_4 r_3^{\gamma_3}(t'-1) r_4^{\gamma_4}(t'-1) \\ \rho_{41}\sigma_4\sigma_1 r_4^{\gamma_4}(t'-1) r_1^{\gamma_1}(t'-1) & \rho_{42}\sigma_4\sigma_2 r_4^{\gamma_4}(t'-1) r_2^{\gamma_2}(t'-1) & \rho_{43}\sigma_4\sigma_3 r_4^{\gamma_4}(t'-1) r_3^{\gamma_3}(t'-1) & \sigma_4^2 r_4^{2\gamma_4}(t'-1) \end{pmatrix} \quad (3.14)$$

The four-factor Vasicek model (hereafter Vasicek4F) for which $\gamma = [0,0,0,0]'$ has the following adjusted time invariant covariance matrix:

$$\Sigma^*_{Vasicek4F}(r) = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \rho_{14}\sigma_1\sigma_4 \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 & \rho_{24}\sigma_2\sigma_4 \\ \rho_{31}\sigma_3\sigma_1 & \rho_{32}\sigma_3\sigma_2 & \sigma_3^2 & \rho_{34}\sigma_3\sigma_4 \\ \rho_{41}\sigma_4\sigma_1 & \rho_{42}\sigma_4\sigma_2 & \rho_{43}\sigma_4\sigma_3 & \sigma_4^2 \end{pmatrix} \quad (3.15)$$

The four-factor CIR model (hereafter CIR4F) for which $\gamma = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]'$ has the following adjusted covariance matrix:

$$\Sigma^*_{CIR4F}(r,t) = \begin{pmatrix} \sigma_1^2 r_1(t'\text{-}1) & \rho_{12}\sigma_1\sigma_2\sqrt{r_1(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \rho_{13}\sigma_1\sigma_3\sqrt{r_1(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \rho_{14}\sigma_1\sigma_4\sqrt{r_1(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} \\ \rho_{21}\sigma_2\sigma_1\sqrt{r_2(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \sigma_2^2 r_2(t'\text{-}1) & \rho_{23}\sigma_2\sigma_3\sqrt{r_2(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \rho_{24}\sigma_2\sigma_4\sqrt{r_2(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} \\ \rho_{31}\sigma_3\sigma_1\sqrt{r_3(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \rho_{32}\sigma_3\sigma_2\sqrt{r_3(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \sigma_3^2 r_3(t'\text{-}1) & \rho_{34}\sigma_3\sigma_4\sqrt{r_3(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} \\ \rho_{41}\sigma_4\sigma_1\sqrt{r_4(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \rho_{42}\sigma_4\sigma_2\sqrt{r_4(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \rho_{43}\sigma_4\sigma_3\sqrt{r_4(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \sigma_4^2 r_4(t'\text{-}1) \end{pmatrix} \tag{3.16}$$

The four-factor Brennan and Schwartz model (hereafter BS4F) for which $\gamma = [1,1,1,1]'$ has the following adjusted covariance matrix:

$$\Sigma^*_{BS4F}(r,t) = \begin{pmatrix} \sigma_1^2 r_1^2(t'-1) & \rho_{12}\sigma_1\sigma_2 r_1(t'-1) r_2(t'-1) & \rho_{13}\sigma_1\sigma_3 r_1(t'-1) r_3(t'-1) & \rho_{14}\sigma_1\sigma_4 r_1(t'-1) r_4(t'-1) \\ \rho_{21}\sigma_2\sigma_1 r_2(t'-1) r_1(t'-1) & \sigma_2^2 r_2^2(t'-1) & \rho_{23}\sigma_2\sigma_3 r_2(t'-1) r_3(t'-1) & \rho_{24}\sigma_2\sigma_4 r_2(t'-1) r_4(t'-1) \\ \rho_{31}\sigma_3\sigma_1 r_3(t'-1) r_1(t'-1) & \rho_{32}\sigma_3\sigma_2 r_3(t'-1) r_2(t'-1) & \sigma_3^2 r_3^2(t'-1) & \rho_{34}\sigma_3\sigma_4 r_3(t'-1) r_4(t'-1) \\ \rho_{41}\sigma_4\sigma_1 r_4(t'-1) r_1(t'-1) & \rho_{42}\sigma_4\sigma_2 r_4(t'-1) r_2(t'-1) & \rho_{43}\sigma_4\sigma_3 r_4(t'-1) r_3(t'-1) & \sigma_4^2 r_4^2(t'-1) \end{pmatrix} \tag{3.17}$$

The continuous-time model 3.14 together with each of the adjusted covariance matrices that follow the Assumption 1 above, constitute the four-factor CKLS4F, Vasicek4F, CIR4F and BS4F approximate continuous-time models that will be estimated using discrete data.

The Five-Factors Continuous-time Term Structure Models

The CKLS, Vasicek, CIR and BS five-factor continuous-time models of the term structure (hereafter CKLS5F, Vasicek5F, CIR5F and BS5F respectively) will have the same mathematical representation given by the generalised system of stochastic differential equations 3.12 for which $n = 5$:

$$\begin{cases} dr_1(t) = [\alpha_1 + \beta_{11}r_1(t) + \beta_{12}r_2(t) + \beta_{13}r_3(t) + \beta_{14}r_4(t) + \beta_{15}r_5(t)]dt + \zeta_1(dt) \\ dr_2(t) = [\alpha_2 + \beta_{21}r_1(t) + \beta_{22}r_2(t) + \beta_{23}r_3(t) + \beta_{24}r_4(t) + \beta_{25}r_5(t)]dt + \zeta_2(dt) \\ dr_3(t) = [\alpha_3 + \beta_{31}r_1(t) + \beta_{32}r_2(t) + \beta_{33}r_3(t) + \beta_{34}r_4(t) + \beta_{35}r_5(t)]dt + \zeta_3(dt) \\ dr_4(t) = [\alpha_4 + \beta_{41}r_1(t) + \beta_{42}r_2(t) + \beta_{43}r_3(t) + \beta_{44}r_4(t) + \beta_{45}r_5(t)]dt + \zeta_4(dt) \\ dr_5(t) = [\alpha_5 + \beta_{51}r_1(t) + \beta_{52}r_2(t) + \beta_{53}r_3(t) + \beta_{54}r_4(t) + \beta_{55}r_5(t)]dt + \zeta_5(dt) \end{cases} \tag{3.18}$$

We differentiate among the classic extended term structure models considered through their specific adjusted matrix extended to $\Sigma^*(r,t) = \{\sigma_{ij}^*\}_{1\le i,j\le 5}$. The new vector of the innovations is $\zeta = \{\zeta_1, \zeta_2, \zeta_3, \zeta_4, \zeta_5\}$. Consequently, the maximum likelihood function $L(\theta)$ will have another different expression for each of the four models as its computation depends on the specific covariance matrix $\Sigma^*(r,t)$ which will be described next.

The five-factor general CKLS model (hereafter CKLS5F) for $\gamma = [\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5]'$ unrestricted has the following adjusted covariance matrix:

$$\Sigma^*_{CKLS5F}(r,t) = \begin{pmatrix} \sigma_1^2 r_1^{2\gamma_1}(t'-1) & \rho_{12}\sigma_1\sigma_2 r_1^{\gamma_1}(t'-1) r_2^{\gamma_2}(t'-1) & \rho_{13}\sigma_1\sigma_3 r_1^{\gamma_1}(t'-1) r_3^{\gamma_3}(t'-1) & \rho_{14}\sigma_1\sigma_4 r_1^{\gamma_1}(t'-1) r_4^{\gamma_4}(t'-1) & \rho_{15}\sigma_1\sigma_5 r_1^{\gamma_1}(t'-1) r_5^{\gamma_5}(t'-1) \\ \rho_{21}\sigma_2\sigma_1 r_2^{\gamma_2}(t'-1) r_1^{\gamma_1}(t'-1) & \sigma_2^2 r_2^{2\gamma_2}(t'-1) & \rho_{23}\sigma_2\sigma_3 r_2^{\gamma_2}(t'-1) r_3^{\gamma_3}(t'-1) & \rho_{24}\sigma_2\sigma_4 r_2^{\gamma_2}(t'-1) r_4^{\gamma_4}(t'-1) & \rho_{25}\sigma_2\sigma_5 r_2^{\gamma_2}(t'-1) r_5^{\gamma_5}(t'-1) \\ \rho_{31}\sigma_3\sigma_1 r_3^{\gamma_3}(t'-1) r_1^{\gamma_1}(t'-1) & \rho_{32}\sigma_3\sigma_2 r_3^{\gamma_3}(t'-1) r_2^{\gamma_2}(t'-1) & \sigma_3^2 r_3^{2\gamma_3}(t'-1) & \rho_{34}\sigma_3\sigma_4 r_3^{\gamma_3}(t'-1) r_4^{\gamma_4}(t'-1) & \rho_{35}\sigma_3\sigma_5 r_3^{\gamma_3}(t'-1) r_5^{\gamma_5}(t'-1) \\ \rho_{41}\sigma_4\sigma_1 r_4^{\gamma_4}(t'-1) r_1^{\gamma_1}(t'-1) & \rho_{42}\sigma_4\sigma_2 r_4^{\gamma_4}(t'-1) r_2^{\gamma_2}(t'-1) & \rho_{43}\sigma_4\sigma_3 r_4^{\gamma_4}(t'-1) r_3^{\gamma_3}(t'-1) & \sigma_4^2 r_4^{2\gamma_4}(t'-1) & \rho_{45}\sigma_4\sigma_5 r_4^{\gamma_4}(t'-1) r_5^{\gamma_5}(t'-1) \\ \rho_{51}\sigma_5\sigma_1 r_5^{\gamma_5}(t'-1) r_1^{\gamma_1}(t'-1) & \rho_{52}\sigma_5\sigma_2 r_5^{\gamma_5}(t'-1) r_2^{\gamma_2}(t'-1) & \rho_{53}\sigma_5\sigma_3 r_5^{\gamma_5}(t'-1) r_3^{\gamma_3}(t'-1) & \rho_{54}\sigma_5\sigma_4 r_5^{\gamma_5}(t'-1) r_4^{\gamma_4}(t'-1) & \sigma_5^2 r_5^{2\gamma_5}(t'-1) \end{pmatrix} \quad (3.19)$$

The five-factor Vasicek model (hereafter Vasicek5F) for which $\gamma = [0,0,0,0,0]'$ has the following adjusted time invariant covariance matrix:

$$\Sigma^*_{Vasicek5F}(r) = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \rho_{14}\sigma_1\sigma_4 & \rho_{15}\sigma_1\sigma_5 \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 & \rho_{24}\sigma_2\sigma_4 & \rho_{25}\sigma_2\sigma_5 \\ \rho_{31}\sigma_3\sigma_1 & \rho_{32}\sigma_3\sigma_2 & \sigma_3^2 & \rho_{34}\sigma_3\sigma_4 & \rho_{35}\sigma_3\sigma_5 \\ \rho_{41}\sigma_4\sigma_1 & \rho_{42}\sigma_4\sigma_2 & \rho_{43}\sigma_4\sigma_3 & \sigma_4^2 & \rho_{45}\sigma_4\sigma_5 \\ \rho_{51}\sigma_5\sigma_1 & \rho_{52}\sigma_5\sigma_2 & \rho_{53}\sigma_5\sigma_3 & \rho_{54}\sigma_5\sigma_4 & \sigma_5^2 \end{pmatrix} \quad (3.20)$$

The five-factor CIR model (hereafter CIR5F) for which $\gamma = \left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right]'$ has the following adjusted covariance matrix:

$$\Sigma^*_{CIR5F}(r,t)=\begin{pmatrix} \sigma_1^2 r_1(t'\text{-}1) & \rho_{12}\sigma_1\sigma_2\sqrt{r_1(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \rho_{13}\sigma_1\sigma_3\sqrt{r_1(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \rho_{14}\sigma_1\sigma_4\sqrt{r_1(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} & \rho_{15}\sigma_1\sigma_5\sqrt{r_1(t'\text{-}1)}\sqrt{r_5(t'\text{-}1)} \\ \rho_{21}\sigma_1\sigma_1\sqrt{r_2(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \sigma_2^2 r_2(t'\text{-}1) & \rho_{23}\sigma_2\sigma_3\sqrt{r_2(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \rho_{24}\sigma_2\sigma_4\sqrt{r_2(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} & \rho_{25}\sigma_2\sigma_5\sqrt{r_2(t'\text{-}1)}\sqrt{r_5(t'\text{-}1)} \\ \rho_{31}\sigma_3\sigma_1\sqrt{r_3(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \rho_{32}\sigma_3\sigma_2\sqrt{r_3(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \sigma_3^2 r_3(t'\text{-}1) & \rho_{34}\sigma_3\sigma_4\sqrt{r_3(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} & \rho_{35}\sigma_3\sigma_5\sqrt{r_3(t'\text{-}1)}\sqrt{r_5(t'\text{-}1)} \\ \rho_{41}\sigma_4\sigma_1\sqrt{r_4(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \rho_{42}\sigma_4\sigma_2\sqrt{r_4(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \rho_{43}\sigma_4\sigma_3\sqrt{r_4(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \sigma_4^2 r_4(t'\text{-}1) & \rho_{45}\sigma_4\sigma_5\sqrt{r_4(t'\text{-}1)}\sqrt{r_5(t'\text{-}1)} \\ \rho_{51}\sigma_5\sigma_1\sqrt{r_5(t'\text{-}1)}\sqrt{r_1(t'\text{-}1)} & \rho_{52}\sigma_5\sigma_2\sqrt{r_5(t'\text{-}1)}\sqrt{r_2(t'\text{-}1)} & \rho_{53}\sigma_5\sigma_3\sqrt{r_5(t'\text{-}1)}\sqrt{r_3(t'\text{-}1)} & \rho_{54}\sigma_5\sigma_4\sqrt{r_5(t'\text{-}1)}\sqrt{r_4(t'\text{-}1)} & \sigma_5^2 r_5(t'\text{-}1) \end{pmatrix} \quad (3.21)$$

The five-factor Brennan and Schwartz model (hereafter BS5F) for which $\gamma=[1,1,1,1,1]$ has the following adjusted covariance matrix:

$$\Sigma^*_{BS5F}(r,t)=\begin{pmatrix} \sigma_1^2 r_1^2(t'-1) & \rho_{12}\sigma_1\sigma_2 r_1(t'-1)r_2(t'-1) & \rho_{13}\sigma_1\sigma_3 r_1(t'-1)r_3(t'-1) & \rho_{14}\sigma_1\sigma_4 r_1(t'-1)r_4(t'-1) & \rho_{15}\sigma_1\sigma_5 r_1(t'-1)r_5(t'-1) \\ \rho_{21}\sigma_2\sigma_1 r_2(t'-1)r_1(t'-1) & \sigma_2^2 r_2^2(t'-1) & \rho_{23}\sigma_2\sigma_3 r_2(t'-1)r_3(t'-1) & \rho_{24}\sigma_2\sigma_4 r_2(t'-1)r_4(t'-1) & \rho_{25}\sigma_2\sigma_5 r_2(t'-1)r_5(t'-1) \\ \rho_{31}\sigma_3\sigma_1 r_3(t'-1)r_1(t'-1) & \rho_{32}\sigma_3\sigma_2 r_3(t'-1)r_2(t'-1) & \sigma_3^2 r_3^2(t'-1) & \rho_{34}\sigma_3\sigma_4 r_3(t'-1)r_4(t'-1) & \rho_{35}\sigma_3\sigma_5 r_3(t'-1)r_5(t'-1) \\ \rho_{41}\sigma_4\sigma_1 r_4(t'-1)r_1(t'-1) & \rho_{42}\sigma_4\sigma_2 r_4(t'-1)r_2(t'-1) & \rho_{43}\sigma_4\sigma_3 r_4(t'-1)r_3(t'-1) & \sigma_4^2 r_4^2(t'-1) & \rho_{45}\sigma_4\sigma_5 r_4(t'-1)r_5(t'-1) \\ \rho_{51}\sigma_5\sigma_1 r_5(t'-1)r_1(t'-1) & \rho_{52}\sigma_5\sigma_2 r_5(t'-1)r_2(t'-1) & \rho_{53}\sigma_5\sigma_3 r_5(t'-1)r_3(t'-1) & \rho_{54}\sigma_5\sigma_4 r_5(t'-1)r_4(t'-1) & \sigma_5^2 r_5^2(t'-1) \end{pmatrix} \quad (3.22)$$

3.3.3 The Discrete -Time Multi-Factor Model

Phillips (1972) and Bergstrom (1984a, Theorem 3) demonstrated that the *basic* continuous-time model has a unique solution that satisfies the following discrete stochastic difference equation:

$$r(t) = e^{\beta} r(t-1) + (e^{\beta} - I)\beta^{-1}\alpha + \varepsilon(t) \qquad t = 1, 2, ..., T \qquad (3.23)$$

where $r(t) = [r_i(t)]'_{1 \le i \le n}$, $\varepsilon(t) = [\varepsilon_i(t)]'_{1 \le i \le n}$, $\alpha = (\alpha_i)'_{1 \le i \le n}$

$$e^{\beta} = I + \sum_{k=1}^{\infty} \frac{1}{k!}\beta^k \quad \text{and} \quad E[\varepsilon(t) \cdot \varepsilon'(t)] = \int_0^1 e^{r\beta} \Sigma^*(r,t) e^{r\beta'} dr = \Omega(r,t)$$

The complete vector of structural parameters is $\theta = (\alpha_i, \beta_{ij}, \sigma_i, \gamma_i, \rho_{ij})_{1 \le i,j \le n}$ comprising a total of $(3n^2 + 5n)/2$ single-value parameters. Following Nowman (2001, 2003, 2006), the elements of θ will be estimated by maximizing the Gaussian likelihood function or minimizing the expression $L(\theta)$ which is equal to minus twice the logarithm of the Gaussian likelihood function:

$$L(\theta) = -2\log(LF(\theta)) = \sum_{t=1}^{T} \log(|\Omega(r,t)|) + \sum_{t=1}^{T} \varepsilon_t' \Omega^{-1}(r,t)\varepsilon_t \qquad (3.24)$$

3.3.4 The Discrete-Time Multi-Factor Interest Rate Models with Feedbacks

The twelve formulations of the EDMs equivalent to the corresponding modified continuous-time models extended to four and five factors will constitute the object of the estimation in this study.

Every time a new factor is added the feedback matrix β will symmetrically expand by the vector of the new feedback coefficients. The elements of the feedback matrix are realistically assumed to be all non-zero, as implied by the theory of close interrelationship between interest rates of different maturities. For each of the twelve distinct cases there will be twelve distinct feedback matrices β, twelve different residual series and implicitly twelve different likelihood functions to be maximised. The general discrete time equation/system (3.24) will be individualised for each case by using the appropriate defining matrices as follows:

EDMs for the four-factor extensions

$CKLS4F:$ $$r(t) = e^{\beta_{CKLS4F}} r(t-1) + (e^{\beta_{CKLS4F}} - I)\beta_{CKLS4F}^{-1}\alpha_{CKLS4F} + \varepsilon_{CKLS4F}(t) \quad (3.25)$$

$Vasicek4F:$ $$r(t) = e^{\beta_{Vasicek4F}} r(t-1) + (e^{\beta_{Vasicek4F}} - I)\beta_{Vasicek4F}^{-1}\alpha_{Vasicek4F} + \varepsilon_{Vasicek4F}(t) \quad (3.26)$$

$CIR4F:$ $$r(t) = e^{\beta_{CIR4F}} r(t-1) + (e^{\beta_{CIR4F}} - I)\beta_{CIR4F}^{-1}\alpha_{CIR4F} + \varepsilon_{CIR4F}(t) \quad (3.27)$$

$BS4F:$ $$r(t) = e^{\beta_{BS4F}} r(t-1) + (e^{\beta_{BS4F}} - I)\beta_{BS4F}^{-1}\alpha_{BS4F} + \varepsilon_{BS4F}(t) \quad (3.28)$$

where $\alpha = [\alpha_1, \alpha_2, \alpha_3, \alpha_4]'$ is the *drift* vector and $\beta \in R_{4\times4}$ is the general *feedback* matrix for the four-factor models above with $\beta = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} \end{pmatrix}$.

EDMs for the five-factor extensions

$CKLS5F:$ $$r(t) = e^{\beta_{CKLS5F}} r(t-1) + (e^{\beta_{CKLS5F}} - I)\beta_{CKLS5F}^{-1}\alpha_{CKLS5F} + \varepsilon_{CKLS5F}(t) \quad (3.29)$$

$Vasicek5F:$ $$r(t) = e^{\beta_{Vasicek5F}} r(t-1) + (e^{\beta_{Vasicek5F}} - I)\beta_{Vasicek5F}^{-1}\alpha_{Vasicek5F} + \varepsilon_{Vasicek5F}(t) \quad (3.30)$$

$CIR5F:$ $$r(t) = e^{\beta_{CIR5F}} r(t-1) + (e^{\beta_{CIR5F}} - I)\beta_{CIR5F}^{-1}\alpha_{CIR5F} + \varepsilon_{CIR5F}(t) \quad (3.31)$$

$BS5F:$ $$r(t) = e^{\beta_{BS5F}} r(t-1) + (e^{\beta_{BS5F}} - I)\beta_{BS5F}^{-1}\alpha_{BS5F} + \varepsilon_{BS5F}(t) \quad (3.32)$$

where $\alpha = [\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5]'$ is the *drift* vector and $\beta \in R_{5\times5}$ is the general *feedback* matrix for the five-factor models above with $\beta = \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} & \beta_{15} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} & \beta_{25} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} & \beta_{35} \\ \beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} & \beta_{45} \\ \beta_{51} & \beta_{52} & \beta_{53} & \beta_{54} & \beta_{55} \end{pmatrix}$.

3.4 Data

The development of theoretical TSIR models involves financial instruments with homogeneous characteristics such as term to maturity and level of credit risk. Therefore, it is important to consider empirical variables that match the conceptual framework of the models proposed. In line with this argument, this study independently employs daily data from the London interbank (LIB) market and the UK government bond market over the period January 2000 – March 2013 inclusively. From the multitude of markets functioning inside any modern financial system the interbank and bond markets play crucial roles. Interbank markets provide a platform for central banks for monitoring their interest rates policies and their liquidity is of paramount importance to financial intermediation efficiency (Furfine, 2002). Bond markets are indispensable to any economy, being a very important mechanism used by governments around the world to meet capital needs and to finance their public debt.

3.4.1 The Interbank Market

The short-term segment of the yield curve could be estimated using various types of data extracted from shot-term instruments traded on the London money market such as general collateral (GC) repo agreements, conventional gilt yields, interbank loans, short futures contracts, forward-rate agreements and swap contracts settled on the sterling overnight interest rate average (SONIA). While the GC repo rates and treasury bills (T-bills) could provide virtual risk-free short-term interest rates, both types of instruments are likely to be affected by factors like small outstanding stock and gilt collateral unavailability (Anderson and Sleath, 1999).

For this study, the interbank segment of the money market will be considered. The interbank market facilitates the transfer of created funds from one bank to another, in order to meet liquidity and reserve requirements. Banks with excess liquidity will offer unsecured short-term loans to banks in need of funds, charging for this service a certain interest rate. Numerous interbank rates are published daily; the most renowned ones include the LIBOR- London Inter Bank Offer Rates (UK), EURIBOR (Eurozone) and FIBOR (Germany).

LIBOR rates are employed in the current empirical analysis. During the sampling period the LIBOR rates were still under the supervision of the British Bankers' Association (BBA) with assistance from the Foreign Exchange and Money Markets Committee (FX & MMC). LIBOR rates were determined using a robust methodology: BBA would select and pool together the panel banks - the most representative financial institutions that actively trade in each currency interbank markets. The offer/lending rates submitted by these banks were used to produce the official LIBOR rates, also called BBA interest settlement rates. LIBOR rates were calculated as an average after the first and last quartiles have been eliminated. Starting with only three currencies (USD, GBP and JPY), the number of LIBOR currencies grew to sixteen prior to 2000, and then dropped to ten, following the creation of the Euro currency. For nearly three decades[11] the BBA was responsible for the complex process of daily calculation of 150 LIBOR rates published by Thomson Reuters on behalf of the BBA. However, after the 2012 LIBOR fixing scandal,

[11] The first LIBOR interest rates were published in 1986.

the BBA was suspended from its governing role over LIBOR. Starting from June 2013 the collection, the calculation and the distribution of the LIBOR rates have been subject to major regulatory reforms. While LIBOR data distribution was unaffected by these statutory amendments, significant changes had to be implemented, including the ceasing of the publication of certain LIBOR currencies and maturities. The Australian and Canadian dollar were the last currencies to be removed from the LIBOR framework with effect from June 2013, with only five currencies being retained: GBP, USD, EUR, JPY and CHF. Also, the nine-month tenor has been excluded from all remaining currencies due to reduced volume of regular transactions. This implementation has considerably affected the market participants[12] whose operations made use of this maturity LIBOR rate, and who have to find different appropriate alternatives such as various interpolation methods or other industry benchmark rates.

Despite these events, LIBOR interest rates are still generally accepted as the lowest interbank lending rates on the London money market. Moreover, they are considered the most important benchmark in the global financial markets for short-term interest rates. Banks use LIBOR as a base rate in calculating their interest rates for loans, mortgages and deposits, whereas financial markets use LIBOR as a base rate in pricing derivatives such as futures, swaps and options.

Given the importance of LIBOR as a benchmark for pricing many financial products, any dysfunctionality in the unsecured interbank lending market will have wide-reaching repercussions on the financial system and on the real economy. During the global financial turmoil of 2007-2009 this aspect was highly relevant, with a starting point in August 2007 when the interbank lending market had to be saved by liquidity injections from both the European Central Bank and the Federal Reserve (see Brunnermeier, 2009). While the crisis has been driven by the problems in the asset-backed securities market in the U.S., other markets such as the Repo and LIBOR markets became also unstable. The interbank market became impaired as banks were reluctant to lend to each other as a cautious measure and as a result of unknown counterparty exposure to asset-backed securities. Consequently, this created a complicated liquidity deadlock, in the sense that the availability of short-term funding was substantially reduced with an immediate decreasing effect on the level of interbank interest rates. This culminated with a credit crunch and a frozen liquidity flow in the interbank markets in the autumn of 2008 when

[12] The nine-month tenor LIBOR rates were initially included in the data for this study; however, because it has been discontinued, the two-month maturity rates have been considered instead.

unsecured interbank lending at 3-month was almost replaced by secured overnight borrowing (Acharya et al. 2009).

Since trading international currencies has become a standard activity in the banking industry, extensive comparative empirical studies for different markets[13] have always provided researchers in the field with valuable insightful information. For this study, the five currencies were carefully chosen based on the importance of the economies worldwide and on the particularities of their financial systems and financial regulatory bodies. In this respect, Japan, as the third power in the global economy, has known considerable uncertainty despite artificial maintenance of extremely low interest rates, while Canada, with very close economic connections to the U.S., but with a conservative banking system closer to the U.K. system, seems to have been least affected by the last global financial crisis. The other three markets the U.K., the US and the Eurozone are the major players in ensuring worldwide financial stability, hence developing interest rate models with improved forecasting power for these markets is of great importance.

The maturity spectrum of the LIBOR rates has been reduced to only seven maturities in the aftermath of the LIBOR scandal in 2012. For estimating the LIBOR curves for the above markets five maturities are utilised, namely one-week, one-, three-, six- and twelve-months. The one, three and six-month LIBOR rates are the most used LIBOR rates, being used to index over $360 trillion of notional financial contracts, from interest rate swaps and other derivatives to floating-rate residential and commercial mortgages. One may argue that the 6-month LIBOR is the most important of the LIBOR rates, being the choice by default reference index rate in most interest rate swap contracts that operate with six-month tenors. Hence, this rate has a direct connection with the swap markets and payments linked to this rate are mainly driven by investors in longer term swap markets. The other important determinant of LIBOR rates is the mortgage and securitization markets. The investors in these latter markets require a quarterly payment structure and in order to avoid interest rate risk exposure, naturally they would like to receive their coupons linked to 3-month LIBOR. Furthermore, the mortgage markets are organised around monthly payments by mortgage borrowers. Hence, mortgage provides are interested in 1-month LIBOR in order to hedge their interest rate risk exposure.

At the very end of the money market rate spectrum, the 12-month LIBOR is a deposit rate but it is also a rate that can be recovered from FRA/futures contracts and it may even be a

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[13] Previous extensive empirical studies include: Tse (1995) who considered money market rates for eleven countries; Dahlquist (1996) who analysed rates for the UK, Germany, Denmark and Sweden; Episcopos (2000) who investigated the dynamics of interbank rates for ten countries.

reference rate in occasional short-term swaps. Last but not least, the one-week LIBOR has more features associated with repo and overnight swap rates.

The LIBOR rates mentioned above correspond to different types of derivatives markets and will respond differently to a specific type of information/shock in the financial markets, albeit there is an obvious common ground. Acharya and Skeie (2011) emphasized that stress and freezes in term inter-bank lending markets may be the result of rollover risk of highly leveraged lenders and illiquidity of assets underlying term loans. They showed that the term inter-bank lending rates and volumes are jointly determined, lenders and aversion of borrowers to trade at high rates of interest playing a very important role. While the levels of the interest rates are highly correlated, the stationarity analysis shows that it is the first difference series that is stationary. Therefore, the relevant correlations are those of first differences. The sample correlations for the first difference series revealed in Table 3.13, that adjacent LIBORs are more correlated but correlations for more distant LIBORs are weaker, therefore it is the changes in the LIBOR rates of different maturities that bring new information into the dynamic continuous interest models.

The consequences of LIBOR manipulation could be very serious, as the LIBOR rates submissions don't portray the true market forces and therefore may result in misallocation of resources and price distortions in the economy (see Abrantes-Metz et al. (2012). Lower LIBOR rates imply lower mortgage and hence the 1-month LIBOR rates could have been most affected in comparison with other maturity LIBOR rates. While there are no comparative studies to assess which of the seven maturity LIBOR rates have been mostly affected, Monticini and Thornton (2013) found that the average of one- and three-month LIBOR – CD spreads declined by nearly 5.5 basis points by mid-2007. In addition, McConnell (2013) provides evidence from the regulators' investigations that followed the LIBOR scandal in 2012, that the one- and three-month LIBOR have been subject to systematic manipulation within and across participating banks. Further, he describes the LIBOR fixing as an example of systemic operational risk, more specifically - people risk, and suggests to banks and regulators some recommendations about how to address the management of systemic people risk.

3.4.2 The U.K. Bond Market

From the UK bond market, the data considered are the nominal UK Government zero coupon (spot) rates of various maturities longer than one year. The instruments used in the construction of the yield curve should have the same risk of default, the same transaction costs, the same coupon rate and the same tax treatment. Hence, government zero coupon bonds (conventional gilts) are the most commonly used type of financial instruments in empirical studies of the term structure of interest rates.

The nominal zero coupon yield (spot) curve is the graphical illustration of the relationship between the maturity of a zero-coupon bond and its yield. The yield curve plays a fundamental role as a discount curve applied to future cash flows in pricing a large number of financial products. The nominal government zero coupon (spot) interest rate for n-years represents the interest rate charged today on a risk free nominal loan with a residual maturity of n-years. It is defined as the yield to maturity of a nominal zero coupon bond (conventional gilt) and it is also the discount rate applicable to future nominal cash flows in order to calculate their present value. The spot rates provided by the Bank of England (BoE) have been estimated using the VRP (Variable Roughness Penalty) model, a spine-based technique specifically designed to obtain a smooth curve for monetary policy analysis (Anderson and Sleath, 2001). The market data used by the BoE in the derivation of the nominal zero coupon yields are the GC (General Collateral) repo rates for maturities under three months and UK conventional gilts for longer maturities.

3.4.3 The Data Sets - A Preliminary Analysis

The complete data for the interbank market will comprise five main LIBOR rates for various currencies across a range of five maturities. They are daily one-week, one-, three-, six- and twelve-month LIBOR rates for the following currencies: The Pound Sterling (GBP), the United States Dollar (USD), the Euro (EUR), the Japanese Yen (JPY) and the Canadian Dollar (CAD). The period covered starts from 3rd of January 2000 to 29th of March 2013 leading to a total of 3,455 daily observations for each currency.

For the bond market data, the study will use the nominal spot rates produced by the BoE. They are the nominal spot rates of tenor one-, seven-, ten-, fifteen- and twenty-years respectively. Starting from 4th of January 2000 to 28th of March 2013 the sample is made of a total of 3,346 daily observations[14]. The empirical literature has found evidence that different frequencies yield different estimation results. This study employs daily observations in order to obtain consistent ML estimates for parameters of diffusion

[14] The difference in the number of observations is the result of how the two data sources Datastream and the BoE have treated the entries of interest rates corresponding to bank holidays. Datastream has equalled the interest rates on bank holidays to the level of the previous day, while the BoE has kept them as unavailable. The difference of 109 observations out of the total of 3,455 is considered insignificant as far as the empirical analysis is concerned.

models as Bergstrom (1984) demonstrated that ML estimators converge to the true values as the sampling interval converges to zero. Also, Wang, Phillips and Yu (2011) have shown that for multivariate diffusion models the consistency of the estimators is achieved by using daily or higher frequency data.

For each segment of the yield curve the choice of the initial factors had been subject to different considerations. Having a limited choice of maturities, compared to the U.K. nominal curve where the maturity spectrum is much larger, for the LIBOR curve the first four factors have been chosen in terms of importance and data accessibility. The two other LIBOR maturities left out of the analysis are the overnight and two-months LIBOR rates. The overnight market can be seen as a particular component within the money-market, with a different scope of maintaining the daily liquidity levels of the banks. Hence, the first maturity directly linked to the lending market is one-week. After that the most important maturities are the one-, six- and twelve- month rates. The fifth factor (three-month rate) was chosen from the two remaining maturities: the two- and three-month rates, because its greater importance. Regarding the U.K. nominal curve, the maturity range can cover from one- to 25- years with data available for each year. Hence, given this availability the initial four factors have been chosen to be evenly spread across the entire spectrum in order to represent a balanced partition of the maturities which was not possible in the LIBOR curve case.

The sample period proves to be quite rich in terms of unanticipated and policy-induced shocks and their complex consequences. For example, the introduction of the common currency - the Euro in January 1999 had a long-term impact on all types of financial markets, with a fundamental change in the structure of European money markets[15], bond markets, equity and foreign exchange markets. Also, the dot.com bubble event of 2000-2001 could be considered a factor with some impact on the financial markets. However, the main shock that the data interval includes is the financial crisis of 2007-2009; one aim of this study is to test if the richness of the proposed term-structure models can satisfactorily explain the dynamics observed during the crisis.

The comparison will be empirically conducted at different levels: across the five different markets and across the multi-factor specifications of four classic models of the term structure of interest rates: Vasicek (1977), CIR (1985), BS (1980) and CKLS (1992).

[15] According to Galati and Tsatsaronis (2003), after 1999 the share of intra-euro area trading in the world's total cross-border interbank market rose from 35% to 50%. Despite a smooth integration, under the new central bank - the Eurosystem, a two-tiered segmentation of the interbank markets occurred with direct cross-borders activity for large banks and restriction of trading at national level for smaller banks.

Prior to the presentation of the summary statistics, a useful preliminary investigation of the data is considered by plotting the time series and their differences. The statistical analysis and all the graphs are reported using the econometric software Eviews. The time characteristics of the data are illustrated below in the multi-panel Figures 3.1 to 3.5 for the five interbank LIBOR data sets and in Figure 3.6 for the U.K. bond market data.

For the data on interbank interest rates, the visual representations illustrate three main features. Firstly, there are two distinctive periods of higher volatility of daily changes as suggested by the volatility clustering pattern in the first difference series. The first period, 2000-2002 corresponds to the impact of a multitude of factors like the introduction of the Euro, the creation and burst of the dot.com bubble, and the 9/11 terrorist attack. The second period, 2007-2009, refers to the recent global financial crisis triggered by the housing bubble and the collapse of the sub-prime mortgage market in the U.S. The sharp decrease in the level of interbank rates appears in October 2008 after the collapse of Lehman Brothers. Secondly, after 2010, markets seem to react to the financial crisis in a convergent way due to implementing similar monetary policies (quantitative easing and currency depreciation) to control their target rate. Thirdly, from an econometric point of view the graphs for the level time-series display the nonstationarity property, while after the first difference transformation they become stationary around the zero-axis. The plots of the LIBOR-GBP rates display similar patterns for all the maturities, indicating a high degree of correlation between interest rates across the yield curve, most pronounced for the shorter maturities. This feature comes to support the choice of a feedback model in which the feedback matrix between interest rates of various maturities constitutes a matrix structural parameter to be estimated. The two periods of higher volatility, also indicated by the volatility clustering pattern in the first difference series are 2000-2003 and 2007-2009 respectively. The first period may correspond to the impact of a multitude of factors such as the introduction of the Euro currency in January 1999, the creation and burst of the dot.com bubble, and the 9/11/2001 terrorist attack in the U.S. Over this period LIBOR-GBP rates decreased by approximately 3% from just above 6% to 3%. A more dramatic decrease can be observed during the last global financial crisis of 2007-2009 when the LIBOR – GBP interest rates had sharply fallen from just above 6% at the end of 2007 to their lowest value of approximately 0.5% in September 2009. The two largest daily negative changes are recorded in December 2007 and November 2008 respectively. However, after 2009 the UK interbank rates seem rather stable, their level oscillating between 0.48% and 1% for shorter term interbank rates, while for longer maturities they are slightly more spread between 0.6% and 1.90% with a single peak in January 2012.

From an econometric point of view, the plots indicate nonstationarity across all different maturities. However, the first difference series appear stationary around a zero mean. Other formal assessments will follow in the form of various hypothesis (normality, auto-correlation, unit root) testing.

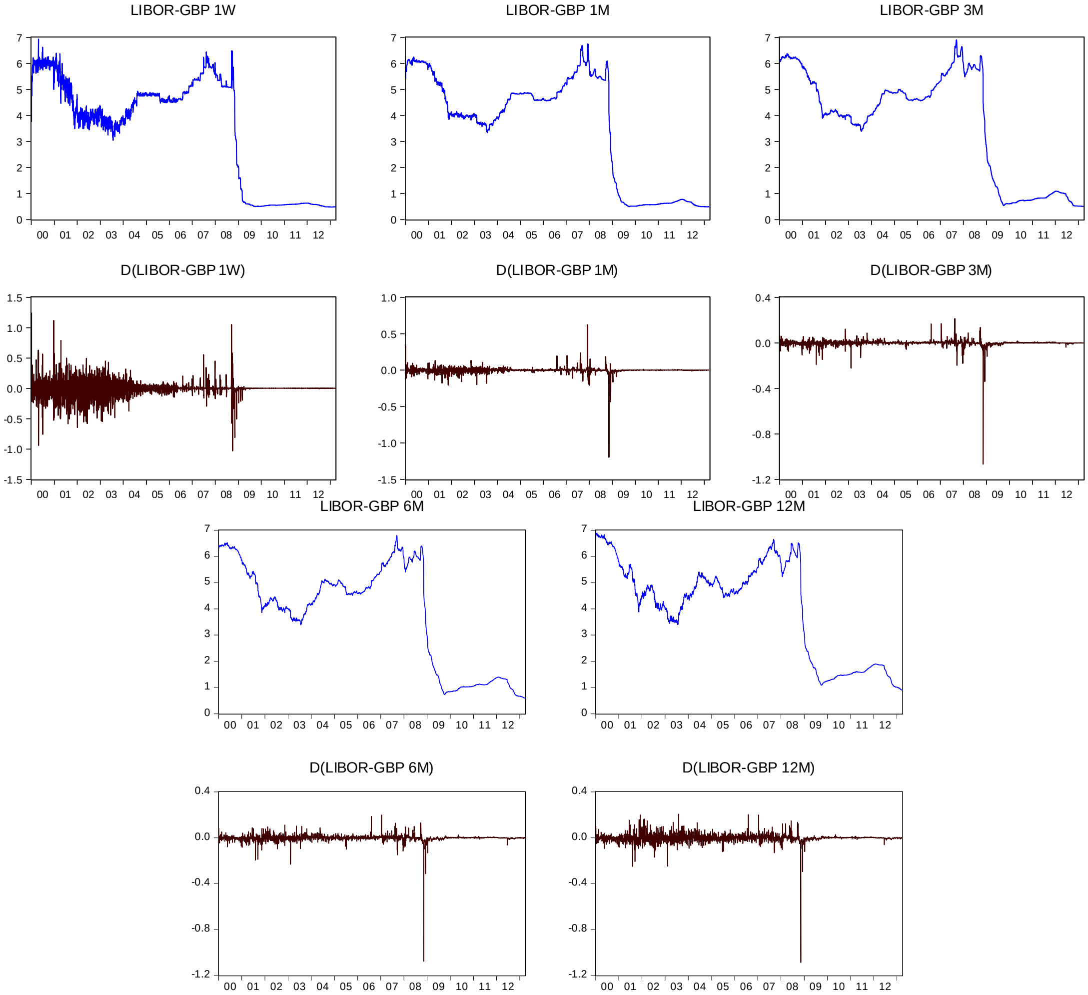


Figure 3.1a)

LIBOR-GBP 2000-2013: Level and First Differences

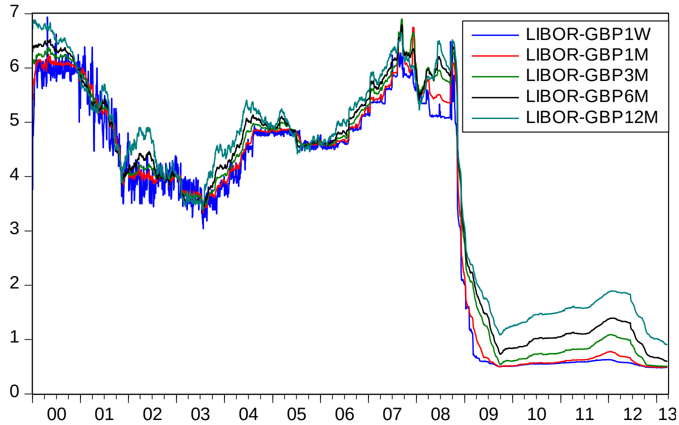
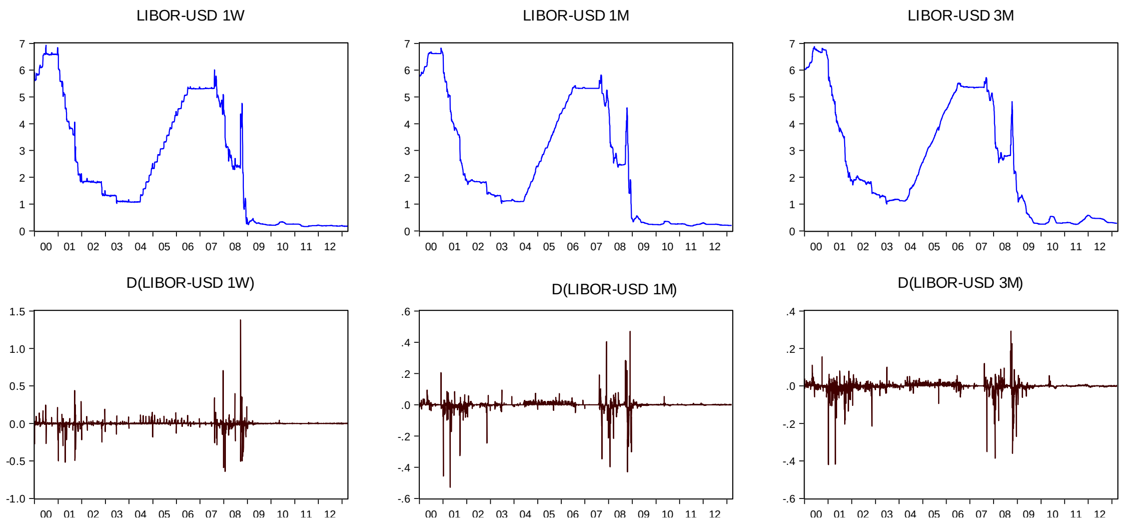


Figure 3.1b)

Multiple graphs for LIBOR-GBP 2000 – 2013: Levels

On inspection of the graphs (Figure 3.2) of the LIBOR-USD rates we can easily identify specific characteristics which are more or less similar to the LIBOR-GBP interbank market. The same two periods of higher volatility are present in the LIBOR-USD interbank market as well, especially because most of the factors/causes originated in the U.S. Compared with other markets where these factors still have a considerable impact, however with some delay and less power, the LIBOR-USD offers the true magnitude of their immediate effect. As can be seen in the panels of Figure 3.2, the impact of the 9/11 terrorist attack is clearly visible, causing a deeper plunge than in the U.K. of the interest rates during the 2000-2003 deflation period of the technology bubble. As for the second major downfall which is related to the 2007-2009 financial crisis, the first signs of distress appear in the summer of 2007[16]. Another substantial decrease observed in Figure 3.2 is realised in November 2008 following the liquidation of the fourth largest American investment bank – Lehman Brothers in September 2008. After 2009 a more stable financial environment is portrayed, however less stable than in the LIBOR-GBP interbank market. Regarding stationarity, one could assume some degree of mean reversion towards the sample mean, but given the daily frequency of the data, deviations from the mean are highly persistent. Therefore, intuitively it can be claimed that the LIBOR–USD rates are nonstationary. In addition, as Figure 3.2 suggests the LIBOR- USD time series are first differenced to stationarity.



[16] A series of major adverse events commenced in June 2007 with the collapse of two hedge funds under Bear Stearns's management as a result of high investment in the subprime ABS (asset-backed securities); followed by the run on the assets of three SIVs (structured investment vehicles) of BNP Paribas in August 2007 (Acharya et al. 2009).

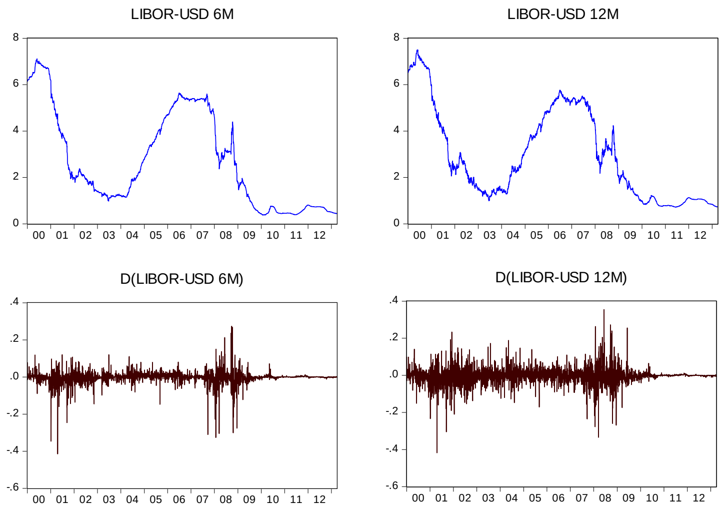


Figure 3.2a)

LIBOR-USD 2000-2013: Level and First Differences

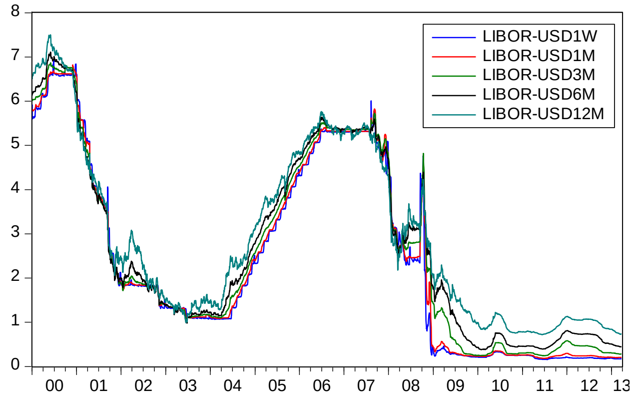


Figure 3.2b)

Multiple graphs for LIBOR-USD 2000 – 2013: Levels

The Eurozone LIBOR-EUR rates (see Figure 3.3) have some particularities as well as certain similarities with other interbank rates. Similarly, the graphs are following each other very closely, indicating high correlations across the maturities. However, the period 2000-2003 presents an interesting feature. The strong upwards trend visible at the beginning of the sample period can reflect the substantial and long-term impact of the introduction of the Euro in January 1999 on LIBOR-EUR rates compared to the other currency LIBOR rates. The benefits of this historical monetary decision may have allowed for a moderate downturn over the 2001-2003 period, only 3% and counteracted a sharper decline as in the LIBOR-USD. In contrast with the other markets studied, the LIBOR-EUR rates seem more volatile after 2009. This can be explained by the Euro crisis, a combination of sovereign debt and banking crises that have developed in 8 countries of the E.U. More generally, the series seem more volatile across the entire sample period, with less evident clustering pattern in the first differenced time series.

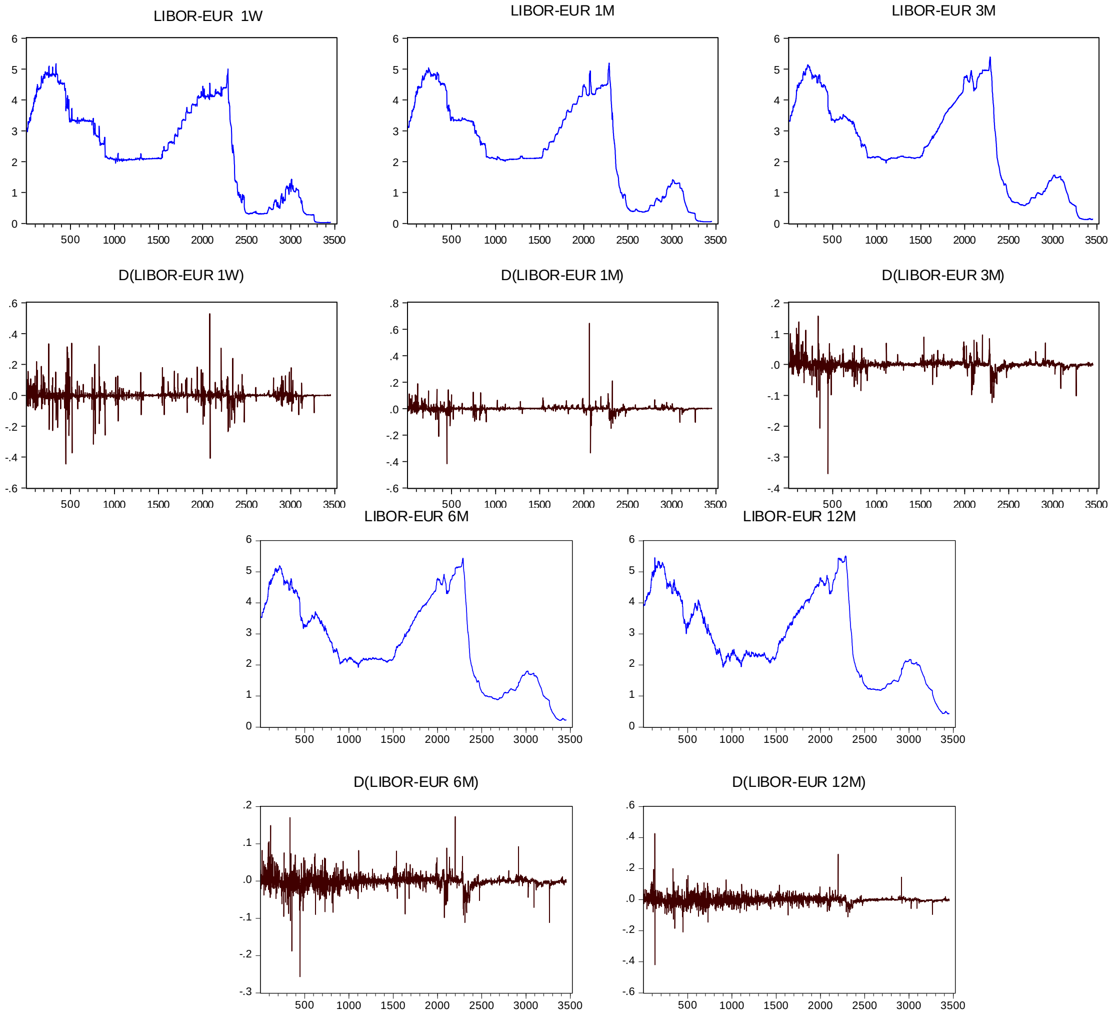


Figure 3.3a)

LIBOR-EUR 2000 -2013: Level and First Differences

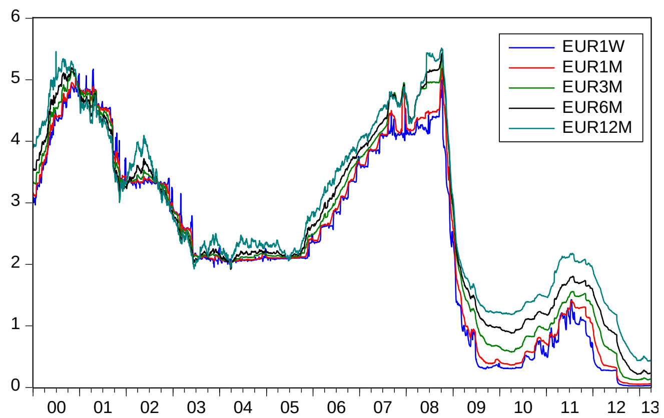


Figure 3.3b)

Multiple graphs for LIBOR-EUR 2000-2013: Levels

The graphs of the Japanese interbank rates (Figure 3.4) suggest some particular features. The first period of higher volatility is much shorter with signs of post-dot.com bubble recovery as early as 2001. This is followed by a prolonged period of almost five years of stable, close to zero interest rates. Regarding the global financial crisis of 2007-2009 the property bubble had double amplitude compared to the dot.com bubble, but of considerably lower dimension when compared with other markets. Another aspect is that following 2009 interest rates of different maturities appear to be moving away from each other indicating less correlation, with interest rates of longer maturity still decreasing gradually, not as abruptly as short-term interest rates of one week and one month maturities.

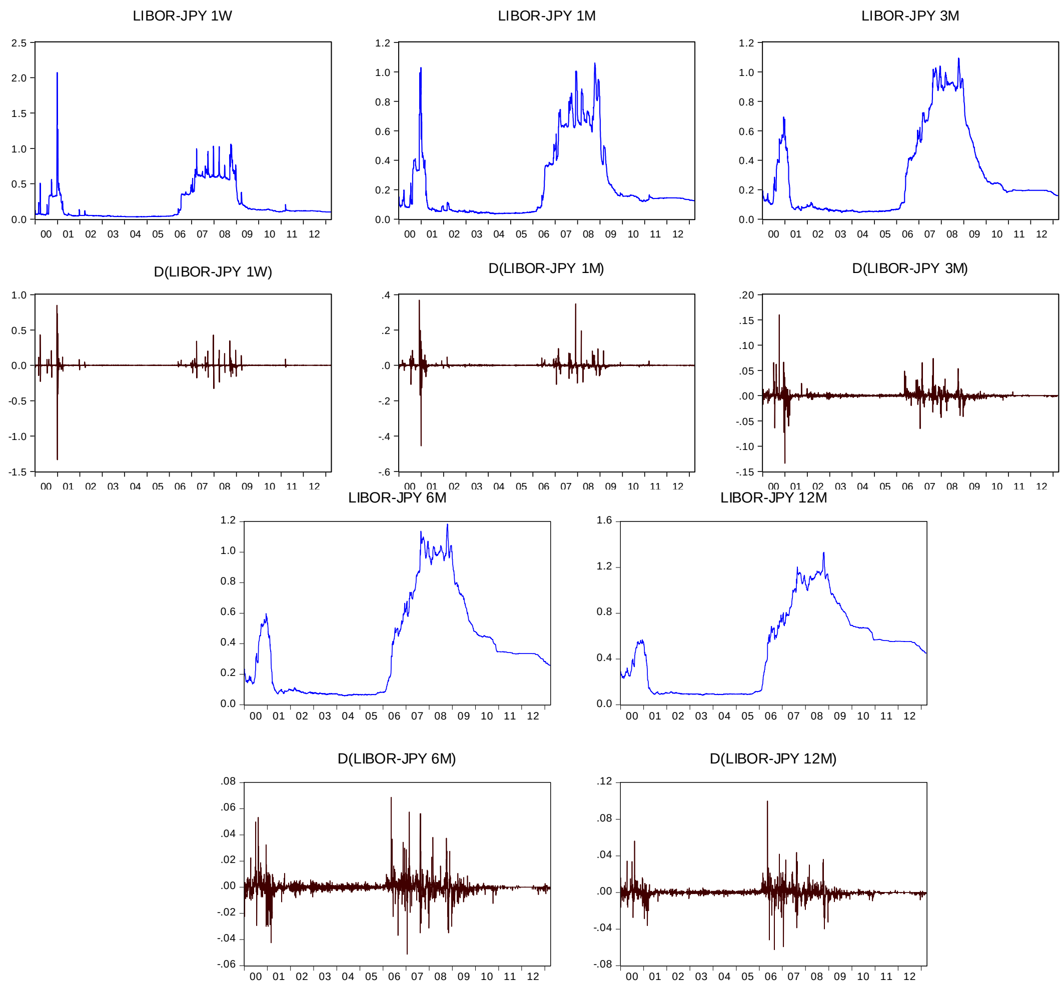


Figure 3.4a)

LIBOR-JPY 2000-2013: Level and First Differences.

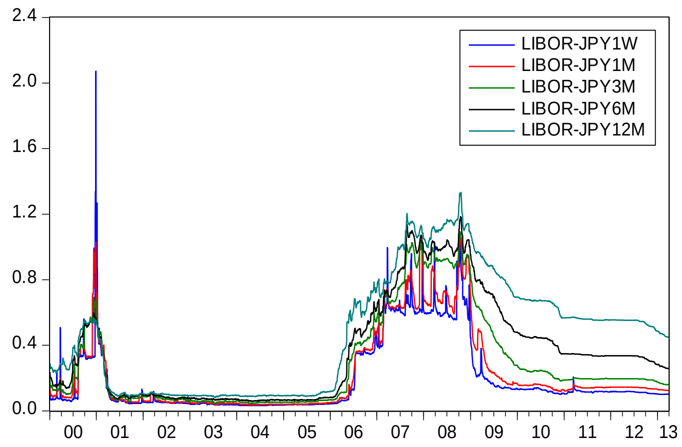


Figure 3.4b)

Multiple graphs for LIBOR-JPY 2000-2013: Levels

The LIBOR-CAD interest rates (see Figure 3.5) appear to follow relatively close to the LIBOR-GBP rates, with similar magnitudes to the downfalls during 2001 and 2009. A plausible explanation can be the similarity between the structure of the financial systems in Canada and the UK. There are also similarities with the U.S. Given its strong economic connections and its geographical border with the United States, Canadian interbank rates seem to suffer almost contemporaneously to those adverse events of the 2001 terrorist attack, the August 2007 BNP Paribas announcement and the September 2008, collapse of Lehman Brothers. However, with a conservative approach and closer supervision the Canadian financial system proved to be more resilient to external unanticipated shocks. Figure 3.5 shows that over the periods 2001-2005 and 2009-2012, the Canadian interbank rates had the most positive evolution compared to their global peers, with interbank rates recovering extremely quickly. The superior health of its financial system made Canada most attractive to international investors, hence its stable, but still vulnerable financial climate after 2009.

In the case of bond market data, as shown in the multi-panels in Figure 3.6, the one-year UK spot rates mimic the one-year LIBOR –GBP rates. For medium-term maturities, the five-, seven- and ten-year spot rates, the series exhibit a clearer decreasing trend than the longer maturity spot rates. As the gravity of the 2007-2009 financial crisis was unfolding and economies around the world failed to convincingly grow, governments were forced to intervene. In order to protect its credibility[17], the UK government had to consider bold fiscal measures such as increase of taxes and spending cuts. With bonds among the safest in the world, hence an increased demand for gilts, at the end of 2012 the

[17] Despite that, in February 2013 Moody and in April 2013 Fitch, downgraded UK credit to Aa1 and AA+ respectively.

UK gilts yields were at their lowest. Another factor contributing to the current low level of gilts yields is the UK's monetary sovereignty. The monetary policy instruments used after the crisis by the UK government, such as quantitative easing, keeping low interest rates level and targeting low inflation can have a significant impact on the term structure of interest rates[18]. All series look nonstationary in levels, while their first differences become stationary. The first difference series show a considerable level of volatility along the entire sample period.

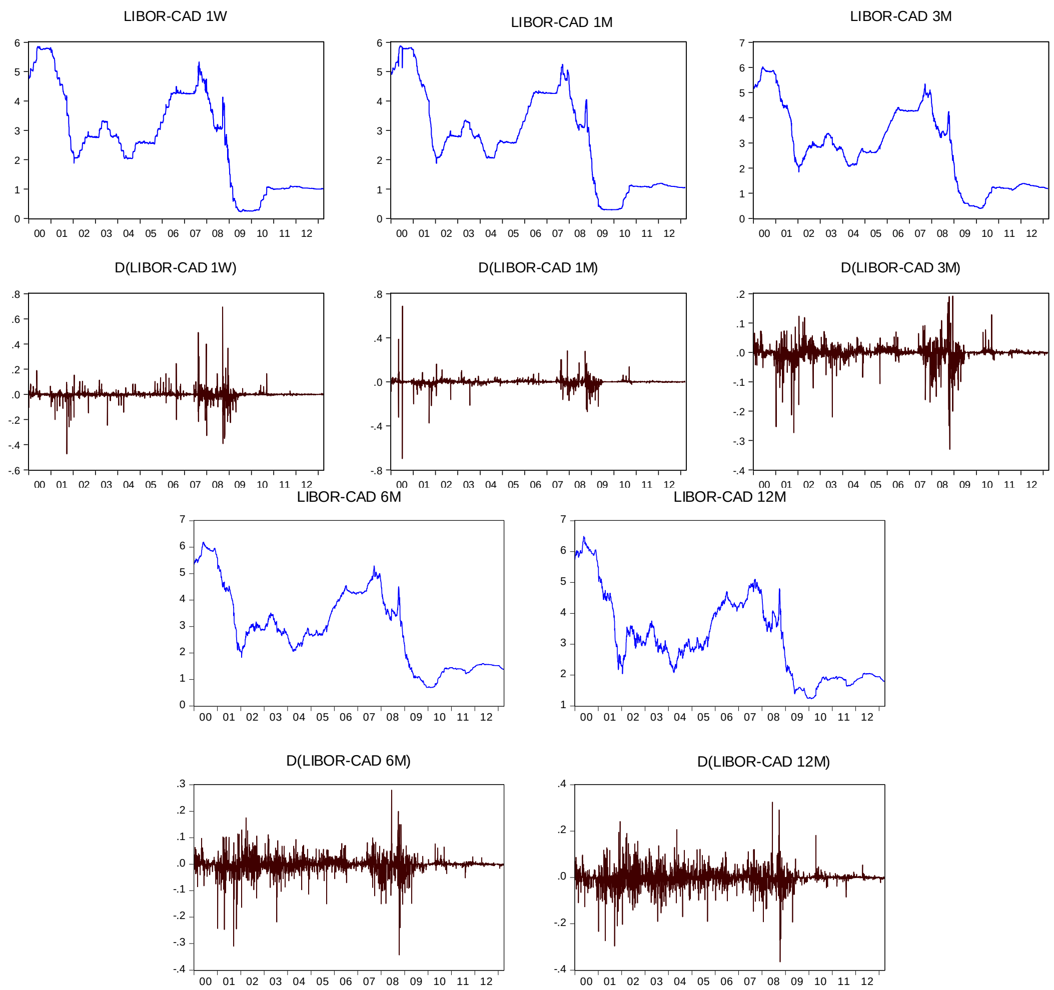


Figure 3.5a)

LIBOR-CAD 2000 -2013: Level and First Differences

[18] Numerous recent studies examine the interaction between macroeconomic variables (inflation, real activity and monetary rules) and bond yields, with some evidence of bidirectional feedback (see Smith and Taylor (2009), Kim and Park (2013)).

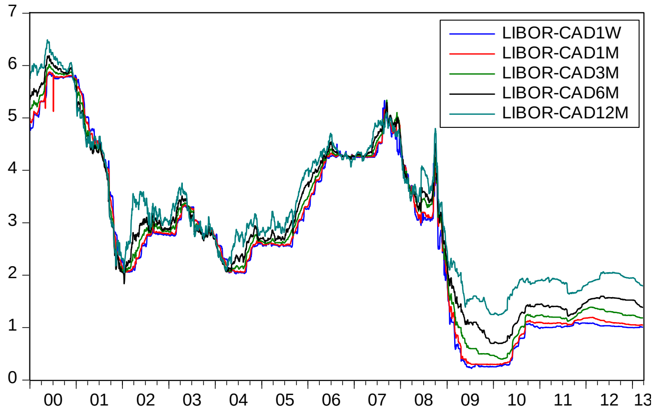


Figure 3.5b)

Multiple graphs for LIBOR-CAD 2000 – 2013: Levels

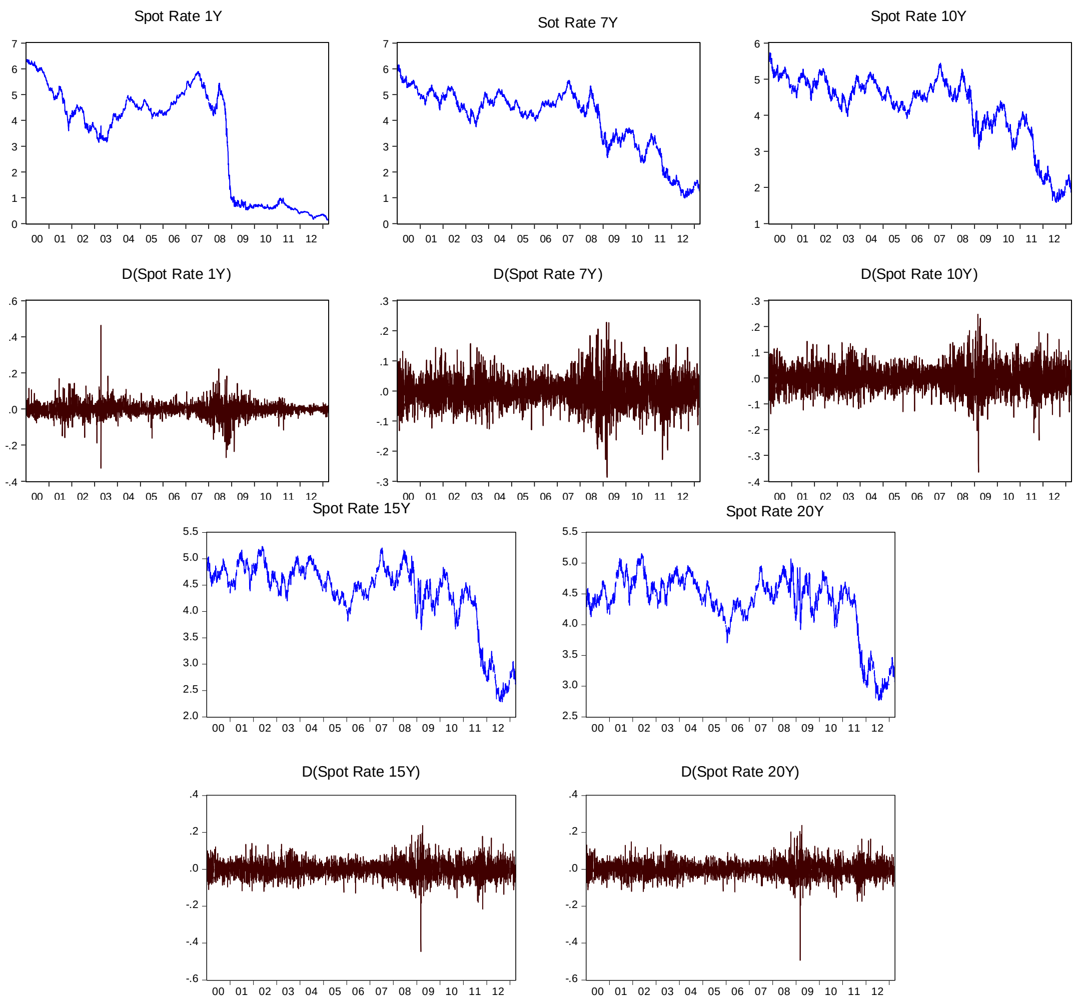


Figure 3.6a)

UK Government Zero Coupon Rates 2000-2013: Level and First Differences

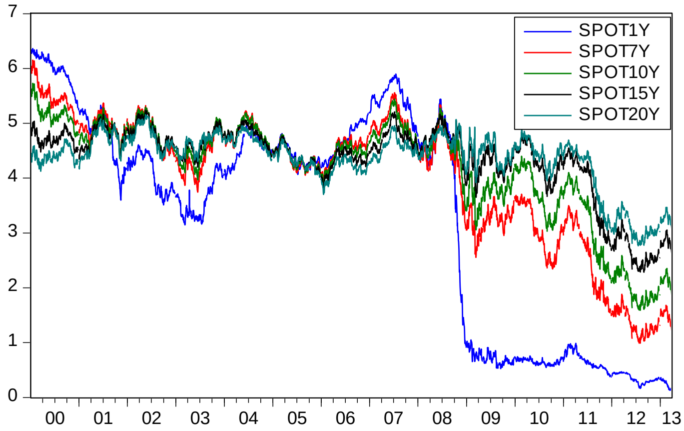


Figure 3.6b)

Multiple graphs for UK Government Zero Coupon Rates 2000-2013: Levels

The section of the summary statistics on the seven data sets is organised in three parts. In the first part the sample estimates for various standard statistical measures are reported for level and first difference series, followed in the second part by the results of testing for autocorrelation. In the final part, the stationarity of the data is formally assessed by conducting various unit root tests.

3.4.4 Summary Statistics

The statistical analysis commences with the computation of central tendency measures such as the mean, median and mode, followed by variability measures like the standard deviation, minimum and maximum values. Further characteristics of the data such as skewness, kurtosis and the results for the Jarque-Bera (1980) normality test are also presented. The sample estimates for all these statistical measures are reported in Table 3.1 to Table 3.5 for the interbank markets and and in Table 3.6 the UK bond market data, respectively.

In addition, the correlogram results are presented individually for each interbank market rates in tables 3.7 to 3.11 and for the UK bond market in Table 3.12. The Sample Autocorrelation Function (ACF) with six lags is computed for both level and first difference of the time series, followed by the results of the Liung-Box (1979) Q-statistic test for autocorrelation. Two lag orders, ten and twenty lags respectively, have been used to compute the sample test statistic, in order to avoid the misinterpretation associated with this test in practice. If the lag order is too small the test may not detect serial correlation at higher-order lags, whereas if it is too large the power of the test may be diminished due to potential dilution of the significant correlation coefficient by insignificant correlation at other lags. For all investigated time-series the sample autocorrelation coefficients decay very slowly in a linear manner, a characteristic of nonstationary time series. For the differenced series, the autocorrelation coefficients seem to cut off straight from the first lag without any discernable pattern (although they are mostly positive), suggesting that the series do not need a higher order differencing. Regarding the Liung-Box tests, all the sample values of the Q-statistic were in excess of the critical value of the corresponding 99% quantile from the $\chi^2(10)$ and $\chi^2(20)$ distributions, respectively. Therefore, the null hypothesis of no autocorrelation has been rejected in all cases for all series, both levels and first-differences.

Table 3.1 Standard Statistics for LIBOR–GBP Interest Rates: 2000-2013.

LIBOR-GBP	LEVEL					FIRST DIFFERENCES				
Interest Rate	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
Observations	3,455	3,455	3,455	3,455	3,455	3,454	3,454	3,454	3,454	3,454
Mean	3.438	3.5237	3.6636	3.7855	3.9983	-0.0009	-0.0014	-0.0016	-0.0016	-0.0017
Median	4.1369	4.0992	4.1891	4.39	4.5663	0	0	0	0	0
Maximum	6.9409	6.75	6.9038	6.7988	6.8877	1.25	0.6238	0.2125	0.1981	0.2056
Minimum	0.48	0.4913	0.5069	0.6013	0.9081	-1.0313	-1.1975	-1.065	-1.0763	-1.0875
Std. Dev.	2.0715	2.0955	2.0537	1.9732	1.8462	0.1132	0.0342	0.0259	0.0272	0.0335
Skewness	-0.4511	-0.4429	-0.4214	-0.3926	-0.3388	0.1083	-11.2208	-21.1702	-18.3976	-9.8028
Kurtosis	1.5768	1.5958	1.6462	1.6562	1.6776	22.6081	483.2458	843.1311	713.5563	327.4826
Jarque-Bera	408.76	396.8128	366.0793	348.702	317.8565	5.53E+04	3.33E+07	1.02E+08	7.29E+07	1.52E+07
Probability	0	0	0	0	0	0	0	0	0	0

Notes: This table reports the standard statistics for both level and first difference of the LIBOR–GBP rates. The statistics comprise measures of central tendency – the mean and the median, measures of variability – maximum, minimum, standard deviation, and measures of normality – skewness, kurtosis and the JB normality test.

The sample estimates of these statistics indicate that the LIBOR-GBP level rates are increasing in the mean as the maturity increases. The opposite is true regarding the volatility, for longer maturity the rates are less volatile. The distributions implied by the data are all asymmetrical and platykurtic $(k < 3)$, therefore they are not normal distributions. This is also confirmed by the JB test - the null of normality is rejected for all the time-series. For the first difference series, the distributions are closely centred around a mean of almost zero, skewed to the left except for the one-week series, leptokurtic $(k > 3)$ and not normal according to the JB test.

Table 3.2 Standard Statistics for LIBOR–USD Interest Rates: 2000-2013.

LIBOR-USD	LEVEL					FIRST DIFFERENCES				
Interest Rate	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
Observations	3,455	3,455	3,455	3,455	3,455	3,454	3,454	3,454	3,454	3,454
Mean	2.3997	2.4367	2.5498	2.6849	2.9042	-0.0017	-0.0016	-0.0017	-0.0016	-0.0017
Median	1.8213	1.8388	1.8794	1.9888	2.385	0	0	0	0	0
Maximum	6.9275	6.8213	6.8688	7.1088	7.5013	1.3813	0.4688	0.2925	0.2738	0.3544
Minimum	0.1585	0.1851	0.245	0.3825	0.7203	-0.6413	-0.5288	-0.42	-0.415	-0.4175
Std. Dev.	2.1309	2.1243	2.0943	2.0328	1.9291	0.0528	0.0326	0.0281	0.0317	0.0425
Skewness	0.5617	0.5591	0.5621	0.5792	0.6362	3.356	-3.4891	-4.7984	-2.415	-0.4388
Kurtosis	1.8875	1.8931	1.9185	1.9766	2.151	186.5144	94.0157	79.4077	38.4838	15.3207
Jarque-Bera	359.8349	356.4129	350.3285	343.9954	336.798	4.85E+06	1.20E+06	8.53E+05	1.85E+05	2.20E+04
Probability	0	0	0	0	0	0	0	0	0	0

Notes: This table reports the standard statistics for both level and first difference of the LIBOR –USD rates. The statistics comprise certain measures of central tendency – the mean and the median, measures of variability – maximum, minimum, standard deviation, and measures of relative normality – skewness, kurtosis and the JB normality test.
The sample estimates of these statistics indicate that the LIBOR-GBP level rates are increasing in the mean as the maturity increases. The opposite is true regarding the volatility, for longer maturity the rates are less volatile. The distributions implied by the data are all asymmetrical and platykurtic $(k < 3)$, therefore they are not normal distributions. This is also confirmed by the JB test - the null of normality is rejected for all the time-series. For the first difference series, the distributions are closely spread around a mean of almost zero; they are skewed to the left and leptokurtic $(k > 3)$ and not normal according to the JB test.

Table 3.3 Standard Statistics for LIBOR–EUR Interest Rates: 2000-2013.

LIBOR-EUR	LEVEL					FIRST DIFFERENCES				
Interest Rate	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
Observations	3,455	3,455	3,455	3,455	3,455	3,454	3,454	3,454	3,454	3,454
Mean	2.3864	2.4561	2.5974	2.7121	2.8761	-0.0009	-0.0009	-0.0009	-0.001	-0.001
Median	2.1559	2.173	2.2754	2.2925	2.4129	0	0	0	0	-0.0007
Maximum	5.1719	5.1863	5.3913	5.4375	5.5138	0.5288	0.6431	0.1569	0.1731	0.4263
Minimum	0.0233	0.0514	0.1207	0.2143	0.4264	-0.445	-0.4166	-0.3539	-0.2568	-0.4194
Std. Dev.	1.5059	1.4964	1.475	1.4088	1.3514	0.0371	0.0228	0.0167	0.0183	0.0269
Skewness	-0.0533	-0.0252	0.0728	0.1459	0.201	0.2174	4.0166	-3.1464	-0.7601	0.6049
Kurtosis	1.756	1.779	1.8369	1.8809	1.9175	47.1072	243.0815	79.9075	28.4649	47.6
Jarque-Bera	224.4304	214.9826	197.8105	192.5621	191.9396	2.80E+05	8.30E+06	8.57E+05	9.37E+04	2.86E+05
Probability	0	0	0	0	0	0	0	0	0	0

Notes: This table reports the standard statistics for both level and first difference of the LIBOR–EUR rates. The statistics comprise certain measures of central tendency – the mean and the median, measures of variability – maximum, minimum, standard deviation and measures of relative normality – skewness, kurtosis and the JB normality test.
The sample estimates of these statistics indicate that the LIBOR-EUR level rates are increasing in the mean as the maturity increases, suggesting an upward shape of the yield curve. The opposite is true regarding the volatility, for longer maturity the rates are less volatile. The distributions implied by the data are slightly asymmetrical and platykurtic $(k < 3)$, therefore they are not normal distributions. This is also confirmed by the JB test - the null of normality is rejected for all the time-series. For the first difference series, the distributions are closely centred around a mean of almost zero, skewed to the left except for the one-week series, leptokurtic $(k > 3)$ and not normal according to the JB normality test.

Table 3.4 Standard Statistics for LIBOR–JPY Interest Rates: 2000-2013.

LIBOR-JPY	LEVEL					FIRST DIFFERENCES				
Interest Rate	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
Observations	3,455	3,455	3,455	3,455	3,455	3,454	3,454	3,454	3,454	3,454
Mean	0.1921	0.2241	0.2887	0.3643	0.4714	-7.19E-06	-8.17E-06	-9.36E-06	7.86E-06	5.16E-05
Median	0.1121	0.138	0.1935	0.3344	0.535	0	0	0	0	0
Maximum	2.0725	1.06	1.0938	1.185	1.3325	0.8488	0.3688	0.16	0.0688	0.1
Minimum	0.0313	0.0363	0.0455	0.0573	0.0831	-1.335	-0.455	-0.1338	-0.0513	-0.0625
Std. Dev.	0.2188	0.2415	0.2851	0.3097	0.3526	0.0389	0.0164	0.0071	0.0054	0.0053
Skewness	1.8253	1.5222	1.2735	0.8986	0.4411	-5.7647	0.8541	2.1448	1.9071	1.5414
Kurtosis	6.5736	4.1784	3.3632	2.7178	2.0335	533.5677	328.6445	143.1974	39.781	65.9803
Jarque-Bera	3757.035	1534.209	952.9288	476.4578	246.5006	4.05E+07	1.53E+07	2.83E+06	1.97E+05	5.72E+05
Probability	0	0	0	0	0	0	0	0	0	0

Notes: This table reports the standard statistics for both level and first difference of the LIBOR–JPY rates. The statistics comprise certain measures of central tendency – the mean and the median, measures of variability – maximum, minimum, standard deviation and measures of relative normality – skewness, kurtosis and the JB normality test. Both mean and standard deviation are increasing with the maturity, reflecting the uncertainty within the Japanese financial system as the result of many policies failing to have any impact on Japanese financial markets. For all the different maturity time series considered, the skewness and the kurtosis estimates indicate non-normality as confirmed by the JB test statistics and its p-values. For the first difference series, the distributions are closely centred around a mean of almost zero, skewed to the left except for the one-week series, leptokurtic $(k > 3)$ and not normal according to the JB normality test.

Table 3.5 Standard Statistics for LIBOR–CAD Interest Rates: 2000-2013.

LIBOR-CAD	LEVEL					FIRST DIFFERENCES				
Interest Rates	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
Observations	3,455	3,455	3,455	3,455	3,455	3,454	3,454	3,454	3,454	3,454
Mean	2.6829	2.7196	2.8085	2.9349	3.193	-0.0011	-0.0011	-0.0011	-0.0011	-0.0011
Median	2.625	2.6867	2.7517	2.8225	3	0	0	0	0	0
Maximum	5.8567	5.8767	6.025	6.1892	6.4933	0.695	0.6866	0.1917	0.28	0.325
Minimum	0.22	0.2917	0.3983	0.6933	1.2333	-0.4733	-0.6987	-0.33	-0.3433	-0.365
Std. Dev.	1.5949	1.5763	1.5272	1.4469	1.3111	0.0352	0.0307	0.0246	0.0293	0.0366
Skewness	0.217	0.2254	0.2777	0.3797	0.5671	1.387	-1.2261	-3.2007	-1.6188	-0.4882
Kurtosis	2.0326	2.0455	2.0907	2.1828	2.4719	98.142	186.6345	41.5035	27.0744	18.4641
Jarque-Bera	161.8163	160.4362	163.4585	179.1474	225.3419	1.30E+06	4.85E+06	2.19E+05	8.49E+04	3.46E+04
Probability	0	0	0	0	0	0	0	0	0	0

Notes: This table reports the standard statistics for both level and first difference of the LIBOR–CAD rates. The statistics comprise certain measures of central tendency – the mean and the median, measures of variability – maximum, minimum, standard deviation and measures of relative normality – skewness, kurtosis and the JB normality test. The mean of the series increases with its maturity suggesting higher on average, interbank rates for longer maturities, therefore an upward interbank yield curve. For all the different maturity time series considered, the skewness and the kurtosis estimates indicate non-normality as confirmed by the JB test statistics and its p-values. For the first difference series, the distributions are closely centred around a mean of almost zero, skewed to the left except for the one-week series, leptokurtic $(k > 3)$ and not normal according to the JB normality test.

Table 3.6 Standard Statistics for UK Spot Rates: 2000-2013.

UK Spot	LEVEL					FIRST DIFFERENCES				
Interest Rates	1Y	7Y	10Y	15Y	25Y	1Y	7Y	10Y	15Y	25Y
Observations	3,346	3,346	3,346	3,346	3,346	3,345	3,345	3,345	3,345	3,345
Mean	3.3274	4.0256	4.208	4.3472	4.3156	-0.0018	-0.0014	-0.0011	-0.0007	-0.0003
Median	4.2342	4.4636	4.5004	4.5239	4.3997	-0.0011	-0.0016	-0.0012	-0.0007	-0.0005
Maximum	6.3652	6.1509	5.7299	5.2352	5.0466	0.4633	0.2278	0.2476	0.2373	0.2347
Minimum	0.1346	0.9909	1.5889	2.2856	3.0762	-0.328	-0.2869	-0.3654	-0.4462	-0.4257
Std. Dev.	2.0136	1.2198	0.9389	0.6596	0.4055	0.037	0.049	0.0492	0.0463	0.0439
Skewness	-0.4447	-0.9565	-1.2547	-1.6127	-1.1542	0.0517	0.0297	-0.0704	-0.2742	-0.2481
Kurtosis	1.5894	2.9242	3.7589	4.8883	3.9158	17.728	4.6897	5.4403	7.7347	7.8254
Jarque-Bera	387.6715	510.9648	958.1979	1947.49	859.8786	30233.9	398.4083	832.7736	3166.269	3279.579
Probability	0	0	0	0	0	0	0	0	0	0

Notes: This table reports the standard statistics for both, level and first difference of the UK nominal interest rates. The statistics comprise certain measures of central tendency the – mean and the median, certain measures of variability – maximum, minimum, standard deviation and measures of relative normality – skewness, kurtosis and the JB normality test. The mean of the series increases with its maturity suggesting higher on average, interbank rates for longer maturities, therefore an upward interbank yield curve. The skewness and the kurtosis estimates indicate non-normality as confirmed by the JB test statistics and its p-values. For the first difference series, the distributions are closely centred around a mean of almost zero, skewed to the left except for the one- and seven-year series, leptokurtic $(k > 3)$ and not normal according to the JB normality test.

Table 3.7 Coefficients of Autocorrelation LIBOR-GBP Interest Rates, 2000-2013.

LIBOR-GBP RATES	LEVEL 1W	LEVEL 1M	LEVEL 3M	LEVEL 6M	LEVEL 12M	FIRST DIFFERENCE 1W	FIRST DIFFERENCE 1M	FIRST DIFFERENCE 3M	FIRST DIFFERENCE 6M	FIRST DIFFERENCE 12M
ρ_1	0.9980	0.9990	0.9990	0.9990	0.9990	-0.0930	0.2120	0.2850	0.2370	0.1800
ρ_2	0.9970	0.9990	0.9990	0.9990	0.9980	-0.1090	0.1180	0.1850	0.1610	0.1000
ρ_3	0.9950	0.9980	0.9980	0.9980	0.9970	-0.0310	0.1050	0.1770	0.1460	0.0880
ρ_4	0.9940	0.9970	0.9970	0.9970	0.9960	-0.0630	0.1010	0.2190	0.1700	0.1110
ρ_5	0.9930	0.9970	0.9960	0.9960	0.9950	-0.0090	0.0960	0.1390	0.1040	0.0530
ρ_6	0.9930	0.9960	0.9960	0.9950	0.9940	0.0030	0.0820	0.1130	0.1020	0.0810
LB1 Q -stat.	34,161.00*	34,362.00*	34,348.00*	34,314.00*	34,236.00*	140.77*	395.63*	925.43*	631.49*	280.12*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB2 Q-stat.	67,827.00*	68,233.00*	68,195.00*	68,061.00*	67,751.00*	180.26*	581.14*	1,336.40*	939.08*	410.45*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

1. In the upper section of this table section we report the values of the first six autocorrelation coefficients for the level and first difference of all LIBOR-GBP time series.
2. In the lower section, the modified Q statistic suggested by Liung-Box (1979) for ten lags (LB1) and twenty lags (LB2) is presented together with its p-values; * indicates 1% level of statistical significance.

Table 3.8 Coefficients of Autocorrelation for LIBOR-USD Interest Rates, 2000-2013.

LIBOR-USD RATES	LEVEL					FIRST DIFFERENCES				
	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
ρ_1	0.999	0.999	0.999	0.999	0.999	0.356	0.456	0.262	0.156	0.106
ρ_2	0.998	0.999	0.998	0.998	0.998	0.102	0.304	0.107	0.042	0.01
ρ_3	0.997	0.998	0.998	0.997	0.997	-0.045	0.228	0.066	0.031	0.02
ρ_4	0.996	0.997	0.997	0.996	0.996	-0.016	0.198	0.115	0.06	0.038
ρ_5	0.995	0.996	0.996	0.996	0.995	-0.07	0.202	0.095	0.053	0.016
ρ_6	0.994	0.995	0.995	0.995	0.994	0.069	0.204	0.157	0.112	0.082
LB1 Q -stat.	34,243.00*	34,303.00*	34,309.00*	34,278.00*	34,237.00*	527.20*	1,788.80*	553.47*	203.68*	93.12*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB2 Q-stat.	67,833.00*	67,957.00*	64,689.00*	67,926.00*	67,781.00*	579.20*	1,897.90*	652.83*	244.01*	116.38*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

1. In the upper section of this table section we report the values of the first six autocorrelation coefficients for the level and first difference of all LIBOR-USD time series.
2. In the lower section, the modified Q statistic suggested by Liung-Box (1979) for ten lags (LB1) and twenty lags (LB2) is presented together with its p-values. * indicates 1% level of statistical significance.

Table 3.9 Coefficients of Autocorrelation for LIBOR-EUR Interest Rates, 2000-2013.

LIBOR-EUR	LEVEL					FIRST DIFFERENCE				
RATES	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
ρ_1	0.999	0.999	0.999	0.999	0.999	0.189	0.267	0.404	0.305	0.065
ρ_2	0.999	0.999	0.999	0.999	0.998	0.065	0.167	0.325	0.25	0.094
ρ_3	0.998	0.998	0.998	0.998	0.998	0.029	0.178	0.273	0.213	0.092
ρ_4	0.997	0.998	0.998	0.997	0.997	0.02	0.115	0.253	0.188	0.083
ρ_5	0.996	0.997	0.997	0.997	0.996	-0.154	0.113	0.181	0.126	0.026
ρ_6	0.995	0.996	0.996	0.996	0.995	0.005	0.126	0.224	0.176	0.099
LB1 Q -stat.	34,323.00*	34,381.00*	34,389.00*	34,358.00*	34,301.00*	227.78*	817.75*	2,258.50*	1,370.70*	223.07*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB2 Q-stat.	68,201.00*	68,285.00*	68,316.00*	68,190.00*	67,966.00*	288.88*	991.33*	3,188.00*	2,080.20*	393.22*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

1. In the upper section of this table section we report the values of the first six autocorrelation coefficients for the level and first difference of all LIBOR-EUR time series.
2. In the lower section, the modified Q statistic suggested by Liung-Box (1979) for ten lags (LB1) and twenty lags (LB2) is presented together with its p-values; *indicates 1% level of statistical significance.

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Table 3.10 Coefficients of Autocorrelation LIBOR-JPY Interest Rates, 2000-2013.

LIBOR-JPY RATES	LEVEL					FIRST DIFFERENCE				
	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
ρ_1	0.984	0.998	1	0.179	1	-0.138	0.033	0.179	0.353	0.34
ρ_2	0.973	0.995	0.999	0.127	1	-0.016	-0.021	0.127	0.244	0.214
ρ_3	0.962	0.993	0.999	0.081	0.999	-0.011	0.049	0.081	0.169	0.139
ρ_4	0.951	0.99	0.998	0.076	0.999	-0.002	-0.057	0.076	0.161	0.144
ρ_5	0.941	0.988	0.997	0.099	0.999	-0.358	0.037	0.099	0.144	0.114
ρ_6	0.941	0.985	0.997	0.103	0.998	0.147	0.115	0.103	0.153	0.116
LB1 Q -stat.	31,135.00*	33,622.00*	34,404.00*	34,469.00*	34,510.00*	636.24*	222.39*	336.64*	1,107.10*	843.94*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB2 Q-stat.	59,618.00*	64,876.00*	68,299.00*	65,189.00*	68,808.00*	672.43*	348.82*	382.83*	1,148.60*	872.91*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

1. In the upper section of this table section we report the values of the first six autocorrelation coefficients for the level and first difference of all LIBOR-JPY time series.
2. In the lower section, the modified Q statistic suggested by Liung-Box (1979) for ten lags (LB1) and twenty lags (LB2) are presented together with their p-values; *indicates 1% level of statistical significance.

Table 3.11 Coefficients of Autocorrelation for LIBOR-CAD Interest Rates, 2000-2013.

LIBOR-CAD	LEVEL					FIRST DIFFERENCE				
RATES	1W	1M	3M	6M	12M	1W	1M	3M	6M	12M
ρ_1	0.999	0.999	0.999	0.999	0.999	0.194	-0.008	0.219	0.222	0.218
ρ_2	0.999	0.999	0.999	0.998	0.998	0.059	0.101	0.156	0.076	0.065
ρ_3	0.998	0.998	0.998	0.997	0.996	0.014	0.074	0.082	0.052	0.041
ρ_4	0.997	0.997	0.997	0.996	0.995	-0.021	0.077	0.129	0.093	0.038
ρ_5	0.996	0.997	0.996	0.996	0.993	0.023	0.074	0.11	0.071	0.039
ρ_6	0.995	0.996	0.996	0.994	0.992	0.034	0.059	0.064	0.043	0.037
LB1 Q -stat.	34,334.00*	34,358.00*	34,339.00*	34,272.00*	34,110.00*	166.84*	157.57*	520.30*	295.26*	219.19*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB2 Q-stat.	68,190.00*	68,219.00*	68,130.00*	67,878.00*	67,231.00*	187.23*	209.12*	653.00*	339.04*	236.26*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

1. In the upper section of this table section we report the values of the first six autocorrelation coefficients for the level and first difference of all LIBOR-CAD time series.
2. In the lower section, the modified Q statistic suggested by Liung-Box (1979) for ten lags (LB1) and twenty lags (LB2) is presented together with its p-values; * indicates 1% level of statistical significance.

Table 3.12 Coefficients of Autocorrelation for UK Spot Interest Rates, 2000-2013.

UK SPOT RATES	LEVEL					FIRST DIFFERENCE				
	1Y	7Y	10Y	15Y	25Y	1Y	7Y	10Y	15Y	25Y
ρ_1	0.999	0.998	0.997	0.996	0.993	0.119	0.038	0.039	0.051	0.071
ρ_2	0.998	0.996	0.995	0.993	0.986	0.028	-0.025	-0.04	-0.055	-0.074
ρ_3	0.997	0.994	0.992	0.989	0.979	-0.003	-0.039	-0.052	-0.065	-0.085
ρ_4	0.996	0.992	0.99	0.986	0.973	0.007	0.038	0.043	0.04	0.028
ρ_5	0.995	0.99	0.987	0.983	0.968	-0.006	-0.022	-0.013	-0.004	-0.007
ρ_6	0.995	0.988	0.985	0.979	0.962	0.029	-0.031	-0.033	-0.035	-0.041
LB1 Q -stat.	33,198.00*	32,820.00*	32,616.00*	32,304.00*	31,301.00*	82.44*	26.25*	38.33*	54.43*	83.88*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LB2 Q-stat.	65,832.00*	64,520.00*	63,830.00*	59,858.00*	59,773.00*	180.56*	50.35*	61.38*	71.60*	93.85*
p-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

1. In the upper section of this table we report the values of the first six autocorrelation coefficients for the level and first difference of all U.K. spot rates.
2. In the lower section, the modified Q statistic suggested by Liung-Box (1979) for ten lags (LB1) and twenty lags (LB2) is presented together with its p-values.

Table 3.13 The Correlations between the first-difference time-series

GBP-LIBOR	1W	1M	3M	6M	12M	USD-LIBOR	1W	1M	3M	6M	12M
1W	1					1W	1				
1M	0.51	1				1M	0.60	1			
3M	0.28	0.85	1			3M	0.49	0.85	1		
6M	0.21	0.75	0.93	1		6M	0.35	0.70	0.88	1	
12M	0.16	0.61	0.79	0.93	1	12M	0.21	0.49	0.67	0.91	1
EUR-LIBOR	1W	1M	3M	6M	12M	JPY-LIBOR	1W	1M	3M	6M	12M
1W	1					1W	1				
1M	0.50	1				1M	0.47	1			
3M	0.43	0.72	1			3M	0.32	0.59	1		
6M	0.33	0.59	0.88	1		6M	0.19	0.45	0.76	1	
12M	0.19	0.39	0.64	0.85	1	12M	0.10	0.33	0.61	0.84	1
CAD-LIBOR	1W	1M	3M	6M	12M	UK-SPOT	1Y	7Y	10Y	15Y	25Y
1W	1					1Y	1				
1M	0.50	1				7Y	0.64	1			
3M	0.46	0.63	1			10Y	0.54	0.98	1		
6M	0.30	0.49	0.84	1		15Y	0.47	0.93	0.97	1	
12M	0.21	0.38	0.70	0.89	1	25Y	0.38	0.81	0.86	0.94	1

Unit Root Testing

As nonstationarity is a dominant characteristic of all the time series under study, a more formal assessment is required. It is assumed that the possibility of the nonstationarity feature is implied by the presence of a single unit root with the time series being I(1). This particular kind of nonstationarity will be tested for consistency with the data. The widely used Augmented Dickey-Fuller (1979) (ADF) test for a single unit root is known to have a low statistical power especially if a structural break is potentially present, see Patterson (2000). In the light of the credit and liquidity crisis within the interbank market during September 2008 this is highly plausible, as indicated by the sharp fall of the level of interest rates at that point in time. Additionally, the ADF can be unreliable if the time series contains a moving average disturbance term. To overcome these problems, the decision regarding the existence or non-existence of a unit root has to be assessed in conjunction with other unit root test statistics. Consequently, another two unit-root tests[19] are considered. They are the Phillips-Perron (PP) (1988) test and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992) test. While the first two tests, ADF and PP, are unit root tests with the null hypothesis H_0: the series has a unit root, the KPSS test is a stationarity test with an opposite null in contrast with former tests.

The ADF test is an extension of the Dickey-Fuller test for higher-order serial correlation in the series. The ADF test tests for the existence of a single unit root in an autoregressive $AR(p)$, $p > 1$ specification. The testing procedure is adjusted by adding more lagged difference terms in the test regression - a parametric correction. While there are three version of the test, we consider the most general augmented specification including two exogeneous variables in the regression model, a constant α and a linear trend βt:

$$AR(p): \qquad \Delta r_t = \alpha + \beta t + \gamma r_{t-1} + \delta_1 \Delta r_{t-1} + ... + \delta_p \Delta r_{t-p} + \varepsilon_t \qquad (3.33)$$

Where the disturbance terms are white noise, $\varepsilon_t \simeq iid(0, \sigma^2)$ independent and identically distributed. The null hypothesis $H_0 : \gamma = 0$ is tested against the one-sided alternative hypothesis $H_1 : \gamma < 0$.

The test statistic under the null hypothesis follows a nonstandard distribution and the critical values are extracted automatically by EViews from the MacKinnon (1996) table which is a larger set of simulations than the original Dickey and Fuller table of critical values. The ADF statistic is always negative. The more negative the sample critical value is, the higher the probability of rejecting the null will be. The number of lagged first difference terms, p, is determined using the Schwarz (1978) Information Criterion (SIC).

An alternative to the ADF test for unit root is the PP test that like the ADF test controls for the higher-order correlation but in a non-parametric way. To account for any serial correlation and heteroskedasticity in the residuals of the regression, therefore allowing for processes ε_t, that are not iid$(0, \sigma^2)$ distributed, the PP test constructs a modified ADF t- statistic using a correction factor. The asymptotic distribution of the PP unit root t-statistic is the same as the ADF t- statistic and the same MacKinnon critical or p-values are used for decision criteria.

[19] Despite the fact that some of the series may indicate structural breaks, in this study we don't consider more general unit root tests that account for parameter instability such as Bai-Perron (2003) and Zivot-Andrews (1992).

The classic ADF and PP unit root tests treat asymmetrically the null hypothesis of "the series is I(1)" against the alternative "the series is I(0)" giving a dominant role to the null. To overcome this shortcoming another class of stationarity tests have been developed such as Leybourne and McCabe (LMc) (1994) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (1992). These tests invert the hypotheses, with the null of stationarity or ARIMA(p,0,0) process against the alternative hypothesis of nonstationarity or an ARIMA(p,1,1) process. In addition, they are more powerful than ADF and PP unit root test when the ARMA processes have a large moving average component[20]. For this study, the KPSS test is employed including both deterministic regressors, the constant and the time trend. The critical values have been simulated and tabulated in Kwiatkowski et al. (1992) and they are identical to the Leybourne and Macabe (1994) test statistics.

The results for the unit root testing obtained using these three tests are very consistent as can be inferred from Tables 3.14 to 3.19.

Table 3.14 LIBOR-GBP Rates: The Unit Root ADF, PP and KPPS Tests.

Unit Root Tests		**ADF**		**PP**		**KPSS**	
LIBOR-GBP		t-Stat.	Prob.*	Adj. t-stat	Prob.*	LM - Stat.	Crit. Val.**
1-week	Level	-1.41538	0.8567	-2.54069	0.3082	0.954851	0.216
LIBOR - GBP	First Diff.	-28.2672	0.0000	-68.8717	0.0000	0.074986	0.216
1-month	Level	-1.1874	0.912	-1.37301	0.8687	0.945881	0.216
LIBOR - GBP	First Diff.	-20.5227	0.0000	-54.6061	0.0000	0.128029	0.216
2-month	Level	-1.83251	0.6888	-1.23469	0.9024	0.961849	0.216
LIBOR - GBP	First Diff.	-7.98846	0.0000	-55.9544	0.0000	0.146846	0.216
3-month	Level	-1.02112	0.9395	-1.1936	0.9108	0.954575	0.216
LIBOR - GBP	First Diff.	-19.7532	0.0000	-56.0292	0.0000	0.152881	0.216
6-month	Level	-0.9729	0.9459	-1.18539	0.9124	0.930066	0.216
LIBOR - GBP	First Diff.	-21.1581	0.0000	-57.1767	0.0000	0.152708	0.216
12-Month	Level	-1.1219	0.9239	-1.32233	0.8821	0.854629	0.216
LIBOR - GBP	First Diff.	-23.4627	0.0000	-55.6623	0.0000	0.135482	0.216

This table presents the sample test statistics and the probabilities for ADF and PP unit root tests; the sample test statistic and the critical values of the KPSS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

[20] Schwert (1989) showed that the ADF and the PP tests could suffer from severe size distortion, results being biased towards rejecting the null when it is true (type I error).

Table 3.15 LIBOR-USD Rates: The Unit Root ADF, PP and KPPS Tests.

Unit Root Tests		ADF		PP		KPSS	
LIBOR-USD		t-Stat.	Prob.*	Adj. t-stat	Prob.*	LM - Stat	Crit. Val.**
1-week	Level	-1.3479	0.8755	-1.3396	0.8776	0.6667	0.216
LIBOR - USD	First Diff.	-21.3138	0.0000	-40.0117	0.0000	0.1896	0.216
1-month	Level	-1.4538	0.8449	-1.296183	0.8885	0.6784	0.216
LIBOR -USD	First Diff.	-15.6319	0.0000	-41.72224	0.0000	0.1956	0.216
2-month	Level	-1.4613	0.8426	-1.2412	0.901	0.6984	0.216
LIBOR - USD	First Diff.	-12.1556	0.0000	-42.2856	0.0000	0.2249	0.216
3-month	Level	-1.4748	0.8382	-1.2460	0.8999	0.7055	0.216
LIBOR - USD	First Diff.	-12.1453	0.0000	-42.9282	0.0000	0.2336	0.216
6-month	Level	-1.3243	0.8816	-1.1986	0.9098	0.7152	0.216
LIBOR - USD	First Diff.	-13.3258	0.0000	-51.3386	0.0000	0.2810	0.216
12-Month	Level	-1.2388	0.9015	-1.3050	0.8864	0.6954	0.216
LIBOR - USD	First Diff.	-52.8442	0.0000	-54.5147	0.0000	0.2831	0.216

This table presents the sample test statistics and the probabilities for ADF and PP unit root tests; the sample test statistic and the critical values of the KPSS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 3.16 LIBOR-EUR Rates: The Unit Root ADF, PP and KPPS Tests.

Unit Root Tests		ADF		PP		KPSS	
LIBOR - EUR		t- stat	Prob.*	Adj. t-stat	Prob.*	LM - Stat.	Crit.val**
1-week	Level	-1.4634	0.8419	-1.4471	0.8470	0.5756	0.216
LIBOR - EUR	First Diff.	-24.2384	0.0000	-48.2664	0.0000	0.1705	0.216
1-month	Level	-1.5206	0.8227	-1.4682	0.8403	0.5868	0.216
LIBOR -EUR	First Diff.	-12.5899	0.0000	-55.5019	0.0000	0.1411	0.216
2-month	Level	-1.3981	0.8617	-1.3659	0.8707	0.6103	0.216
LIBOR -EUR	First Diff.	-12.7543	0.0000	-58.0599	0.0000	0.1492	0.216
3-month	Level	-1.7327	0.7366	-1.2872	0.8906	0.6215	0.216
LIBOR - EUR	First Diff.	-8.0070	0.0000	-56.0441	0.0000	0.1534	0.216
6-month	Level	-1.1539	0.9182	-1.1922	0.9111	0.6326	0.216
LIBOR - EUR	First Diff.	-13.0708	0.0000	-60.9614	0.0000	0.1538	0.216
12-Month	Level	-0.9613	0.9474	-1.1491	0.9191	0.6140	0.216
LIBOR - EUR	First Diff.	-16.8580	0.0000	-64.5011	0.0000	0.1497	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; the sample test statistic and the critical values of the KPSS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 3.17 LIBOR –JPY Rates: The Unit Root ADF, PP and KPPS Tests.

Unit Root Tests		**ADF**		**PP**		**KPSS**	
LIBOR - JPY		t-Stat.	Prob.*	Adj. t-stat	Prob.*	LM - Stat.	Crit.Val.**
1-week	Level	-3.1112	0.1036	-3.8276	0.0152	0.7309	0.216
LIBOR - JPY	First Diff.	-24.9361	0.0000	-91.0419	0.0000	0.0447	0.216
1-month	Level	-1.9296	0.6386	-2.2471	0.4625	0.7404	0.216
LIBOR - JPY	First Diff.	-15.7659	0.0000	-57.1410	0.0000	0.0477	0.216
2-month	Level	-1.6243	0.7835	-1.3951	0.8626	0.8018	0.216
LIBOR - JPY	First Diff.	-13.5408	0.0000	-52.5393	0.0000	0.1081	0.216
3-month	Level	-1.1410	0.9206	-1.0501	0.9354	0.8286	0.216
LIBOR - JPY	First Diff.	-16.7229	0.0000	-54.0182	0.0000	0.1685	0.216
6-month	Level	-0.8975	0.9548	-0.8429	0.9604	0.8589	0.216
LIBOR - JPY	First Diff.	-16.8506	0.0000	-47.6673	0.0000	0.2177	0.216
12-Month	Level	-0.5921	0.9790	-0.6885	0.973	0.8598	0.216
LIBOR - JPY	First Diff.	-21.8245	0.0000	-46.7694	0.0000	0.2899	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; the sample test statistic and the critical values of the KPSS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 3.18 LIBOR – CAD Rates: The Unit Root ADF, PP and KPPS Tests.

Unit Root Tests		**ADF**		**PP**		**KPSS**	
LIBOR - CAD		t-Stat.	Prob.*	Adj. t-stat	Prob.*	LM - Stat.	Crit. Val.**
1-week	Level	-1.1041	0.9269	-1.1886	0.9117	0.5094	0.216
LIBOR - CAD	First Diff.	-48.3407	0.0000	-48.9854	0.0000	0.1809	0.216
1-month	Level	-1.0835	0.9302	-1.2755	0.8933	0.5148	0.216
LIBOR - CAD	First Diff.	-21.2242	0.0000	-65.6230	0.0000	0.1626	0.216
2-month	Level	-1.3380	0.8781	-1.2708	0.8944	0.5214	0.216
LIBOR - CAD	First Diff.	-13.6088	0.0000	-71.4236	0.0001	0.1679	0.216
3-month	Level	-1.3426	0.8769	-1.2999	0.8876	0.5263	0.216
LIBOR - CAD	First Diff.	-14.0961	0.0000	-55.6726	0.0000	0.1643	0.216
6-month	Level	-1.2805	0.8922	-1.3831	0.8659	0.5397	0.216
LIBOR - CAD	First Diff.	-24.4136	0.0000	-50.8584	0.0000	0.1593	0.216
12-Month	Level	-1.6112	0.7888	-1.7290	0.7384	0.5268	0.216
LIBOR - CAD	First Diff.	-47.0961	0.0000	-48.7181	0.0000	0.1187	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; the sample test statistic and the critical values of the KPSS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 3.19 UK Spot Rates: The Unit Root ADF, PP and KPPS Tests.

Unit Root Tests		ADF		PP		KPSS	
UK Spot		t-Stat.	Prob.*	Adj. t-stat	Prob.*	LM	Crit. Val.**
1Y Nominal	Level	-0.91893	0.9524	-1.167207	0.9158	0.982913	0.216
Spot Rate	First Diff.	-36.3994	0	-51.22148	0	0.194957	0.216
5Y Nominal	Level	-1.54899	0.8125	-1.62033	0.7851	1.226648	0.216
Spot Rate	First Diff.	-54.1928	0	-54.1545	0	0.067427	0.216
7Y Nominal	Level	-1.84338	0.6833	-1.802913	0.7034	1.176754	0.216
Spot Rate	First Diff.	-54.5107	0	-54.51919	0	0.053061	0.216
10Y Nominal	Level	-2.51627	0.3201	-2.009306	0.5955	1.058845	0.216
Spot Rate	First Diff.	-53.7537	0	-54.60421	0	0.040864	0.216
15Y Nominal	Level	-2.51627	0.3201	-2.206746	0.4851	0.891395	0.216
Spot Rate	First Diff.	-53.7537	0	-53.90468	0	0.030382	0.216
20Y Nominal	Level	-3.13102	0.0992	-2.584089	0.2877	0.719565	0.216
Spot Rate	First Diff.	-40.6086	0	-52.76153	0	0.026933	0.216
25Y Nominal	Level	-3.37868	0.0544	-2.971168	0.1405	0.556694	0.216
Spot Rate	First Diff.	-40.735	0	-52.94229	0	0.026808	0.216

This table presents the sample test-statistics and the probabilities for the ADF and the PP unit root tests; also the sample test statistic and the critical values of the KPSS test, computed using EViews, for the UK spot rates time series with 1, 5, 7, 10, 15 and 25 year maturities.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

All the time series in levels have a unit root, failing to reject the null hypothesis in ADF, PP tests at all common significance levels, while the null of stationarity is rejected in the case of KPPS test. Similarly, the tests confirm that the time series can be differenced to stationarity at the same level of significance. The entire statistical analysis had been conducted in Eviews, and the intermediary regressions for all the unit root/stationarity tests included an intercept and a linear trend. For most of the series, forty-two in total, the intercept and the trend were both statistically insignificant. Even when they were excluded one by one, the final conclusion remained unchanged: all time-series under study are I(1).

3.5 The Estimation Results

The econometric estimation of the proposed continuous-time models is conducted in two stages corresponding to the two extensions, four- and five-factor specifications. Four multi-factor continuous-time models, namely CKLS, Vasicek, CIR and BS are estimated for each of the multivariate time-series. All the models incorporate a linear mean-reversion drift by recognising feedback effects in all directions among the factors, which are interest rates of various maturities. Another way to explain the connection between different maturity rates along the yield curve is by assuming that the stochastic components, more specifically the individual Brownian motions are correlated as defined by the covariance matrices presented in section 3.5. Therefore, the parameters of most interest are the level effect vector-parameter γ, the feedback matrix β and the correlation coefficients $(\rho_{ij})_{1\le i,j\le 4(5)}$.

3.5.1 Estimation Results for the Four-Factor Continuous-Time Models

For the first stage twenty-four four-factor models are estimated, four models for each of the five LIBOR curves and another four models for the UK spot curve. The QMLE estimates of the parameters are grouped in the vector solution θ to the respective optimization problem of maximizing the objective function given in equation (3.25) and are presented together with their standard errors entered in the next column in Tables 3.20-3.25. The vector parameter to be estimated has thirty-four components under the general model CKLS and thirty under any of the restricted models. Given the high dimension of the vector of parameters, each table consists of two parts: part a) for the drift parameters and part b) for the diffusion parameters, respectively.

The estimation results are interpreted separately based on the money-market and bond market data sets, respectively. The parameters of interest are the vector of the level effects, the feedback matrix and the covariance matrix. For three out the five LIBOR time-series, namely GBP-LIBOR, USD-LIBOR and JPY-LIBOR, the level effect estimates are close to unity. This suggests a strong dependence of the volatility of the interest rate changes on the level of the interest rate itself. For the EUR-LIBOR and CAD-LIBOR multivariate time series the estimates regarding the elasticity of the volatility parameter are situated in the vicinity of 0.5. The restricted models are tested for their explanatory power against the general CKLS model using the likelihood ratio test (LR). Based on the corresponding $\chi^2(4\text{df})$ tests, under the null hypothesis of the validity of the nested model, the results indicate rejection at the 1% level of significance of all of the restricted models. According to the values of log-maximum likelihood functions, the best performing restricted specifications are: the BS model for GBP-LIBOR, USD-LIBOR and JPY-LIBOR rates, while the CIR model for EUR-LIBOR and CAD-LIBOR rates. The drift function of the models proposed is defined by the four-dimensional intercept vector α and the feedback matrix β of sixteen components. The estimates of all intercept elements are almost zero, most of them being however statistically significant.

The majority of the feedback estimates are significant indicating evidence of feedback in most directions. For the GBP-LIBOR time series, the matrix β can be re-specified by assuming $\beta_{13} = \beta_{31} = 0$ while all the other elements of the feedback matrix are significantly different from zero; in the case of the USD-LIBOR time series there is no feedback evidence from the six-month USD-LIBOR rate to the twelve-month USD-LIBOR rate in either direction as $\beta_{34} = \beta_{43} = 0$, both being statistically insignificant. For the EUR-LIBOR time series some elements can be considered zero: $\beta_{11} = \beta_{12} = \beta_{43} = \beta_{44} = 0$; for the JPY-LIBOR rates the inference suggests that $\beta_{23} = \beta_{24} = \beta_{41} = \beta_{42} = 0$; for the CAD-LIBOR rates only one feedback coefficient is insignificant $\beta_{32} = 0$ implying that there is no feedback from the one-month rate to the six- month rate. Finally, for the U.K. spot rates the estimation results for the feedback matrix imply $\beta_{12} = \beta_{24} = \beta_{34} = \beta_{41} = \beta_{44} = 0$. When analysed in comparison with the corresponding best restricted models, there is always a higher degree of significant elements in the feedback matrix in the more general CKLS model. Hence, we can argue that the increased flexibility provided by the CKLS specification by not restricting the elasticity of the variance parameter γ, may render higher degree of significance in the feedback matrix reflecting a stronger correlation among the factors explicitly modelled via a more complex deterministic drift.

The estimates for the correlation coefficients are all positive under the CKLS model for all LIBOR currencies. A ranking in terms of the degree of correlation among the factors can be observed across all the LIBOR data sets. The estimation results for the correlation coefficients indicate that the six-month and twelve-month rates are most highly correlated with the value of the correlation coefficient ρ_{34} between 0.75 (JPY-LIBOR) and 0.98 (USD-LIBOR). The other pairs of highly correlated short-term interest rates are for the maturities of one-month with six-month and one-week with one-month.

In the case of the bond market data, the estimation results are rather different, with a much lower level effect estimates and another correlation structure. As can be seen in the Table 3.25, the components of vector γ are estimated within the range (0.000004, 0.22), suggesting a much weaker sensitivity of the conditional variance with respect to the level of interest rate; and only $\gamma_1 = 0.22$ is statistically different from zero. The Vasicek model supports the data best with the highest value of the restricted log-likelihood functions $LogLF =$ 105,776.12. All the restricted models are rejected against the unrestricted CKLS model. Regarding the drift components, under the CKLS, the intercept estimates are very close to zero, whereas the feedback matrix has only five elements statistically insignificant and they degenerate to zero $(\beta_{12} = \beta_{24} = \beta_{34} = \beta_{41} = \beta_{44} = 0)$. As expected, the correlation coefficients are higher between the spot rates corresponding to the flatter end of the term structure with $\rho_{34} = 0.95,\ \rho_{23} = 0.94$ and $\rho_{24} = 0.82$.

Table 3.20a) LIBOR-GBP, The Drift Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	SE	**Vasicek**	SE TR	**CIR**	SE	**BR&SC**	SE
Alpha1	**0**	0	**-0.00066**	0.00011	**-0.00007**	0	**0.00004**	0
Alpha2	**0.00003**	0	**-0.00011**	0.00002	**-0.0001**	0	**0**	0
Alpha3	**0.00003**	0	**-0.00003**	0	**-0.00011**	0	**-0.00002**	0
Alpha4	**0.00005**	0	**0.00003**	0	**-0.00008**	0	**-0.00004**	0
B11	**0.01438**	0.00496	**-0.0175**	0.00948	**-0.19935**	0	**0.05997**	0.0067
B12	**-0.01732**	0.00644	**-0.00322**	0.0162	**0.26642**	0	**-0.04241**	0.00873
B13	**-0.00395**	0.00312	**-0.08595**	0.03195	**-0.14338**	0	**-0.04315**	0.00531
B14	**0.00439**	0.00148	**0.11546**	0.02602	**0.07316**	0	**0.02383**	0.00266
B21	**0.02164**	0.00352	**0.03063**	0.00207	**0.01596**	0.00229	**0.03622**	0.00212
B22	**-0.01966**	0.00457	**-0.05358**	0.00193	**-0.00678**	0.00308	**-0.03477**	0.00268
B23	**0.00147**	0.00215	**0.03197**	0.00445	**-0.03782**	0.00261	**-0.00769**	0.00149
B24	**-0.00399**	0.00095	**-0.00712**	0.00377	**0.0301**	0.00131	**0.00594**	0.00072
B31	**-0.00293**	0.0007	**0.00492**	0.00165	**0.01713**	0.0017	**0.00726**	0.0017
B32	**0.01334**	0.00061	**-0.00347**	0.00132	**-0.00426**	0.00224	**-0.00761**	0.00221
B33	**-0.0094**	0.00135	**0.00027**	0.00187	**-0.04474**	0.00215	**-0.00342**	0.00155
B34	**-0.00157**	0.00074	**-0.00125**	0.00151	**0.03364**	0.00107	**0.00356**	0.00075
B41	**-0.01358**	0.00133	**0.0022**	0.00213	**0.01106**	0.00209	**-0.0168**	0.0022
B42	**0.0235**	0.00134	**-0.00726**	0.00202	**0.00295**	0.0027	**0.01639**	0.00294
B43	**-0.00556**	0.0014	**0.02532**	0.00195	**-0.04132**	0.00216	**-0.0051**	0.00202
B44	**-0.00528**	0.00078	**-0.02085**	0.00133	**0.02843**	0.00083	**0.00528**	0.00087

Table 3.20b) LIBOR-GBP, The Diffusion Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	SE	**Vasicek**	SE	**CIR**	SE	**BR&SC**	SE TR
Gamma1	**1.59404**	0.00696	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**1.22368**	0.01377	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**1.03083**	0.00288	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**1.39513**	0.00476	**0**	N/A	**0.5**	N/A	**1**	N/A

Sigma1	**0.20586**	0.0062	**0.00114**	0.00002	**0.00557**	0.00003	**0.02878**	0.00023
Sigma2	**0.01809**	0.00099	**0.00034**	0	**0.0016**	0.00799	**0.00785**	0.00006
Sigma3	**0.0074**	0.0001	**0.00027**	0	**0.00126**	0.00842	**0.00568**	0.00004
Sigma4	**0.02825**	0.00056	**0.00034**	0	**0.00155**	0.00826	**0.00724**	0.00006
Corr12	**0.54379**	0.0091	**0.49558**	0.01111	**0.53476**	0.00795	**0.59934**	0.00754
Corr13	**0.25767**	0.01148	**0.182**	0.01305	**0.2214**	0.01132	**0.04943**	0.01248
Corr14	**0.21541**	0.0118	**0.13789**	0.01333	**0.17669**	0.0117	**-0.09881**	0.01349
Corr23	**0.77737**	0.0065	**0.73874**	0.00525	**0.75729**	0.0048	**0.53113**	0.00948
Corr24	**0.66319**	0.00845	**0.60368**	0.00741	**0.61929**	0.00695	**0.26443**	0.01247
Corr34	**0.93308**	0.00145	**0.92748**	0.00139	**0.92822**	0.00135	**0.87946**	0.00238
LogLF	**112,577.66**	N/A	**105,903.42**	13,348.48+	**109,627.13**	5,901.06+	**110,947.21**	3,260.91+

Note The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the corresponding values of the LR test statistics ($\chi^2_{crit}(4df, 1\%) = 13.28$).

Table 3.21a) LIBOR-USD, The Drift Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	SE	**VASICEK**	SE	**CIR**	SE	**BS**	SE
Alpha1	**0.00002**	0*	**-0.00011**	0.00003	**-0.00003**	0*	**0.00001**	0*
Alpha2	**0.00005**	0*	**-0.00005**	0.00001	**-0.00003**	0*	**0.00001**	0*
Alpha3	**0.00004**	0*	**-0.00003**	0	**-0.00002**	0*	**0.00001**	0*
Alpha4	**0.00006**	0*	**0.00001**	0	**0**	0*	**0.00001**	0*
Beta11	**0.03357**	0.00277	**0.03177**	0.00464	**0.0575**	0.00348	**0.04982**	0.00227
Beta12	**-0.03421**	0.00304	**-0.0439**	0.00594	**-0.06642**	0.00403	**-0.05548**	0.0025
Beta13	**-0.00621**	0.00196	**-0.00292**	0.01063	**0.00318**	0.0019	**0.00594**	0.00082
Beta14	**0.00443**	0.00153	**0.01642**	0.00879	**0.00546**	0.00046	**-0.00209**	0.00043
Beta21	**0.0228**	0.00291	**0.07734**	0.00215	**0.09286**	0.00259	**0.08553**	0.00255
Beta22	**-0.01928**	0.00316	**-0.09453**	0.00251	**-0.0997**	0.00297	**-0.08928**	0.00277
Beta23	**0.0017**	0.0018	**0.01827**	0.00465	**-0.00238**	0.00137	**0.00057**	0.00067
Beta24	**-0.00696**	0.00135	**-0.00038**	0.00347	**0.00949**	0.0007	**0.00249**	0.00036
Beta31	**-0.00896**	0.00252	**0.02701**	0.00215	**0.0324**	0.002	**-0.00123**	0.00345
Beta32	**0.02317**	0.00186	**-0.03339**	0.00173	**-0.03079**	0.0028	**0.00369**	0.00387
Beta33	**-0.01592**	0.00359	**0.00812**	0.00246	**-0.01049**	0.00205	**0.00103**	0.00132
Beta34	**0.00134**	0.00291	**-0.00162**	0.0019	**0.00875**	0.00101	**-0.00379**	0.00058
Beta41	**-0.01595**	0.00314	**0.00829**	0.00307	**0.01634**	0.00274	**-0.01586**	0.00514
Beta42	**0.03148**	0.00237	**-0.01322**	0.00267	**-0.01401**	0.00407	**0.0183**	0.00593
Beta43	**-0.0006**	0.00434	**0.01727**	0.00252	**-0.00435**	0.00338	**0.00749**	0.0026
Beta44	**-0.01558**	0.00348	**-0.01304**	0.00174	**0.0012**	0.00186	**-0.01004**	0.00112

Table 3.21b) LIBOR-USD, The Diffusion Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	**SE**	**VASICEK**	**SE**	**CIR**	**SE**	**BS**	**SE**
Gamma1	**0.97513**	0.00419	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.80915**	0.0054	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.99445**	0.00117	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.9881**	0.00267	**0**	N/A	**0.5**	N/A	**1**	N/A

Sigma1	**0.01945**	0.00047	**0.00052**	0.01421	**0.00289**	0.00403	**0.02139**	0.00605
Sigma2	**0.007**	0.00019	**0.00031**	0.00897	**0.00187**	0.00765	**0.01461**	0.00667
Sigma3	**0.01557**	0.00016	**0.00031**	0.00856	**0.00183**	0.00834	**0.01223**	0.00819
Sigma4	**0.01704**	0.00026	**0.00043**	0.00832	**0.00252**	0.00819	**0.01718**	0.00843
Corr12	**0.55108**	0.00999	**0.57393**	0.00974	**0.55132**	0.00746	**0.48312**	0.00855
Corr13	**0.24234**	0.01213	**0.32574**	0.01286	**0.3246**	0.01043	**0.13464**	0.01244
Corr14	**0.19591**	0.01238	**0.20007**	0.01293	**0.19688**	0.01143	**-0.03379**	0.01301
Corr23	**0.5616**	0.0087	**0.68931**	0.0061	**0.66626**	0.00629	**0.4416**	0.00999
Corr24	**0.49847**	0.00953	**0.47818**	0.00912	**0.46016**	0.0092	**0.21256**	0.01208
Corr34	**0.98207**	0.00037	**0.91024**	0.00183	**0.90904**	0.00173	**0.89437**	0.00211
Log LF	**112,669.38**	N/A	**107,366.63**	10,605.51+	**111,026.43**	3,285.90+	**111,329.85**	2,679.06+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the corresponding values of the LR test statistics ($\chi^2_{crit}(4df,1\%)=13.28$).

Table 3.22a) LIBOR-EUR, The Drift Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	S.E.	**VASICEK**	S.E.	**CIR**	S.E.	**BS**	S.E.
Alpha1	**0**	0*	**-0.00001**	0.00002	**0.00006**	0*	**0**	0*
Alpha2	**-0.00003**	0*	**-0.00003**	0.00001	**0.00004**	0.000001	**0.00004**	0*
Alpha3	**-0.00003**	0*	**0**	0	**0.00002**	0.000002	**0**	0*
Alpha4	**-0.00002**	0*	**0.00007**	0	**0.00006**	0.000004	**0.00001**	0
Beta11	**0.00559**	0.00406	**-0.0032**	0.00792	**-0.00405**	0.00364	**0.01012**	0.00293
Beta12	**-0.00415**	0.00459	**0.00322**	0.00819	**-0.00369**	0.0046	**-0.00432**	0.00314
Beta13	**-0.0045**	0.00194	**0.00316**	0.008	**0.03634**	0.00267	**-0.01373**	0.00099
Beta14	**0.00236**	0.00103	**-0.00313**	0.00548	**-0.03042**	0.0012	**0.00767**	0.00049
Beta21	**0.02631**	0.00316	**0.04727**	0.00267	**0.02565**	0.00229	**0.04537**	0.00218
Beta22	**-0.0228**	0.0035	**-0.06318**	0.00286	**-0.03559**	0.00283	**-0.0525**	0.00243
Beta23	**-0.02009**	0.00146	**0.02516**	0.0032	**0.03275**	0.00167	**0.02346**	0.00085
Beta24	**0.01709**	0.00082	**-0.00825**	0.00268	**-0.02374**	0.00089	**-0.01696**	0.00043
Beta31	**0.01791**	0.00115	**0.00533**	0.00137	**0.00417**	0.00206	**0.00036**	0.002
Beta32	**-0.01345**	0.00199	**-0.00083**	0.00233	**-0.00016**	0.00264	**0.00176**	0.00248
Beta33	**-0.01827**	0.00177	**-0.00377**	0.00285	**0.00006**	0.00179	**-0.00016**	0.00142
Beta34	**0.01454**	0.00092	**-0.00057**	0.00176	**-0.00441**	0.001	**-0.00199**	0.00075
Beta41	**0.01306**	0.0021	**-0.00281**	0.00235	**0.00156**	0.00309	**-0.01864**	0.00311
Beta42	**-0.01182**	0.00312	**0.00315**	0.00408	**-0.00176**	0.00398	**0.01808**	0.00404
Beta43	**-0.00301**	0.00249	**0.02722**	0.00433	**0.02561**	0.00272	**0.01339**	0.00277
Beta44	**0.0021**	0.00133	**-0.02871**	0.0023	**-0.02672**	0.00153	**-0.01327**	0.00146

Table 3.22b) LIBOR-EUR, The Diffusion Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	S.E.	**VASICEK**	S.E.	**CIR**	S.E.	**BS**	S.E.
Gamma1	**0.69498**	0.007182	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.66565**	0.015356	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.75755**	0.001991	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**1.04215**	0.015731	**0**	N/A	**0.5**	N/A	**1**	N/A

Sigma1	**0.00526**	0.01721	**0.00037**	0.02025	**0.00236**	0.00593	**0.02429**	0.00906
Sigma2	**0.00249**	0.02845	**0.00023**	0.00738	**0.00132**	0.00754	**0.01214**	0.00735
Sigma3	**0.00257**	0.00992	**0.00018**	0.00846	**0.00103**	0.00843	**0.00675**	0.00882
Sigma4	**0.0098**	0.02541	**0.00028**	0.00767	**0.00148**	0.00821	**0.00934**	0.01032
Corr12	**0.6222**	0.00803	**0.4761**	0.0149	**0.58779**	0.01107	**0.67471**	0.01204
Corr13	**0.38564**	0.01021	**0.2897**	0.012	**0.37323**	0.01032	**-0.05124**	0.01421
Corr14	**0.24264**	0.01133	**0.1513**	0.01233	**0.23817**	0.01144	**-0.24354**	0.01366
Corr23	**0.61826**	0.00776	**0.5441**	0.00877	**0.61991**	0.00712	**0.26422**	0.01395
Corr24	**0.40621**	0.01007	**0.3471**	0.01127	**0.42326**	0.00981	**-0.00937**	0.01477
Corr34	**0.86406**	0.00273	**0.8584**	0.00273	**0.86823**	0.00257	**0.84559**	0.00333
Log LF	**115,424.04**	N/A	**111,188.22**	8,471.63+	**114,586.98**	1,674.13+	**112,955.44**	4,937.20+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the corresponding values of the LR test statistics ($\chi^2_{crit}(4df, 1\%) = 13.28$).

Table 3.23a) LIBOR-JPY, The Diffusion Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	S.E.	**VASICEK**	S.E.	**CIR**	S.E.	**BS**	S.E.
Alpha1	**-0.00001**	0*	**0***	0*	**0***	0*	**0**	0*
Alpha2	**-0.00001**	0*	**0***	0*	**0**	0*	**0***	0*
Alpha3	**0***	0*	**0***	0*	**0**	0*	**-0.00001**	0*
Alpha4	**0**	0*	**0***	0*	**0***	0*	**0**	0*
Beta11	**0.04638**	0.00519	**-0.0493**	0.0082	**-0.10204**	0.00568	**0.00606**	0.00579
Beta12	**-0.03782**	0.00502	**-0.0018**	0.0104	**0.04321**	0.00706	**-0.00905**	0.00569
Beta13	**0.01662**	0.00346	**0.0746**	0.0135	**0.04654**	0.00605	**-0.02784**	0.00397
Beta14	**-0.00954**	0.00183	**-0.0414**	0.0082	**-0.02002**	0.003	**0.01993**	0.00215
Beta21	**0.05453**	0.00473	**0.0004**	0.0036	**0.02624**	0.0038	**0.08036**	0.00446
Beta22	**-0.03377**	0.0048	**-0.0299**	0.0043	**-0.05356**	0.00447	**-0.09728**	0.00477
Beta23	**-0.00067**	0.00269	**0.0448**	0.0049	**0.02751**	0.00381	**0.01564**	0.00286
Beta24	**-0.00209**	0.00141	**-0.0237**	0.0025	**-0.00847**	0.00205	**-0.0012**	0.0015
Beta31	**-0.00821**	0.00189	**0.0117**	0.0012	**0.00484**	0.00145	**0.00114**	0.00213
Beta32	**0.00977**	0.00216	**-0.0099**	0.0015	**-0.0051**	0.00176	**0.00714**	0.00254
Beta33	**0.00772**	0.00237	**0.0008**	0.0018	**0.0005**	0.0019	**0.00001**	0.00245
Beta34	**-0.0076**	0.00129	**-0.0022**	0.001	**-0.0002**	0.00105	**-0.00055**	0.00128
Beta41	**0.00169**	0.00146	**0.0032**	0.0011	**0.00046**	0.00132	**-0.01146**	0.00176
Beta42	**0.00154**	0.00177	**-0.0069**	0.0014	**-0.00314**	0.00163	**0.01221**	0.00215
Beta43	**-0.00669**	0.00229	**0.011**	0.0017	**0.00797**	0.00189	**0.0041**	0.00245
Beta44	**0.00321**	0.0013	**-0.0087**	0.001	**-0.00522**	0.00107	**-0.00282**	0.00136

Table 3.23b) LIBOR-JPY, The Diffusion Coefficients Estimates, Four-Factor Models

Param.	**CKLS**	**S.E.**	**VASICEK**	**S.E.**	**CIR**	**S.E.**	**BS**	**S.E.**
Gamma1	**1.30589**	0.00614	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**1.20374**	0.00643	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.87365**	0.00817	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.80425**	0.00698	**0**	N/A	**0.5**	N/A	**1**	N/A

Sigma1	**0.54749**	0.02519	**0.0004**	0	**0.0047**	0.00614	**0.07925**	0.00064
Sigma2	**0.1552**	0.00726	**0.0002**	0	**0.00232**	0.00643	**0.04243**	0.0003
Sigma3	**0.00807**	0.00044	**0.0001**	0	**0.00085**	0.00817	**0.0181**	0.00014
Sigma4	**0.00413**	0.00018	**0.0001**	0	**0.00075**	0.00698	**0.01383**	0.00012
Corr12	**0.57337**	0.00796	**0.4966**	0.00859	**0.55043**	0	**0.60882**	0.00773
Corr13	**0.21099**	0.01198	**0.2129**	0.01174	**0.29174**	0.00002	**-0.04755**	0.01362
Corr14	**0.12127**	0.01215	**0.1209**	0.01212	**0.19937**	0.00001	**-0.16739**	0.0134
Corr23	**0.42502**	0.01021	**0.4625**	0.00943	**0.52402**	0.00001	**0.13845**	0.01422
Corr24	**0.34108**	0.01088	**0.3368**	0.01082	**0.42389**	0.0062	**-0.01224**	0.01425
Corr34	**0.75093**	0.00511	**0.8379**	0.00292	**0.81611**	0.01093	**0.72776**	0.00532
LogLF	**131,915.25**	N/A	**121,197.90**	21,434.71+	**128,664.45**	6,501.60+	**130,785.48**	2,259.54+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the corresponding values of the LR test statistics ($\chi^2_{crit}(4df, 1\%) = 13.28$).

Table 3.24a) LIBOR-CAD, The Drift Coefficients Estimates, Four-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**-0.00012**	0	**-0.00011**	0.00001	**0.00005**	0*	**-0.00003**	0*
Alpha2	**-0.00012**	0	**-0.00011**	0.00001	**0**	0	**-0.00005**	0*
Alpha3	**-0.00013**	0*	**0.00003**	0.00001	**-0.00008**	0	**-0.00001**	0*
Alpha4	**-0.00007**	0*	**0.00007**	0.00001	**-0.00003**	0*	**0.00005**	0
Beta11	**-0.03696**	0.01663	**-0.01253**	0.00405	**-0.01063**	0.00493	**0.02391**	0.00491
Beta12	**0.04398**	0.01811	**0.00589**	0.00431	**0.00581**	0.00554	**-0.0225**	0.00521
Beta13	**-0.02876**	0.00295	**-0.00535**	0.00359	**0.01239**	0.00304	**-0.01054**	0.00156
Beta14	**0.02321**	0.00141	**0.01364**	0.00173	**-0.00945**	0.00135	**0.00878**	0.00078
Beta21	**0.0265**	0.01615	**0.0671**	0.00378	**0.05256**	0.00428	**0.07785**	0.0046
Beta22	**-0.02453**	0.01676	**-0.07678**	0.00541	**-0.06578**	0.00479	**-0.07882**	0.00484
Beta23	**-0.02337**	0.00264	**-0.00961**	0.00458	**0.01758**	0.00237	**-0.01193**	0.00151
Beta24	**0.02328**	0.00131	**0.02094**	0.00201	**-0.00493**	0.00115	**0.01378**	0.00075
Beta31	**0.01834**	0.00179	**0.01831**	0.00403	**-0.00009**	0.00482	**-0.00002**	0.00481
Beta32	**-0.00615**	0.00349	**-0.02359**	0.00463	**-0.00706**	0.00548	**-0.00034**	0.00546
Beta33	**-0.04321**	0.00336	**0.00603**	0.00336	**-0.00106**	0.00238	**-0.00198**	0.00245
Beta34	**0.03307**	0.00151	**-0.00185**	0.0017	**0.00906**	0.00091	**0.00181**	0.00108
Beta41	**0.02556**	0.00367	**0.00374**	0.00527	**-0.00008**	0.00621	**-0.00443**	0.00636
Beta42	**-0.01484**	0.00547	**-0.01401**	0.00599	**-0.00703**	0.00725	**-0.0031**	0.00744
Beta43	**-0.03409**	0.0046	**0.02427**	0.00389	**0.00307**	0.00459	**0.01927**	0.00352
Beta44	**0.02402**	0.0021	**-0.01607**	0.00194	**0.00364**	0.00258	**-0.01362**	0.00138

Table 3.24b) LIBOR-CAD, The Diffusion Coefficients Estimates, Four-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.65518**	0.01285	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.59215**	0.02102	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.67015**	0.00389	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.77973**	0.01837	**0**	N/A	**0.5**	N/A	**1**	N/A

Sigma1	**0.0038**	0.00025	**0.00035**	0	**0.00209**	0.00001	**0.01625**	0.00011
Sigma2	**0.00247**	0.00023	**0.00031**	0	**0.00174**	0.00001	**0.01366**	0.00008
Sigma3	**0.00306**	0.00006	**0.00029**	0	**0.00165**	0.00001	**0.0103**	0.00009
Sigma4	**0.00534**	0.00036	**0.00037**	0	**0.00201**	0.00002	**0.01189**	0.0001
Corr12	**0.59118**	0.00832	**0.48123**	0.00836	**0.56093**	0.00747	**0.59482**	0.00714
Corr13	**0.33732**	0.01092	**0.29445**	0.01084	**0.29695**	0.01086	**0.02271**	0.01258
Corr14	**0.23498**	0.0118	**0.21001**	0.01151	**0.19524**	0.01157	**-0.03793**	0.01268
Corr23	**0.51084**	0.01025	**0.4683**	0.00893	**0.49047**	0.00866	**0.30095**	0.01161
Corr24	**0.40887**	0.01039	**0.36166**	0.01013	**0.38595**	0.01002	**0.22973**	0.01203
Corr34	**0.87807**	0.00227	**0.88707**	0.00202	**0.88185**	0.00203	**0.87228**	0.00226
Log LF	**109,831.29**	N/A	**107,760.40**	4,141.78+	**109,639.74**	383.1+	**108,492.75**	2,677.08 +

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the corresponding values of the LR test statistics ($\chi^2_{crit}(4df, 1\%) = 13.28$).

Table 3.25a) U.K. Spot Rates, The Drift Coefficients Estimates, Four-Factor Models

Param	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**-0.00026**	0.00001	**-0.00018**	0.00001	**0.00038**	0*	**0**	0*
Alpha2	**-0.00007**	0.00001	**-0.00009**	0.00001	**0**	0*	**0.00014**	0*
Alpha3	**0**	0.00002	**0.00012**	0.00002	**0.00013**	0*	**-0.00004**	0*
Alpha4	**0.00006**	0.00001	**0.00029**	0.00002	**0.00013**	0*	**0.00001**	0*
Beta11	**0.00337**	0.00096	**0.00723**	0.00091	**0.00363**	0.0008	**0.00692**	0.0009
Beta12	**-0.00505**	0.00333	**-0.01849**	0.00305	**-0.02133**	0.0027	**-0.00737**	0.0025
Beta13	**-0.01483**	0.00514	**0.01199**	0.00509	**0.04502**	0.0041	**-0.00383**	0.0038
Beta14	**0.02255**	0.00262	**0.00339**	0.00282	**-0.03791**	0.0019	**0.0057**	0.0019
Beta21	**0.0087**	0.00124	**0.01143**	0.00092	**0.00814**	0.0012	**0.02493**	0.0017
Beta22	**-0.03025**	0.00373	**-0.0286**	0.00238	**-0.01911**	0.0042	**-0.07331**	0.005
Beta23	**0.02288**	0.00586	**0.02075**	0.00442	**0.00623**	0.0071	**0.09573**	0.007
Beta24	**0.00005**	0.00383	**-0.00135**	0.00328	**0.00419**	0.0044	**-0.05147**	0.0032
Beta31	**0.00531**	0.00106	**0.0073**	0.00096	**0.00282**	0.0012	**-0.00062**	0.0009
Beta32	**-0.02127**	0.0026	**-0.01658**	0.00255	**-0.00659**	0.0038	**0.00199**	0.0031
Beta33	**0.0158**	0.00343	**0.01081**	0.00397	**-0.00153**	0.0064	**-0.00061**	0.0051
Beta34	**0**	0.00262	**-0.00387**	0.0024	**0.00178**	0.0042	**-0.00053**	0.0029
Beta41	**0.00204**	0.00102	**0.00568**	0.00101	**-0.00009**	0.0011	**-0.01389**	0.001
Beta42	**-0.01332**	0.00238	**-0.01665**	0.00298	**-0.00159**	0.0037	**0.02947**	0.0032
Beta43	**0.01183**	0.00261	**0.02224**	0.00469	**-0.00164**	0.0059	**-0.01138**	0.006
Beta44	**-0.00223**	0.00169	**-0.01789**	0.00258	**-0.00025**	0.0035	**-0.0059**	0.004

Table 3.25b) U.K. Spot Rates, The Diffusion Coefficients Estimates, Four-Factor Models

Param	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.21811**	0.04274	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.02441**	0.34311	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0***	28.65801	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.09501**	0.26365	**0**	N/A	**0.5**	N/A	**1**	N/A

Sigma1	**0.00083**	0.03155	**0.00037**	0.00739	**0.0026**	0.0077	**0.02843**	0.01235
Sigma2	**0.00055**	0.0295	**0.00049**	0.00864	**0.0029**	0.0092	**0.01762**	0.01221
Sigma3	**0.00048**	0.00907	**0.00046**	0.00867	**0.00241**	0.0092	**0.00914**	0.00666
Sigma4	**0.00061**	0.07252	**0.00044**	0.00858	**0.00221**	0.009	**0.01013**	0.01081
Corr12	**0.66703**	0.00649	**0.63917**	0.0067	**0.65498**	0.0119	**0.61379**	0.01016
Corr13	**0.51899**	0.00891	**0.47226**	0.00927	**0.54301**	0.0123	**0.29423**	0.0106
Corr14	**0.42898**	0.0101	**0.37866**	0.0104	**0.4678**	0.0123	**-0.09382**	0.01903
Corr23	**0.93557**	0.00151	**0.92943**	0.00158	**0.94274**	0.0117	**0.67929**	0.00932
Corr24	**0.82032**	0.00385	**0.80745**	0.00396	**0.83423**	0.0116	**0.07866**	0.02137
Corr34	**0.94505**	0.00113	**0.94214**	0.00114	**0.94757**	0.0106	**0.74792**	0.00749
Log LF	**105,776.12**	**N/A**	**105,661.29**	229.66+	**104,941.82**	1,668.60+	**100,376.30**	10,799.62+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the corresponding values of the LR test statistics ($\chi^2_{crit}(4df,1\%) = 13.28$).

The estimates for the level effect parameters in the unrestricted CKLS model across all the data sets are also presented in Table 3.26 below. Within the money-market context there are similarities between the U.K and Japan on one side, and between the U.S., the Eurozone and Canada on the other side. For the U.K and Japan, the shortest maturity rates (one-week and one-month) exhibit the highest dependence of the volatility on the level of interest rates, while for the U.S, Japan and Canada this happens for the six- and twelve-month LIBOR rates. In the case of the U.K. spot rates, only the first component of the level effect parameter significant, indicating that from the restricted models then Vasicek specification may explain the dynamics of the data as close as the general CKLS model. The level effect parameter for the 15-year U.K spot rates is $\gamma = 0.000004$ suggesting a constant conditional volatility for the process of these time-series.

Table 3.26 The Estimates for the Level - Effect Parameter for Four-Factor CKLS models.

CKLS	GBP-LIBOR	USD-LIBOR	EURLIBOR	JPY-LIBOR	CAD-LIBOR	UK Spot
Gamma1	1.5940388	0.9751338	0.6949800	1.3058910	0.6551755	0.218114
Gamma2	1.2236800	0.8091528	0.6656462	1.2037392	0.5921455	0.024406
Gamma3	1.0308315	0.9944530	0.7575482	0.8736540	0.6701456	0.000004
Gamma4	1.3951253	0.9880993	1.0421511	0.8042529	0.7797263	0.095013

3.5.2 Estimation Results for the Five-Factor Continuous-Time Models

The second stage in the estimation corresponds to the extension to five-factor continuous-time models presented in Section 3.5. The fifth factor added to the previous four-factor specifications is chosen as the three-month LIBOR and the 10-year UK nominal rate time series for the short end and the long end of the yield curve, respectively. As a result, the number of parameters increases to fifty for the CKLS model and to forty-five for the restricted models. The QMLE estimates of the parameters for all the continuous-time models and the benchmark models are presented in Tables 3.27-3.32. Relative to the four-factor specifications, the five-factor models gain naturally more explanatory power and the estimation results confirmed that with considerably higher maxima of the log-likelihood functions compared to the four-factor models. The ranking among the continuous-time models has remained unchanged for each dataset, with the same nested models being designated as the best match to the data.

The new estimates for the level effect in all the unrestricted CKLS models are presented in Table 3.33. In comparison with the four-factor models these values seem to suggest a slightly lower degree of dependence of the variance on the level of the interest rate. The highest estimates for the level effect parameters are recorded for the GBP-LIBOR, USD-LIBOR and JPY-LIBOR time series, followed by the EUR-LIBOR and CAD-LIBOR rates., whereas for the U.K. spot rates the models do not support such a dependence. Based on the likelihood ratio tests (with a $\chi^2(5\text{df})$ distribution) all the restricted models are rejected against the general model CKLS. Under the CKLS model the drift parameters are the five-dimensional intercept vector α with most of its components being significant and the feedback matrix β of twenty-five components whose estimates produce evidence of feedback in most directions. With regard to the correlation between the five factors, the new factor appears to be of significant influence as the degree of its positive correlation with the six-month and twelve-month rates respectively is very high relative to the correlation coefficient between the two factors at the very short-term of the yield curve, namely the one-week and the one-month. In conclusion the last three factors, the three-, six- and twelve-month LIBOR rates move closely together implying that if any twists were to exist in the term structure of interest rates over the period 2000-2013, they should have occurred outside this three to twelve-month maturity zone.

For the U.K. spot rates in Table 3.32, the estimation results for the five-factor models consolidate the findings from the four-factor framework. The estimates of the level effect parameters are very close to zero implying a homoscedastic conditional variance for all the factors. Out of the five level effect parameters only $\gamma_1 = 0.20$ is statistically significant. Therefore, the Vasicek model is the most appropriate restricted model, a fact indicated by its second highest log-likelihood function value and a close to acceptance LR statistic value. The drift coefficients $\alpha_i (i = 1,...,5)$ are all insignificant, while among the elements of the feedback matrix there is evidence of highly significant feedbacks in both directions between three pairs of factors. They are the (7-year, 10-year) pair with a negative feedback coefficient from the 10-year to 7-year spot rates of $\beta_{23} = -0.11326$; the (7-year, 15-year) pair with bidirectional effects $\beta_{24} = 0.05517$ > $\beta_{42} = 0.01013$; the third pair is the (10-year, 25-year) pair with a stronger positive feedback coefficient from the 10-year interest rate to the 25-year interest rate than the one from the 25-year to the 10-year interest rates $\beta_{53} = 0.04617 > \beta_{35} = 0.01884$. The correlation coefficients estimates are all highly significant and positive with the highest values ($\rho_{23} = 0.98312$ and $\rho_{34} = 0.97802$) being realised consistently across the models for two pairs of maturities: (7-year, 10-year) and (10-year, 15-year), respectively. This observation is consistent with the feedback results and highlights the importance of the new factor introduced in the models - the 10-year maturity spot rates, which corresponds to a crucial position on the term structure of interest rates given the fact that the 10-year U.K. discount bond market is one of the most liquid ones.

Table 3.27a) GBP-LIBOR, The Diffusion Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**-0.00002**	0.00000	**-0.00010**	0.00006	**-0.00010**	0.00003	**0.00014**	0.00001
Alpha2	**-0.00001**	0.00000	**-0.00004**	0.00001	**0.00001**	0.00001	**0.00003**	0.00000
Alpha3	**-0.00003**	0.00000	**-0.00006**	0.00001	**0.00001**	0.00000	**0.00000**	0.00000
Alpha4	**-0.00001**	0.00000	**0.00001**	0.00001	**0.00012**	0.00000	**0.00009**	0.00000
Alpha5	**-0.00001**	0.00000	**0.00009**	0.00001	**0.00029**	0.00001	**0.00017**	0.00000
Beta11	**-0.02037**	0.00301	**-0.19926**	0.00959	**-0.36694**	0.00623	**-0.18496**	0.00768
Beta12	**0.03718**	0.00431	**0.32199**	0.01385	**0.61832**	0.00335	**0.25828**	0.01088
Beta13	**-0.06067**	0.00532	**-0.36988**	0.00000	**-0.44243**	0.03253	**-0.08394**	0.01404
Beta14	**0.04843**	0.00635	**0.32819**	0.00000	**0.18706**	0.05246	**0.01521**	0.01646
Beta15	**-0.00838**	0.00239	**-0.08213**	0.00834	**0.00030**	0.02286	**-0.01118**	0.00494
Beta21	**0.01926**	0.00119	**-0.00370**	0.00302	**-0.01687**	0.00222	**-0.00215**	0.00282
Beta22	**-0.01013**	0.00167	**0.01747**	0.00487	**0.06387**	0.00304	**0.01744**	0.00377
Beta23	**-0.02973**	0.00086	**-0.04805**	0.00267	**-0.05254**	0.00724	**-0.01527**	0.00334
Beta24	**0.02498**	0.00000	**0.04754**	0.00497	**-0.00678**	0.01036	**0.00096**	0.00390
Beta25	**-0.00483**	0.00000	**-0.01260**	0.00291	**0.01210**	0.00444	**-0.00175**	0.00123
Beta31	**0.02411**	0.00084	**0.01384**	0.00223	**0.00512**	0.00198	**-0.00060**	0.00194
Beta32	**-0.01673**	0.00101	**-0.00832**	0.00369	**0.02117**	0.00284	**0.01064**	0.00272
Beta33	**0.00151**	0.00047	**-0.00905**	0.00361	**0.00929**	0.00171	**0.02111**	0.00233
Beta34	**-0.02309**	0.00079	**-0.00018**	0.00440	**-0.06084**	0.00270	**-0.05020**	0.00288
Beta35	**0.01462**	0.00034	**0.00481**	0.00174	**0.02522**	0.00166	**0.01883**	0.00120

Beta35	**0.01462**	0.00034	**0.00481**	0.00174	**0.02522**	0.00166	**0.01883**	0.00120
Beta41	**0.01781**	0.00109	**0.01056**	0.00237	**0.00243**	0.00239	**-0.00274**	0.00203
Beta42	**-0.00388**	0.00142	**-0.00756**	0.00378	**0.02066**	0.00316	**0.01327**	0.00268
Beta43	**-0.02408**	0.00145	**0.00923**	0.00216	**-0.00187**	0.00216	**-0.01403**	0.00161
Beta44	**0.00958**	0.00134	**-0.01287**	0.00373	**-0.01579**	0.00427	**0.02190**	0.00204
Beta45	**0.00073**	0.00038	**0.00071**	0.00203	**-0.00710**	0.00200	**-0.01992**	0.00079
Beta51	**0.01170**	0.00177	**0.00273**	0.00289	**0.00492**	0.00346	**0.01474**	0.00323
Beta52	**0.00130**	0.00232	**-0.00021**	0.00433	**-0.00182**	0.00423	**-0.02395**	0.00411
Beta53	**-0.01405**	0.00355	**0.03247**	0.00428	**0.03635**	0.00486	**0.03401**	0.00491
Beta54	**-0.00432**	0.00339	**-0.03419**	0.00815	**-0.00305**	0.00970	**-0.00078**	0.00723
Beta55	**0.00540**	0.00091	**-0.00222**	0.00401	**-0.04090**	0.00441	**-0.02682**	0.00291

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.27b) GBP-LIBOR, The Diffusion Coefficients Estimates, Five-Factor Models

Param.	**CKLS**	S.E.	**VASICEK**	S.E.	**CIR**	S.E.	**BS**	S.E.
Gamma1	**1.58807**	0.00299	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**1.22215**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.86831**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.92924**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma5	**1.27792**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Sigma1	**0.20505**	0.02381	**0.00118**	0.00001	**0.00668**	0.00005	**0.02832**	0.00032
Sigma2	**0.01763**	0.00267	**0.00035**	0.00000	**0.00158**	0.00001	**0.00865**	0.00009
Sigma3	**0.00397**	0.00824	**0.00026**	0.00000	**0.00105**	0.00001	**0.00540**	0.00004
Sigma4	**0.00506**	0.00838	**0.00027**	0.00000	**0.00120**	0.00001	**0.00563**	0.00004
Sigma5	**0.01849**	0.00000	**0.00034**	0.00000	**0.00171**	0.00002	**0.00794**	0.00009
Corr12	**0.55962**	0.01203	**0.54664**	0.00834	**0.62411**	0.00826	**0.58063**	0.01513
Corr13	**0.30440**	0.01230	**0.32261**	0.01112	**0.17589**	0.01331	**0.35995**	0.01051
Corr14	**0.24132**	0.01230	**0.24304**	0.01183	**-0.06537**	0.01537	**0.21677**	0.01302
Corr15	**0.18952**	0.01239	**0.19756**	0.01213	**-0.22488**	0.01598	**0.04715**	0.02028
Corr23	**0.83978**	0.01102	**0.84476**	0.00316	**0.69596**	0.00819	**0.73144**	0.00642
Corr24	**0.75200**	0.01173	**0.74675**	0.00522	**0.40506**	0.01396	**0.39348**	0.01295
Corr25	**0.62118**	0.01207	**0.60850**	0.00768	**0.12289**	0.01625	**-0.00006**	0.01639
Corr34	**0.92663**	0.01126	**0.93030**	0.00156	**0.86624**	0.00390	**0.80420**	0.00607
Corr35	**0.79402**	0.01120	**0.78430**	0.00448	**0.63241**	0.00895	**0.45625**	0.01324
Corr45	**0.92734**	0.00906	**0.92838**	0.00144	**0.90443**	0.00224	**0.84470**	0.00377
LogLF	**145,178.45**	N/A	**137,767.67**	14,821.57+	**141,026.26**	8,304.39+	**142,591.04**	5,174.82+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.28a) USD-LIBOR, The Drift Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**0.00002**	0.00000	**0.00009**	0.00001	**0.00000**	0.00000	**-0.00001**	0.00000
Alpha2	**0.00003**	0.00000	**0.00002**	0.00001	**-0.00005**	0.00000	**0.00000**	0.00000
Alpha3	**0.00002**	0.00000	**0.00000**	0.00000	**-0.00004**	0.00000	**0.00001**	0.00000
Alpha4	**0.00003**	0.00000	**-0.00004**	0.00001	**-0.00002**	0.00000	**0.00005**	0.00000
Alpha5	**0.00006**	0.00000	**-0.00008**	0.00001	**0.00002**	0.00000	**0.00012**	0.00000
Beta11	**0.04633**	0.00291	**-0.01089**	0.00404	**0.05643**	0.00369	**0.05212**	0.00243
Beta12	**-0.05349**	0.00322	**0.01760**	0.00382	**-0.05905**	0.00536	**-0.06412**	0.00312
Beta13	**0.00121**	0.00300	**-0.03192**	0.00424	**-0.02191**	0.00618	**0.02118**	0.00268
Beta14	**0.00890**	0.00308	**0.04608**	0.00330	**0.03423**	0.00507	**-0.01901**	0.00241
Beta15	**-0.00520**	0.00101	**-0.02327**	0.00212	**-0.01044**	0.00099	**0.00881**	0.00076
Beta21	**0.07786**	0.00423	**0.05280**	0.00239	**0.07930**	0.00292	**0.07869**	0.00277
Beta22	**-0.07926**	0.00385	**-0.04865**	0.00482	**-0.07199**	0.00397	**-0.08283**	0.00339
Beta23	**-0.01049**	0.00362	**-0.01631**	0.00563	**-0.03574**	0.00420	**0.00875**	0.00219
Beta24	**0.01841**	0.00425	**0.01333**	0.00372	**0.02592**	0.00400	**-0.01151**	0.00200
Beta25	**-0.00768**	0.00137	**-0.00181**	0.00113	**0.00316**	0.00146	**0.00662**	0.00072
Beta31	**0.02815**	0.00188	**0.03741**	0.00201	**0.03272**	0.00234	**0.00696**	0.00283
Beta32	**-0.02676**	0.00086	**-0.03386**	0.00360	**-0.03449**	0.00322	**0.00442**	0.00377
Beta33	**0.00238**	0.00247	**-0.00339**	0.00460	**-0.00027**	0.00322	**-0.00048**	0.00275
Beta34	**-0.00402**	0.00327	**-0.01458**	0.00385	**-0.00671**	0.00282	**-0.01735**	0.00215
Beta35	**-0.00092**	0.00116	**0.01395**	0.00132	**0.00895**	0.00089	**0.00604**	0.00068
Beta41	**0.01011**	0.00106	**0.02187**	0.00257	**-0.00087**	0.00295	**0.03290**	0.00383
Beta42	**-0.01486**	0.00144	**-0.02491**	0.00338	**-0.00321**	0.00369	**-0.01795**	0.00535
Beta43	**0.01874**	0.00154	**0.01596**	0.00192	**0.00331**	0.00262	**-0.01728**	0.00477
Beta44	**-0.01372**	0.00335	**-0.04384**	0.00283	**-0.00091**	0.00270	**0.01040**	0.00367
Beta45	**-0.00186**	0.00142	**0.03071**	0.00161	**0.00143**	0.00089	**-0.00912**	0.00101
Beta51	**-0.00162**	0.00000	**0.00062**	0.00392	**-0.01149**	0.00471	**0.06487**	0.00565
Beta52	**0.00334**	0.00000	**0.02873**	0.00489	**-0.00527**	0.00621	**-0.03701**	0.00835
Beta53	**0.00045**	0.00384	**-0.06087**	0.00281	**0.01171**	0.00660	**-0.06965**	0.00902
Beta54	**0.01035**	0.00508	**0.00311**	0.00530	**0.02403**	0.00659	**0.08471**	0.00699
Beta55	**-0.01461**	0.00191	**0.02871**	0.00295	**-0.02024**	0.00213	**-0.04526**	0.00181

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.28b) USD-USD, The Diffusion Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.97443**	0.00450	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.74230**	0.01073	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.77727**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.72288**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma5	**0.62682**	0.00001	**0**	N/A	**0.5**	N/A	**1**	N/A
Sigma1	**0.01907**	0.00048	**0.00053**	0.00000	**0.00291**	0.00002	**0.02110**	0.00012
Sigma2	**0.00493**	0.00025	**0.00031**	0.00000	**0.00186**	0.00002	**0.01496**	0.00012
Sigma3	**0.00441**	0.00002	**0.00027**	0.00000	**0.00137**	0.00001	**0.01096**	0.00010
Sigma4	**0.00426**	0.00003	**0.00031**	0.00000	**0.00164**	0.00001	**0.01297**	0.00011
Sigma5	**0.00408**	0.00000	**0.00042**	0.00000	**0.00255**	0.00002	**0.01758**	0.00015
Corr12	**0.54332**	0.01004	**0.59918**	0.00712	**0.56286**	0.00787	**0.48783**	0.00861
Corr13	**0.45298**	0.00948	**0.47752**	0.00894	**0.42129**	0.00892	**0.40063**	0.01002
Corr14	**0.33784**	0.01072	**0.33543**	0.01069	**0.17508**	0.01168	**0.27185**	0.01156
Corr15	**0.20874**	0.01157	**0.19556**	0.01190	**-0.02912**	0.01337	**0.14946**	0.01235
Corr23	**0.79032**	0.00559	**0.83575**	0.00312	**0.72634**	0.00549	**0.73763**	0.00488
Corr24	**0.64449**	0.00876	**0.67366**	0.00642	**0.36762**	0.01134	**0.58553**	0.00797
Corr25	**0.44871**	0.01066	**0.44350**	0.00978	**0.02147**	0.01353	**0.40191**	0.01036
Corr34	**0.86513**	0.00285	**0.87022**	0.00284	**0.76658**	0.00541	**0.84247**	0.00353
Corr35	**0.67174**	0.00634	**0.65536**	0.00680	**0.44530**	0.01063	**0.64478**	0.00708
Corr45	**0.91200**	0.00175	**0.90093**	0.00202	**0.86997**	0.00284	**0.91023**	0.00179
LogLF	**143,684.87**	N/A	**137,729.50**	11,910.75	**142,015.79**	3,338.16	**142,784.54**	1,800.68

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.29a) EUR-LIBOR, The Drift Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**0.00001**	0.00000	**-0.00007**	0.00001	**0.00001**	0.00000	**-0.00001**	0.00000
Alpha2	**-0.00001**	0.00000	**-0.00007**	0.00001	**0.00000**	0.00000	**0.00000**	0.00000
Alpha3	**-0.00002**	0.00000	**-0.00007**	0.00000	**-0.00003**	0.00000	**-0.00004**	0.00000
Alpha4	**-0.00002**	0.00000	**-0.00006**	0.00000	**-0.00007**	0.00000	**0.00000**	0.00000
Alpha5	**0.00001**	0.00000	**-0.00001**	0.00001	**-0.00006**	0.00000	**0.00002**	0.00000
Beta11	**-0.00784**	0.00423	**-0.04455**	0.00329	**0.01707**	0.00395	**-0.04740**	0.00385
Beta12	**0.01909**	0.00647	**0.07386**	0.00588	**-0.01992**	0.00663	**0.07865**	0.00523
Beta13	**-0.02390**	0.00492	**-0.03834**	0.01331	**0.00163**	0.00644	**-0.04632**	0.00246
Beta14	**0.01990**	0.00312	**-0.00675**	0.01495	**0.00064**	0.00528	**0.01004**	0.00143
Beta15	**-0.00800**	0.00116	**0.01706**	0.00525	**-0.00011**	0.00178	**0.00310**	0.00058
Beta21	**0.02220**	0.00195	**0.01219**	0.00231	**0.03511**	0.00234	**-0.01554**	0.00291
Beta22	**-0.01604**	0.00185	**-0.01142**	0.00343	**-0.02049**	0.00389	**0.03999**	0.00409
Beta23	**-0.01694**	0.00249	**0.00343**	0.00325	**-0.03650**	0.00397	**-0.03872**	0.00233
Beta24	**0.01038**	0.00165	**-0.02478**	0.00405	**0.02368**	0.00420	**0.01462**	0.00158
Beta25	**0.00066**	0.00086	**0.02197**	0.00194	**-0.00141**	0.00201	**-0.00089**	0.00060
Beta31	**0.01074**	0.00000	**0.01037**	0.00150	**0.02863**	0.00180	**0.02082**	0.00195
Beta32	**0.00604**	0.00000	**-0.00185**	0.00271	**-0.00710**	0.00285	**-0.02375**	0.00290
Beta33	**-0.03212**	0.00000	**-0.00893**	0.00381	**-0.03440**	0.00260	**0.01979**	0.00208
Beta34	**0.01346**	0.00201	**-0.02017**	0.00417	**-0.00200**	0.00249	**-0.04219**	0.00150
Beta35	**0.00267**	0.00136	**0.02224**	0.00160	**0.01580**	0.00101	**0.02578**	0.00053
Beta41	**0.01738**	0.00141	**0.00372**	0.00177	**0.03441**	0.00197	**0.00018**	0.00195
Beta42	**-0.01523**	0.00178	**0.00340**	0.00305	**-0.02616**	0.00292	**0.01415**	0.00310
Beta43	**-0.01413**	0.00177	**0.00815**	0.00279	**0.00743**	0.00212	**-0.02487**	0.00286
Beta44	**0.01802**	0.00243	**-0.04247**	0.00209	**-0.05789**	0.00247	**0.01663**	0.00212
Beta45	**-0.00544**	0.00165	**0.02832**	0.00103	**0.04350**	0.00121	**-0.00610**	0.00070
Beta51	**0.02408**	0.00379	**-0.00407**	0.00283	**0.03623**	0.00299	**0.04432**	0.00316
Beta52	**-0.03643**	0.00448	**0.01088**	0.00439	**-0.02665**	0.00489	**-0.06520**	0.00531
Beta53	**-0.00229**	0.00666	**-0.00677**	0.00283	**-0.00789**	0.00567	**0.02773**	0.00610
Beta54	**0.03945**	0.00473	**0.00056**	0.00359	**-0.03255**	0.00624	**0.00173**	0.00558
Beta55	**-0.02486**	0.00113	**-0.00027**	0.00256	**0.03220**	0.00244	**-0.00888**	0.00205

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.29b) EUR-LIBOR, The Diffusion Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.67597**	0.00339	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.65892**	0.00400	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.62624**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.72557**	0.00201	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma5	**1.02395**	0.01769	**0**	N/A	**0.5**	N/A	**1**	N/A
Sigma1	**0.00487**	0.00012	**0.00038**	0.00000	**0.00232**	0.00001	**0.02578**	0.00026
Sigma2	**0.00244**	0.00005	**0.00022**	0.00000	**0.00129**	0.00001	**0.01367**	0.00013
Sigma3	**0.00152**	0.00001	**0.00016**	0.00000	**0.00093**	0.00001	**0.00721**	0.00006
Sigma4	**0.00231**	0.00004	**0.00018**	0.00000	**0.00100**	0.00001	**0.00628**	0.00005
Sigma5	**0.00932**	0.00066	**0.00027**	0.00000	**0.00143**	0.00001	**0.00857**	0.00007
Corr12	**0.62617**	0.00739	**0.49311**	0.00850	**0.55880**	0.00753	**0.78655**	0.00469
Corr13	**0.50961**	0.00888	**0.43169**	0.00957	**0.44953**	0.00934	**0.57559**	0.00847
Corr14	**0.39580**	0.01019	**0.32943**	0.01083	**0.32874**	0.01069	**0.35471**	0.01145
Corr15	**0.25590**	0.01153	**0.20143**	0.01177	**0.19033**	0.01161	**-0.04313**	0.01380
Corr23	**0.76720**	0.00593	**0.70774**	0.00539	**0.75117**	0.00471	**0.72627**	0.00615
Corr24	**0.62560**	0.00704	**0.57580**	0.00778	**0.59530**	0.00758	**0.45690**	0.01075
Corr25	**0.42005**	0.01057	**0.37850**	0.01027	**0.38705**	0.01017	**-0.00830**	0.01375
Corr34	**0.86467**	0.00287	**0.87440**	0.00259	**0.85894**	0.00297	**0.76083**	0.00525
Corr35	**0.64241**	0.00858	**0.64260**	0.00675	**0.63044**	0.00693	**0.36155**	0.01166
Corr45	**0.86715**	0.00412	**0.85490**	0.00284	**0.85916**	0.00267	**0.74360**	0.00554
LogLF	**147,889.62**	N/A	**142,985.74**	9,807.75+	**147,122.62**	1,534.00+	**144,380.65**	7,017.94+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df,1\%) = 15.09$).

Table 3.30a) JPY-IBOR: The Drift Coefficients Estimates, Five-Factor Models

Param.	**CKLS**	**S.E.**	**VASICEK**	**S.E.**	**CIR**	**S.E.**	BS	**S.E.**
Alpha1	**0.00002**	0.00000	**0.00002**	0.00001	**0.00002**	0.00000	**0.00001**	0.00000
Alpha2	**0.00000**	0.00000	**-0.00001**	0.00000	**0.00002**	0.00000	**0.00001**	0.00000
Alpha3	**0.00000**	0.00000	**0.00000**	0.00000	**0.00001**	0.00000	**0.00000**	0.00000
Alpha4	**0.00000**	0.00000	**0.00000**	0.00000	**0.00001**	0.00000	**0.00000**	0.00000
Alpha5	**0.00000**	0.00000	**0.00000**	0.00000	**0.00001**	0.00000	**0.00000**	0.00000
Beta11	**-0.14052**	0.00491	**-0.15649**	0.00888	**-0.17006**	0.00664	**-0.11679**	0.00530
Beta12	**0.04135**	0.00548	**0.16344**	0.01259	**0.17347**	0.00860	**0.05253**	0.00629
Beta13	**0.00758**	0.00648	**-0.26951**	0.02642	**-0.09971**	0.01221	**-0.00198**	0.00716
Beta14	**0.01397**	0.00631	**0.40466**	0.03730	**0.05110**	0.01216	**0.01393**	0.00695
Beta15	**0.00222**	0.00240	**-0.16592**	0.01672	**-0.00014**	0.00461	**-0.00060**	0.00281
Beta21	**-0.00293**	0.00445	**-0.02522**	0.00358	**-0.03223**	0.00404	**0.00748**	0.00402
Beta22	**-0.04205**	0.00514	**0.00407**	0.00496	**0.08508**	0.00534	**-0.04409**	0.00501
Beta23	**0.01959**	0.00661	**-0.00426**	0.00990	**-0.07300**	0.00686	**0.00845**	0.00458
Beta24	**0.00073**	0.00731	**0.01576**	0.01334	**-0.00452**	0.00771	**0.00776**	0.00444
Beta25	**0.00236**	0.00270	**0.00022**	0.00557	**0.01619**	0.00317	**-0.00012**	0.00181
Beta31	**0.00331**	0.00251	**0.00320**	0.00145	**0.01884**	0.00225	**0.00893**	0.00246
Beta32	**-0.00720**	0.00285	**-0.00205**	0.00198	**0.00392**	0.00306	**0.00199**	0.00330
Beta33	**-0.01670**	0.00829	**0.00363**	0.00360	**-0.00550**	0.00443	**-0.02167**	0.00369
Beta34	**0.02059**	0.00978	**-0.01577**	0.00460	**-0.01440**	0.00501	**-0.00182**	0.00369
Beta35	**-0.00549**	0.00354	**0.01044**	0.00189	**0.00177**	0.00205	**0.00625**	0.00148
Beta41	**0.00290**	0.00187	**0.00565**	0.00114	**0.01470**	0.00190	**0.00854**	0.00198
Beta42	**-0.00393**	0.00201	**0.00134**	0.00157	**0.01071**	0.00262	**-0.01794**	0.00277
Beta43	**-0.00505**	0.00829	**-0.00385**	0.00287	**-0.00443**	0.00419	**0.00733**	0.00372
Beta44	**0.00271**	0.01023	**-0.00679**	0.00370	**-0.00802**	0.00490	**-0.00272**	0.00404
Beta45	**0.00104**	0.00373	**0.00497**	0.00154	**-0.00547**	0.00200	**0.00076**	0.00160
Beta51	**0.00389**	0.00176	**0.00163**	0.00118	**0.01379**	0.00168	**0.00105**	0.00163
Beta52	**-0.00588**	0.00193	**-0.00027**	0.00162	**-0.00308**	0.00234	**-0.01977**	0.00228
Beta53	**-0.00421**	0.00846	**0.00356**	0.00304	**0.00247**	0.00413	**0.01271**	0.00367
Beta54	**0.00303**	0.01063	**-0.00834**	0.00408	**0.00226**	0.00502	**0.01149**	0.00439
Beta55	**0.00090**	0.00391	**0.00361**	0.00177	**-0.01071**	0.00208	**-0.00899**	0.00177

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.30b) JPY-LIBOR, The Diffusion Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**1.37158**	0.00645	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**1.11087**	0.00578	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.77445**	0.01178	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.56626**	0.01499	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma5	**0.63635**	0.01416	**0**	N/A	**0.5**	N/A	**1**	N/A
Sigma1	**0.82750**	0.03968	**0.00040**	0.00000	**0.00490**	0.00000	**0.07714**	0.00050
Sigma2	**0.08341**	0.00344	**0.00016**	0.00000	**0.00235**	0.00002	**0.03997**	0.00027
Sigma3	**0.00576**	0.00046	**0.00007**	0.00000	**0.00129**	0.00001	**0.02334**	0.00017
Sigma4	**0.00123**	0.00012	**0.00005**	0.00000	**0.00107**	0.00001	**0.01790**	0.00014
Sigma5	**0.00159**	0.00014	**0.00005**	0.00000	**0.00092**	0.00001	**0.01351**	0.00011
Corr12	**0.56133**	0.00826	**0.45298**	0.01089	**0.50598**	0.01082	**0.53260**	0.00860
Corr13	**0.33453**	0.01107	**0.19795**	0.01202	**0.11057**	0.01774	**0.22528**	0.01205
Corr14	**0.23346**	0.01176	**0.02378**	0.01298	**-0.06732**	0.01838	**-0.00661**	0.01265
Corr15	**0.15754**	0.01220	**-0.03859**	0.01371	**-0.08078**	0.01790	**-0.07636**	0.01280
Corr23	**0.61076**	0.00756	**0.42977**	0.00988	**0.56975**	0.01154	**0.54858**	0.00798
Corr24	**0.48595**	0.00953	**0.19456**	0.01232	**0.40697**	0.01549	**0.36286**	0.01095
Corr25	**0.41054**	0.01049	**-0.00022**	0.01268	**0.37029**	0.01521	**0.30088**	0.01172
Corr34	**0.75046**	0.00527	**0.73137**	0.00506	**0.82975**	0.00458	**0.65563**	0.00665
Corr35	**0.62742**	0.00716	**0.55077**	0.00828	**0.76917**	0.00577	**0.51283**	0.00891
Corr45	**0.80587**	0.00536	**0.81863**	0.00347	**0.89141**	0.00281	**0.72757**	0.00490
LogLF	**167,763.45**	N/A	**154,236.80**	27,053.31+	**162,272.20**	10,982.50+	**165,906.16**	3,714.59+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df,1\%)=15.09$).

Table 3.31a) CAD-LIBOR, The Drift Coefficients Estimates, Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**-0.00022**	0.00000	**-0.00019**	0.00002	**-0.00003**	0.00001	**0.00004**	0.00000
Alpha2	**-0.00015**	0.00000	**-0.00007**	0.00001	**-0.00003**	0.00001	**0.00003**	0.00000
Alpha3	**-0.00015**	0.00000	**0.00005**	0.00001	**-0.00005**	0.00001	**-0.00006**	0.00000
Alpha4	**-0.00008**	0.00000	**0.00009**	0.00001	**0.00016**	0.00000	**0.00003**	0.00000
Alpha5	**0.00000**	0.00000	**0.00019**	0.00001	**0.00026**	0.00001	**0.00013**	0.00001
Beta11	**-0.01226**	0.01037	**-0.11595**	0.00497	**-0.13022**	0.00577	**-0.07274**	0.00546
Beta12	**0.01062**	0.01128	**0.19845**	0.00532	**0.21271**	0.00835	**0.09864**	0.00730
Beta13	**0.00440**	0.00796	**-0.14192**	0.00829	**-0.11261**	0.00782	**-0.03231**	0.00539
Beta14	**-0.03467**	0.00589	**0.05277**	0.00957	**0.02787**	0.00567	**0.00882**	0.00301
Beta15	**0.03549**	0.00138	**0.01028**	0.00430	**0.00215**	0.00236	**-0.00524**	0.00084
Beta21	**0.12865**	0.00402	**0.02030**	0.00419	**-0.06513**	0.00460	**-0.01636**	0.00531
Beta22	**-0.23533**	0.00172	**-0.02470**	0.00859	**0.10165**	0.00675	**0.02748**	0.00706
Beta23	**0.16053**	0.00144	**-0.03050**	0.00911	**-0.02311**	0.00661	**-0.01342**	0.00524
Beta24	**-0.08475**	0.00300	**0.04259**	0.00642	**-0.03222**	0.00444	**0.00207**	0.00297
Beta25	**0.03318**	0.00145	**-0.00659**	0.00331	**0.01860**	0.00149	**-0.00148**	0.00083
Beta31	**0.07693**	0.00341	**-0.00406**	0.00371	**0.01287**	0.00324	**0.00002**	0.00376
Beta32	**-0.08896**	0.00368	**-0.00323**	0.00540	**-0.00726**	0.00467	**0.01470**	0.00560
Beta33	**-0.00196**	0.00493	**0.00388**	0.00602	**0.02702**	0.00438	**-0.01530**	0.00474
Beta34	**-0.00466**	0.00373	**0.00853**	0.00449	**-0.05917**	0.00270	**-0.01256**	0.00284
Beta35	**0.02132**	0.00134	**-0.00681**	0.00124	**0.02696**	0.00116	**0.01384**	0.00083
Beta41	**0.05756**	0.00153	**0.00554**	0.00459	**-0.00347**	0.00419	**-0.01973**	0.00405
Beta42	**-0.09612**	0.00256	**-0.01681**	0.00572	**0.02793**	0.00776	**0.01850**	0.00669
Beta43	**0.05719**	0.00370	**0.03596**	0.00503	**0.00016**	0.00718	**0.01818**	0.00699
Beta44	**-0.03568**	0.00217	**-0.03058**	0.00468	**-0.02360**	0.00431	**-0.02117**	0.00454
Beta45	**0.01777**	0.00145	**0.00320**	0.00214	**-0.00488**	0.00172	**0.00259**	0.00154
Beta51	**0.03118**	0.00221	**0.02224**	0.00592	**0.03278**	0.00561	**0.00021**	0.00540
Beta52	**-0.03247**	0.00627	**-0.03672**	0.00713	**-0.00079**	0.01102	**-0.00520**	0.00919
Beta53	**-0.00718**	0.00548	**0.04023**	0.00805	**-0.01908**	0.01066	**0.01487**	0.01094
Beta54	**0.00442**	0.00230	**-0.02217**	0.00833	**0.00155**	0.00579	**0.00120**	0.00742
Beta55	**0.00320**	0.00155	**-0.00821**	0.00347	**-0.02035**	0.00203	**-0.01475**	0.00231

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.31b) CAD-LIBOR, The Diffusion Coefficients Estimates, Five-Factor Models

Param	**CKLS**	**S.E.**	**VASICEK**	**S.E.**	**CIR**	**S.E.**	**BS**	**S.E.**
Gamma1	**0.656942**	0.014533	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.515675**	0.000000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.424173**	0.000000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.423111**	0.000000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma5	**0.479316**	0.000000	**0**	N/A	**0.5**	N/A	**1**	N/A
Sigma1	**0.003856**	0.000260	**0.000366**	0.000002	**0.002175**	0.000015	**0.016834**	0.000130
Sigma2	**0.001920**	0.000000	**0.000304**	0.000002	**0.001753**	0.000013	**0.014500**	0.000114
Sigma3	**0.001085**	0.000008	**0.000225**	0.000002	**0.001413**	0.000012	**0.009024**	0.000064
Sigma4	**0.001253**	0.000005	**0.000281**	0.000002	**0.001725**	0.000015	**0.009830**	0.000078
Sigma5	**0.001863**	0.000000	**0.000371**	0.000003	**0.002116**	0.000018	**0.011689**	0.000097
Corr12	**0.552287**	0.009283	**0.482036**	0.000002	**0.564749**	0.008167	**0.665071**	0.006305
Corr13	**0.492859**	0.011125	**0.379991**	0.000002	**0.406618**	0.010188	**0.452773**	0.009345
Corr14	**0.334355**	0.010991	**0.123280**	0.000002	**0.195941**	0.012250	**0.212895**	0.011976
Corr15	**0.241445**	0.011421	**-0.004915**	0.000002	**0.085582**	0.012700	**0.039235**	0.013081
Corr23	**0.651413**	0.006311	**0.438119**	0.000003	**0.649942**	0.006486	**0.513225**	0.009010
Corr24	**0.504028**	0.008624	**0.162689**	0.009325	**0.470072**	0.009633	**0.213147**	0.012161
Corr25	**0.391218**	0.010052	**-0.001304**	0.010321	**0.357145**	0.010916	**0.030215**	0.012801
Corr34	**0.808215**	0.003627	**0.780547**	0.012736	**0.796744**	0.004201	**0.695670**	0.005964
Corr35	**0.681028**	0.005894	**0.608534**	0.013483	**0.674240**	0.006432	**0.526868**	0.008950
Corr45	**0.881205**	0.002084	**0.877181**	0.009636	**0.891189**	0.002062	**0.847125**	0.002916
LogLF	**139,554.87**	N/A	**137,049.16**	0.012210	**139,055.08**	1,426.10	**137,668.73**	4,198.79

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

Table 3.32a) U.K. Spot Rates, The Drift Coefficients Estimates, Five-Factor Models

Param	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**-0.00001**	0.00000	**-0.00003**	0.00003	**0.00002**	0.00003	**0.00014**	0.00001
Alpha2	**0.00000**	0.00001	**0.00000**	0.00001	**0.00000**	0.00002	**-0.00003**	0.00001
Alpha3	**-0.00001**	0.00001	**0.00000**	0.00001	**0.00000**	0.00001	**-0.00011**	0.00000
Alpha4	**0.00023**	0.00000	**0.00017**	0.00002	**0.00030**	0.00003	**-0.00004**	0.00000
Alpha5	**0.00039**	0.00002	**0.00034**	0.00004	**0.00065**	0.00006	**0.00003**	0.00001
Beta11	**0.00043**	0.00084	**-0.00281**	0.00104	**-0.00231**	0.00065	**0.00026**	0.00077
Beta12	**-0.00180**	0.00105	**0.00139**	0.00684	**0.00091**	0.00549	**-0.01884**	0.00461
Beta13	**0.00600**	0.00000	**0.00398**	0.01127	**0.00256**	0.01131	**0.01847**	0.00913
Beta14	**-0.00615**	0.00290	**-0.00364**	0.00785	**-0.00330**	0.01040	**0.00816**	0.00723
Beta15	**0.00136**	0.00253	**0.00092**	0.00269	**0.00033**	0.00472	**-0.01398**	0.00210
Beta21	**0.00133**	0.00105	**-0.00065**	0.00111	**-0.00199**	0.00094	**0.00175**	0.00094
Beta22	**0.05825**	0.00241	**0.05796**	0.00392	**0.06238**	0.00277	**0.02277**	0.00000
Beta23	**-0.11326**	0.00244	**-0.10905**	0.00305	**-0.12070**	0.00473	**-0.06412**	0.00000
Beta24	**0.05517**	0.00325	**0.05248**	0.00534	**0.05999**	0.00399	**0.04116**	0.00369
Beta25	**-0.00153**	0.00186	**-0.00046**	0.00327	**-0.00027**	0.00168	**-0.00288**	0.00197
Beta31	**0.00121**	0.00107	**-0.00116**	0.00105	**-0.00121**	0.00093	**0.00084**	0.00084
Beta32	**0.02568**	0.00289	**0.03812**	0.00278	**0.02618**	0.00432	**0.01251**	0.00000
Beta33	**-0.03146**	0.00354	**-0.05404**	0.00264	**-0.03184**	0.00799	**-0.02681**	0.00000
Beta34	**-0.01381**	0.00260	**0.00358**	0.00670	**-0.01531**	0.00543	**-0.00042**	0.00415
Beta35	**0.01884**	0.00150	**0.01416**	0.00390	**0.02229**	0.00179	**0.01518**	0.00174
Beta41	**0.00009**	0.00100	**-0.00189**	0.00092	**-0.00109**	0.00091	**0.00001**	0.00077
Beta42	**0.01013**	0.00247	**0.01238**	0.00164	**0.00938**	0.00612	**0.00069**	0.00000
Beta43	**0.00029**	0.00176	**-0.00002**	0.00530	**0.00005**	0.01176	**-0.00065**	0.00415
Beta44	**-0.02287**	0.00000	**-0.02237**	0.00893	**-0.02389**	0.00859	**-0.01298**	0.00692
Beta45	**0.00706**	0.00132	**0.00827**	0.00481	**0.00865**	0.00349	**0.01284**	0.00258
Beta51	**0.00119**	0.00101	**-0.00008**	0.00091	**-0.00039**	0.00103	**0.00001**	0.00088
Beta52	**-0.01976**	0.00328	**-0.01747**	0.00385	**-0.02410**	0.00820	**-0.01001**	0.00454
Beta53	**0.04617**	0.00400	**0.04049**	0.00967	**0.05648**	0.01586	**0.00765**	0.01092
Beta54	**-0.03178**	0.00409	**-0.02323**	0.01180	**-0.03571**	0.01264	**0.00276**	0.01009
Beta55	**-0.00515**	0.00253	**-0.00772**	0.00599	**-0.01178**	0.00562	**-0.00238**	0.00350

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df,1\%)=15.09$).

Table 3.32b) U.K. Spot Rates, The Diffusion Coefficients Estimates, Five-Factor Models

Param	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.19900**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma2	**0.00000**	0.00000	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma3	**0.00016**	4.14475	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma4	**0.00952**	0.20682	**0**	N/A	**0.5**	N/A	**1**	N/A
Gamma5	**0.04091**	0.30036	**0**	N/A	**0.5**	N/A	**1**	N/A
Sigma1	**0.00074**	0.00000	**0.00037**	0.00000	**0.00249**	0.00002	**0.02633**	0.00018
Sigma2	**0.00051**	0.00856	**0.00046**	0.00000	**0.00275**	0.00002	**0.01735**	0.00015
Sigma3	**0.00052**	0.00902	**0.00046**	0.00000	**0.00260**	0.00002	**0.01443**	0.00012
Sigma4	**0.00051**	0.01085	**0.00044**	0.00000	**0.00234**	0.00002	**0.01180**	0.00010
Sigma5	**0.00053**	0.03971	**0.00042**	0.00000	**0.00219**	0.00002	**0.01056**	0.00841
Corr12	**0.56647**	0.01311	**0.57325**	0.00782	**0.58149**	0.00787	**0.54885**	0.00803
Corr13	**0.46949**	0.01353	**0.45310**	0.00966	**0.49937**	0.00934	**0.48992**	0.00900
Corr14	**0.40554**	0.01355	**0.34479**	0.01118	**0.43797**	0.01034	**0.44471**	0.00969
Corr15	**0.33187**	0.01334	**0.21527**	0.01311	**0.35463**	0.01141	**0.36821**	0.01072
Corr23	**0.98311**	0.01173	**0.97620**	0.00059	**0.98211**	0.00042	**0.98018**	0.00044
Corr24	**0.93958**	0.01297	**0.90329**	0.00284	**0.93015**	0.00183	**0.91998**	0.00207
Corr25	**0.83443**	0.01348	**0.73928**	0.00733	**0.80731**	0.00487	**0.77940**	0.00554
Corr34	**0.97802**	0.01351	**0.96401**	0.00113	**0.97382**	0.00072	**0.97061**	0.00082
Corr35	**0.88136**	0.01334	**0.81766**	0.00510	**0.86356**	0.00345	**0.84591**	0.00391
Corr45	**0.94914**	0.01201	**0.92690**	0.00179	**0.94364**	0.00130	**0.93683**	0.00155
LogLF	**138,495.64**	N/A	**138,482.50**	30.47+	**137,511.71**	1,972.05+	**134,136.69**	8,722.10+

Note: The cells marked with * contain values smaller than 10^{-6}; the cells marked with + contain the values of the LR test statistics, corresponding to the three restricted models ($\chi^2_{crit}(5df, 1\%) = 15.09$).

The table 3.33 bellow presents all the estimates of the level-effect parameters implied by the general five-factor CKLS model. The patterns observed previously for the four-factor framework are not as clear, quite reversing with regards to the one-year GBP- and USD -LIBOR rates. Therefore, it seems that the volatility structure across short-term maturities differs from the four- to the five-factor CKLS models.

Table 3.33 The Estimates for the Level -Effect Parameter Five-Factor CKLS Models

CKLS	GBP-LIBOR	USD-LIBOR	EURLIBOR	JPY-LIBOR	CAD-LIBOR	UK Spot
Gamma1	1.58807	0.97443	0.67597	1.37158	0.65694	0.19900
Gamma2	1.22215	0.74230	0.65892	1.11087	0.51568	0.00000
Gamma3	0.86831	0.77727	0.62624	0.77445	0.42417	0.00016
Gamma4	0.92924	0.72288	0.72557	0.56626	0.42311	0.00952
Gamma5	1.27792	0.62682	1.02395	0.63635	0.47932	0.04091

The results reported in the Table 3.34 show that for both extended specifications the ranking among the four continuous-time models in terms of goodness of fit remains unchanged. However, a consistent pattern can be observed for the money-market segment: the heterosckedasticity is strongly present and there is evidence of a substantially higher dependence of the conditional volatility on the actual level of the interest rates. For the longer maturity segment, represented by the U.K government nominal rates, the Vasicek is the first nested model against the other two nested models, CIR and BS respectively. As a conclusion, we can say that regardless the number of factors in the model, the money-market segment of interest rates is more elastic than the long-term segment.

Table 3.34 The Model Ranking in Terms of the Highest Value of the Likelihood Function.

FOUR-FACTOR CONTINUOUS-TIME MODELS						
Best Model	GBP-LIBOR	USD-LIBOR	EURLIBOR	JPY-LIBOR	CAD-LIBOR	UK Spot
Model1	CKLS	CKLS	CKLS	CKLS	CKLS	CKLS
Model2	BS	BS	CIR	BS	CIR	VASICEK
Model3	CIR	CIR	BS	CIR	BS	CIR
Model4	VASICEK	VASICEK	VASICEK	VASICEK	VASICEK	BS
FIVE-FOUR CONTINUOUS-TIME MODELS						
Best Model	GBP-LIBOR	USD-LIBOR	EURLIBOR	JPY-LIBOR	CAD-LIBOR	UK Spot
Model1	CKLS	CKLS	CKLS	CKLS	CKLS	CKLS
Model2	BS	BS	CIR	BS	CIR	VASICEK
Model3	CIR	CIR	BS	CIR	BS	CIR
Model4	VASICEK	VASICEK	VASICEK	VASICEK	VASICEK	BS

3.5.3 The Impact of the Financial Crisis on the U.K. Nominal Yield Curve

The impact of the last financial crisis on the level of the U.K. nominal interest rates is assessed by dividing the whole sample period into the pre-crisis and post-crisis period samples. Based on previous studies (see Cheung et al. (2010) and Dontis-Charitos et al. (2013)) the cut-off point is the beginning of the third quarter, in July 2007 when the first substantial signs of financial distress were observed in the US subprime market. The four continuous-time models have been estimated for four- and five-factors over the pre-crisis period (4 January 2000 to 29 June 2007) and post-crisis period (2 July 2007 to 28 March

2013), respectively. The estimation results are organised in the Table 3.35 and 3.36 for the four-factor extension and in Table 3.37 and 3.38 for the five-factor extension.

The estimation results provided by the four-factor continuous-time show that some of the parameter estimates have substantially changed as a result of the crisis. The most affected parameters are the level-effect parameters, while the remaining parameters have relatively unchanged values between the two sub-periods. More specifically, in the pre-crisis period the estimates of the level-effect parameters are very low (between 0 and 0.21), whereas after the crisis the volatility is highly dependent on the level of the interest rates (with values between 0.43 and 0.55). These results may suggest that different specifications should fit the data best for each sub-period, with the Vasicek model as a most appropriate modelling choice for the pre-crisis period and the CIR formulation for the post-crisis period. Moreover, the other diffusion component, the volatility scale-factor parameter, has been estimated at values ten times larger in the post-crisis period than in the pre-crisis period. A similar impact can be observed in the case of the five-factor models, where the change in the parameter values as a result of the crisis is realised only in the level-effect parameter component of the volatility. The rest of the parameters seem to remain unaffected. These findings confirm the conclusion from the CKLS paper that the level-effect parameter is of a much greater importance than the drift parameters.

A very important result is that for the pre-crisis period for the U.K. spot rates the Vasicek model cannot be rejected against the more general CKLS model for both the four- and five-factor models. More specifically the LR test values (12.56 in Table 3.35b and 11.06 in Table 3.37b, respectively) are smaller than the critical value of 15.09. We conclude that for both extensions, the particular feature of the Vasicek model of homoscedasticity is most appropriate for explaining the data during calm periods. For the post-crisis period the best nested model is the CIR model, which is reflected in higher value of the level-effect parameter, i.e. after the crisis the interest rates become more elastic. Overall, these findings emphasise two important aspects when modelling interest rates. One is the intrinsic feature of less volatility for longer maturity interest rates and second the higher level of volatility that exists during turbulent periods of time.

Table 3.35a) U.K. Spot Rates Pre-Crisis Period; The Drift Coefficients Estimates for the Four-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**0.0003**	0.0000	**0.0001**	0.0000	**0.0000**	0.0000	**0.0001**	0.0000
Alpha2	**0.0004**	0.0000	**0.0001**	0.0000	**0.0001**	0.0001	**0.0002**	0.0001
Alpha3	**0.0005**	0.0000	**0.0002**	0.0000	**0.0002**	0.0001	**0.0004**	0.0001
Alpha4	**0.0005**	0.0000	**0.0002**	0.0000	**0.0003**	0.0001	**0.0004**	0.0001
Beta11	**-0.0062**	0.0018	**0.0016**	0.0016	**0.0014**	0.0018	**-0.0016**	0.0018
Beta12	**0.0077**	0.0072	**-0.0171**	0.0053	**-0.0175**	0.0047	**-0.0161**	0.0047
Beta13	**0.0002**	0.0151	**0.0334**	0.0128	**0.0373**	0.0103	**0.0489**	0.0103
Beta14	**-0.0087**	0.0091	**-0.0204**	0.0094	**-0.0219**	0.0083	**-0.0354**	0.0083
Beta21	**-0.0036**	0.0018	**0.0077**	0.0019	**0.0060**	0.0031	**0.0011**	0.0031
Beta22	**0.0010**	0.0067	**-0.0250**	0.0034	**-0.0208**	0.0137	**-0.0178**	0.0137
Beta23	**0.0063**	0.0118	**0.0225**	0.0077	**0.0190**	0.0292	**0.0361**	0.0292
Beta24	**-0.0131**	0.0066	**-0.0072**	0.0065	**-0.0060**	0.0162	**-0.0252**	0.0162
Beta31	**-0.0049**	0.0015	**0.0054**	0.0018	**0.0033**	0.0032	**-0.0026**	0.0032
Beta32	**0.0056**	0.0051	**-0.0142**	0.0041	**-0.0083**	0.0148	**0.0010**	0.0148
Beta33	**0.0054**	0.0084	**0.0074**	0.0085	**0.0009**	0.0320	**0.0043**	0.0320
Beta34	**-0.0175**	0.0046	**-0.0032**	0.0060	**-0.0008**	0.0177	**-0.0120**	0.0177
Beta41	**-0.0043**	0.0016	**0.0044**	0.0019	**0.0026**	0.0031	**-0.0032**	0.0031
Beta42	**-0.0018**	0.0047	**-0.0156**	0.0052	**-0.0105**	0.0143	**0.0012**	0.0143
Beta43	**0.0271**	0.0080	**0.0196**	0.0110	**0.0137**	0.0311	**0.0096**	0.0311
Beta44	**-0.0326**	0.0044	**-0.0138**	0.0070	**-0.0116**	0.0173	**-0.0175**	0.0173

Table 3.35b) U.K. Spot Rates Pre-Crisis Period, The Diffusion Coefficients Estimates for the Four-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.0000**	0.0001	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma2	**0.2166**	0.0319	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma3	**0.1899**	0.0477	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma4	**0.1193**	0.0113	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Sigma1	**0.0004**	0.0000	**0.0004**	0.0074	**0.0017**	0.0000	**0.0085**	0.0001
Sigma2	**0.0008**	0.0001	**0.0004**	0.0086	**0.0019**	0.0000	**0.0086**	0.0001
Sigma3	**0.0007**	0.0000	**0.0004**	0.0087	**0.0018**	0.0000	**0.0082**	0.0001
Sigma4	**0.0005**	0.0000	**0.0004**	0.0086	**0.0017**	0.0000	**0.0083**	0.0001
Corr12	**0.6726**	0.0005	**0.6724**	0.0067	**0.6625**	0.0002	**0.6529**	0.0005
Corr13	**0.5135**	0.0004	**0.5152**	0.0093	**0.5051**	0.0002	**0.4949**	0.0004
Corr14	**0.4144**	0.0133	**0.4172**	0.0104	**0.4071**	0.0161	**0.3973**	0.0136
Corr23	**0.9287**	0.0021	**0.9291**	0.0016	**0.9289**	0.0153	**0.9273**	0.0022
Corr24	**0.8172**	0.0050	**0.8186**	0.0040	**0.8173**	0.0152	**0.8141**	0.0051
Corr34	**0.9546**	0.0012	**0.9551**	0.0011	**0.9546**	0.0136	**0.9540**	0.0012
Log LF	**61,371.96**	**N/A**	**61,365.68**	12.56+	**61,256.61**	230.70+	**61,078.48**	586.96+

Table 3.36a) U.K. Spot Rates Post-Crisis Period; The Drift Coefficients Estimates for the Four-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**-0.0002**	0.0000	**-0.0002**	0.0000	**-0.0002**	0.0000	**0.0000**	0.0000
Alpha2	**-0.0003**	0.0000	**-0.0001**	0.0001	**-0.0002**	0.0000	**0.0001**	0.0000
Alpha3	**0.0001**	0.0000	**0.0004**	0.0000	**0.0001**	0.0000	**0.0005**	0.0000
Alpha4	**0.0004**	0.0000	**0.0007**	0.0000	**0.0004**	0.0000	**0.0007**	0.0000
Beta11	**0.0015**	0.0010	**0.0044**	0.0011	**0.0010**	0.0010	**-0.0038**	0.0017
Beta12	**-0.0207**	0.0042	**-0.0255**	0.0055	**-0.0175**	0.0036	**-0.0122**	0.0032
Beta13	**0.0268**	0.0079	**0.0293**	0.0112	**0.0202**	0.0067	**0.0199**	0.0058
Beta14	**-0.0085**	0.0052	**-0.0064**	0.0080	**-0.0036**	0.0045	**-0.0101**	0.0037
Beta21	**0.0048**	0.0015	**0.0036**	0.0016	**0.0045**	0.0017	**0.0030**	0.0024
Beta22	**-0.0167**	0.0044	**-0.0164**	0.0074	**-0.0148**	0.0060	**-0.0116**	0.0085
Beta23	**0.0104**	0.0073	**0.0180**	0.0148	**0.0082**	0.0100	**0.0171**	0.0126
Beta24	**0.0056**	0.0046	**-0.0052**	0.0107	**0.0060**	0.0060	**-0.0136**	0.0060
Beta31	**-0.0007**	0.0010	**-0.0027**	0.0016	**-0.0003**	0.0014	**-0.0023**	0.0019
Beta32	**0.0076**	0.0031	**0.0091**	0.0069	**0.0066**	0.0046	**0.0091**	0.0080
Beta33	**-0.0124**	0.0055	**-0.0026**	0.0125	**-0.0114**	0.0079	**0.0018**	0.0126
Beta34	**0.0029**	0.0012	**-0.0132**	0.0080	**0.0030**	0.0055	**-0.0198**	0.0065
Beta41	**-0.0018**	0.0010	**-0.0034**	0.0016	**-0.0013**	0.0014	**-0.0034**	0.0018
Beta42	**0.0057**	0.0029	**0.0043**	0.0068	**0.0037**	0.0048	**0.0062**	0.0076
Beta43	**0.0054**	0.0052	**0.0210**	0.0110	**0.0087**	0.0070	**0.0206**	0.0123
Beta44	**-0.0192**	0.0032	**-0.0394**	0.0057	**-0.0210**	0.0036	**-0.0412**	0.0063

Table 3.36b) U.K. Spot Rates Post-Crisis Period, The Diffusion Coefficients Estimates for the Four-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.4840**	0.0135	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma2	**0.4307**	0.0179	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma3	**0.5276**	35.7919	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma4	**0.5522**	0.0441	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Sigma1	**0.0031**	0.0002	**0.0004**	0.0000	**0.0033**	0.0000	**0.0386**	0.0004
Sigma2	**0.0028**	0.0002	**0.0006**	0.0000	**0.0036**	0.0000	**0.0242**	0.0003
Sigma3	**0.0031**	0.0001	**0.0006**	0.0000	**0.0029**	0.0000	**0.0150**	0.0002
Sigma4	**0.0031**	0.0004	**0.0005**	0.0000	**0.0026**	0.0000	**0.0129**	0.0002
Corr12	**0.6116**	0.0007	**0.6416**	0.0006	**0.6032**	0.0006	**0.5362**	0.0005
Corr13	**0.4447**	0.0005	**0.3835**	0.0005	**0.4465**	0.0005	**0.4403**	0.0005
Corr14	**0.3510**	0.0161	**0.2726**	0.0171	**0.3521**	0.0161	**0.3609**	0.0158
Corr23	**0.9052**	0.0032	**0.8761**	0.0041	**0.9066**	0.0031	**0.9160**	0.0027
Corr24	**0.7586**	0.0074	**0.7284**	0.0083	**0.7591**	0.0073	**0.7594**	0.0073
Corr34	**0.9316**	0.0021	**0.9331**	0.0020	**0.9316**	0.0021	**0.9276**	0.0022
Log LF	**44,871.65**	**N/A**	**44,467.96**	807.38+	**44,861.84**	19.62+	**44,404.27**	934.76+

Table 3.37a) U.K. Spot Rates Pre-Crisis Period; The Drift Coefficients Estimates for the Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**0.0000**	0.0001	**0.0000**	0.0001	**0.0000**	0.0000	**0.0000**	0.0001
Alpha2	**0.0000**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000
Alpha3	**0.0000**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000
Alpha4	**0.0000**	0.0000	**0.0002**	0.0000	**0.0001**	0.0000	**0.0003**	0.0000
Alpha5	**0.0000**	0.0000	**0.0006**	0.0000	**0.0001**	0.0001	**0.0005**	0.0000
Beta11	**0.0026**	0.0022	**-0.0022**	0.0020	**-0.0015**	0.0006	**-0.0025**	0.0021
Beta12	**0.0133**	0.0169	**0.0006**	0.0156	**0.0009**	0.0055	**0.0003**	0.0167
Beta13	**-0.0066**	0.0253	**0.0018**	0.0214	**-0.0044**	0.0113	**0.0019**	0.0220
Beta14	**-0.0042**	0.0201	**-0.0011**	0.0117	**0.0168**	0.0104	**-0.0030**	0.0122
Beta15	**-0.0056**	0.0108	**0.0005**	0.0087	**-0.0134**	0.0047	**0.0005**	0.0093
Beta21	**0.0127**	0.0008	**-0.0019**	0.0019	**-0.0015**	0.0009	**-0.0027**	0.0020
Beta22	**-0.0202**	0.0061	**0.0643**	0.0034	**0.0746**	0.0028	**0.0643**	0.0035
Beta23	**-0.0003**	0.0065	**-0.1161**	0.0050	**-0.1383**	0.0047	**-0.1217**	0.0037
Beta24	**0.0039**	0.0073	**0.0544**	0.0048	**0.0640**	0.0040	**0.0587**	0.0079
Beta25	**0.0054**	0.0074	**-0.0004**	0.0039	**0.0008**	0.0017	**-0.0003**	0.0057
Beta31	**0.0121**	0.0007	**-0.0003**	0.0018	**-0.0005**	0.0009	**-0.0022**	0.0017
Beta32	**-0.0084**	0.0040	**0.0259**	0.0006	**0.0269**	0.0043	**0.0286**	0.0034
Beta33	**-0.0221**	0.0016	**-0.0334**	0.0000	**-0.0321**	0.0080	**-0.0312**	0.0000
Beta34	**0.0163**	0.0069	**-0.0117**	0.0000	**-0.0176**	0.0054	**-0.0139**	0.0019
Beta35	**0.0027**	0.0069	**0.0203**	0.0000	**0.0233**	0.0018	**0.0175**	0.0050
Beta41	**0.0134**	0.0009	**-0.0002**	0.0014	**0.0003**	0.0009	**-0.0010**	0.0025
Beta42	**-0.0094**	0.0066	**0.0069**	0.0000	**0.0072**	0.0061	**0.0130**	0.0000
Beta43	**-0.0262**	0.0066	**0.0001**	0.0000	**-0.0034**	0.0118	**0.0001**	0.0000
Beta44	**0.0229**	0.0073	**-0.0250**	0.0000	**-0.0185**	0.0086	**-0.0289**	0.0000
Beta45	**0.0004**	0.0064	**0.0136**	0.0000	**0.0125**	0.0035	**0.0092**	0.0046
Beta51	**0.0161**	0.0014	**-0.0002**	0.0008	**-0.0001**	0.0010	**-0.0004**	0.0026
Beta52	**-0.0030**	0.0064	**-0.0293**	0.0000	**-0.0201**	0.0082	**-0.0137**	0.0000
Beta53	**-0.0518**	0.0104	**0.0475**	0.0000	**0.0507**	0.0159	**0.0394**	0.0000
Beta54	**0.0428**	0.0078	**-0.0179**	0.0000	**-0.0464**	0.0126	**-0.0356**	0.0000
Beta55	**-0.0033**	0.0062	**-0.0129**	0.0000	**0.0126**	0.0056	**-0.0022**	0.0052

Table 3.37b) U.K. Spot Rates Pre-Crisis Period, The Diffusion Coefficients Estimates for the Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.4593**	0.0715	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma2	**0.4642**	0.0220	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma3	**0.5473**	0.0076	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma4	**0.5481**	0.0074	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma5	**0.3344**	0.0279	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Sigma1	**0.0035**	0.1607	**0.0003**	0.0108	**0.0018**	0.0074	**0.0083**	0.0092
Sigma2	**0.0016**	0.0321	**0.0004**	0.0099	**0.0023**	0.0084	**0.0081**	0.0106
Sigma3	**0.0034**	0.0192	**0.0004**	0.0101	**0.0023**	0.0085	**0.0082**	0.0105
Sigma4	**0.0044**	0.0179	**0.0004**	0.0103	**0.0021**	0.0085	**0.0079**	0.0105
Sigma5	**0.0097**	0.0915	**0.0004**	0.0108	**0.0020**	0.0085	**0.0080**	0.0106
Corr12	**0.6743**	0.0148	**0.5717**	0.0162	**0.4762**	0.0119	**0.5601**	0.0152
Corr13	**0.5946**	0.0159	**0.4568**	0.0175	**0.3698**	0.0124	**0.4753**	0.0162
Corr14	**0.5064**	0.0176	**0.3391**	0.0192	**0.2551**	0.0128	**0.4032**	0.0167
Corr15	**0.4031**	0.0193	**0.2186**	0.0209	**0.1210**	0.0131	**0.3258**	0.0168
Corr23	**0.9817**	0.0179	**0.9773**	0.0200	**0.9839**	0.0119	**0.9791**	0.0155
Corr24	**0.9185**	0.0218	**0.9057**	0.0242	**0.9209**	0.0136	**0.9195**	0.0171
Corr25	**0.7989**	0.0221	**0.7780**	0.0241	**0.7412**	0.0140	**0.7969**	0.0171
Corr34	**0.9699**	0.0221	**0.9672**	0.0241	**0.9697**	0.0140	**0.9725**	0.0165
Corr35	**0.8680**	0.0207	**0.8605**	0.0222	**0.8188**	0.0136	**0.8667**	0.0158
Corr45	**0.9532**	0.0164	**0.9519**	0.0171	**0.9256**	0.0118	**0.9483**	0.0138
LogLF	**80,402.56**	N/A	**80,397.01**	11.10+	**79,749.79**	1,305.54+	**79,415.56**	1,974.00+

Table 3.38a) U.K. Spot Rates Post-Crisis Period; The Drift Coefficients Estimates for the Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Alpha1	**0.0000**	0.0001	**0.0000**	0.0001	**0.0000**	0.0000	**0.0002**	0.0000
Alpha2	**0.0000**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000	**0.0000**	0.0000
Alpha3	**-0.0001**	0.0000	**0.0000**	0.0000	**0.0002**	0.0000	**0.0000**	0.0000
Alpha4	**0.0000**	0.0000	**0.0001**	0.0000	**0.0006**	0.0000	**0.0003**	0.0000
Alpha5	**0.0000**	0.0000	**0.0003**	0.0000	**0.0012**	0.0000	**0.0005**	0.0001
Beta11	**-0.0030**	0.0013	**-0.0027**	0.0013	**-0.0070**	0.0011	**-0.0118**	0.0018
Beta12	**0.0209**	0.0163	**-0.0005**	0.0099	**0.0101**	0.0082	**0.0027**	0.0065
Beta13	**-0.0326**	0.0408	**0.0028**	0.0184	**-0.0069**	0.0167	**-0.0363**	0.0124
Beta14	**0.0063**	0.0382	**-0.0042**	0.0193	**0.0004**	0.0153	**0.0628**	0.0099
Beta15	**0.0070**	0.0145	**0.0014**	0.0103	**0.0002**	0.0061	**-0.0351**	0.0028
Beta21	**0.0004**	0.0016	**-0.0018**	0.0017	**-0.0002**	0.0017	**-0.0021**	0.0022
Beta22	**0.0138**	0.0083	**0.0573**	0.0059	**-0.0020**	0.0122	**0.0125**	0.0183

Beta23	**-0.0261**	0.0056	**-0.1290**	0.0051	**-0.0194**	0.0179	**-0.0159**	0.0330
Beta24	**0.0098**	0.0108	**0.0703**	0.0107	**0.0182**	0.0077	**0.0029**	0.0230
Beta25	**0.0023**	0.0075	**0.0006**	0.0059	**0.0000**	0.0033	**0.0006**	0.0076
Beta31	**0.0005**	0.0016	**-0.0019**	0.0016	**0.0003**	0.0016	**-0.0002**	0.0017
Beta32	**-0.0029**	0.0078	**0.0362**	0.0072	**-0.0018**	0.0134	**0.0014**	0.0142
Beta33	**0.0168**	0.0038	**-0.0675**	0.0100	**-0.0025**	0.0208	**0.0013**	0.0237
Beta34	**-0.0270**	0.0092	**0.0120**	0.0106	**-0.0078**	0.0098	**-0.0045**	0.0152
Beta35	**0.0165**	0.0061	**0.0198**	0.0055	**0.0063**	0.0031	**0.0006**	0.0057
Beta41	**-0.0004**	0.0014	**-0.0017**	0.0015	**0.0011**	0.0017	**0.0013**	0.0014
Beta42	**0.0013**	0.0050	**0.0026**	0.0094	**-0.0018**	0.0150	**-0.0014**	0.0092
Beta43	**0.0006**	0.0103	**0.0001**	0.0157	**0.0000**	0.0250	**-0.0033**	0.0105
Beta44	**0.0007**	0.0141	**-0.0210**	0.0113	**0.0021**	0.0145	**0.0132**	0.0045
Beta45	**-0.0022**	0.0057	**0.0156**	0.0053	**-0.0164**	0.0036	**-0.0167**	0.0032
Beta51	**0.0002**	0.0013	**0.0002**	0.0015	**-0.0011**	0.0017	**0.0024**	0.0018
Beta52	**-0.0194**	0.0092	**-0.0317**	0.0112	**-0.0158**	0.0157	**-0.0110**	0.0132
Beta53	**0.0280**	0.0223	**0.0341**	0.0197	**0.0340**	0.0277	**0.0171**	0.0215
Beta54	**0.0012**	0.0210	**0.0035**	0.0126	**0.0001**	0.0191	**-0.0002**	0.0202
Beta55	**-0.0108**	0.0069	**-0.0170**	0.0036	**-0.0443**	0.0056	**-0.0193**	0.0093

Table 3.38b) U.K. Spot Rates Post-Crisis Period, The Diffusion Coefficients Estimates for the Five-Factor Models

Param.	CKLS	S.E.	VASICEK	S.E.	CIR	S.E.	BS	S.E.
Gamma1	**0.7155**	0.0610	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma2	**0.4544**	0.0168	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma3	**0.7071**	0.0045	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma4	**0.8001**	0.0168	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Gamma5	**1.0483**	0.1385	**0.0000**	N/A	**0.5000**	N/A	**1.0000**	N/A
Sigma1	**0.0027**	0.1385	**0.0004**	0.0126	**0.0033**	0.0117	**0.0390**	0.0150
Sigma2	**0.0031**	0.0295	**0.0006**	0.0135	**0.0034**	0.0117	**0.0240**	0.0025
Sigma3	**0.0038**	0.0083	**0.0006**	0.0136	**0.0031**	0.0118	**0.0169**	0.0095
Sigma4	**0.0032**	0.0373	**0.0006**	0.0137	**0.0027**	0.0118	**0.0114**	0.0082
Sigma5	**0.0015**	0.0971	**0.0006**	0.0137	**0.0025**	0.0122	**0.0129**	0.0000
Corr12	**0.5504**	0.0215	**0.6784**	0.0192	**0.5666**	0.0170	**0.5405**	0.0271
Corr13	**0.4498**	0.0110	**0.5694**	0.0201	**0.4534**	0.0181	**0.4817**	0.0211
Corr14	**0.3749**	0.0176	**0.4590**	0.0209	**0.3263**	0.0201	**0.3205**	0.0163
Corr15	**0.2855**	0.0189	**0.3666**	0.0213	**0.2067**	0.0219	**-0.0034**	0.0250
Corr23	**0.9778**	0.0134	**0.9777**	0.0203	**0.9717**	0.0212	**0.9660**	0.0222
Corr24	**0.8958**	0.0187	**0.9010**	0.0224	**0.8586**	0.0257	**0.6545**	0.0000

Corr25	**0.7285**	0.0209	**0.7831**	0.0227	**0.6778**	0.0256	**0.0000**	0.0000
Corr34	**0.9564**	0.0209	**0.9628**	0.0226	**0.9430**	0.0259	**0.8070**	0.0000
Corr35	**0.8019**	0.0202	**0.8549**	0.0217	**0.7809**	0.0236	**0.1959**	0.0000
Corr45	**0.9223**	0.0170	**0.9462**	0.0181	**0.9237**	0.0175	**0.7167**	0.0167
LogLF	**58,673.30**	N/A	**58,333.06**	680.48+	**58,559.25**	228.10+	**57,407.42**	2,531.76+

3.6 The Forecasting Analysis

The forecasting analysis is conducted along three dimensions, across six different forecasting methods, three horizon lengths and using various measures of forecasting accuracy and formal statistical tests. Four continuous-time models (CKLS, Vasicek, CIR and BS) and two benchmark discrete time models (VAR(1) and AR(1)) are estimated based on the six time series described in section 3.4. The choice of these discrete-time models as benchmarks is consistent with the specification of the discrete analogue model implied by Bergstrom's methodology, where for a $k-$th order linear stochastic differential system the discrete analogue model is a $VARMA\left(k,k-1\right)$ model. The continuous-time models considered for estimation in this study correspond to the particular case of $k=1$, hence their discrete analogues are VAR (1), with the following vector-specification:

$$r(t+1)=e^{\beta}r(t)+(e^{\beta}-I)\beta^{-1}\alpha+\varepsilon(t+1) \quad (3.34)$$

It is important to note that, while in the basic continuous model the coefficients are linear in the elements of the feedback matrix β, the coefficients of the discrete time model are exponential functions of the feedback matrix β, carrying some potential causal predictive value from the other factors, which is consistent with the financial theory of correlation among interest rates of different maturities. The corresponding VAR(1) models have been estimated in Eviews by the OLS method together the univariate AR(1) models for each individual time series. Once all six types of models have been estimated for each extension, the corresponding optimal ex-post point forecasts[21] are also evaluated. A robust forecasting comparison is conducted using dynamic forecasting, where the daily optimal forecasts are computed for out-of-sample periods based only on information from the fitting period. The forecast horizon (H) is a vital component in the forecasting analysis, as the conclusions regarding the forecasting accuracy may vary across different horizons and/or different loss functions (Diebold and Lopez, 1996). In this regard, the

[21] Other types of forecasts include the probability forecast, direction-of-change forecasts and volatility forecasts (Diebold and Lopez (1996)).

out-of-sample performance is evaluated over different horizons of 22, 44 and 66 steps ahead. For all the LIBOR time series, the out-of-sample periods are: from 01 April 2013 to 30 April 2013 for the first horizon $H_1 = 22$ days; from 01 May 2013 to 30 May 2013 for the second horizon $H_2 = 44$ days and from 01 June 2013 to 01 July 2013 for the third horizon $H_3 = 66$ days, respectively. For the U.K. nominal rates, the out-of-sample periods are: from 02 April 2013 to 01 May 2013 for the first horizon H_1 days; from 02 May 2013 to 04 June 2013 for the second horizon H_2 days and from 05 June 2013 to 04 July 2013 for the third horizon H_3 days, respectively.

The evaluation of the out-of-sample forecasts is based on several forecasting accuracy metrics and on two formal statistical tests, the Diebold and Mariano (1995) (hereafter D-M) test for non-nested models and the Clark and West (2007) (hereafter C-W) for nested models.

3.6.1 The Dynamic Forecasting Algorithm

Assuming parameter stability and given the property of infinite memory of the general autoregressive models, the dynamic optimal forecasts are generated by “the chain rule”. Accordingly, for an AR(1) model the h-step-ahead optimal forecast is given by the intercept plus the coefficient of the one-period lagged variable multiplied by the previous ($h-1$)-step-ahead optimal forecast (Brooks, 2008). The origin observation used in the forecasting analysis is the last observation r_T $(T = 3{,}455)$ from the in-the-sample data set. The one-step-ahead optimal forecast is defined as $f_{T,1} = E(r_{T+1|T})$, i.e. the conditional expectation of r at time $T+1$ given all the information available up to and including time T. The discrete-time analogues at time $T+1$ have the general equation:

$$r(T+1) = e^{\beta} r(T) + (e^{\beta} - I)\beta^{-1}\alpha + \varepsilon(T+1) \tag{3.35}$$

Therefore, by applying the conditional expectation operator, the one-, two- and the h-step-ahead optimal forecasts are derived as follows:

$$\begin{aligned} f_{T,1} &= E(r_{T+1|T}) = e^{\beta} r(T) + (e^{\beta} - I)\beta^{-1}\alpha \\ f_{T,2} &= E(r_{T+2|T}) = e^{\beta} f_{T,1} + (e^{\beta} - I)\beta^{-1}\alpha \\ \vdots \quad & \quad \vdots \qquad\qquad \vdots \\ f_{T,h} &= E(r_{T+h|T}) = e^{\beta} f_{T,h-1} + (e^{\beta} - I)\beta^{-1}\alpha \end{aligned} \tag{3.36}$$

In order to determine the forecasting accuracy of the models, the forecast errors are aggregated using various statistical and economic forecasting metrics. Over the last two decades the literature on measures of forecast error still portrays a controversial picture documenting their various limitations[22] (Hyndman and Koehler (2005)). Acknowledging the controversy around the choice of a suitable forecasting accuracy measure, this forecasting analysis employs a range of stylized statistical and economic metrics: the ME (Mean Error) and the VARE (Variance Error) have been chosen to test for bias in the forecasts, while the MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error), RMSE (Root Mean Squared Error) and the CDIR (Correct Direction Change Percentage Prediction) have been employed to evaluate the accuracy of the forecasts cross the models considered. These metrics have been computed using the following formulae:

$$ME_i = \frac{1}{H_i}\sum_{t=T+1}^{T+H_i}(r^f(t) - r^a(t)) \tag{3.37}$$

$$MAE_i = \frac{1}{H_i}\sum_{t=T+1}^{T+H_i}| r^f(t) - r^a(t)| \tag{3.38}$$

$$MAPE_i = \frac{100}{H_i}\sum_{t=T+1}^{T+H_i}\left|\frac{r^f(t) - r^a(t)}{r^a(t)}\right| \tag{3.39}$$

$$RMSE_i = \sqrt{\frac{1}{H_i}\sum_{t=T+1}^{T+H_i}(r^f(t) - r^a(t))^2} \tag{3.40}$$

$$CDCP_i = \frac{1}{H_i}\sum_{t=T+1}^{T+H_i} z_t, \quad \text{where } z_t = \begin{cases} 1, & \text{if } \quad [r^a(t) - r^a(t-1)][r^f(t) - r^a(\text{t}-1)] > 0 \\ 0, & \text{otherwise} \end{cases} \tag{3.41}$$

and where $r^a(t)$ and $r^f(t)$ are the actual and the forecasted value at time t, respectively.

Despite their sensitivity to the presence of outliers (Armstrong, 2001), the loss functions MAE and RMSE are still the most commonly used scale-dependent measures and have been used in this forecasting analysis mainly due to their relevance in statistical modelling. The MAPE metric has the advantage of scale-independence, and is widely used for forecasting comparison across data sets. A general disadvantage of percentage based measures is their asymmetry as they penalise positive errors more than negative errors and this motivated the introduction of “symmetric” measures by Makridakis

[22] The forecasting measures can often become infinite or undefined given the nature of real data.

(1993). Also, if the data contains frequent zero or negative values the MAPE figure tends to explode or to have a strongly skewed distribution if out-of-sample data is consistently close to zero (Coleman and Swanson, 2004).

3.6.2 The Forecasting Results for the Four- and Five-Factor Models

The forecasting results produced by the four- and five-factor models are organised across the forecasting methods and horizons for each maturity. In order to compare the predictive performance of the two extensions, the forecasting accuracy measures are combinedly presented in the Tables 3.39 to 3.44.

The results from the forecasting analysis for the four-factor specifications are rather mixed and complex with considerable differences from one data set to another. For GBP-LIBOR rates (see Table 3.39) the forecasting results indicate that the CKLS model performs best (dominating also the benchmark models) for one-week and one-month GBP-LIBOR rates based on the standard criteria of producing the smallest statistical accuracy measures and a higher percentage in predicting the sign changes. However, for the longer maturity rates of 6-month and 12-month GBP-LIBOR rates, the CIR model have the best prediction relative to the other models used in this forecasting comparison.

In the case of the USD-LIBOR rates (see Table 3.40) the forecasting results are rather different from those in the case of GBP-LIBOR, with the discrete models VAR(1) and AR(1) outperforming all the continuous models for all maturities interest rates. While the BRSC model was best in terms of explanatory power, the Vasicek model provides the best forecasts across the continuous-time models.

The forecasting results for the EUR-LIBOR rates (see Table 3.41) indicate a particularly different situation that keeps the models with the best goodness of fit from the estimation stage also as the best in forecasting performance; two of the continuous-time models, CKLS and CIR outperform the benchmark models VAR(1) and AR(1) especially in the case of 1-week, 1month- and 6-month EUR-LIBOR time series.

For the JPY-LIBOR interest rates (see Table 3.41) the forecasting analysis provide similar conclusions as in the case of the USD-LIBOR rates, with VAR(1), AR(1) and Vasicek models as the best three models in terms of forecasting power.

The CAD-LIBOR rates data set (Table 3.43) offers in terms of forecasting comparison among models some mixed results with the AR(1) and VAR(1) discrete time models performing better than the continuous-time models. From the continuous-time models the CIR model produces the best forecasts for one-week and one-month time-series, while the Vasicek model forecasts best the future six-month and twelve-month CAD-LIBOR rates.

Finally, in the case of the U.K. nominal spot rates (see Table 3.44) the overall forecasting performance is dominated by the discrete time benchmark model the VAR1 model, while among the continuous-time models the best forecasts are obtained under the Vasicek model for one-year and fifteen-year maturity spot rates and the CIR model for the remaining interest rates of seven and twenty-five years. Regarding the accuracy of the forecasts across different horizons, the findings suggest that the predictability power of all models diminishes for all the forecasting statistics as the horizon increases. As anticipated, in the shorter run (one-month) the predictive power of the models considered is at its highest; however, one could consider that the error forecasts over longer horizons (two and three months) are only marginally higher, hence the forecasting performance of the models may be considered satisfactory also in the long run.

Table 3.39 Forecasting accuracy measures for the individual LIBOR-GBP time-series for the four- and five-factor models. There are 4 panels A, B, C and D for each maturity 1-week, 1-, 6- and 12-months, respectively.

Panel A GBP-LIBOR 1W	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	-0.00010	-0.00230	0.00080	0.00140	0.00020	0.00030
4F-ME2	-0.00020	-0.00400	0.00110	0.00380	-0.00010	0.00060
4E-ME3	-0.00030	-0.00510	0.00140	0.00770	-0.00030	0.00090
4F-MAE1	0.00010	0.00230	0.00080	0.00140	0.00020	0.00030
4F-MAE2	0.00020	0.00400	0.00110	0.00380	0.00020	0.00060
4F-MAE3	0.00030	0.00510	0.00140	0.00770	0.00040	0.00090
4F-MAPE1	1.65%	47.54%	16.50%	27.57%	3.31%	5.85%
4F-MAPE2	3.27%	81.24%	23.47%	76.99%	4.36%	12.04%
4F-MAPE3	5.60%	105.54%	28.96%	158.91%	7.78%	17.87%
4F-RMSE1	0.00010	0.00260	0.00090	0.00160	0.00020	0.00030
4F-RMSE2	0.00020	0.00440	0.00120	0.00470	0.00020	0.00070
4F-RMSE3	0.00030	0.00560	0.00150	0.01010	0.00050	0.00100
4F-CDCP1	27.27%	27.27%	4.55%	4.55%	27.27%	27.27%
4F-CDCP2	15.91%	15.91%	4.55%	4.55%	15.91%	15.91%
4F-CDCP3	10.61%	10.61%	3.03%	3.03%	10.61%	10.61%
5F-ME1	-0.00013	-0.00074	0.00051	0.00037	0.00028	0.00031
5F-ME2	-0.00022	-0.00116	0.00083	0.00032	0.00023	0.00061
5F-ME3	-0.00032	-0.00161	0.00085	-0.00044	0.00013	0.00089
5F-MAE1	0.00013	0.00074	0.00051	0.00037	0.00028	0.00031
5F-MAE2	0.00022	0.00116	0.00083	0.00034	0.00023	0.00061
5F-MAE3	0.00032	0.00161	0.00085	0.00087	0.00018	0.00089

5F-MAPE1	2.74%	15.11%	10.36%	7.51%	0.0570521	6.37%
5F-MAPE2	4.57%	23.76%	17.07%	6.93%	0.0475214	12.55%
5F-MAPE3	6.49%	33.06%	17.53%	17.96%	0.0376137	18.37%
5F-RMSE1	0.00015	0.00080	0.00057	0.00039	0.00029	0.00036
5F-RMSE2	0.00025	0.00126	0.00092	0.00037	0.00025	0.00070
5F-RMSE3	0.00035	0.00179	0.00092	0.00137	0.00021	0.00102
5F-CDIR1	22.73%	22.73%	4.55%	4.55%	4.55%	4.55%
5F-CDIR2	13.64%	13.64%	4.55%	4.55%	4.55%	4.55%
5F-CDIR3	9.09%	9.09%	3.03%	3.03%	3.03%	3.03%

Panel B **GBP LIBOR 1M**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	-0.00009	-0.00045	0.00057	0.00023	-0.00011	-0.00015
4F-ME2	-0.00020	-0.00088	0.00104	0.00033	-0.00024	-0.00032
4E-ME3	-0.00032	-0.00132	0.00137	0.00012	-0.00037	-0.00049
4F-MAE1	0.00009	0.00045	0.00057	0.00023	0.00011	0.00015
4F-MAE2	0.00020	0.00088	0.00104	0.00033	0.00024	0.00032
4F-MAE3	0.00032	0.00132	0.00137	0.00037	0.00037	0.00049
4F-MAPE1	1.85%	9.10%	11.47%	4.73%	2.29%	3.09%
4F-MAPE2	4.05%	17.83%	21.15%	6.80%	4.80%	6.43%
4F-MAPE3	6.43%	26.85%	27.88%	7.47%	7.48%	10.00%
4F-RMSE1	0.00011	0.00051	0.00064	0.00026	0.00013	0.00017
4F-RMSE2	0.00024	0.00101	0.00117	0.00036	0.00028	0.00037
4F-RMSE3	0.00037	0.00153	0.00152	0.00044	0.00043	0.00058
4F-CDCP1	18.18%	18.18%	4.55%	4.55%	18.18%	18.18%
4F-CDCP2	11.36%	11.36%	9.09%	9.09%	11.36%	11.36%
4F-CDCP3	7.58%	7.58%	7.58%	6.06%	7.58%	7.58%
5F-ME1	-0.00009	-0.00045	0.00057	0.00023	-0.00011	-0.00015
5F-ME2	-0.00020	-0.00088	0.00104	0.00033	-0.00024	-0.00032
5F-ME3	-0.00032	-0.00132	0.00137	0.00012	-0.00037	-0.00049
5F-MAE1	0.00009	0.00045	0.00057	0.00023	0.00011	0.00015
5F-MAE2	0.00020	0.00088	0.00104	0.00033	0.00024	0.00032
5F-MAE3	0.00032	0.00132	0.00137	0.00037	0.00037	0.00049
5F-MAPE1	1.85%	9.10%	11.47%	4.73%	2.29%	3.09%
5F-MAPE2	4.05%	17.83%	21.15%	6.80%	4.80%	6.43%
5F-MAPE3	6.43%	26.85%	27.88%	7.47%	7.48%	10.00%
5F-RMSE1	0.00011	0.00051	0.00064	0.00026	0.00013	0.00017
5F-RMSE2	0.00024	0.00101	0.00117	0.00036	0.00028	0.00037
5F-RMSE3	0.00037	0.00153	0.00152	0.00044	0.00043	0.00058
5F-CDIR1	18.18%	18.18%	4.55%	4.55%	18.18%	18.18%
5F-CDIR2	11.36%	11.36%	9.09%	9.09%	11.36%	11.36%
5F-CDIR3	7.58%	7.58%	7.58%	6.06%	7.58%	7.58%

Panel C **GBP-LIBOR 6M**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00020	-0.00030	-0.00010	0.00000	-0.00010	-0.00010

4F-ME2	0.00040	-0.00080	-0.00010	0.00000	-0.00010	-0.00030
4E-ME3	0.00050	-0.00130	-0.00010	0.00020	-0.00020	-0.00050
4F-MAE1	0.00020	0.00030	0.00010	0.00000	0.00010	0.00010
4F-MAE2	0.00040	0.00080	0.00010	0.00010	0.00010	0.00030
4F-MAE3	0.00050	0.00130	0.00010	0.00020	0.00020	0.00050
4F-MAPE1	3.85%	5.88%	0.91%	0.41%	1.18%	2.18%
4F-MAPE2	6.94%	13.13%	1.39%	0.87%	1.97%	5.11%
4F-MAPE3	9.09%	21.33%	2.24%	3.09%	2.93%	8.57%
4F-RMSE1	0.00030	0.00040	0.00010	0.00000	0.00010	0.00020
4F-RMSE2	0.00050	0.00090	0.00010	0.00010	0.00010	0.00040
4F-RMSE3	0.00060	0.00150	0.00020	0.00030	0.00020	0.00060
4F-CDCP1	4.55%	40.91%	40.91%	40.91%	40.91%	40.91%
4F-CDCP2	11.36%	34.09%	34.09%	34.09%	34.09%	34.09%
4F-CDCP3	18.18%	34.85%	34.85%	33.33%	34.85%	34.85%
5F-ME1	-0.00017	-0.00003	0.00075	0.00024	0.00002	-0.00013
5F-ME2	-0.00037	-0.00015	0.00133	0.00019	0.00004	-0.0003
5F-ME3	-0.00061	-0.00034	0.0018	-0.00048	0.00004	-0.00051
5F-MAE1	0.00017	0.00004	0.00075	0.00024	0.00002	0.00013
5F-MAE2	0.00037	0.00016	0.00133	0.00023	0.00004	0.0003
5F-MAE3	0.00061	0.00034	0.0018	0.00076	0.00005	0.00051
5F-MAPE1	2.84%	0.69%	12.61%	4.10%	0.41%	2.18%
5F-MAPE2	6.35%	2.69%	22.59%	3.84%	0.71%	5.11%
5F-MAPE3	10.30%	5.79%	30.32%	12.72%	0.78%	8.57%
5F-RMSE1	0.0002	0.00005	0.00084	0.00026	0.000028	0.000155
5F-RMSE2	0.00044	0.00021	0.0015	0.00025	0.000049	0.00036
5F-RMSE3	0.00073	0.00046	0.00199	0.00125	0.000053	0.000615
5F-CDIR1	40.91%	36.36%	4.55%	4.55%	13.64%	40.91%
5F-CDIR2	34.09%	31.82%	11.36%	6.82%	15.91%	34.09%
5F-CDIR3	34.85%	33.33%	18.18%	16.67%	22.73%	34.85%

Panel D **GBP-LIBOR 12M**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00030	-0.00020	0.00010	-0.00030	0.00010	-0.00010
4F-ME2	0.00050	-0.00050	0.00010	-0.00090	0.00020	-0.00010
4E-ME3	0.00070	-0.00080	0.00000	-0.00230	0.00030	-0.00030
4F-MAE1	0.00030	0.00020	0.00010	0.00030	0.00010	0.00010
4F-MAE2	0.00050	0.00050	0.00010	0.00090	0.00020	0.00010
4F-MAE3	0.00070	0.00080	0.00010	0.00230	0.00030	0.00030
4F-MAPE1	3.31%	2.38%	0.67%	3.14%	1.35%	0.61%
4F-MAPE2	6.15%	5.09%	0.87%	10.69%	2.75%	1.64%
4F-MAPE3	7.94%	8.72%	1.03%	25.30%	3.28%	3.60%
4F-RMSE1	0.00030	0.00020	0.00010	0.00030	0.00010	0.00010
4F-RMSE2	0.00060	0.00050	0.00010	0.00120	0.00030	0.00020
4F-RMSE3	0.00080	0.00090	0.00010	0.00310	0.00030	0.00040

4F-CDCP1	0.00%	59.09%	4.55%	59.09%	54.55%	54.55%
4F-CDCP2	6.82%	43.18%	9.09%	43.18%	40.91%	40.91%
4F-CDCP3	19.70%	43.94%	21.21%	43.94%	42.42%	42.42%
5F-ME1	-0.00016	0.00047	0.00110	0.00065	0.00020	-0.00006
5F-ME2	-0.00035	0.00075	0.00195	0.00126	0.00038	-0.00015
5F-ME3	-0.00063	0.00085	0.00264	0.00206	0.00049	-0.00032
5F-MAE1	0.00016	0.00047	0.00110	0.00065	0.00020	0.00006
5F-MAE2	0.00035	0.00075	0.00195	0.00126	0.00038	0.00015
5F-MAE3	0.00063	0.00085	0.00264	0.00206	0.00049	0.00032
5F-MAPE1	1.82%	5.23%	12.33%	7.26%	2.25%	0.61%
5F-MAPE2	3.99%	8.44%	21.99%	14.26%	4.32%	1.64%
5F-MAPE3	7.01%	9.60%	29.56%	23.04%	5.45%	3.60%
5F-RMSE1	0.00018	0.00053	0.00125	0.00074	0.000238	0.000061
5F-RMSE2	0.00042	0.00082	0.00219	0.00145	0.000437	0.000184
5F-RMSE3	0.00077	0.00091	0.00293	0.00245	0.000536	0.000427
5F-CDIR1	54.55%	0.00%	0.00%	0.00%	0.00%	54.55%
5F-CDIR2	40.91%	6.82%	6.82%	6.82%	6.82%	40.91%
5F-CDIR3	42.42%	19.70%	19.70%	19.70%	19.70%	42.42%

Table 3.40 Forecasting accuracy measures for the individual LIBOR-USD time-series for the four- and five-factor models. There are 4 panels A, B, C and D for each maturity 1-week, 1-, 6- and 12-months, respectively.

Panel A USD-LIBOR 1W	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00300	0.00260	0.00300	0.00310	-0.00030	0.00000
4F-ME2	0.00290	0.00220	0.00280	0.00300	-0.00050	0.00000
4E-ME3	0.00280	0.00170	0.00260	0.00300	-0.00070	-0.00010
4F-MAE1	0.00300	0.00260	0.00300	0.00310	0.00030	0.00000
4F-MAE2	0.00290	0.00220	0.00280	0.00300	0.00050	0.00000
4F-MAE3	0.00280	0.00170	0.00260	0.00300	0.00070	0.00010
4F-MAPE1	176.82%	153.32%	173.14%	179.97%	18.00%	1.68%
4F-MAPE2	174.65%	128.64%	166.50%	180.64%	32.46%	2.00%
4F-MAPE3	169.83%	102.05%	156.97%	178.63%	45.25%	3.24%
4F-RMSE1	0.00300	0.00270	0.00300	0.00310	0.00040	0.00000
4F-RMSE2	0.00290	0.00220	0.00280	0.00300	0.00060	0.00000
4F-RMSE3	0.00280	0.00190	0.00260	0.00300	0.00080	0.00010
4F-CDCP1	22.73%	27.27%	27.27%	22.73%	40.91%	40.91%
4F-CDCP2	22.73%	25.00%	25.00%	22.73%	40.91%	40.91%
4F-CDCP3	24.24%	25.76%	25.76%	24.24%	36.36%	36.36%
5F-ME1	0.00003	0.00058	0.00001	-0.00005	-0.00023	-0.00002
5F-ME2	0.00008	0.00109	0.00009	-0.00008	-0.00041	-0.00003
5F-ME3	0.00011	0.00157	0.00018	-0.00012	-0.00057	-0.00005
5F-MAE1	0.00003	0.00058	0.00003	0.00006	0.00023	0.00003

5F-MAE2	0.00008	0.00109	0.0001	0.00008	0.00041	0.00003
5F-MAE3	0.00011	0.00157	0.00019	0.00012	0.00057	0.00005
5F-MAPE1	1.88%	33.73%	1.47%	3.26%	13.68%	1.68%
5F-MAPE2	4.78%	66.13%	6.07%	5.00%	24.81%	2.00%
5F-MAPE3	7.01%	96.96%	11.72%	7.12%	35.18%	3.24%
5F-RMSE1	0.00004	0.00065	0.00003	0.00006	0.00027	0.00003
5F-RMSE2	0.00010	0.00125	0.00013	0.00009	0.00046	0.00004
5F-RMSE3	0.00013	0.00179	0.00024	0.00013	0.00064	0.00006
5F-CDIR1	22.73%	22.73%	27.27%	36.36%	40.91%	50.00%
5F-CDIR2	22.73%	22.73%	25.00%	38.64%	40.91%	45.45%
5F-CDIR3	`	24.24%	25.76%	34.85%	36.36%	39.39%

Panel B USD-LIBOR 1M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00300	0.00250	0.00290	0.00300	-0.00030	-0.00010
4F-ME2	0.00290	0.00210	0.00270	0.00290	-0.00060	-0.00010
4E-ME3	0.00280	0.00160	0.00250	0.00290	-0.00080	-0.00020
4F-MAE1	0.00300	0.00250	0.00290	0.00300	0.00030	0.00010
4F-MAE2	0.00290	0.00210	0.00270	0.00290	0.00060	0.00010
4F-MAE3	0.00280	0.00160	0.00250	0.00290	0.00080	0.00020
4F-MAPE1	150.52%	126.63%	144.70%	149.80%	15.53%	3.18%
4F-MAPE2	148.52%	104.42%	137.68%	148.07%	28.96%	7.44%
4F-MAPE3	144.75%	82.03%	129.33%	145.20%	40.23%	11.69%
4F-RMSE1	0.00300	0.00250	0.00290	0.00300	0.00040	0.00010
4F-RMSE2	0.00290	0.00210	0.00270	0.00290	0.00060	0.00020
4F-RMSE3	0.00290	0.00180	0.00260	0.00290	0.00090	0.00030
4F-CDCP1	13.64%	18.18%	18.18%	13.64%	36.36%	36.36%
4F-CDCP2	13.64%	15.91%	15.91%	13.64%	31.82%	31.82%
4F-CDCP3	18.18%	19.70%	19.70%	18.18%	33.33%	33.33%
5F-ME1	0.00003	0.00034	-0.00009	-0.00004	-0.00027	-0.00006
5F-ME2	0.00004	0.00073	-0.00011	-0.00009	-0.00048	-0.00015
5F-ME3	0.00006	0.00116	-0.00009	-0.00014	-0.00066	-0.00023
5F-MAE1	0.00003	0.00034	0.00009	0.00004	0.00027	0.00006
5F-MAE2	0.00004	0.00073	0.00011	0.00009	0.00048	0.00015
5F-MAE3	0.00006	0.00116	0.00009	0.00014	0.00066	0.00023
5F-MAPE1	1.58%	16.87%	4.51%	1.97%	13.41%	3.18%
5F-MAPE2	2.26%	37.27%	5.74%	4.74%	24.41%	7.44%
5F-MAPE3	3.19%	59.52%	4.80%	7.18%	33.66%	11.69%
5F-RMSE1	0.00003	0.00039	0.0001	0.00005	0.0003	0.00008
5F-RMSE2	0.00005	0.00087	0.00012	0.00011	0.00054	0.00017
5F-RMSE3	0.00007	0.00137	0.00011	0.00016	0.00073	0.00027
5F-CDIR1	18.18%	13.64%	31.82%	31.82%	36.36%	36.36%
5F-CDIR2	15.91%	13.64%	29.55%	29.55%	31.82%	31.82%
5F-CDIR3	19.70%	18.18%	31.82%	31.82%	33.33%	33.33%

Panel C USD-LIBOR 6M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00180	0.00130	0.00160	0.00150	-0.00010	0.00000
4F-ME2	0.00200	0.00110	0.00160	0.00150	-0.00030	0.00000
4E-ME3	0.00210	0.00080	0.00160	0.00150	-0.00040	-0.00010

4F-MAE1	0.00180	0.00130	0.00160	0.00150	0.00010	0.00000
4F-MAE2	0.00200	0.00110	0.00160	0.00150	0.00030	0.00000
4F-MAE3	0.00210	0.00080	0.00160	0.00150	0.00040	0.00010
4F-MAPE1	41.29%	29.83%	36.55%	35.27%	3.43%	0.31%
4F-MAPE2	46.60%	24.43%	37.36%	35.65%	6.08%	0.71%
4F-MAPE3	50.31%	18.40%	37.10%	35.41%	8.52%	1.68%
4F-RMSE1	0.00180	0.00130	0.00160	0.00150	0.00020	0.00000
4F-RMSE2	0.00200	0.00110	0.00160	0.00150	0.00030	0.00000
4F-RMSE3	0.00210	0.00090	0.00160	0.00150	0.00040	0.00010
4F-CDCP1	4.55%	9.09%	9.09%	4.55%	63.64%	63.64%
4F-CDCP2	6.82%	9.09%	9.09%	6.82%	52.27%	52.27%
4F-CDCP3	18.18%	24.24%	19.70%	18.18%	50.00%	50.00%
5F-ME1	0.00007	0.00045	-0.00011	0.00007	-0.00012	-0.00001
5F-ME2	0.00013	0.00093	-0.00022	0.00012	-0.0002	-0.00003
5F-ME3	0.00016	0.00139	-0.00037	0.00013	-0.00028	-0.00007
5F-MAE1	0.00007	0.00045	0.00011	0.00007	0.00012	0.00001
5F-MAE2	0.00013	0.00093	0.00022	0.00012	0.0002	0.00003
5F-MAE3	0.00016	0.00139	0.00037	0.00013	0.00028	0.00007
5F-MAPE1	1.56%	10.33%	2.48%	1.67%	2.83%	0.31%
5F-MAPE2	2.95%	21.84%	5.27%	2.80%	4.76%	0.71%
5F-MAPE3	3.83%	33.18%	8.86%	3.19%	6.60%	1.68%
5F-RMSE1	0.00008	0.00052	0.00012	0.00008	0.00013	0.00002
5F-RMSE2	0.00014	0.00108	0.00026	0.00013	0.00022	0.00004
5F-RMSE3	0.00018	0.0016	0.00045	0.00015	0.00031	0.0001
5F-CDIR1	9.09%	4.55%	59.09%	9.09%	59.09%	59.09%
5F-CDIR2	9.09%	6.82%	50.00%	9.09%	50.00%	50.00%
5F-CDIR3	19.70%	18.18%	48.48%	19.70%	48.48%	48.48%

Panel D **USD- LIBOR 12M**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00220	0.00160	0.00190	0.00170	0.00000	0.00010
4F-ME2	0.00260	0.00140	0.00200	0.00160	0.00010	0.00020
4E-ME3	0.00290	0.00120	0.00200	0.00150	0.00010	0.00020
4F-MAE1	0.00220	0.00160	0.00190	0.00170	0.00000	0.00010
4F-MAE2	0.00260	0.00140	0.00200	0.00160	0.00010	0.00020
4F-MAE3	0.00290	0.00120	0.00200	0.00150	0.00010	0.00020
4F-MAPE1	31.35%	22.43%	26.51%	23.65%	0.32%	1.25%
4F-MAPE2	37.25%	20.23%	28.10%	23.03%	0.81%	2.27%
4F-MAPE3	41.43%	17.23%	28.70%	21.81%	1.09%	2.62%
4F-RMSE1	0.00230	0.00160	0.00190	0.00170	0.00000	0.00010
4F-RMSE2	0.00270	0.00140	0.00200	0.00160	0.00010	0.00020
4F-RMSE3	0.00290	0.00130	0.00200	0.00150	0.00010	0.00020
4F-CDCP1	9.09%	13.64%	13.64%	9.09%	63.64%	63.64%
4F-CDCP2	9.09%	11.36%	11.36%	9.09%	59.09%	59.09%
4F-CDCP3	16.67%	18.18%	18.18%	16.67%	57.58%	57.58%
5F-ME1	0.00015	0.00057	-0.00006	0.00023	0.00005	0.00009
5F-ME2	0.00028	0.0011	-0.00016	0.00039	0.00012	0.00016
5F-ME3	0.00035	0.00156	-0.00033	0.00048	0.00015	0.00018
5F-MAE1	0.00015	0.00057	0.00007	0.00023	0.00006	0.00009

5F-MAE2	0.00028	0.0011	0.00016	0.00039	0.00012	0.00016
5F-MAE3	0.00035	0.00156	0.00033	0.00048	0.00016	0.00018
5F-MAPE1	2.13%	8.03%	0.93%	3.23%	0.78%	1.25%
5F-MAPE2	3.98%	15.80%	2.34%	5.59%	1.73%	2.27%
5F-MAPE3	5.09%	22.64%	4.86%	6.87%	2.25%	2.62%
5F-RMSE1	0.00017	0.00065	0.00008	0.00025	0.00006	0.0001
5F-RMSE2	0.00031	0.00126	0.0002	0.00043	0.00014	0.00018
5F-RMSE3	0.00039	0.00177	0.00044	0.00052	0.00018	0.0002
5F-CDIR1	9.09%	9.09%	54.55%	9.09%	13.64%	13.64%
5F-CDIR2	9.09%	9.09%	54.55%	9.09%	11.36%	11.36%
5F-CDIR3	16.67%	16.67%	54.55%	16.67%	18.18%	18.18%

Table 3.41 Forecasting accuracy measures for the individual EUR-LIBOR time-series for the four- and five-factor models. There are 4 panels A, B, C and D for each maturity 1-week, 1-, 6- and 12-months, respectively.

Panel A **EUR-LIBOR 1W**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00000	0.00020	-0.00010	0.00000	-0.00020	-0.00010
4F-ME2	0.00000	0.00030	-0.00010	0.00000	-0.00040	-0.00020
4E-ME3	0.00000	0.00050	-0.00010	0.00000	-0.00060	-0.00030
4F-MAE1	0.00000	0.00020	0.00010	0.00000	0.00020	0.00010
4F-MAE2	0.00000	0.00030	0.00010	0.00000	0.00040	0.00020
4F-MAE3	0.00000	0.00050	0.00010	0.00000	0.00060	0.00030
4F-MAPE1	1.36%	44.01%	14.57%	4.78%	61.15%	33.12%
4F-MAPE2	3.80%	86.09%	23.45%	5.67%	114.07%	62.33%
4F-MAPE3	5.28%	124.05%	22.40%	8.75%	154.98%	89.03%
4F-RMSE1	0.00000	0.00020	0.00010	0.00000	0.00020	0.00010
4F-RMSE2	0.00000	0.00030	0.00010	0.00000	0.00050	0.00030
4F-RMSE3	0.00000	0.00060	0.00010	0.00000	0.00070	0.00040
4F-CDCP1	9.09%	4.55%	4.55%	4.55%	4.55%	4.55%
4F-CDCP2	11.36%	9.09%	9.09%	9.09%	9.09%	9.09%
4F-CDCP3	21.21%	10.61%	25.76%	10.61%	10.61%	10.61%
5F-ME1	-0.00006	-0.00038	0.00006	-0.00008	-0.00050	-0.00012
5F-ME2	-0.00010	-0.00070	0.00013	-0.00016	-0.00089	-0.00022
5F-ME3	-0.00016	-0.00102	0.00020	-0.00036	-0.00125	-0.00034
5F-MAE1	0.00006	0.00038	0.00006	0.00008	0.00050	0.00012
5F-MAE2	0.00010	0.00070	0.00013	0.00016	0.00089	0.00022
5F-MAE3	0.00016	0.00102	0.00020	0.00036	0.00125	0.00034
5F-MAPE1	16.60%	107.89%	15.75%	21.54%	141.98%	33.12%
5F-MAPE2	29.00%	199.33%	37.12%	44.38%	253.41%	62.33%
5F-MAPE3	41.63%	270.74%	52.30%	92.31%	330.87%	89.03%
5F-RMSE1	0.00007	0.00043	0.00007	0.00009	0.00057	0.00013
5F-RMSE2	0.00011	0.00080	0.00016	0.00018	0.00100	0.00025
5F-RMSE3	0.00019	0.00117	0.00023	0.00051	0.00140	0.00040

5F-CDIR1	4.55%	4.55%	4.55%	4.55%	4.55%	4.55%
5F-CDIR2	9.09%	9.09%	9.09%	9.09%	9.09%	9.09%
5F-CDIR3	10.61%	10.61%	24.24%	10.61%	10.61%	10.61%

Panel B EUR-LIBOR 1M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00000	0.00030	0.00000	0.00000	-0.00030	-0.00020
4F-ME2	0.00010	0.00050	-0.00010	-0.00010	-0.00060	-0.00030
4E-ME3	0.00010	0.00070	0.00000	0.00000	-0.00080	-0.00050
4F-MAE1	0.00000	0.00030	0.00000	0.00000	0.00030	0.00020
4F-MAE2	0.00010	0.00050	0.00010	0.00010	0.00060	0.00030
4F-MAE3	0.00010	0.00070	0.00000	0.00000	0.00080	0.00050
4F-MAPE1	8.03%	46.44%	4.59%	5.00%	57.62%	26.18%
4F-MAPE2	11.71%	79.71%	8.80%	9.11%	99.51%	51.72%
4F-MAPE3	17.30%	106.35%	8.21%	8.21%	129.21%	77.02%
4F-RMSE1	0.00010	0.00030	0.00000	0.00000	0.00040	0.00020
4F-RMSE2	0.00010	0.00050	0.00010	0.00010	0.00060	0.00030
4F-RMSE3	0.00010	0.00070	0.00010	0.00010	0.00090	0.00060
4F-CDCP1	27.27%	27.27%	27.27%	27.27%	27.27%	27.27%
4F-CDCP2	20.45%	20.45%	20.45%	20.45%	20.45%	20.45%
4F-CDCP3	16.67%	16.67%	24.24%	25.76%	16.67%	16.67%
5F-ME1	-0.00009	-0.00034	0.00000	-0.00005	-0.00046	-0.00016
5F-ME2	-0.00016	-0.00065	0.00004	-0.00011	-0.00085	-0.00030
5F-ME3	-0.00026	-0.00099	0.00007	-0.00031	-0.00123	-0.00048
5F-MAE1	0.00009	0.00034	0.00001	0.00005	0.00046	0.00016
5F-MAE2	0.00016	0.00065	0.00004	0.00011	0.00085	0.00030
5F-MAE3	0.00026	0.00099	0.00007	0.00031	0.00123	0.00048
5F-MAPE1	15.34%	55.68%	1.11%	8.10%	76.08%	26.18%
5F-MAPE2	27.91%	111.12%	7.52%	18.87%	145.51%	51.72%
5F-MAPE3	41.64%	158.95%	11.03%	48.09%	198.19%	77.02%
5F-RMSE1	0.00011	0.00039	0.00001	0.00006	0.00052	0.00018
5F-RMSE2	0.00018	0.00075	0.00006	0.00013	0.00097	0.00035
5F-RMSE3	0.00031	0.00115	0.00008	0.00046	0.00140	0.00057
5F-CDIR1	27.27%	27.27%	27.27%	27.27%	27.27%	27.27%
5F-CDIR2	20.45%	20.45%	20.45%	20.45%	20.45%	20.45%
5F-CDIR3	16.67%	16.67%	28.79%	16.67%	16.67%	16.67%

Panel C EUR-LIBOR 6M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00010	0.00000	0.00000	0.00010	-0.00020	-0.00010
4F-ME2	0.00010	0.00000	-0.00010	0.00010	-0.00030	-0.00020
4E-ME3	0.00030	0.00020	-0.00010	0.00020	-0.00050	-0.00050
4F-MAE1	0.00010	0.00000	0.00010	0.00010	0.00020	0.00010
4F-MAE2	0.00010	0.00000	0.00010	0.00010	0.00030	0.00020
4F-MAE3	0.00030	0.00020	0.00020	0.00020	0.00050	0.00050
4F-MAPE1	4.62%	1.51%	2.44%	2.92%	9.15%	6.84%

4F-MAPE2	6.52%	1.59%	6.96%	3.42%	12.80%	11.87%
4F-MAPE3	14.47%	8.12%	7.82%	10.61%	20.39%	22.68%
4F-RMSE1	0.00010	0.00000	0.00010	0.00010	0.00020	0.00020
4F-RMSE2	0.00010	0.00000	0.00020	0.00010	0.00030	0.00030
4F-RMSE3	0.00050	0.00030	0.00020	0.00040	0.00060	0.00070
4F-CDCP1	36.36%	36.36%	18.18%	36.36%	36.36%	36.36%
4F-CDCP2	36.36%	40.91%	27.27%	36.36%	36.36%	36.36%
4F-CDCP3	30.30%	33.33%	28.79%	30.30%	30.30%	30.30%
5F-ME1	-0.00021	-0.00017	0.00000	-0.00006	-0.00027	-0.00015
5F-ME2	-0.00036	-0.00029	0.00004	-0.00009	-0.00045	-0.00024
5F-ME3	-0.00068	-0.00059	-0.00010	-0.00038	-0.00078	-0.00051
5F-MAE1	0.00021	0.00017	0.00003	0.00006	0.00027	0.00015
5F-MAE2	0.00036	0.00029	0.00005	0.00009	0.00045	0.00024
5F-MAE3	0.00068	0.00059	0.00017	0.00038	0.00078	0.00051
5F-MAPE1	9.60%	7.58%	1.28%	2.73%	12.38%	6.84%
5F-MAPE2	17.47%	14.02%	2.62%	4.33%	22.19%	11.87%
5F-MAPE3	30.34%	26.05%	7.03%	15.95%	35.03%	22.68%
5F-RMSE1	0.00023	0.00018	0.00003	0.00006	0.00030	0.00016
5F-RMSE2	0.00040	0.00033	0.00006	0.00011	0.00051	0.00027
5F-RMSE3	0.00086	0.00077	0.00028	0.00061	0.00095	0.00068
5F-CDIR1	36.36%	36.36%	27.27%	36.36%	36.36%	36.36%
5F-CDIR2	36.36%	36.36%	31.82%	36.36%	36.36%	36.36%
5F-CDIR3	30.30%	30.30%	28.79%	30.30%	30.30%	30.30%

Panel D EUR-LIBOR 12M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00000	-0.00020	-0.00020	0.00000	0.00000	0.00000
4F-ME2	0.00010	-0.00030	-0.00040	0.00000	0.00000	-0.00010
4E-ME3	0.00030	-0.00020	-0.00030	0.00030	-0.00010	-0.00030
4F-MAE1	0.00000	0.00020	0.00020	0.00000	0.00000	0.00000
4F-MAE2	0.00010	0.00030	0.00040	0.00010	0.00010	0.00010
4F-MAE3	0.00030	0.00030	0.00030	0.00030	0.00020	0.00040
4F-MAPE1	1.13%	4.40%	4.73%	0.85%	0.90%	1.02%
4F-MAPE2	1.66%	8.17%	9.50%	1.28%	1.74%	1.67%
4F-MAPE3	7.69%	7.61%	8.27%	6.93%	4.74%	8.02%
4F-RMSE1	0.00010	0.00020	0.00020	0.00000	0.00000	0.00000
4F-RMSE2	0.00010	0.00040	0.00040	0.00010	0.00010	0.00010
4F-RMSE3	0.00060	0.00040	0.00040	0.00050	0.00030	0.00060
4F-CDCP1	59.09%	27.27%	27.27%	63.64%	54.55%	54.55%
4F-CDCP2	47.73%	31.82%	31.82%	54.55%	43.18%	43.18%
4F-CDCP3	43.94%	36.36%	37.88%	48.48%	40.91%	40.91%
5F-ME1	-0.00016	-0.00005	0.00000	0.00007	-0.00013	-0.00004
5F-ME2	-0.00030	-0.00008	0.00000	0.00015	-0.00021	-0.00006
5F-ME3	-0.00071	-0.00038	-0.00026	0.00001	-0.00052	-0.00035
5F-MAE1	0.00016	0.00005	0.00003	0.00007	0.00013	0.00004

5F-MAE2	0.00030	0.00008	0.00005	0.00015	0.00021	0.00007
5F-MAE3	0.00071	0.00038	0.00029	0.00019	0.00052	0.00035
5F-MAPE1	3.78%	1.28%	0.68%	1.64%	3.18%	1.02%
5F-MAPE2	7.54%	2.11%	1.25%	3.76%	5.36%	1.67%
5F-MAPE3	16.54%	8.74%	6.63%	4.65%	12.15%	8.02%
5F-RMSE1	0.00018	0.00006	0.00003	0.00009	0.00015	0.00005
5F-RMSE2	0.00035	0.00012	0.00007	0.00018	0.00024	0.00010
5F-RMSE3	0.00097	0.00062	0.00050	0.00025	0.00072	0.00058
5F-CDIR1	54.55%	59.09%	54.55%	31.82%	54.55%	63.64%
5F-CDIR2	43.18%	45.45%	47.73%	34.09%	43.18%	50.00%
5F-CDIR3	40.91%	42.42%	43.94%	34.85%	40.91%	45.45%

Table 3.42 Forecasting accuracy measures for the individual JPY-LIBOR time-series for the four- and five-factor models. There are 4 panels A, B, C and D for each maturity 1-week, 1-, 6- and 12-months, respectively.

Panel A JPY-LIBOR	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00420	0.00240	0.00260	0.00380	0.00000	0.00010
4F-ME2	0.00440	0.00170	0.00210	0.00350	-0.00010	0.00020
4E-ME3	0.00410	0.00140	0.00180	0.00320	-0.00010	0.00030
4F-MAE1	0.00420	0.00240	0.00260	0.00380	0.00010	0.00010
4F-MAE2	0.00440	0.00170	0.00210	0.00350	0.00010	0.00020
4F-MAE3	0.00410	0.00140	0.00180	0.00320	0.00010	0.00030
4F-MAPE1	407.72%	234.17%	251.07%	361.65%	5.32%	11.51%
4F-MAPE2	424.51%	168.21%	205.72%	345.34%	8.84%	23.95%
4F-MAPE3	404.54%	28.99%	179.46%	315.44%	11.87%	33.40%
4F-RMSE1	0.00420	0.00250	0.00270	0.00380	0.00010	0.00010
4F-RMSE2	0.00440	0.00190	0.00220	0.00360	0.00010	0.00030
4F-RMSE3	0.00420	0.00160	0.00190	0.00330	0.00010	0.00040
4F-CDCP1	27.27%	27.27%	27.27%	27.27%	36.36%	36.36%
4F-CDCP2	22.73%	22.73%	22.73%	22.73%	29.55%	29.55%
4F-CDCP3	18.18%	3.03%	18.18%	18.18%	25.76%	25.76%
5F-ME1	-0.00011	-0.00036	0.00043	-0.0001	0.00016	0.00012
5F-ME2	-0.00013	-0.00032	0.00134	-0.00012	0.0002	0.00024
5F-ME3	-0.00013	-0.00024	0.00318	-0.00014	0.00021	0.00034
5F-MAE1	0.00011	0.00036	0.00043	0.0001	0.00016	0.00012
5F-MAE2	0.00013	0.00032	0.00134	0.00012	0.0002	0.00024
5F-MAE3	0.00013	0.00024	0.00318	0.00014	0.00021	0.00034
5F-MAPE1	10.61%	33.86%	41.29%	9.51%	15.30%	11.51%
5F-MAPE2	12.14%	30.64%	131.44%	11.90%	20.01%	23.95%

5F-MAPE3	13.00%	23.58%	313.90%	13.61%	20.76%	33.40%
5F-RMSE1	0.00013	0.00037	0.00051	0.00012	0.00017	0.00013
5F-RMSE2	0.00014	0.00033	0.00173	0.00013	0.00021	0.00028
5F-RMSE3	0.00014	0.00028	0.00436	0.00015	0.00022	0.00038
5F-CDIR1	31.82%	31.82%	27.27%	31.82%	27.27%	27.27%
5F-CDIR2	27.27%	27.27%	22.73%	27.27%	22.73%	22.73%
5F-CDIR3	24.24%	24.24%	18.18%	24.24%	18.18%	18.18%

Panel B **JPY-LIBOR 1M**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00450	0.00290	0.00310	0.00370	-0.00010	0.00360
4F-ME2	0.00510	0.00240	0.00270	0.00360	-0.00010	0.00340
4E-ME3	0.00540	0.00070	0.00240	0.00350	-0.00010	0.00320
4F-MAE1	0.00450	0.00290	0.00310	0.00370	0.00010	0.00360
4F-MAE2	0.00510	0.00240	0.00270	0.00360	0.00010	0.00340
4F-MAE3	0.00540	0.00070	0.00240	0.00350	0.00010	0.00320
4F-MAPE1	371.80%	237.99%	257.54%	303.32%	5.03%	291.30%
4F-MAPE2	424.30%	194.73%	222.78%	299.26%	8.70%	280.67%
4F-MAPE3	450.93%	14.24%	198.67%	287.31%	11.03%	268.53%
4F-RMSE1	0.00460	0.00290	0.00320	0.00370	0.00010	0.00360
4F-RMSE2	0.00520	0.00240	0.00280	0.00360	0.00010	0.00340
4F-RMSE3	0.00550	0.00090	0.00250	0.00350	0.00010	0.00320
4F-CDCP1	9.09%	9.09%	9.09%	9.09%	22.73%	27.27%
4F-CDCP2	9.09%	9.09%	9.09%	9.09%	15.91%	18.18%
4F-CDCP3	6.06%	9.09%	6.06%	6.06%	13.64%	15.15%
5F-ME1	-0.00004	0.0002	0.00061	-0.00005	0.00014	0.00005
5F-ME2	-0.00007	0.00041	0.0018	-0.00008	0.00018	0.00008
5F-ME3	-0.00008	0.0006	0.00419	-0.0001	0.0002	0.00011
5F-MAE1	0.00004	0.0002	0.00061	0.00005	0.00014	0.00005
5F-MAE2	0.00007	0.00041	0.0018	0.00008	0.00018	0.00008
5F-MAE3	0.00008	0.0006	0.00419	0.0001	0.0002	0.00011
5F-MAPE1	3.32%	16.07%	49.92%	3.96%	11.17%	3.80%
5F-MAPE2	5.44%	33.87%	150.18%	6.41%	15.05%	6.76%
5F-MAPE3	6.85%	50.35%	352.49%	8.09%	16.53%	9.35%
5F-RMSE1	0.00005	0.00023	0.00073	0.00006	0.00015	0.00005
5F-RMSE2	0.00007	0.00047	0.00231	0.00009	0.00019	0.00009
5F-RMSE3	0.00009	0.00069	0.00570	0.00011	0.00021	0.00012
5F-CDIR1	22.73%	9.09%	9.09%	22.73%	9.09%	9.09%
5F-CDIR2	15.91%	9.09%	9.09%	15.91%	9.09%	9.09%
5F-CDIR3	13.64%	6.06%	6.06%	13.64%	6.06%	6.06%

Panel C **JPY-LIBOR 6M**	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00330	0.00330	0.00340	0.00380	0.00010	0.00010
4F-ME2	0.00330	0.00320	0.00340	0.00420	0.00020	0.00010
4E-ME3	0.00330	0.00110	0.00340	0.00450	0.00020	0.00010

4F-MAE1	0.00330	0.00330	0.00340	0.00380	0.00010	0.00010
4F-MAE2	0.00330	0.00320	0.00340	0.00420	0.00020	0.00010
4F-MAE3	0.00330	0.00110	0.00340	0.00450	0.00020	0.00010
4F-MAPE1	133.48%	133.21%	137.38%	152.50%	3.97%	2.64%
4F-MAPE2	133.19%	128.62%	138.77%	168.86%	6.95%	4.31%
4F-MAPE3	134.90%	17.92%	140.43%	185.04%	9.83%	6.06%
4F-RMSE1	0.00330	0.00330	0.00340	0.00380	0.00010	0.00010
4F-RMSE2	0.00330	0.00320	0.00340	0.00420	0.00020	0.00010
4F-RMSE3	0.00330	0.00120	0.00340	0.00450	0.00030	0.00020
4F-CDCP1	0.00%	0.00%	0.00%	0.00%	31.82%	36.36%
4F-CDCP2	0.00%	0.00%	0.00%	0.00%	22.73%	25.00%
4F-CDCP3	6.06%	18.18%	6.06%	6.06%	25.76%	25.76%
5F-ME1	0.00007	0.00011	0	0.00003	0.00016	0.00007
5F-ME2	0.00012	0.00018	0.00026	0.00004	0.00027	0.00011
5F-ME3	0.00017	0.00025	0.00105	0.00006	0.00037	0.00015
5F-MAE1	0.00007	0.00011	0.00002	0.00003	0.00016	0.00007
5F-MAE2	0.00012	0.00018	0.00027	0.00004	0.00027	0.00011
5F-MAE3	0.00017	0.00025	0.00105	0.00006	0.00037	0.00015
5F-MAPE1	2.83%	4.23%	0.93%	1.24%	6.48%	2.64%
5F-MAPE2	4.85%	7.23%	11.28%	1.76%	11.19%	4.31%
5F-MAPE3	7.09%	10.52%	44.67%	2.50%	15.55%	6.06%
5F-RMSE1	0.00008	0.00012	0.00003	0.00003	0.00018	0.00007
5F-RMSE2	0.00013	0.00020	0.00043	0.00005	0.00030	0.00012
5F-RMSE3	0.00019	0.00028	0.00165	0.00007	0.00041	0.00016
5F-CDIR1	0.00%	0.00%	27.27%	4.55%	0.00%	0.00%
5F-CDIR2	0.00%	0.00%	13.64%	2.27%	0.00%	0.00%
5F-CDIR3	6.06%	6.06%	15.15%	7.58%	6.06%	6.06%

Panel D JPY-LIBOR 12M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00470	0.00430	0.00450	0.00470	0.00010	0.00010
4F-ME2	0.00490	0.00400	0.00440	0.00480	0.00010	0.00010
4E-ME3	0.00510	0.00190	0.00440	0.00490	0.00020	0.00020
4F-MAE1	0.00470	0.00430	0.00450	0.00470	0.00010	0.00010
4F-MAE2	0.00490	0.00400	0.00440	0.00480	0.00010	0.00010
4F-MAE3	0.00510	0.00190	0.00440	0.00490	0.00020	0.00020
4F-MAPE1	106.41%	96.60%	101.34%	104.81%	1.85%	1.38%
4F-MAPE2	110.88%	91.42%	100.55%	107.65%	3.30%	2.36%
4F-MAPE3	117.02%	20.90%	101.10%	112.56%	5.24%	3.89%
4F-RMSE1	0.00470	0.00430	0.00450	0.00470	0.00010	0.00010
4F-RMSE2	0.00490	0.00410	0.00440	0.00480	0.00020	0.00010
4F-RMSE3	0.00510	0.00200	0.00440	0.00490	0.00030	0.00020
4F-CDCP1	0.00%	0.00%	0.00%	0.00%	36.36%	36.36%
4F-CDCP2	0.00%	0.00%	0.00%	0.00%	25.00%	27.27%
4F-CDCP3	4.55%	19.70%	4.55%	4.55%	30.30%	30.30%

5F-ME1	0.00007	0.00009	-0.00011	-0.00007	0.00013	0.00006
5F-ME2	0.00013	0.00016	-0.00011	-0.00012	0.00023	0.0001
5F-ME3	0.00021	0.00026	0.00015	-0.00014	0.00033	0.00017
5F-MAE1	0.00007	0.00009	0.00011	0.00007	0.00013	0.00006
5F-MAE2	0.00013	0.00016	0.00011	0.00012	0.00023	0.0001
5F-MAE3	0.00021	0.00026	0.00029	0.00014	0.00033	0.00017
5F-MAPE1	1.63%	2.06%	2.47%	1.47%	2.97%	1.38%
5F-MAPE2	2.94%	3.70%	2.51%	2.73%	5.16%	2.36%
5F-MAPE3	4.91%	6.02%	6.90%	3.21%	7.74%	3.89%
5F-RMSE1	0.00008	0.00010	0.00012	0.00008	0.00015	0.00007
5F-RMSE2	0.00014	0.00018	0.00012	0.00014	0.00025	0.00011
5F-RMSE3	0.00025	0.00030	0.00046	0.00015	0.00038	0.00020
5F-CDIR1	0.00%	0.00%	31.82%	31.82%	0.00%	0.00%
5F-CDIR2	0.00%	0.00%	22.73%	25.00%	0.00%	0.00%
5F-CDIR3	4.55%	4.55%	19.70%	28.79%	4.55%	4.55%

Table 3.43 Forecasting accuracy measures for the individual CAD-LIBOR time-series for the four- and five-factor models. There are 4 panels A, B, C and D for each maturity 1-week, 1-, 6- and 12-months, respectively.

Panel A CAD-LIBOR 1W	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00560	-0.00570	-0.00510	-0.00520	-0.00020	-0.00010
4F-ME2	0.00610	-0.00620	-0.00510	-0.00530	-0.00030	-0.00020
4F-MAE1	0.00560	0.00570	0.00510	0.00520	0.00020	0.00010
4F-MAE2	0.00610	0.00620	0.00510	0.00530	0.00030	0.00020
4F-MAPE1	56.02%	56.60%	50.28%	51.64%	1.51%	0.54%
4F-MAPE2	59.90%	60.87%	50.25%	51.99%	3.09%	1.53%
4F-RMSE1	0.00560	0.00570	0.00510	0.00520	0.00020	0.00010
4F-RMSE2	0.00610	0.00620	0.00510	0.00530	0.00040	0.00020
4F-CDIR1	9.09%	13.64%	13.64%	13.64%	9.09%	9.09%
4F-CDIR2	11.36%	13.64%	13.64%	13.64%	11.36%	11.36%
5F-ME1	-0.00019	-0.0004	-0.00006	-0.00007	-0.00015	-0.00005
5F-ME2	-0.00041	-0.0008	-0.00002	-0.00017	-0.00031	-0.00016
5F-MAE1	0.00019	0.0004	0.00006	0.00007	0.00015	0.00005
5F-MAE2	0.00041	0.0008	0.00008	0.00017	0.00031	0.00016
5F-MAPE1	1.91%	3.93%	0.60%	0.71%	1.48%	0.54%
5F-MAPE2	4.02%	7.83%	0.77%	1.64%	3.06%	1.53%
5F-RMSE1	0.00022	0.00044	0.00007	0.00008	0.00017	0.00006
5F-RMSE2	0.0005	0.00094	0.00009	0.00023	0.00038	0.00023
5F-CDIR1	9.09%	9.09%	9.09%	9.09%	9.09%	9.09%

5F-CDIR2	11.36%	11.36%	15.91%	11.36%	11.36%	11.36%

Panel B CAD-LIBOR 1M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00590	-0.00580	-0.00550	-0.00560	-0.00010	-0.00010
4F-ME2	0.00630	-0.00620	-0.00560	-0.00560	-0.00020	-0.00010
4F-MAE1	0.00590	0.00580	0.00550	0.00560	0.00010	0.00010
4F-MAE2	0.00630	0.00620	0.00560	0.00560	0.00020	0.00010
4F-MAPE1	56.48%	55.81%	52.90%	53.10%	0.95%	0.65%
4F-MAPE2	59.89%	59.09%	53.59%	53.46%	1.98%	1.37%
4F-RMSE1	0.00590	0.00590	0.00550	0.00560	0.00010	0.00010
4F-RMSE2	0.00630	0.00620	0.00560	0.00560	0.00020	0.00020
4F-CDIR1	0.00%	4.55%	4.55%	4.55%	0.00%	0.00%
4F-CDIR2	0.00%	2.27%	2.27%	2.27%	0.00%	0.00%
5F-ME1	-0.00017	-0.00012	-0.00002	-0.00008	-0.00009	-0.00007
5F-ME2	-0.00036	-0.00026	-0.00003	-0.00016	-0.0002	-0.00014
5F-MAE1	0.00017	0.00012	0.00002	0.00008	0.00009	0.00007
5F-MAE2	0.00036	0.00026	0.00003	0.00016	0.0002	0.00014
5F-MAPE1	1.63%	1.19%	0.19%	0.76%	0.81%	0.65%
5F-MAPE2	3.42%	2.49%	0.25%	1.48%	1.91%	1.37%
5F-RMSE1	0.0002	0.00014	0.00002	0.00009	0.0001	0.00008
5F-RMSE2	0.00042	0.00031	0.00003	0.00018	0.00024	0.00017
5F-CDIR1	4.55%	0.00%	0.00%	0.00%	0.00%	0.00%
5F-CDIR2	2.27%	0.00%	0.00%	0.00%	0.00%	0.00%

Panel C CAD-LIBOR 6M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00810	-0.00760	-0.00820	-0.00790	0.00000	0.00000
4F-ME2	0.00830	-0.00740	-0.00870	-0.00800	0.00000	0.00000
4F-MAE1	0.00810	0.00760	0.00820	0.00790	0.00000	0.00000
4F-MAE2	0.00830	0.00740	0.00870	0.00800	0.00000	0.00010
4F-MAPE1	58.77%	55.12%	59.82%	57.11%	0.28%	0.22%
4F-MAPE2	60.33%	53.76%	63.06%	57.67%	0.35%	0.48%
4F-RMSE1	0.00810	0.00760	0.00830	0.00790	0.00010	0.00000
4F-RMSE2	0.00830	0.00740	0.00870	0.00800	0.00010	0.00010
4F-CDIR1	31.82%	31.82%	31.82%	31.82%	31.82%	31.82%
4F-CDIR2	20.45%	20.45%	20.45%	20.45%	20.45%	20.45%
5F-ME1	-0.0001	0.00041	0.00012	-0.00002	0.00003	0.00001
5F-ME2	-0.00026	0.00075	0.00015	-0.00012	0	-0.00004
5F-MAE1	0.0001	0.00041	0.00012	0.00003	0.00004	0.00003
5F-MAE2	0.00026	0.00075	0.00015	0.00012	0.00005	0.00007
5F-MAPE1	0.73%	2.99%	0.87%	0.19%	0.27%	0.22%
5F-MAPE2	1.91%	5.44%	1.06%	0.88%	0.36%	0.48%
5F-RMSE1	0.00011	0.00048	0.00015	0.00003	0.00005	0.00004
5F-RMSE2	0.00032	0.00085	0.00016	0.00017	0.00006	0.00009
5F-CDIR1	31.82%	9.09%	9.09%	31.82%	22.73%	22.73%

5F-CDIR2	20.45%	20.45%	20.45%	20.45%	20.45%	18.18%

Panel D CAD-LIBOR 12M	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00870	-0.00840	-0.00880	-0.00870	0.00020	0.00010
4F-ME2	0.00870	-0.00810	-0.00900	-0.00860	0.00020	0.00020
4F-MAE1	0.00870	0.00840	0.00880	0.00870	0.00020	0.00010
4F-MAE2	0.00870	0.00810	0.00900	0.00860	0.00020	0.00020
4F-MAPE1	49.01%	47.32%	49.75%	48.67%	1.10%	0.84%
4F-MAPE2	48.82%	45.57%	50.71%	48.59%	1.38%	0.91%
4F-RMSE1	0.00870	0.00840	0.00880	0.00870	0.00020	0.00020
4F-RMSE2	0.00870	0.00810	0.00900	0.00860	0.00030	0.00020
4F-CDIR1	45.45%	45.45%	45.45%	45.45%	50.00%	45.45%
4F-CDIR2	31.82%	31.82%	31.82%	31.82%	34.09%	31.82%
5F-ME1	0.00005	0.00063	0.0003	0.00015	0.00019	0.00015
5F-ME2	-0.00004	0.00104	0.00048	0.00014	0.00024	0.00016
5F-MAE1	0.00006	0.00063	0.0003	0.00015	0.00019	0.00015
5F-MAE2	0.00011	0.00104	0.00048	0.00014	0.00024	0.00016
5F-MAPE1	0.31%	3.55%	1.72%	0.85%	1.08%	0.84%
5F-MAPE2	0.62%	5.88%	2.72%	0.81%	1.34%	0.91%
5F-RMSE1	0.00007	0.00072	0.00036	0.00018	0.00023	0.00018
5F-RMSE2	0.00015	0.00115	0.00053	0.00018	0.00026	0.00019
5F-CDIR1	18.18%	4.55%	4.55%	4.55%	4.55%	9.09%
5F-CDIR2	15.91%	22.73%	22.73%	25.00%	22.73%	25.00%

Table 3.44 Forecasting accuracy measures for the U.K. spot rates time-series for the four- and five-factor models. There are 4 panels A, B, C and D for each maturity 1-,7-, 15- and 25-years, respectively.

Panel A UK SPOT 1Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00130	0.00150	0.00550	0.00330	-0.00020	-0.00040
4F-ME2	-0.00050	0.00010	0.00790	0.00330	-0.00050	-0.00100
4E-ME3	-0.00220	-0.00100	0.01050	0.00340	-0.00070	-0.00140
4F-MAE1	0.00130	0.00150	0.00550	0.00330	0.00020	0.00040
4F-MAE2	0.00190	0.00150	0.00790	0.00330	0.00050	0.00100
4F-MAE3	0.00300	0.00210	0.01050	0.00340	0.00070	0.00140
4F-MAPE1	72.78%	82.99%	280.59%	169.32%	10.04%	21.70%
4F-MAPE2	78.06%	65.75%	326.13%	144.76%	19.44%	38.37%
4F-MAPE3	108.43%	78.31%	383.03%	134.57%	24.19%	49.63%
4F-RMSE1	0.00170	0.00180	0.00570	0.00330	0.00020	0.00050
4F-RMSE2	0.00220	0.00170	0.00830	0.00330	0.00060	0.00120
4F-RMSE3	0.00360	0.00240	0.01140	0.00340	0.00080	0.00170
4F-CDCP1	63.64%	54.55%	54.55%	54.55%	45.45%	40.91%
4F-CDCP2	50.00%	45.45%	59.09%	59.09%	40.91%	38.64%

4F-CDCP3	50.00%	46.97%	56.06%	56.06%	43.94%	42.42%
5F-ME1	-0.00071	-0.00030	-0.00029	-0.00054	-0.00030	-0.00044
5F-ME2	-0.00148	-0.00071	-0.00070	-0.00106	-0.00068	-0.00099
5F-ME3	-0.00210	-0.00103	-0.00104	-0.00139	-0.00094	-0.00144
5F-MAE1	0.00071	0.00031	0.00029	0.00054	0.00031	0.00044
5F-MAE2	0.00148	0.00072	0.00070	0.00106	0.00069	0.00099
5F-MAE3	0.00210	0.00104	0.00104	0.00139	0.00094	0.00144
5F-MAPE1	34.78%	14.92%	14.33%	26.37%	14.92%	21.70%
5F-MAPE2	57.90%	27.48%	26.90%	41.79%	26.43%	38.37%
5F-MAPE3	73.40%	35.56%	35.58%	48.88%	32.33%	49.64%
5F-RMSE1	0.00081	0.00036	0.00035	0.00061	0.00036	0.00051
5F-RMSE2	0.00173	0.00087	0.00086	0.00123	0.00082	0.00118
5F-RMSE3	0.00240	0.00121	0.00122	0.00156	0.00109	0.00166
5F-CDIR1	40.91%	45.45%	45.45%	40.91%	59.09%	40.91%
5F-CDIR2	38.64%	40.91%	40.91%	38.64%	61.36%	38.64%
5F-CDIR3	42.42%	43.94%	43.94%	42.42%	57.58%	42.42%

Panel B UKSPOT 7Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	-0.00750	-0.00760	-0.00660	-0.00710	0.00060	0.00030
4F-ME2	-0.00890	-0.00880	-0.00690	-0.00820	-0.00020	-0.00070
4E-ME3	-0.01080	-0.01060	-0.00760	-0.01000	-0.00160	-0.00240
4F-MAE1	0.00750	0.00760	0.00660	0.00710	0.00060	0.00040
4F-MAE2	0.00890	0.00880	0.00690	0.00820	0.00090	0.00110
4F-MAE3	0.01080	0.01060	0.00760	0.01000	0.00210	0.00270
4F-MAPE1	62.16%	0.629876	54.97%	58.90%	5.20%	3.12%
4F-MAPE2	67.34%	0.67347	52.88%	62.46%	6.50%	7.97%
4F-MAPE3	72.10%	0.707222	51.59%	66.53%	12.42%	15.89%
4F-RMSE1	0.00750	0.00760	0.00670	0.00710	0.00070	0.00040
4F-RMSE2	0.00900	0.00900	0.00690	0.00830	0.00100	0.00150
4F-RMSE3	0.01130	0.01100	0.00770	0.01040	0.00290	0.00370
4F-CDCP1	59.09%	0.590909	59.09%	59.09%	63.64%	59.09%
4F-CDCP2	47.73%	0.477273	47.73%	47.73%	50.00%	47.73%
4F-CDCP3	48.48%	0.484848	48.48%	48.48%	50.00%	48.48%
5F-ME1	-0.00023	0.00043	0.00068	0.00062	0.00069	0.00030
5F-ME2	-0.00263	-0.00081	-0.00097	-0.00025	0.00000	-0.00073
5F-ME3	-0.00708	-0.00307	-0.00494	-0.00190	-0.00142	-0.00244
5F-MAE1	0.00039	0.00047	0.00068	0.00064	0.00071	0.00037
5F-MAE2	0.00271	0.00131	0.00171	0.00096	0.00083	0.00111
5F-MAE3	0.00713	0.00340	0.00544	0.00238	0.00197	0.00269
5F-MAPE1	3.24%	3.99%	5.72%	5.39%	5.97%	3.12%
5F-MAPE2	19.08%	9.45%	12.31%	7.18%	6.34%	7.97%
5F-MAPE3	41.68%	20.00%	31.44%	14.14%	11.83%	15.89%
5F-RMSE1	0.00050	0.00055	0.00077	0.00074	0.00082	0.00045
5F-RMSE2	0.00389	0.00174	0.00232	0.00116	0.00096	0.00146

5F-RMSE3	0.01008	0.00479	0.00810	0.00332	0.00275	0.00374
5F-CDIR1	59.09%	54.55%	40.91%	54.55%	36.36%	54.55%
5F-CDIR2	47.73%	45.45%	38.64%	50.00%	50.00%	45.45%
5F-CDIR3	48.48%	46.97%	42.42%	50.00%	50.00%	46.97%

Panel C UKSPOT 15Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	-0.01930	-0.01840	-0.01810	-0.02000	0.00120	0.00120
4F-ME2	-0.02000	-0.01810	-0.01770	-0.02130	0.00040	0.00050
4E-ME3	-0.02110	-0.01840	-0.01770	-0.02310	-0.00080	-0.00070
4F-MAE1	0.01930	0.01840	0.01810	0.02000	0.00120	0.00120
4F-MAE2	0.02000	0.01810	0.01770	0.02130	0.00100	0.00100
4F-MAE3	0.02110	0.01840	0.01770	0.02310	0.00180	0.00170
4F-MAPE1	75.76%	0.719196	71.01%	78.18%	4.58%	4.63%
4F-MAPE2	75.71%	0.687364	66.98%	80.38%	3.86%	3.81%
4F-MAPE3	75.88%	0.66446	63.92%	82.64%	6.12%	5.93%
4F-RMSE1	0.01930	0.01840	0.01820	0.02000	0.00130	0.00130
4F-RMSE2	0.02000	0.01820	0.01770	0.02130	0.00120	0.00120
4F-RMSE3	0.02120	0.01840	0.01770	0.02330	0.00230	0.00220
4F-CDCP1	59.09%	0.590909	59.09%	59.09%	63.64%	63.64%
4F-CDCP2	45.45%	0.454545	45.45%	45.45%	47.73%	47.73%
4F-CDCP3	45.45%	0.454545	45.45%	45.45%	46.97%	46.97%
5F-ME1	0.00079	0.00104	0.00173	0.00138	0.00098	0.00071
5F-ME2	-0.00039	0.00020	0.00137	0.00097	0.00027	-0.00023
5F-ME3	-0.00267	-0.00153	-0.00018	-0.00018	-0.00114	-0.00182
5F-MAE1	0.00080	0.00105	0.00173	0.00138	0.00099	0.00073
5F-MAE2	0.00130	0.00108	0.00140	0.00105	0.00095	0.00106
5F-MAE3	0.00328	0.00238	0.00203	0.00152	0.00195	0.00237
5F-MAPE1	4.43%	5.79%	9.53%	7.64%	5.47%	4.04%
5F-MAPE2	6.53%	5.62%	7.53%	5.66%	5.00%	5.38%
5F-MAPE3	14.10%	10.43%	9.35%	7.03%	8.62%	10.31%
5F-RMSE1	0.00092	0.00119	0.00193	0.00156	0.00113	0.00084
5F-RMSE2	0.00159	0.00123	0.00167	0.00131	0.00109	0.00126
5F-RMSE3	0.00460	0.00324	0.00264	0.00200	0.00263	0.00322
5F-CDIR1	40.91%	36.36%	36.36%	36.36%	31.82%	50.00%
5F-CDIR2	43.18%	40.91%	50.00%	52.27%	47.73%	47.73%
5F-CDIR3	43.94%	42.42%	48.48%	50.00%	50.00%	46.97%

Panel D UKSPOT 25Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	-0.02290	-0.02130	-0.02230	-0.02390	0.00120	0.00150
4F-ME2	-0.02300	-0.02020	-0.02200	-0.02510	0.00060	0.00110
4E-ME3	-0.02330	-0.01930	-0.02190	-0.02650	-0.00010	0.00060
4F-MAE1	0.02290	0.02130	0.02230	0.02390	0.00120	0.00150
4F-MAE2	0.02300	0.02020	0.02200	0.02510	0.00090	0.00110

4F-MAE3	0.02330	0.01930	0.02190	0.02650	0.00120	0.00090
4F-MAPE1	70.16%	0.653085	68.41%	73.35%	3.72%	4.53%
4F-MAPE2	68.95%	0.604682	65.96%	75.12%	2.74%	3.45%
4F-MAPE3	67.81%	0.565316	63.99%	76.94%	3.35%	2.83%
4F-RMSE1	0.02290	0.02140	0.02230	0.02390	0.00130	0.00160
4F-RMSE2	0.02300	0.02020	0.02200	0.02520	0.00110	0.00140
4F-RMSE3	0.02330	0.01940	0.02190	0.02660	0.00140	0.00120
4F-CDCP1	54.55%	54.55%	54.55%	54.55%	59.09%	59.09%
4F-CDCP2	45.45%	45.45%	45.45%	45.45%	47.73%	47.73%
4F-CDCP3	45.45%	45.45%	45.45%	45.45%	46.97%	46.97%
5F-ME1	0.00098	0.00121	0.00190	0.00172	0.00125	0.00117
5F-ME2	-0.00002	0.00048	0.00170	0.00151	0.00058	0.00048
5F-ME3	-0.00173	-0.00086	0.00070	0.00075	-0.00064	-0.00074
5F-MAE1	0.00098	0.00121	0.00190	0.00172	0.00125	0.00117
5F-MAE2	0.00118	0.00103	0.00170	0.00151	0.00099	0.00099
5F-MAE3	0.00251	0.00187	0.00161	0.00135	0.00169	0.00172
5F-MAPE1	3.88%	4.79%	7.50%	6.79%	4.93%	4.63%
5F-MAPE2	4.44%	3.97%	6.55%	5.82%	3.84%	3.81%
5F-MAPE3	8.49%	6.44%	5.86%	4.97%	5.85%	5.93%
5F-RMSE1	0.00109	0.00134	0.00209	0.00190	0.00138	0.00130
5F-RMSE2	0.00135	0.00120	0.00190	0.00171	0.00119	0.00116
5F-RMSE3	0.00332	0.00241	0.00189	0.00159	0.00216	0.00219
5F-CDIR1	45.45%	40.91%	40.91%	40.91%	36.36%	40.91%
5F-CDIR2	43.18%	40.91%	54.55%	54.55%	52.27%	40.91%
5F-CDIR3	43.94%	42.42%	57.58%	56.06%	53.03%	42.42%

The evaluation of the forecasting metrics for the first five-factor extension is presented for each maturity interest rate time series of each data set in the panel-Tables 3.39 - 3.44. In terms of forecasting performance, the results are very mixed with considerable differences from one data set to another.

For GBP-LIBOR rates (see Table 3.39) two continuous-time models (CKLS and Vasicek) seem to produce consistently very good forecasts particularly for the shorter maturities (one-week and one-month); for the six-month GBP-LIBOR rates however, the VAR(1) benchmark model performs marginally better across all forecasting measures, while the AR(1) appears to perform best out of all the models for the last factor - the twelve-month GBP-LIBOR time series. In term of the economic measure CDCP the continuous models are superior to their rival models VAR(1) and AR(1).

The forecasting results for the USD-LIBOR rates (see Table 3.40) indicate a balanced conclusion regarding continuous-time versus discrete time modelling issue. The

CKLS and BS continuous-time models are the best predictive models for interest rates of shorter maturities (one- and three-month), while the benchmark models AR(1) and VAR(1) surpass the continuous-time models for maturities such as six-month and twelve-month, respectively. The evidence regarding the prediction of the change in direction of movements in the interest rates is quite mixed with a slight advantage towards the continuous-time models.

Based on the analysis of the financial forecasting metrics, in the case of the EUR-LIBOR data, the CIR model is consistently the best model in terms of forecasting performance. This finding is not surprising given the homogeneity of the estimates for the level effect vector parameter with all values close to 0.5.

Moving to the JPY-LIBOR rates, the forecasting results suggest that again the continuous-time models are clearly superior to the discrete time benchmarks. However, as the estimation results indicate, it is no clear cut which of the continuous models should be considered as a winner over all the factors. For example, the CKLS model produces the best forecasts in the case of one-and three- month JPY-LIBOR rates, while the BS model predicts best for one-week and six-month rates sometimes interchangeably with the CIR model. It is only for the last factor of twelve-month maturity rates where arguably the AR(1) outperforms the BS and CIR models. Additionally, the results illustrate the superiority of the continuous models with regards to the economic predictive power measured by CDCP (see Table 3.42).

Despite the more compact level effect estimates, as in the case of the EUR-LIBOR rates, the forecasting results regarding the CAD-LIBOR rates are not that clear-cut, with the best predictions alternating between the CIR, BS, CKLS and AR(1) models across both different maturities and horizons (see Table 3.43).

Finally, turning to the time series of interest rates on U.K. zero-coupon bonds, the six methods of forecasting produce rather more complex results, that are more difficult to draw conclusions from. However, following a closer examination of Table 2.44, one could observe that, over the short horizon of one month, the CKLS model forecasts best the future spot rates of seven-, ten- and fifteen-year maturity. Focussing on the same one-month horizon, it is the CIR and the VAR (1) models that predict best the one- and twenty-five-year future spot rates, respectively. Once the forecasting is projected further into the future, other models become superior, more specifically the discrete time benchmark model VAR(1) and the continuous-time BS. When examined across the three length horizons (one, two and three months) the quality of the forecasts deteriorates as the forecasting horizon is more distant in time with the best prediction of future interest rates being realized in the first horizon of one month.

3.6.3 Formal Tests for the Statistical Significance of the Model Forecasts

The statistical significance of the out-of-sample forecasts can be tested formally using the Diebold and Mariano test (1995) for any two sets of forecasts and the Clark and West (2007) test for nested models. The Diebold-Mariano tests is carried out under the quadratic error loss, following the approach outlined in Diebold (2015) where the forecasts produced by the various models are compared and not the models themselves[23]. Hence, we are interested in comparing the forecasts and test for significance between different series of 66 forecasts (3 months horizon).

Diebold (2015) discussed why the D-M test works well when we compare the forecasts and not the models as data generating processes. If one takes into consideration models as well some corrections may provide a better insight. For nested models, one technical problem with the D-M test is that under the null hypothesis that the parsimonious model is assumed to generate the data and therefore the larger model, in finite samples, is contaminated in terms of estimation because of additional unnecessary parameters. Clark and West (2007) provided an adjustment for the D-M tests such that their test statistic had approximately zero mean under the null hypothesis. It is necessary to observe that the C-W test is a one-side test while the Diebold-Mariano is a two-side test. We are going to employ the C-W test for the nested models in the CKLS family as well as for the four-factor versus five-factor models of the same specification (e.g. four-factor Vasicek versus five-factor Vasicek) and the D-M test for the remaining pairs of non-nested models.

The results of these two tests are reported in Table 3.34 - 3.38 for both model extensions, the four- and five -factor models, respectively. The results from the Clark-West test have a straightforward standard interpretation: any test-statistic larger than the appropriate critical values will reject the null hypothesis of equal predictive performance and conclude that the general CKLS model yields better forecasts. For the Diebold-Mariano test statistic, a negative number outside the critical area indicates that the first

[23] Another line of inquiry would be to compare the models themselves on the basis of pseudo-out-of-sample forecasts. Clark and McCracken (2001) and Clark and McCracken (2013) highlight that the distribution of the test statistic can be very different when the null hypothesis makes use of the model specification and parameter estimation uncertainty is taken into consideration. The testing based on model specification needs then to distinguish between nested versus non-nested models.

series forecasts (produced by the model on the vertical column of the table) yield a significantly lower loss error than the second forecast series. The opposite interpretation is true for the positive values and significance is evidently subject to a threshold comparison with a two-sided normal test constructed appropriately.

Table 3.45 The GBP-LIBOR Rates: Diebold-Mariano and Clark-West tests, for the four- and five-factor continuous-time models and VAR(1) and AR(1) models. The Clark-West results for the nested continuous-time models are underlined.

	GBP-LIBOR 4F					GBP-LIBOR 5F				
1W	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	12.22	13.03	6.04	-57.18	-9.70	76.88	51.64	36.23	-45.18	-11.89
Vasicek		11.99	-4.85	-19.16	12.12		61.92	50.79	48.27	77.00
CIR			-6.12	-57.12	-8.21			-41.94	-47.74	18.32
BS				1.29	6.10				-7.97	68.79
VAR					57.56					55.49
1M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	9.85	7.98	5.63	-30.37	-7.33	66.76	66.06	27.54	-21.85	-8.89
Vasicek		9.64	-1.81	1.76	9.66		-1.08	20.60	43.02	38.36
CIR			-5.64	-30.35	-8.77			44.53	15.69	30.10
BS				1.92	5.61				-5.05	10.44
VAR					29.93					20.33
3M						Vasicek	CIR	BS	VAR	AR1
CKLS						50.97	60.47	24.85	-15.61	-0.84
Vasicek							-7.28	12.05	43.45	21.04
CIR								42.60	12.39	34.07
BS									-6.31	14.92
VAR										14.89
6M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	8.38	7.82	3.48	-5.32	-0.86	7.98	108.66	157.62	20.10	21.94
Vasicek		7.91	8.07	-4.05	8.01		-50.40	-86.59	2.13	3.62
CIR			-3.00	-5.51	-7.36			-133.11	55.54	57.02
BS				-5.50	-8.34				91.50	92.57
VAR					5.31					4.12
12M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	8.09	9.50	5.79	-2.77	12.21	9.26	-51.48	-58.74	6.95	6.97
Vasicek		5.68	-5.43	-2.70	7.72		-58.80	-28.06	-0.69	2.81
CIR			-5.46	-2.84	8.46			88.78	61.00	62.86
BS				-0.19	5.57				30.24	32.37
VAR					2.88					3.87

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

Table 3.46 The USD-LIBOR Rates Diebold-Mariano and Clark-West tests for the forecasts generated by four- and five-factor continuous-time models and VAR(1) and AR(1) models. The Clark-West results for the nested continuous-time models are underlined.

	USD-LIBOR 4F					USD-LIBOR 5F				
1W	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	23.69	17.37	-8.53	1.27	1.30	9.25	-7.90	7.70	-10.39	-5.80
Vasicek		7.50	9.58	1.01	1.16		9.61	9.43	9.32	9.42
CIR			8.70	-1.18	1.12			6.31	-14.14	6.85
BS				-1.17	1.10				-10.31	9.03
VAR					1.16					10.17
1M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	23.40	11.10	-6.42	23.12	34.63	8.29	-2.81	6.96	-10.82	-9.26
Vasicek		8.35	9.85	6.32	9.64		8.24	8.25	7.43	8.26
CIR			7.29	-13.37	6.74			-7.66	-10.58	-7.51
BS				-10.82	-10.99				-10.92	-6.15
VAR					10.78					11.04
3M						Vasicek	CIR	BS	VAR	AR1
CKLS						8.12	9.81	-7.64	-10.09	-8.24
Vasicek							9.07	9.13	8.95	9.15
CIR								9.95	-10.31	11.66
BS									-10.16	-8.41
VAR										10.70
6M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	15.88	7.71	12.75	19.88	19.61	9.74	7.82	11.71	-8.92	-2.43
Vasicek		8.50	8.09	6.43	9.07		10.17	9.89	9.96	10.02
CIR			-10.13	-11.81	4.94			3.18	-1.00	6.43
BS				-12.13	6.82				-5.40	7.70
VAR					10.20					10.56
12M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	21.89	-5.49	8.41	21.54	21.57	13.18	7.73	10. 32	11.96	11.37
Vasicek		14.73	9.38	14.79	14.27		11.94	13.25	13.04	12.97
CIR			-6.41	2.85				-0.87	4.93	4.65
BS									11.80	11.32
VAR										-8.38

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

Table 3.47 The EUR-LIBOR Rates: Diebold-Mariano and Clark-West tests results for the forecasts generated by four- and five-factor continuous-time models and VAR(1) and AR(1) models. The Clark-West results for the nested continuous-time models are underlined.

	EUR-LIBOR 4F					EUR-LIBOR 5F				
1W	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	1.15	4.95	8.05	-9.17	-8.07	9.23	9.12	4.86	-10.19	-8.90
Vasicek		7.93	8.19	-11.84	8.22		9.06	0.56	-12.36	9.26
CIR			5.60	-9.02	-7.64			-3.70	-10.10	-6.12
BS				-9.21	-8.15				-3.98	3.59
VAR					9.82					10.28
1M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	0.75	-2.73	5.37	-10.50	-8.00	9.94	8.47	4.70	-9.71	-8.41
Vasicek		9.61	9.60	-12.25	13.13		8.75	1.02	-11.02	9.07
CIR			0.44	-10.38	-7.88			-3.85	-9.55	-7.83
BS				-10.38	-7.87				-5.60	3.11
VAR					12.67					9.90
	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS						9.77	-1.29	4.36	-9.69	-8.74
Vasicek							7.99	3.41	-10.60	8.78
CIR								-4.13	-9.06	-6.89
BS									-10.67	2.74
VAR										9.76
6M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	5.55	4.36	5.41	-12.16	-6.08	6.51	-2.18	4.36	3.25	6.80
Vasicek		2.98	-5.46	-7.95	-6.02		5.83	-3.56	-8.99	7.38
CIR			-3.51	-5.56	-5.13			-4.21	-6.81	-5.52
BS				-8.81	-6.09				2.57	3.79
VAR					-4.45					8.54
12M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	3.73	8.11	5.10	4.58	-4.53	-4.89	-2.38	4.98	5.57	5.92
Vasicek		-1.72	-2.39	0.57	-2.90		4.06	2.94	-6.76	-5.35
CIR			-1.75	0.96	-2.30			-1.56	-5.13	-4.22
BS				4.43	-4.92				-4.29	-3.19
VAR					-4.58					7.12

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

Table 3.48 The JPY-LIBOR Rates: Diebold-Mariano and Clark-West tests results for the forecasts generated by four- and five-factor continuous-time models and VAR(1) and AR(1) models. The Clark-West results for the nested continuous-time models are entered in bold and italic font.

	JPY-LIBOR 4F					JPY-LIBOR 5F				
1W	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	<u>18.01</u>	<u>19.98</u>	<u>25.80</u>	1.28	1.28	<u>20.40</u>	<u>5.37</u>	<u>8.44</u>	-11.57	-9.06
Vasicek		-22.17	-28.63	1.10	1.05		-5.33	18.49	14.80	0.89
CIR			-26.85	1.26	1.24			5.37	5.36	5.36
BS				1.28	1.28				-8.17	-8.97
VAR					-1.14					-7.49
1M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	**BS**	VAR	AR1
CKLS	<u>29.60</u>	<u>35.78</u>	<u>22.89</u>	34.18	34.17	<u>9.38</u>	<u>5.44</u>	<u>7.25</u>	-17.56	9.18
Vasicek		-42.57	-27.26	10.34	10.36		-5.42	8.63	8.00	8.60
CIR			-24.27	15.46	15.48			5.44	5.44	5.44
BS				54.37	54.37				-21.49	-9.27
VAR					5.95					20.01
3M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	**BS**	VAR	AR1
CKLS						<u>10.18</u>	<u>4.52</u>	<u>4.12</u>	-13.96	-20.12
Vasicek							-4.68	10.18	9.63	10.05
CIR								4.97	4.91	4.96
BS									-13.87	-8.72
VAR										13.17
6M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	<u>-8.61</u>	<u>12.18</u>	<u>12.84</u>	192.15	199.45	<u>19.23</u>	<u>4.42</u>	<u>6.24</u>	-11.34	8.26
Vasicek		-12.89	-13.42	40.96	41.96		-4.51	9.76	-10.60	10.32
CIR			-13.59	380.19	424.30			4.56	4.42	4.55
BS				32.82	32.66				-10.30	-8.52
VAR					9.73					10.52
12M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	**BS**	VAR	AR1
CKLS	<u>-16.28</u>	<u>14.36</u>	<u>12.95</u>	63.64	63.17	<u>-7.51</u>	<u>3.72</u>	<u>5.95</u>	-11.07	7.46
Vasicek		-15.76	-14.10	37.78	38.17		-3.42	-3.79	-10.06	7.35
CIR			-12.67	190.44	195.64			2.97	2.39	3.58
BS				84.40	83.46				-5.53	7.34
VAR					7.98					9.50

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

Table 3.49 The CAD-LIBOR Rates: Diebold-Mariano and Clark-West tests results for the forecasts generated by four- and five-factor continuous-time models and VAR(1) and AR(1) models. The Clark-West results for the nested continuous-time models are entered in bold and italic font.

	CAD-LIBOR 4F					CAD-LIBOR 5F				
1W	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	12.35	-13.76	19.68	3.59	3.06	5.95	-3.48	4.73	-13.10	7.94
Vasicek		12.72	11.76	37.00	36.56		5.89	6.09	6.25	6.22
CIR			-19.87	145.19	136.73			-1.77	-4.23	-2.33
BS				155.51	143.60				-5.15	-3.06
VAR					6.02					5.94
1M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-21.28	-11.37	-1.88	49.49	49.28	4.81	-5.40	-4.10	10.16	8.72
Vasicek		11.56	10.62	51.11	50.88		7.15	6.97	7.22	7.08
CIR			1.90	205.05	199.94			3.88	-7.00	-7.40
BS				370.13	354.85				-6.63	-6.54
VAR					7.37					6.70
3M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS						7.01	6.84	-6.46	8.72	8.67
Vasicek							7.66	7.27	7.33	7.35
CIR								6.55	6.72	6.78
BS									10.95	9.44
VAR										8.01
6M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-13.63	10.74	-11.35	110.43	110.61	7.32	-10.95	4.82	5.57	6.02
Vasicek		-11.14	-11.53	105.71	105.55		7.97	8.25	8.17	8.22
CIR			10.90	55.09	55.13			1.44	5.86	3.03
BS				258.24	259.56				3.71	4.35
VAR					-3.13					-3.20
12M	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-12.09	10.08	-11.65	275.39	278.85	8.80	-9.66	-9.02	9.15	10.02
Vasicek		-10.76	-10.38	68.72	68.98		9.35	9.10	9.15	9.21
CIR			11.31	132.96	132.81			7.76	8.02	8.50
BS				235.50	237.71				0.33	10.27
VAR					10.60					10.71

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

Table 3.50 The U.K. Spot Rates Full Sample Results: Diebold-Mariano and Clark-West tests results for the forecasts generated by four- and five-factor continuous-time models and VAR(1) and AR(1) models. The Clark-West results for the nested continuous-time models are underlined.

	UK SPOT 4F					UK SPOT 5F				
1Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-3.96	5.32	8.50	-1.33	-1.34	-0.34	-1.59	-0.14	13.10	7.94
Vasicek		-0.31	-7.54	-1.33	-1.34		5.89	6.09	6.25	6.22
CIR			-8.20	-1.33	-1.34			-1.77	-4.23	-2.33
BS				-1.33	-1.34				-5.15	-3.06
VAR					-0.17					5.94
7Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	1.71	4.32	4.82	-3.96	-5.92	-1.53	-1.21	-2.73	10.16	8.72
Vasicek		-4.33	-4.14	-3.97	-5.92		7.15	6.97	7.22	7.08
CIR			-4.12	-3.97	-5.92			3.88	-7.00	-7.40
BS				-3.97	-5.92				-6.63	-6.54
VAR					-5.87					6.70
10Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS						-1.52	-0.89	1.96	8.72	8.67
Vasicek							7.66	7.27	7.33	7.35
CIR								6.55	6.72	6.78
BS									10.95	9.44
VAR										8.01
15Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-3.15	-0.68	6.84	-5.17	-9.97	-1.32	-0.95	2.04	5.57	6.02
Vasicek		1.76	-4.83	-5.20	-9.98		7.97	8.25	8.17	8.22
CIR			-5.08	-5.20	-9.98			1.44	5.86	3.03
BS				-5.17	-9.95				3.71	4.35
VAR					-7.68					-3.20
25Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-14.97	-10.89	8.78	-4.24	-19.87	-0.60	0.59	0.47	9.15	10.02
Vasicek		13.02	-6.49	-4.27	-19.87		9.35	9.10	9.15	9.21
CIR			-6.85	-4.28	-19.87			7.76	8.02	8.50
BS				-4.21	-19.86				0.33	10.27
VAR					-19.95					10.71

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

The results provided by the D-M test suggest that for the money-market segment the CKLS model is superior to VAR(1) and AR(1) models only for one-week, one- and three-month maturities, as the negative values of the test-statistics are negative enough to reject the null hypothesis and conclude that forecasting errors produced by CKLS are smaller than those yielded by VAR(1) and AR(1). For the U.K. nominal rates the Diebold-Mariano test-statistics are positive enough to again reject the null, however with a opposite conclusion. These results are consistent with the evidence from the analysis of the forecasting accuracy measures that for longer maturities it is the parsimonious specifications that produce better forecasts in general. Regarding the Clark-West (2007) test, any negative value is interpreted as a failure to reject the null hypothesis of equal forecasting performance, while positive values higher than the critical values will result in the rejection of the null and the conclusion that the more general (here CKLS) model is superior in terms of predictive power relative to the nested model. In the money-market segment, for LIBOR-GBP series the CKLS is categorically superior to the nested models for all the maturities apart from the 12-month maturity where none of the nested models can be considered inferior to five-factor CKLS model.

In the case of the LIBOR-USD rates, the five-factor specification seems to outperform the four-factor models, relative to the benchmark models for all the maturities up to six-months inclusively, as all the D-M test statistics are negative. In terms of the nested models according to the C-W statistics the null is always rejected, implying the predictive superiority of the CKLS five-factor model (see Table 3.46). With regard to the LIBOR-EUR rates, according to the D-M test the CKLS model is superior to the benchmark models for the first three maturities. the C-W test results emphasise the high performance of the CIR model that is at least as good as the CKLS (five-factor) for the 3-, 6- and 12-month maturities. Similar results are obtained for the LIBOR-JPY rates in general, however the AR(1) discrete-time model appears to outperform few times the CKLS model. Slightly different results are observed for the LIBOR-CAD interest rates where the five-factor CIR model is superior over all models, both continuous- and discrete-time models. Finally, for the U.K nominal interest rates the Vasicek models equals at least the forecasting performance of the more general CKLS model for all the five long maturities, but underperforms relative to both the discrete-time models.

These general results are consistent with most of the evaluations of the model forecasting performance based on the accuracy measures analysis.

In addition, the C-W test has been used to assess the forecasting performance of all the four-factor models against their extensions to five-factor specifications. The results are reported for each maturity in Table 3.51 below.

Table 3.51 Four-Factor versus Five-Factor Models: The Clark and West Test Results

GBP-LIBOR	1Y	7Y	15Y	25Y
CKLS	7.16	14.80	-15.87	9.97
VASICEK	-25.58	0.28	0.51	0.13
CIR	-21.68	-10.20	-10.79	-16.02
BS	4.09	4.90	-5.97	7.79
USD	1Y	7Y	15Y	25Y
CKLS	15.34	9.42	-2.80	-13.79
VASICEK	-22.42	-19.10	-14.27	-21.02
CIR	7.48	3.61	-5.18	-0.63
BS	-8.41	-6.03	9.74	9.17
EUR	1Y	7Y	15Y	25Y
CKLS	-2.31	6.98	5.28	4.75
VASICEK	-8.68	-9.23	-4.35	-2.59
CIR	10.49	3.51	4.41	6.36
BS	3.87	-0.12	4.26	-4.46
JPY	1Y	7Y	15Y	25Y
CKLS	-16.87	-14.11	17.08	13.47
VASICEK	-12.94	25.23	23.82	18.43
CIR	10.23	10.33	6.89	4.03
BS	-18.42	-5.50	0.72	-12.69
CAD-LIBOR	1Y	7Y	15Y	25Y
CKLS	11.06	13.00	12.83	-10.26
VASICEK	9.90	11.94	-11.37	-13.33
CIR	0.49	13.75	-2.85	-10.66
BS	9.37	40.44	11.12	-8.09
UK SPOT	1Y	7Y	15Y	25Y
CKLS	3.83	9.31	11.32	12.31
VASICEK	2.04	3.77	5.44	6.27
CIR	3.44	3.68	5.27	4.85
BS	5.38	3.17	4.01	5.27

A general and important finding is that for the U.K. spot interest rates for all maturities and all the continuous-time models the five-factor models do not provide more reliable forecasts than their respective less complex four-factor counterparts. Therefore, according to this forecasting analysis the benefit from adding new factors is realised only for the LIBOR curve where the more complex models are necessary to capture the higher volatility of the short-term interest rates.

3.6.4 The Forecasting Analysis for the Post-Crisis Period

It is important to conduct the same comparative analysis for the forecasting results provided by the same models based on the post-crisis sample for the nominal U.K. interest rates. In addition, the forecasting performance of the full-sample versus the post-crisis results are formally tested using the Diebold-Mariano test. If the post-crisis forecasts are found to be superior then this could be interpreted as supportive evidence for a structural break in the data.

As in the full-sample case, the new estimates for the post-crisis period are used to compute the forecasts for all six types of models based on this latest data. The models predictive performance is assessed in two ways: first, using the same five forecasting accuracy measures and second by implementing the formal statistical tests of Diebold-Mariano (1995) and Clark-West (2007) for non-nested and nested models, respectively.

Table 3.52 The Forecasting accuracy measures for the U.K. spot rates time-series for the four- and five-factor models.

UK SPOT 1Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	-0.00063	0.00032	-0.00037	0.00014	-0.01456	-0.00153
4F-ME2	-0.00137	0.00051	-0.00098	0.00022	-0.04375	-0.00474
4F-ME3	-0.00199	0.00081	-0.00157	0.00046	-0.06365	-0.00367
4F-MAE1	0.00063	0.00033	0.00038	0.00017	0.01600	0.01464
4F-MAE2	0.00137	0.00051	0.00099	0.00024	0.04447	0.01537
4F-MAE3	0.00199	0.00082	0.00157	0.00047	0.06413	0.01867
4F-MAPE1	30.76%	16.64%	18.22%	8.76%	7.73%	8.45%
4F-MAPE2	53.18%	21.30%	37.58%	10.19%	16.71%	7.36%
4F-MAPE3	69.16%	29.53%	53.62%	17.17%	21.62%	7.22%
4F-RMSE1	0.00072	0.00039	0.00045	0.00021	0.02018	0.01763
4F-RMSE2	0.00162	0.00057	0.00122	0.00028	0.05701	0.01852
4F-RMSE3	0.00230	0.00100	0.00188	0.00067	0.07821	0.02347
4F-CDCP1	45.45%	59.09%	50.00%	50.00%	45.45%	54.55%
4F-CDCP2	40.91%	61.36%	43.18%	56.82%	40.91%	43.18%
4F-CDCP3	43.94%	57.58%	45.45%	54.55%	43.94%	45.45%
5F-ME1	0.00032	-0.00021	-0.00004	-0.00109	0.00228	-0.00153
5F-ME2	-0.00058	-0.00086	-0.00061	-0.00244	0.00279	-0.00474
5F-ME3	-0.00270	-0.00153	-0.00134	-0.00365	0.00427	-0.00367
5F-MAE1	0.00032	0.00028	0.00018	0.00111	0.10697	0.01464
5F-MAE2	0.00091	0.00090	0.00069	0.00244	0.14594	0.01537
5F-MAE3	0.00292	0.00155	0.00139	0.00365	0.17189	0.01867

5F-MAPE1	19.59%	15.34%	10.62%	61.12%	61.74%	8.45%
5F-MAPE2	36.37%	35.57%	26.73%	101.21%	66.70%	7.36%
5F-MAPE3	90.00%	50.37%	43.80%	124.50%	66.17%	7.22%
5F-RMSE1	0.00036	0.00035	0.00022	0.00131	0.11670	0.01763
5F-RMSE2	0.00129	0.00116	0.00092	0.00289	0.15500	0.01852
5F-RMSE3	0.00432	0.00192	0.00180	0.00423	0.18370	0.02347
5F-CDIR1	45.45%	59.09%	50.00%	59.09%	45.45%	54.55%
5F-CDIR2	38.64%	45.45%	40.91%	45.45%	56.82%	43.18%
5F-CDIR3	42.42%	46.97%	43.94%	46.97%	54.55%	45.45%

UK SPOT 7Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00002	0.00049	0.00085	0.00004	0.05632	-0.00243
4F-ME2	-0.00139	-0.00043	0.00013	-0.00118	-0.03160	-0.01016
4F-ME3	-0.00349	-0.00203	-0.00138	-0.00297	-0.18794	-0.01272
4F-MAE1	0.00025	0.00052	0.00085	0.00024	0.05854	0.03575
4F-MAE2	0.00154	0.00100	0.00089	0.00134	0.09662	0.03740
4F-MAE3	0.00359	0.00241	0.00205	0.00308	0.23129	0.04214
4F-MAPE1	2.10%	4.42%	7.17%	1.98%	4.93%	2.71%
4F-MAPE2	10.90%	7.38%	6.86%	9.50%	7.16%	2.66%
4F-MAPE3	21.23%	14.31%	12.42%	18.21%	13.78%	2.65%
4F-RMSE1	0.00034	0.00060	0.00096	0.00033	0.06709	0.04233
4F-RMSE2	0.00217	0.00124	0.00101	0.00187	0.11644	0.04529
4F-RMSE3	0.00492	0.00333	0.00285	0.00421	0.32019	0.05522
4F-CDCP1	68.18%	50.00%	40.91%	68.18%	50.00%	63.64%
4F-CDCP2	54.55%	45.45%	43.18%	54.55%	45.45%	50.00%
4F-CDCP3	53.03%	46.97%	45.45%	53.03%	46.97%	50.00%
5F-ME1	-0.00071	0.00013	0.00009	-0.00127	0.00198	-0.00243
5F-ME2	-0.00238	-0.00097	-0.00064	-0.00281	0.02584	-0.01016
5F-ME3	-0.00576	-0.00329	-0.00252	-0.00498	0.10366	-0.01272
5F-MAE1	0.00072	0.00042	0.00047	0.00127	0.11554	0.03575
5F-MAE2	0.00238	0.00130	0.00099	0.00281	0.24972	0.03740
5F-MAE3	0.00576	0.00351	0.00275	0.00498	0.44168	0.04214
5F-MAPE1	5.44%	3.23%	3.55%	9.57%	8.70%	2.71%
5F-MAPE2	15.69%	8.59%	6.62%	18.88%	16.77%	2.66%
5F-MAPE3	31.73%	19.14%	15.00%	28.52%	25.27%	2.65%
5F-RMSE1	0.00080	0.00050	0.00055	0.00135	0.12347	0.04233
5F-RMSE2	0.00314	0.00176	0.00126	0.00335	0.29599	0.04529
5F-RMSE3	0.00795	0.00499	0.00398	0.00615	0.54513	0.05522
5F-CDIR1	63.64%	68.18%	63.64%	54.55%	54.55%	63.64%
5F-CDIR2	50.00%	52.27%	52.27%	45.45%	45.45%	50.00%
5F-CDIR3	50.00%	51.52%	51.52%	46.97%	46.97%	50.00%

UK SPOT 15Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00056	0.00094	0.00079	0.00093	0.10469	0.00561
4F-ME2	-0.00074	0.00001	-0.00026	0.00006	0.01965	-0.00610

4F-ME3	-0.00253	-0.00142	-0.00178	-0.00129	-0.11759	-0.00819
4F-MAE1	0.00056	0.00094	0.00079	0.00093	0.10472	0.03287
4F-MAE2	0.00136	0.00110	0.00116	0.00105	0.10742	0.03579
4F-MAE3	0.00295	0.00216	0.00238	0.00203	0.20231	0.03570
4F-MAPE1	2.23%	3.71%	3.12%	3.67%	4.14%	1.27%
4F-MAPE2	4.98%	4.14%	4.31%	3.98%	4.08%	1.34%
4F-MAPE3	9.91%	7.35%	8.07%	6.92%	6.92%	1.27%
4F-RMSE1	0.00068	0.00104	0.00089	0.00104	0.11591	0.04063
4F-RMSE2	0.00176	0.00125	0.00137	0.00119	0.12013	0.04386
4F-RMSE3	0.00392	0.00279	0.00312	0.00261	0.25861	0.04524
4F-CDCP1	45.45%	40.91%	40.91%	40.91%	40.91%	40.91%
4F-CDCP2	40.91%	40.91%	40.91%	40.91%	40.91%	54.55%
4F-CDCP3	42.42%	42.42%	42.42%	42.42%	42.42%	56.06%
5F-ME1	0.00010	0.00200	0.00082	0.00002	0.00175	0.00561
5F-ME2	-0.00117	0.00234	0.00005	-0.00134	0.02709	-0.00610
5F-ME3	-0.00297	0.00209	-0.00145	-0.00320	0.08263	-0.00819
5F-MAE1	0.00031	0.00200	0.00084	0.00029	0.03849	0.03287
5F-MAE2	0.00141	0.00234	0.00099	0.00153	0.16717	0.03579
5F-MAE3	0.00313	0.00210	0.00215	0.00332	0.33353	0.03570
5F-MAPE1	1.18%	7.74%	3.24%	1.12%	1.47%	1.27%
5F-MAPE2	5.04%	8.72%	3.67%	5.44%	5.97%	1.34%
5F-MAPE3	10.32%	7.58%	7.15%	10.98%	11.06%	1.27%
5F-RMSE1	0.00040	0.00228	0.00098	0.00038	0.04779	0.04063
5F-RMSE2	0.00197	0.00252	0.00115	0.00214	0.22750	0.04386
5F-RMSE3	0.00421	0.00230	0.00288	0.00445	0.43476	0.04524
5F-CDIR1	59.09%	45.45%	50.00%	63.64%	63.64%	40.91%
5F-CDIR2	45.45%	56.82%	45.45%	47.73%	50.00%	54.55%
5F-CDIR3	45.45%	57.58%	45.45%	46.97%	48.48%	56.06%

UK SPOT 25Y	CKLS	VASICEK	CIR	BRSC	VAR1	AR1
4F-ME1	0.00070	0.00086	0.00076	0.00044	0.12436	0.01050
4F-ME2	-0.00041	-0.00007	-0.00026	-0.00072	0.05599	-0.00097
4F-ME3	-0.00164	-0.00110	-0.00136	-0.00189	-0.02972	-0.00251
4F-MAE1	0.00070	0.00086	0.00076	0.00046	0.12436	0.03333
4F-MAE2	0.00119	0.00106	0.00111	0.00122	0.09637	0.03413
4F-MAE3	0.00216	0.00176	0.00193	0.00222	0.13129	0.03087
4F-MAPE1	2.16%	2.67%	2.34%	1.42%	3.84%	1.03%
4F-MAPE2	3.53%	3.15%	3.29%	3.58%	2.93%	1.03%
4F-MAPE3	6.11%	5.01%	5.46%	6.27%	3.79%	0.91%
4F-RMSE1	0.00084	0.00098	0.00089	0.00066	0.13484	0.04380
4F-RMSE2	0.00144	0.00119	0.00130	0.00159	0.11326	0.04304
4F-RMSE3	0.00269	0.00213	0.00237	0.00280	0.15478	0.03930
4F-CDCP1	50.00%	50.00%	50.00%	54.55%	45.45%	40.91%
4F-CDCP2	45.45%	45.45%	45.45%	45.45%	45.45%	52.27%

4F-CDCP3	45.45%	45.45%	45.45%	45.45%	45.45%	53.03%
5F-ME1	0.00346	0.00457	0.00456	0.00378	0.00664	0.01050
5F-ME2	0.00279	0.00474	0.00458	0.00276	0.04110	-0.00097
5F-ME3	0.00301	0.00511	0.00465	0.00158	0.07682	-0.00251
5F-MAE1	0.00346	0.00457	0.00456	0.00378	0.23515	0.03333
5F-MAE2	0.00279	0.00474	0.00458	0.00276	0.19124	0.03413
5F-MAE3	0.00301	0.00511	0.00465	0.00216	0.25830	0.03087
5F-MAPE1	10.62%	14.03%	14.00%	11.60%	7.22%	1.03%
5F-MAPE2	8.45%	14.23%	13.74%	8.40%	5.75%	1.03%
5F-MAPE3	8.80%	14.87%	13.60%	6.48%	7.43%	0.91%
5F-RMSE1	0.00349	0.00462	0.00461	0.00381	0.24825	0.04380
5F-RMSE2	0.00293	0.00479	0.00462	0.00305	0.21007	0.04304
5F-RMSE3	0.00317	0.00517	0.00469	0.00259	0.28904	0.03930
5F-CDIR1	40.91%	36.36%	36.36%	40.91%	40.91%	40.91%
5F-CDIR2	52.27%	50.00%	50.00%	52.27%	40.91%	52.27%
5F-CDIR3	53.03%	51.52%	51.52%	50.00%	42.42%	53.03%

Table 3.53 The U.K. Spot rates Post-Crisis: Diebold-Mariano and Clark-West tests results.

	Four-Factor Models					Five-Factor Models				
1Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	-1.58	-6.32	2.35	1.12	1.11	2.65	-9.41	-3.85	-1.12	0.96
Vasicek		-7.25	8.06	1.03	1.14		4.64	2.96	-0.93	1.4
CIR			8.15	1.09	1.08			-5.58	-1.01	0.73
BS				-0.59	1.05				-1.01	1.07
VAR					1.05					1.07
7Y	Vasicek	CIR	**BS**	VAR	AR1	Vasicek	CIR	**BS**	VAR	AR1
CKLS	6.51	-6.97	2.01	6.5	5.84	-1.98	-5.96	5.54	4.65	4.61
Vasicek		5.95	-7.02	6.28	5.09		-27.08	-3.7	-6.22	-6.56
CIR			-6.65	-5.83	4.72			14.58	-1.43	-0.76
BS				6.93	5.75				-6.84	-7.41
VAR					4.99					4.17
10Y	Vasicek	CIR	**BS**	VAR	AR1	Vasicek	CIR	**BS**	VAR	AR1
CKLS						-4.56	-2.34	2.25	-5.24	-5.37
Vasicek							5.07	0.61	3.07	1.24
CIR								-23.32	-6.1	-6.11
BS									0.67	-0.4
VAR										-5.75
15Y	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
CKLS	6.46	-1.38	5.83	5.84	6.53	4.79	-3.01	0.13	3.24	-5.41

	Vasicek	CIR	BS	VAR	AR1	Vasicek	CIR	BS	VAR	AR1
Vasicek		-6.53	-7.16	5.19	6.47		-3.8	-6.92	-3.1	-5.28
CIR			-7.46	5.57	6.55			-1.07	2.53	-3.78
BS				6.25	6.93				4.29	-0.76
VAR					6.88					-3.6
25Y	Vasicek	CIR	**BS**	VAR	AR1	Vasicek	CIR	**BS**	VAR	AR1
CKLS	0.88	1.22	1.26	-4.24	-6.3	0.78	0.11	0.26	-4.24	-6.3
Vasicek		1.67	0.78	-4.24	-6.3		-0.02	-0.23	-4.24	-6.3
CIR			0.26	-4.84	-9.27			1.72	-4.84	-9.27
BS				-4.84	-9.27			0.78	-4.84	-9.27
VAR				-4.84	-9.27			0.26	-4.84	-9.27

The critical values are 1.645, 1.96 for Diebold-Mariano test and 1.282, 1.645 for Clark-West test at the 90%, 95% confidence level, respectively.

Table 3.54 Diebold-Mariano test for the forecasts series based on the two sample periods: the post-crisis and the full-sample data.

4F Full-sample versus post-crisis-sample forecasts					
CKLS	VAS	CIR	BS	VAR	AR1
-9.51	-5.67	-8.49	-4.06	-7.04	-0.49
-5.88	-5.16	-4.82	-5.80	-4.99	5.87
-5.94	-5.56	-5.69	-5.55	-5.39	7.55
-6.70	-5.07	-6.77	-7.10	-6.88	19.95
5F Full-sample versus post-crisis-sample forecasts					
CKLS	VAS	CIR	BS	VAR	AR1
-5.08	-7.62	-6.73	-9.31	-11.63	-0.49
-5.51	-5.30	-4.95	-6.85	-6.73	5.87
-6.77	-8.23	-12.42	-6.43	-6.20	-4.63
-6.12	-10.95	-5.24	-6.28	-6.59	7.55
-11.93	-26.71	-33.47	-8.01	-8.68	19.95

The critical values for Diebold-Mariano test are 1.645 and 1.96 at the 90%, 95% confidence level, respectively.

The results reported in Table 3.54 are mostly negative and higher in absolute value than the critical values for both 90% and 95% confidence levels, hence the null hypothesis id rejected on the left side, so the forecast errors from the full-sample period are smaller than those generated by the post-crisis period data. This could be interpreted as evidence for the importance of a longer in-sample estimation period as it may include information that is reflected in more reliable forecasts.

3.7 Conclusions

The empirical study conducted in this chapter tries to shed more light over two questions which are still unanswered in the TSIR literature. Firstly, it asks how many factors should be included in a good model. To answer this question, the performance of two model-extensions (four- and five-factors) is compared and analysed across different segments of yield curve. The empirical results of the dynamic estimation favour the five-factor models over the four-factor models with the former models consolidating the findings in the case of the latter. The addition of the fifth factor increased substantially the goodness of fit of the more complex models to the data, with some of the restricted models being very close to failing rejection against the general CKLS model. After a closer examination, the transition between the extensions from four to five-factor specification suggests that the level effect parameters are overestimated when only four factors are used.

Another benefit of increasing the model flexibility is that one could observe the change in the structure of the variance-covariance matrix between the two extensions. This allows for a clearer identification of where the strongest connections among the factors are situated along the term structure. This feature of the analysis has important implications for the investment decision making process; investors who focus on certain segments of the term structure of interest rates could determine, given the structure of the estimated covariance matrix, the regions where a twist/inversion in the shape of the yield curve may occur or be absent.

Second, empirical studies have always emphasized the importance of the trade-off between the level of realism and parsimony of the models employed. To elucidate further the "continuous-time versus discrete time modelling" debate both modelling approaches are brought together and compared within a complex setting in terms of their forecasting performance.

While the forecasting performance of the four-factor continuous models was inconclusive relative to the discrete time benchmark models VAR(1) and AR(1) that perform better overall, the evidence found in the forecasting analysis of the five-factor term structure models reveals a pattern in their predictive power. For shorter maturity (up to six months) interest rates, the continuous-time models nested in the CKLS framework outperform consistently the discrete time models. However, once the model involves interest rates of longer maturities, the situation reverses. Hence in order to optimize the forecasts it is necessary to include the "easy to implement" discrete time alternatives given their better forecasting performance for these longer term rates. These findings could have great implications for financial areas where the accuracy of interest rate forecasting is crucial. In conclusion, the forecasting results suggest that the availability of alternative forecasting methods should become an intrinsic feature of any forecasting analysis of the short end of the yield curve and the typical averaged forecasts from multiple methods could be improved even further by considering weighted average forecasts that reflect the empirical results.

Another aim of this chapter was to investigate the impact of the last GFC on the U.K. nominal interest rates by considering the pre- and post-crisis subperiods. The estimation results reveal that only the volatility parameters have been affected by the event of the crisis, especially the level-effect parameter that was significantly higher in the post-crisis period relative to the pre-crisis period. Moreover, it was found that from the nested models it is a different model that explains best the data for each subperiod: the Vasicek model could not be rejected against the more general CKLS model for both model-extensions in the pre-crisis period, while the CIR model could not be rejected against the CKLS model for the post-crisis period. Moreover, there is evidence for the importance of a longer in-sample estimation period as it may include information that is reflected in more reliable forecasts, as it was found that the forecast-errors from the full-sample period are smaller than those generated by the post-crisis period data.

Regarding the general forecasting performance, an important result is that for the U.K. spot interest rates across all the maturities and all the continuous-time models the five-factor specifications do not provide more reliable forecasts than their less complex four-factor counterparts. Therefore, according to our forecasting analysis the benefit from adding new factors is realised only for the LIBOR curve where the more complex models are necessary to capture the higher volatility of the short-term interest rates.

Overall, these findings seem to suggest two important aspects when modelling interest rates. One is the intrinsic feature of less volatility for longer maturity interest rates and the second one is the higher level of volatility that exists during turbulent periods of time.

Chapter 4

Dynamic Modelling and Forecasting of Scandinavian Interest Rates

4.1 Introduction

The theoretical literature on the term structure of interest rates (TSIR) models is well established providing researchers and practitioners with a myriad of dynamic specifications[1]. There are two main categories of TSIR models: equilibrium and no-arbitrage models. In the previous chapter an equilibrium based framework has been used to model the TSIR of several major economies. In this chapter, a no-arbitrage multi-factor model will be applied to a new set of interest rates. Given the current environment of persistent negative interest rates, the positivity restriction on a TSIR model such as the CIR model is not justified anymore. The general multi-factor linear Gaussian model of Babbs and Nowman (1999) (BN hereafter) that admits negative interest rates is employed to model the TSIR of three Scandinavian countries, namely Denmark, Norway and Sweden. Moreover, by contrast with the multi-factor CKLS framework used in the previous study the BN model treats the factors more realistically as they enter the model in latent form. As a result, by conducting a factor analysis one can conclude on the nature of these factors in terms of the attributes described by Litterman and Scheinkman (1991): the level, the slope and the curvature. This is not possible in the case of the CKLS framework where the factors are directly observable. Another important feature of the BN multi-factor model that the CKLS framework does not possess is tractability, providing closed formulae for the theoretical spot rates and the zero-coupon bond prices, therefore it is very useful for pricing interest rate contingent claims. Moreover, the BN model uses explicitly and therefore provides direct estimates for the market price of risk.

[1] See the numerous TSIR models presented in Chapter1

On empirical grounds, the choice of the estimation method is rather a complex consideration given the two dimensions of the yield curve. Despite a large volume of empirical studies, it is still not clear which is the optimal estimation method of such complex modelling frameworks. Initially, the estimation methodologies focused either on the time (dynamic) dimension using time-series (e.g. Chan et al.,1992; Dalquist, 1996; Nowman, 1997; Christiansen, 2008; Hong et al., 2010) or on the maturity dimension using cross-sectional data (e.g. Brown and Schaefer, 1994; De Munnik and Schotman, 1994). One econometric tool, that can take into account both, the dynamic and the cross-sectional aspect of the yield curve, is the Kalman filter a conditional moment estimator for linear Gaussian systems.

In general, the empirical literature on the estimation of TSIR models distinguishes between three main types of estimation methods, the maximum likelihood (ML), moment-based and simulation methods. In a recent comparative study, Duffee and Stanton (2012) concluded that when the finite-sample properties of the estimators are analysed, using standard methods on their own can introduce severe bias in the parameter-estimates. However, their accuracy can be improved by implementing a Kalman filter through the state-space approach, by choosing a discrete time model analogue to the original continuous time model.

In this chapter, the Kalman filter method is combined with the ML estimator towards the estimation of one-, two- and three-factor versions of the BN TSIR model. The models are estimated using panel data formed of daily spot yields with a cross-section of eight maturities over the January 2000 - September 2014. This econometric method yields estimates with desirable econometric properties – efficient and consistent, and it has been successfully applied before to model the term structure of the nominal rates of the U.S., the U.K., Japan and the Eurozone.

This study employs new data sets, by comparing the TSIR of Denmark, Sweden and Norway - three Scandinavian countries that have historically important economic and financial connections. As part of the Scandinavian political movement in the 19-th century, Denmark, Sweden and Norway formed in 1875 the Scandinavian monetary union by pegging their currencies at the same level to gold. Despite being considered the most successful of all the European currency unions, the World War I was the main factor that caused the union to dissolve.

Currently, despite their close geographical position, they have rather different status in relation to the E.U. and EMU and these differentials may be reflected in the final empirical results. While only Denmark and Sweden are part of the EU, none of these countries adhered yet to the EMU and hence they have their own currency. All three countries follow an inflation-targeting monetary policy but their mechanism of implementing it is not the same. On one hand, Sweden and Norway have a floating exchange rate in relation to the euro and formulate their monetary policy by explicitly targeting a low level of inflation of approximately 2% and 2.5% respectively. For Sweden, the key policy rate is the repo rate, while for Norway the main monetary policy tool is the interest rate on banks' deposits (sight deposit rate). On the other hand, Denmark has its currency pegged to the euro and therefore its monetary measures have to support directly the stability of the exchange rate through the exchange rate mechanism (ERM2); a stable nominal DKK-exchange rate assumes that the inflation in Denmark has to follow closely the inflation rate in the Eurozone. The main monetary policy instruments used by the Danish central bank to control short term interest are the day-to-day current account interest rate and the 14-day deposit rate (see Christiansen et al., 2004).

The main aim of this investigation is to analyse and compare the in-sample and out-of-sample performance of the three model specifications, between the pre-crisis and post-crisis sub-periods and among the three Scandinavian countries. In addition, the theoretical latent factors implied by the model are extracted using the Kalman filter technique for the two- and three-factor models and then they are compared to the empirical factors as in Diebold and Rudebusch (2013). The factor-loadings are assessed in order to determine the nature of the factors in terms of the three classical attributes suggested by Litterman and Scheinkman (1991).

This chapter is organized as follows. Section 4.2 provides a succinct literature review on Kalman filtering applications to interest rate modelling. Section 4.3 presents the theoretical framework, including the state-space form for the BN model and the Kalman filtering algorithm. The data is described and analysed in section 4.4. The estimation results and the empirical residuals are analysed in Section 4.5. The factors implied by the Kalman filter are characterised in terms of level, slope and curvature in Section 4.6. The forecasting comparative analysis is conducted in Section 4.7, while the conclusions are drawn in Section 4.8.

4.2 Empirical Applications of the Kalman Filter Technique

Originally, the Kalman filter (Kalman, 1960) was developed for engineers trying to estimate the state of a system from noisy measurements and only later it was applied in empirical economics and finance as a generalization of latent factor models. Early applications of the Kalman filter technique include Chow (1975) and Engle and Watson (1981) who analysed dynamic economic models. In finance, the Kalman filter technique was applied in several contexts: for calibrating and forecasting the term structure of interest rates (see Pennacchi, 1991; Duan and Simonato, 1999; Koopman et al., 2010), for pricing futures on commodities (see Schwartz, 1997; Manoliou and Tompaidis, 2002 and Lautier and Galli, 2004) or for the estimation of the volatility of stock prices based on intra-day data as in Barndorff-Nielsen and Shephard (2002). More recently, Kalman filtering has been chosen in other financial areas such as credit risk models (e.g. Chen et al., 2008; Carr and Wu, 2010) and equity options (e.g. Carr and Wu, 2007; Forbes et al., 2007 and Bakshi et al., 2008). Several reviews on the concept of the Kalman filter are contained in James and Webber (2000) (see chapter 18), Date and Ponomareva (2011) and Prokopczuk and Wu (2013).

An important feature and advantage of the state-space approach is the allowance for measurement errors, explicitly contained in the measurement equation that is one of the two equations defining the state-space form of the continuous-time model. The noise is the error in the measurement/calculation of the "observed" data. Possible sources for this type of error include the discount bond pricing methods from average between bid and ask prices or from coupon bonds, the rounding involved in bond pricing and the non-synchronous trading (Chen and Scott, 1995).

Given their tractability, the affine-type interest rate models allow for a straightforward derivation of a filtering algorithm since the theoretical prices can be expressed in terms of the unobservable short rate. The measurement equation is a linear or non-linear multivariate regression equation where the explanatory variables are the latent factors and the observed/measured yields are the endogeneous variables. Consequently, numerous affine term-structure models have been estimated using different Kalman techniques (Pennacchi, 1991; Chen and Scott, 1995; Duan and Simonato, 1999; Lund 1997).

Different variants of the Kalman filter can be applied depending on whether the model is linearly or non-linearly specified. Linear filtering is mostly applied in financial modelling to linear Gaussian interest rates models and stochastic volatility models (SV). If the dynamics of the state variables are Gaussian and the noises have normal distribution then the Kalman filter is straightforwardly enhanced by the maximum likelihood (Duffee and Stanton, 2012) and the resulting exact linear KF estimator possesses the best statistical properties. For non-linear term structure models (e.g. CIR, 1985), the literature distinguishes between two types of non-linear filters. First is the exact non-linear filter developed by Kitagawa (1987) which is unfortunately impractical given its high computational cost, and second is the approximate non-linear filtering for which numerous variants have been developed such as the extended KF (EKF), particle filters and sigma point filters (see Date and Ponomareva, 2011).

Duan and Simonato (1999) extended the transformed-data maximum likelihood method of estimation presented in Duan (1994) to the whole class of exponential-affine model class. Using their own filtering method, the authors estimated the one-factor Vasicek and CIR models and the two-factor Chen and Scott (1992) model, based on monthly data of U.S. treasury securities with four maturities over the period 1964 - 1997. The data was analysed in two sub-samples by eliminating the period of 1979 - 1982 containing a shift in the monetary policy of the Federal Reserve. For all three models, the estimation results support previous evidence (Hamilton, 1988) of a structural break as the variances of the two sub-samples are considerably smaller than the variance of the short rate from the whole period estimation. However, based on a Lagrange multiplier test, all three specifications are rejected, implying that these affine models are not explaining satisfactorily the dynamics of the term structure of interest rates. In the case of one-factor specifications, the estimate of the market price of risk parameter was positive for the Vasicek model and negative for the CIR model, implying a positive risk premium in bond prices for the unobservable factor. For the Chen and Scott (1992) model the results are consistent with the findings reported by Chen and Scot (1993b) in the sense that the second factor seems to be insignificant.

Multi-factor versions of the equilibrium asset pricing CIR (1985b) model were estimated by Chen and Scott (2003) using a non-linear Kalman filter to generate estimates for the unobservable state variables: the short-term rate, the long-term interest rate and the interest rate volatility. The instantaneous interest rate was modelled as a sum of these factors, with each factor following a univariate square root diffusion process. The authors found that while the three-factor formulation is superior to one- and two-factor versions, the resulting quasi maximum likelihood (QML) estimators are significantly conditionally biased.

Jegadeesh and Pennacchi (1996) estimated a two-factor dynamic model for the TSIR using panel data on Eurodollar futures prices. They derived the associate space-state form model and showed that the futures prices can be written as a linear combination of both factors, the short rate and the stochastic long-term mean, respectively. The estimation results implied by the Kalman filter were highly significant and represent a substantial improvement in fitting the yield curve over the single-factor version and other estimation techniques.

Lund (1997) and Babbs and Nowman (1999) estimated multi-factor versions of the generalized Vasicek model using different Kalman filter methods. Lund (1997) developed an iterative extended Kalman filtering (IEKF) algorithm that can be applied to the exponential-affine class models of the term structure based on directly observable market data. The analytical log-likelihood function was derived and the finite sample properties of the QML estimator were explored using Monte Carlo simulations. Most of the parameter estimates were found to be on average very close to their true values, except for the risk premia. A possible reason for the efficiency loss can be the approximations techniques involved in the non-linear filters.

Babbs and Nowman (1999) proposed a more general model of the term structure with multiple latent factors underlying the dynamics of the short rate. More specifically, their model represents a subclass of Langetieg's (1980) models where the short rate is a particular combination of unobservable variables. These latent variables can be interpreted as a stream of news affecting different segments of the yield curve. Moreover, it was shown that the no-arbitrage version of Babbs and Nowman (1999) general model is equivalent to the generalized Vasicek multi-factor models studied by Babbs (1993) and includes the "double decay" model of Beaglehole and Tenney (1991) as a particular case. Kalman filtering was applied to US weekly interest rates of eight different maturities covering the 1987-1996 period, in order to empirically investigate one-, two- and three-factor models. The estimation results suggested that the three-factor model had statistically fitted the data better than the one- and two-factor models. While most of the parameter estimates were statistically significant the two- and three-factor models did not yield significant estimates for the market prices of risk associated with the unobservable factors. The same subclass of Gaussian models has been employed by Nowman (2010) in order to analyse the evolution of the UK and Euro yield curves over a period including the last GFC of 2007-2009.

Another related empirical study on US yield curves is Geyer and Pichler (1999), who employed the state-space form and the non-linear Kalman filter framework used by Chen and Scott (1995) and found substantial evidence to reject the CIR multi-factor specifications. Their diagnostic tests on the residuals resulted in biased and autocorrelated prediction errors, the main reason for rejection being the non-negativity constraint of the classic CIR model. De Rosi (2004) efficiently estimated a two-factor Gaussian model for the forward curve by Kalman filter using time series of eight UK weekly interest rates spanning the LIBOR swap curve. The state space form allowed for time-varying intercepts and only the market price of risk associated with the first factor – the short rate, changed over time as a linear functional of the short rate. Another innovation in De Rossi (2004) is the presentation of the Kalman filter under both risk-neutral and physical probability measure, respectively. However, the residuals analysis rejects the model, suggesting that other functional forms should be investigated.

4.2.1 Kalman Filter Estimation of Linkages between Macroeconomics and Yield Curves

Numerous studies employed Kalman filtering to estimate the yield curve in a macroeconomic context. An early study by Pennacchi (1991) explored the relationship between the real interest rates and the expected rate of inflation by implementing the state-space approach for a two-factor continuous-time equilibrium asset-pricing model. Based on prices of different maturity U.S. Treasury bills and on survey of inflation forecasts, Pennacchi (1991) identified the latent factors as the real interest rate and expected inflation. In contrast with previous studies such as Fama (1975) and Fama and Gibbons (1982), Pennacchi (1991) relaxed the assumption of independence between the two state variables and found that they were negatively correlated during the 1968-1988 period. In addition, the two factors exhibited rather different dynamics, with the real interest rates being more volatile and exhibiting a weaker mean reversion feature than the expected rate of inflation. These findings support what economic theory suggests, that there are many exogenous variables (such as technological change and output change) that may affect simultaneously both real interest rates and the rate of inflation.

Fendel (2004) proposed a no-arbitrage Gaussian macroeconomic affine TSIR model that incorporates two observable factors that affect the dynamics of the interest rates, namely the inflation and output gap, and one latent factor without a clear economic interpretation. The estimation results implied by the Kalman filter indicated a very good characterization of the German yield curve between 1979 and 1998, as interest rates movements were explained very well by the expected variations in the macroeconomic factors. The interpretation of the factors was consistent with the monetary policy rules. The "inflation factor" acted as the level factor with nearly equal impact on all maturities; the impact of the "output factor" declined as the maturity increased and could be interpreted as the curvature, while the third latent factor can play the role of the slope factor as it influences only the short end of the yield curve. Diebold et al. (2006) explored the dynamic interactions between the yield curve and the economy by extracting three latent yield curve factors (level, slope, and curvature) under the Nelson-Siegel (1987) parameterization and also by including three observable macroeconomic variables (real activity, inflation, and the monetary policy instrument). The analysis of different impulse response functions showed some impact of the yields on the three main fundamentals while in the other direction the slope factor seemed to be highly responsive to shocks in all three macro variables.

Joyce et al. (2012) examined the behaviour of the U.K. real interest rates during the conundrum of 2004-2005 when long horizon interest rates had fallen globally. Using index-linked bonds and survey data, they modelled the real forward rates within a flexible affine framework of Duffee (2002) and derived a three-factor state-space form by implementing Backus et al. (1998) discretization. While exploring several model specifications[2], some in a macroeconomic context, the authors identified the main factor that explained the declining long-term real forward rates as the time-varying term premia, a conclusion that supports the "excess liquidity" explanation during turbulent financial times.

4.2.2 Numerical Issues Related to Kalman Filtering

Numerous empirical studies using the Kalman filter technique (e.g. De Jong, 2000 and Chen and Scott, 2003) are based on the strong assumption that the measurement errors are identically and independently distributed (i.i.d.). In a Monte Carlo study, Dempster and Tang (2011) explore different multifactor exponentially affine models (EAM) showing that these errors are serially and cross-sectionally correlated and therefore their specification affects the estimation results. While the cross-sectional correlation is less detrimental the serial correlation results in poor estimates for the mean-reversion parameters as the Kalman filter procedure fails to recognise the mean-reversion of the underlying process from the mean reversion of the measurement error. Dempster and Tang (2011) proposed an augmented state-space form that seems to improve the estimation results by reducing the biases introduced by a particular state-space form.

In a comparative study, Duffee and Stanton (2012) analysed the finite-sample properties of three estimation methods widely used in empirical finance, arguing that previous work had relied mostly on the asymptotic properties of such estimators and ignored the finite-sample properties of such estimators. They claimed that when finite-

[2] In Joyce et al. (2012) there are three specifications including a baseline model that incorporates only real yields and then a survey model (that includes also long-term GDP growth forecasts) and a policy rate model (that incorporates the 1-month policy rate.

sample properties are considered the results are surprizing rejecting standard methods such as maximum likelihood (ML) or efficient method of moments (EMM). Most importantly, in the context of complex TSIR models and relatively small sized samples, they concluded that the Kalman filter method and its variants are the most appropriate methods. When the standard Kalman filter cannot be implemented Duffee and Stanton (2012) proposed a modified filter that despite being inconsistent produced finite-sample biases like those obtained by ML methods.

Despite their complexity, latent factor models in general are more realistic than specified factor models, hence their revival in analysing the dynamics of economic and financial variables that are driven by unobservable factors. The state-space approach together with advanced filtering algorithms constitute a powerful econometric tool in the estimation of such complex models.

4.3 Methodology

This section presents the complex modelling framework used in this investigation towards the estimation of the term structure of nominal interest rates based on panel data from three Scandinavian countries: Denmark, Norway and Sweden.

For clarity, the methodology is presented in three sections. In the first section, the multi-factor generalised Vasicek model developed by Babbs and Nowman (1999) is presented, followed in the second section by the derivation of an appropriate state-space form (see Babbs and Nowman, 1999). The third section describes the linear Kalman filter algorithm implied by the multi-factor BN TSIR model and the respectively augmented ML estimator. Given the linearity and the Gaussianity of the BN model, the Kalman filter is linear and exact, ensuring desirable properties such as efficiency and consistency for the ML estimators.

4.3.1 The Babbs and Nowman (1999) TSIR Model

In this model, the short rate $r(t)$ is determined as a particular combination of one or more correlated unobservable factors $X(t) = (X_1(t), X_2(t), ..., X_J(t))$ that can be interpreted as streams of positive or negative economic news with current impact on different segments of the yield curve. The BN model is a particular case of the general Gaussian model of Langetieg (1980) in the sense that state variables enter the short rate specification with equal weights of minus unity, while the underlying latent factors $X_j(t)$ follow a zero mean Vasicek process[3]:

$$r(t) = \mu - \sum_{j=1}^{J} X_j(t) \tag{4.1}$$

$$dX_j(t) = -\xi_j X_j dt + \sigma_j dW_j(t) \tag{4.2}$$

The dynamic processes (eq. 4.2) are driven by the Brownian motions W_j which are correlated with correlation coefficients ρ_{ij}. At this stage, the vector of constant parameters consists of $\theta = (\mu, \xi_j, \sigma_j, \rho_{ij}, \lambda_j)$, where μ is the long-run average rate, ξ_j and σ_j are the mean reversion and the diffusion parameters, respectively. With each random state factor, there is an associated market price of risk parameter λ_j that is not yet explicitly identified in the model. The BN model is a general n-factor model that possesses closed formulae for the theoretical spot rates and the zero-coupon bond prices, therefore it is very useful for pricing interest rate contingent claims.

4.3.2 The State-Space Form for the Babbs and Nowman TSIR Model

In order to apply the Kalman filter algorithm to a continuous-time dynamic model, we need an analogue discrete-time state-space form. In general, the state-space form consists of a system of two types of equations, the measurement and the transition equation, respectively. The measurement equation considers the measurement errors as the difference between the observed variables and their predicted (filtered) values. The transition equation involves the unobserved variables and it is usually derived as a discretization of a continuous dynamic model. In the case of Babbs and Nowman (1999) model an appropriate state-space form in derived in two steps.

First, the tractability of the model provides the theoretical spot rates used in the measurement equation and second, the continuous dynamic processes describing the underlying state factors are discretized following Bergstrom (1984) to obtain the transition equation. The measurement equation relates the theoretical spot rates of different maturities τ_m $(m = 1,...,N)$ denoted by $R_m(t)$ with the corresponding continuously compounded interest rates $Y_m(t)$ extracted from the observed yield curve at time t. In other words, the observed rates (the panel data sample) $Y(t) = (Y_1(t), Y_2(t), ..., Y_N(t)) = (Y_m(t))_m$ with $(t = 1,...,T)$ are assumed imperfect, i.e. they are

[3] These particularities are not that restrictive; they reduce the number of estimated parameters, otherwise redundant (see Babbs and Nowman, 1999).

sampled with error, and are modelled as a simple multivariate regression on the theoretical spot rates:

$$Y(t) = R(t) + \varepsilon(t) \tag{4.3}$$

where the disturbances $\varepsilon(t)$ are the measurement errors assumed to be independently and identically distributed (i.i.d.) $\varepsilon \sim N(0, H(\omega))$. For empirical reasons. the variance-covariance matrix $H(N \times N)$ is restricted to diagonal form[4] with maturity-specific variances, i.e. $diag(H) = (h_1, ..., h_N)$.

It is known (Duffie and Khan, 1996) that in the case of affine interest rate models the continuously compounded spot rates are also affine combinations of the short (instantaneous) rate $r(t)$:

$$\begin{aligned} R_m(t) \equiv R(t+\tau_m) &= -\frac{\log(B(t+\tau_m, t)}{\tau_m} = A_0(\tau_m \mid \theta) + A_1(\tau_m \mid \theta)' X(t) \\ &= d_m(\theta) - \sum_{j=1}^{J} c_{mj}(\theta)' X_j(t) \end{aligned} \tag{4.4}$$

where $d_m(\theta) = R(\infty) - \alpha(\tau_m)$ and $c_{mj} = (1 - e^{-\xi_j \tau_m}) / \xi_j \tau_m$ are calculated as in Babbs and Nowman (1999) (p.121). Therefore, for each maturity m the equation (4.3) can be projected as:

$$Y_m(t) = R(t+\tau_m, t) + \varepsilon_m(t) = d_m(\theta) - \sum_{j=1}^{J} c_{mj}(\theta)' X_j(t) + \varepsilon_m(t) \tag{4.5}$$

In vector-format the measurement equation used in the state-space form can be written as:

$$Y(t) = R(t) + \varepsilon(t) = d(\theta) + C(\theta) X(t) + \varepsilon(t) \tag{4.6}$$

where d is a $N \times 1$ vector, C is a $N \times J$ matrix and ε is a $T \times 1$ vector.

For the transition-equation we return to the continuous time stochastic process assumed for the state variables that need to be appropriately discretised. Following Bergstrom (1984) the exact discrete analogue model of the continuous time specification (4.2) is a VAR(1) model without feedbacks given by:

$$X(t_k) = B(\omega) X(t_{k-1}) + \eta_{t_k} \tag{4.7}$$

where $B(\omega)$ is a diagonal $J \times J$ matrix called the transition matrix and is given by $\beta_{jj}(\omega) = e^{-\xi_j (t_k - t_{k-1})} = e^{-\xi_j \Delta t_k}$ and the disturbances $\eta_t \sim N(0, V_{t_k})$. The elements of V_{t_k} are calculated as in Bergstrom (1984). Equation (4.7) above represents the transition equation needed for the state-space form.

[4] By doing so we recognize the differences in trading at different maturities (see Babbs and Nowman 1999)

Together the equations (4.6) and (4.7) represent the state-space formulation of the Babbs and Nowman (1999) model which is a linear Gaussian system that will be estimated by combining the Kalman filter with the maximum likelihood (ML) estimator.

4.3.3 The Kalman Filter Algorithm

The Kalman filter is an iterative method involving a sequence of steps that will be presented in the following. At the end of iteration k the Kalman filter will provide an improved filtered estimate for the state vector $X(t_k)$ based on all the information up to time t_k. Therefore, at the end of all the iterations the Kalman filter generates time series of estimates for both, observable and unobservable variables, Y_{t_k} and X_{t_k}, respectively. In the following equations, the index t_k will be written as k to reduce the complexity (see Babbs and Nowman, 1999) of the mathematical expressions. The two indices m and k are distinct. The index m is denotes the ranking of the maturity in the whole selection of N maturities (in our case $N=8$) .More precisely, the maturity τ_m is the m-th maturity, $(m=1,...,N)$ (see page 197). The index k is a time-index, which represents the ranking of one observation out of the total of T observation. Because k is a discrete time index and our models are in continuous time, the theoretical model distinguishes between the t (which covers an interval) and t_k (which is a value in the daily observation point k).

A very important aspect is the initialization step at the beginning of all iterations. To start the Kalman filter technique the initial values for the state vector and its covariance matrix should be chosen. For practical reasons, it assumed that the VAR(1) model in the transition equation (4.7) is stationary[5] and therefore, the starting values are set as the unconditional moments of X_k, the unconditional mean and covariance matrix (see, Martin et al., 2013):

$$\begin{cases} X_{1|0} = E(X_i) = 0 \\ \text{var}_{1|0}(\text{X}_i) = \dfrac{\sigma_i^2}{2\xi_i}, \quad \text{cov}_{1|0}(X_i, X_j) = \dfrac{\rho_{ij}\sigma_i\sigma_j}{(\xi_i + \xi_j)} \end{cases} \tag{4.8}$$

The implementation of the Kalman filter involves four main steps inside a particular k-th iteration (see Hamilton, 1994)

[5] This implies that all eigenvalues of parameter matrix B are negative.

1) The prediction of the state variable X_k

We use as inputs from the previous iteration the updated estimates $\hat{X}_{k-1|k-1}$ and its covariance matrix $\Sigma_{k-1|k-1}$, to predict X_k and its mean square error (MSE) matrix $\Sigma_{k|k-1}$ conditional to all information up to t_{k-1} :

$$\hat{X}_{k|k-1} = E(\mathrm{X}_k \mid (\mathrm{Y}_{k-1}, \mathrm{Y}_{k-2}, ..., \mathrm{Y}_1)) = E_{k-1}(BX_{k-1} + \eta_k) = BE_{k-1}(X_{k-1}) = B\hat{X}_{k-1|k-1} \qquad (4.9)$$

$$\Sigma_{k|k-1} = E[(\mathrm{X}_k - \hat{X}_{k|k-1})(\mathrm{X}_k - \hat{X}_{k|k-1})'] = B\Sigma_{k-1|k-1}B' + V \qquad (4.10)$$

2) Forecasting Y_k

The best forecast for Y_k is given by its conditional mean based on all the information up to time t_{k-1}:

$$\hat{Y}_{k|k-1} = E(Y_k \mid Y_{k-1}, Y_{k-2}, ..., Y_1) = E_{k-1}(CX_k + d) = C\hat{X}_{k|k-1} + d \qquad (4.11)$$

The availability of a new observation Y_k allows us to calculate the vector of prediction errors ν_k and their covariance matrix F_k:

$$\begin{aligned} \nu_k &= Y_k - \hat{Y}_{k|k-1} = C(\mathrm{X}_k - \hat{X}_{k|k-1}) + \varepsilon_k \\ F_k &= E[(Y_k - \hat{Y}_{k|k-1})(Y_k - \hat{Y}_{k|k-1})'] = C\Sigma_{k|k-1}C' + H \end{aligned} \qquad (4.12)$$

3) Updating the inference about X_k

This is a very important step that combines optimally the past information with new measurements; more precisely, the new observation Y_k is used to improve the forecast of X_k by considering the joint conditional normal distribution of Y_k and X_k. The new filtered estimate for the unobservable variables is obtained as follows:

$$\hat{X}_{k|k} = \hat{E}(X_k \mid Y_k \text{ and } (Y_{k-1}, Y_{k-2}, ..., Y_1)) = \hat{X}_{k|k-1} + \Sigma_{k|k-1}C'\mathrm{F}_k^{-1}\nu_k = \hat{X}_{k|k-1} + K_k\nu_k \qquad (4.13)$$

where the matrix $K_k = \Sigma_{k|k-1}C'F_k^{-1}$ is called the *gain matrix* (see Hamilton, 1994, p.380)

$$\Sigma_{k|k} = \Sigma_{k|k-1} - \Sigma_{k|k-1}C'F_k^{-1}C\Sigma_{k|k-1}$$

4) Producing a forecast of X_{k+1}

The one-step-ahead forecast for the unobservable variables is derived from the state equation based on the improved estimate $\hat{X}_{k|k}$:

$$\hat{X}_{k+1|k} = B\hat{X}_{k|k} = B(\hat{\mathrm{X}}_{k|k-1} + K_k\nu_k) \qquad (4.14)$$

Its MSE (mean square error) $P_{k+1|k}$ involves all matrices present in the current iteration: the coefficient matrices of the state-space system B and C, the variance matrices of the two uncorrelated disturbances, V and H and finally the MSE of the Y_k forecast error, Σ_k :

$$\Sigma_{k+1|k} = B\Sigma_{k|k}B' + V = B\Sigma_{k|k-1}B' + V - B\Sigma_{k|k-1}C'F_k^{-1}C\Sigma_{k|k-1}B' \quad (4.15)$$

To calculate these values, the time step depends on the frequency of the data, for example $\Delta t_k = 1/52$ for weekly observations or $\Delta t_k = 1/252$ for daily observations.

The Kalman filter is an algorithm that works under the hypothesis that the population parameters are given. Each step aims to produce state estimators for the state vector X_t which enter the log-likelihood function $L(X_t, \theta)$. In the case of linear Gaussian models, the log-likelihood function has a closed formula (see Babbs and Nowman, 1999) given by:

$$L(X_t;\theta) = -\frac{NT}{2}\ln 2\pi - \frac{1}{2}\sum_{t=1}^{T}(\ln|F_t| + \nu_t' F_t^{-1}\nu_t) \quad (4.16)$$

The optimisation problem is transformed from the maximization of the log-likelihood function to the minimization of the expression minus twice of the log-likelihood with respect to the vector of parameters θ. The optimal solution θ^* is used recursively (iteration by iteration) back in the Kalman filter algorithm to produce the state estimates for the observable and unobservable variables.

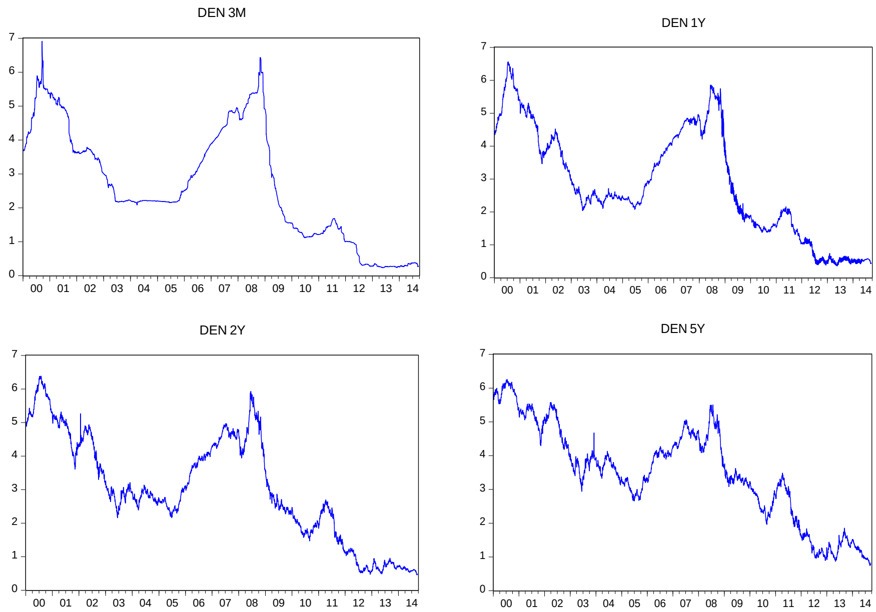
Despite the fact that the BN modelling framework is among the few theoretically generalized models for interest rates, when it comes to empirical studies no estimation of such models has been conducted for more than three factors. This can be justified in two ways based on empirical evidence and also on theoretical grounds. Empirically, previous studies (Scheinkman and Litterman, 1991; Babs and Nowman, 1999) have shown by using principal component analysis (PCA) that three factors are sufficient to explain up to 95% of the variations observed in the data. In another empirical investigation Geyer and Pichler (1999) have estimated and tested also four- and five-factor model of Cox, Ingersoll, and Ross (1985). Their estimation results showed that for four- and five-factor models the ML objective function had multiple local maxima making impossible to choose between different sets of parameter-estimates. Another short-coming of the higher number of factors was that the fourth and fifth factors behaved like pure random walks, therefore their simulation for future realisations is of no benefit to risk management measures.

On theoretical grounds, while the level, slope and curvature attributes are practical interpretations within the mathematical derivative calculus, there is no yet attribute for the third derivative in the way slope is for the first derivative and the curvature is for the second derivative. These findings had important implications in making the decision to employ only one-, two- and three-factor BN models.

4.4 Data

The data used for this empirical analysis is a panel of daily spaced time series of zero coupon (spot) Government yields for three Scandinavian countries, respectively Denmark, Norway and Sweden. The cross-sectional dimension of the yield curve involves eight points of the following maturities: three-month, one-, two-, five-, seven-, ten-, fifteen- and twenty-years. The data have been collected from Bloomberg and covers the period from January 2000 to September 2014, inclusive, with a total of 3,847 daily observations. The full data set is divided into two sub-periods: the pre-crisis period from 03 January 2000 to 31 June 2007, and post-crisis period from 02 July 2007 to 30 September 2014[6]. For the forecasting analysis over three horizons, the out-of-sample daily data are the last three months October, November and December 2014.

The graphs of the eight time-series over the entire period are presented in Figures 4.1- 4.3 with eight panels for each country, respectively.



[66] To be consistent with the other investigations in this thesis the same breaking point in time is recognized for the start of the crisis, namely the third quarter of 2007.

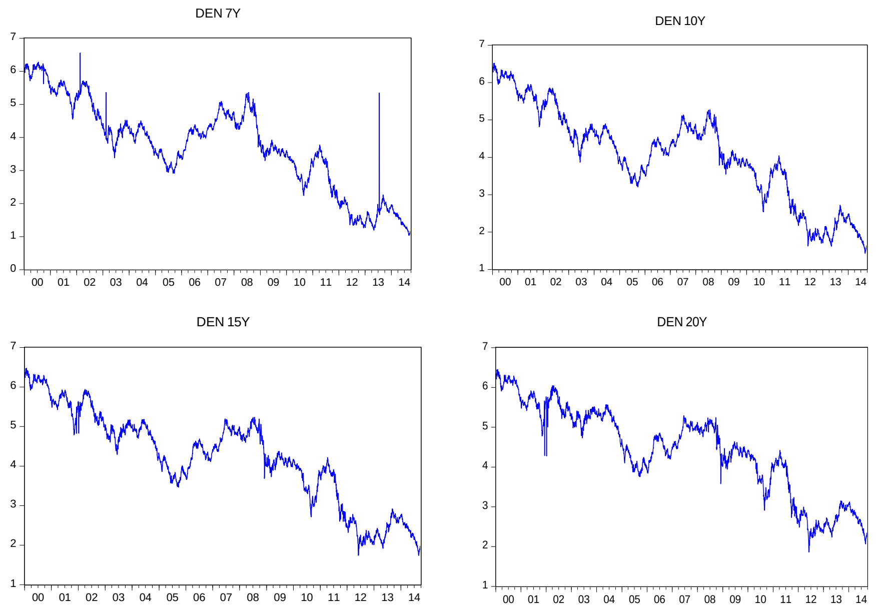
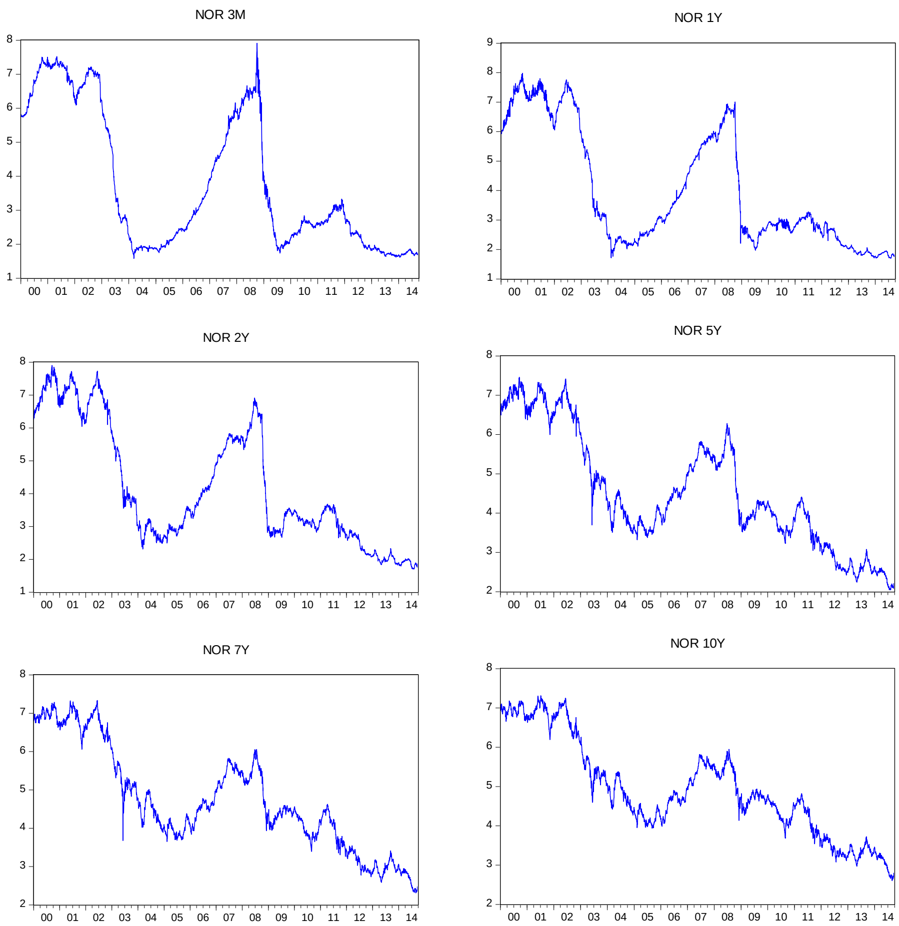


Figure 4.1 DENMARK: The individual daily time-series of interest rates of eight maturities over the period 3/1/2000 – 30/9/2014.



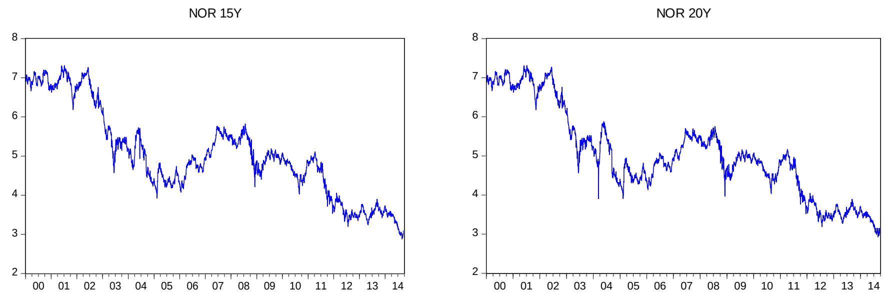


Figure 4.2 NORWAY: The individual daily time-series of interest rates of eight maturities over the period 3/1/2000 – 30/9/2014.

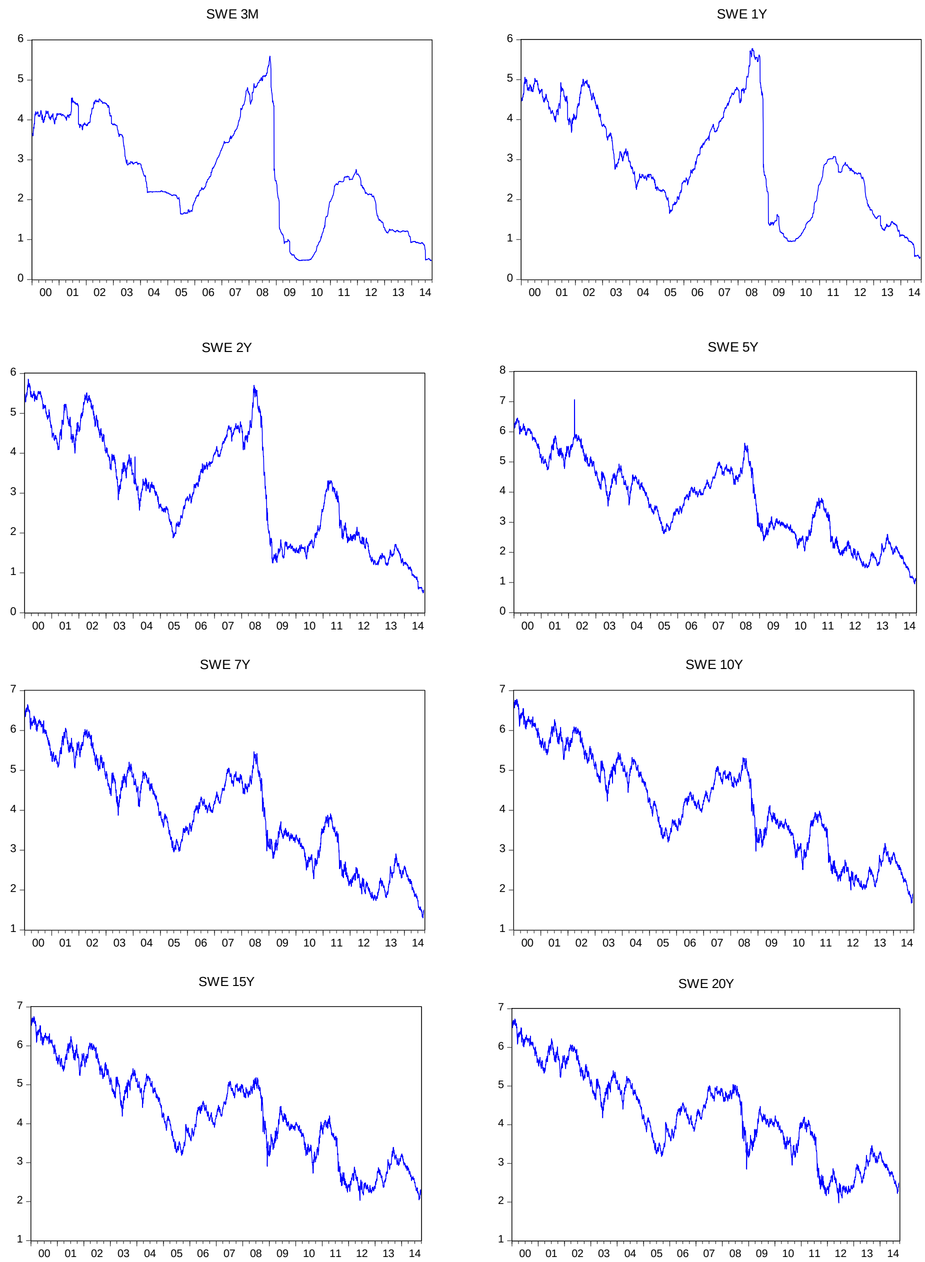


Figure 4.3 SWEDEN: The individual daily time-series of interest rates of eight maturities over the period 3/1/2000 – 30/9/2014.

In order to observe the evolution of the term structure over the entire period, five out of eight series (three-month, one-, five-, ten- and twenty-year interest rates) are plotted in Fig. 4.4-4.6 for each country. The shape of the yield curve is defined by the relationship between the short term and long-term interest rates, and the former can be greatly influenced by the monetary policy of each country. Indeed, the paths of the 3-month and 1-year interest rates differ mostly from one country to another.

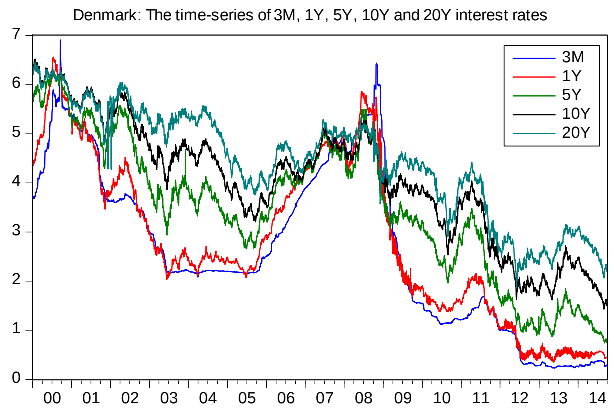


Figure 4.4 Daily time-series of interest rates for Denmark from 2/1/2000 to 29/9 /2014.

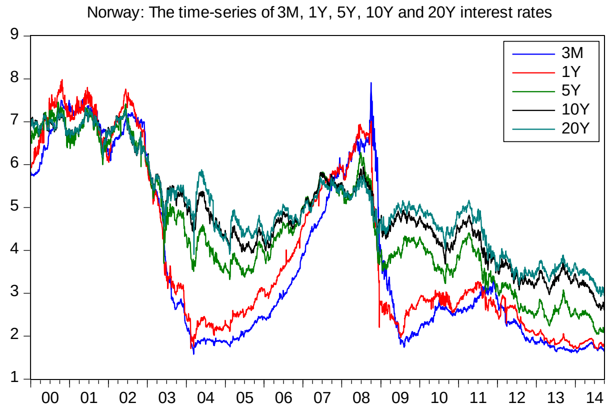


Figure 4.5 Daily time-series of interest rates for Norway from 2/1/2000 to 29/9 /2014.

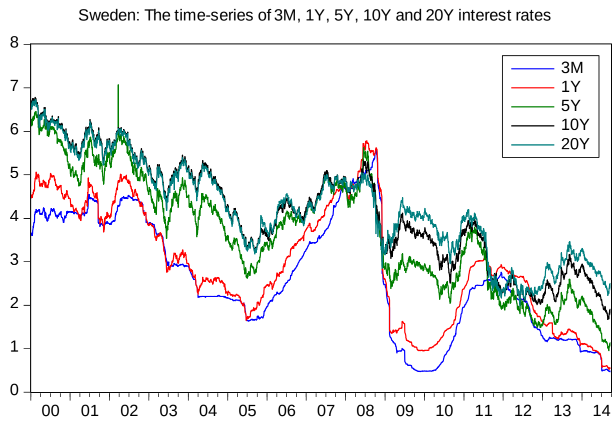


Figure 4.6 Daily time-series of interest rates for Sweden from 2/1/2000 to 29/9 /2014.

The shape of the TSIR above seems to change over time with upward sloping yield curves most of the time, but also with some downward sloping shapes especially during the global financial crisis of 2007-2009 when the money-market liquidity crisis prompted a substantial decrease in the short-term interest rates for all three countries. Overall, three periods of turbulence implied by an inverted/humped yield curve can be observed. The first period starts during 2000, reflecting the uncertainty in the financial markets following the introduction of the euro. The most affected country seems to be Norway with a negative slope yield curve present over two years until 2003, followed by Denmark, while for Sweden the shape of the yield remains upward. The second most persistent crisis corresponds to the global financial crisis GFC between 2007 and 2009, and again Sweden seems to be least affected. In general, downward sloping yield curves are associated with financial crisis, and it can be observed that in all three graphs the yield curves change shapes in the third quarter of 2007, suggesting this point in time as a plausible beginning of the crisis[7]. Compared with Denmark and Norway where the yield curve is clearly downward slopping over this period, for Sweden the shape is more flattening reflecting the more positive expectations for long-term interest rates. However, the third crisis period of 2011-2012 presents a totally different situation with clear signs of recession for Sweden which was mostly affected by the European downturn and an appreciating domestic currency. For the other two countries, the sovereign crisis period has a less impact, with a short-lived humped shape in the case of Norway and only some flattening patterns of the yield curve at very low levels in the case of Denmark.

[7] Several empirical studies have used the beginning of the third quarter of 2007 to mark the start of the GFC (see Cheung et al., 2010 and Dontis-Charitos et al., 2013)

In Tables 4.1 - 4.3 we report the standard summary statistics of the three data samples for each country.

Table 4.1: Descriptive Statistics of daily yields at various maturities;
DENMARK: Pre-Crisis, Post-Crisis and Full Sample Period

Pre-crisis	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	1,955	1,955	1,955	1,955	1,955	1,955	1,955	1,955
Mean	3.3778	3.6204	3.8402	4.3330	4.5632	4.8185	4.9703	5.1549
Median	3.3230	3.4950	3.7550	4.0880	4.3260	4.6590	4.9290	5.2740
Maximum	6.9070	6.5600	6.3830	6.2520	6.5520	6.5190	6.4740	6.4520
Minimum	2.0850	2.0400	2.1610	2.6590	2.9120	3.2070	3.4650	3.7560
Std. Dev.	1.1337	1.1861	1.1089	0.9449	0.8877	0.8341	0.7443	0.6673
Skewness	0.5900	0.5086	0.4039	0.3079	0.2615	0.1864	0.0431	-0.2107
Kurtosis	2.2373	2.1899	2.0342	2.0089	2.0356	2.0465	2.1199	2.1188
Jarque-Bera	160.81	137.75	129.14	110.89	98.05	85.37	63.70	77.71
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Post-crisis	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	1,892	1,892	1,892	1,892	1,892	1,892	1,892	1,892
Mean	1.920	2.035	2.159	2.614	2.899	3.222	3.422	3.647
Median	1.253	1.555	1.859	2.488	2.867	3.240	3.462	3.699
Maximum	6.437	5.858	5.928	5.501	5.362	5.273	5.234	5.243
Minimum	0.233	0.352	0.457	0.741	1.035	1.429	1.737	1.863
Std. Dev.	1.813	1.663	1.525	1.320	1.202	1.078	1.013	0.950
Skewness	0.992	0.974	0.797	0.396	0.246	0.150	0.115	0.064
Kurtosis	2.490	2.564	2.439	1.971	1.823	1.717	1.648	1.583
Jarque-Bera	331.08	313.88	225.35	132.96	128.29	136.80	148.37	159.66
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
Full Sample	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	3,847	3,847	3,847	3,847	3,847	3,847	3,847	3,847
Mean	2.6606	2.8406	3.0136	3.4876	3.7449	4.0333	4.2087	4.4132
Median	2.2150	2.4870	2.8290	3.5210	3.8730	4.1450	4.3160	4.5060
Maximum	6.9070	6.5600	6.3830	6.2520	6.5520	6.5190	6.4740	6.4520
Minimum	0.2330	0.3520	0.4570	0.7410	1.0350	1.4290	1.7370	1.8630
Std. Dev.	1.6735	1.6443	1.5730	1.4312	1.3428	1.2497	1.1772	1.1130
Skewness	0.2440	0.2251	0.0693	-0.1825	-0.2214	-0.2189	-0.3040	-0.3890
Kurtosis	1.9583	1.9699	2.0070	2.1559	2.2286	2.2492	2.2088	2.1546
Jarque-Bera	212.11	202.58	161.13	135.56	126.78	121.07	159.59	211.60
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4.2: Descriptive Statistics of daily yields at various maturities;
NORWAY: Pre-Crisis, Post-Crisis and Full Sample Period

Pre-crisis	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	1,955	1,955	1,955	1,955	1,955	1,955	1,955	1,955
Mean	4.4969	4.8193	5.0399	5.3663	5.4986	5.6242	5.6824	5.7037
Median	4.3200	4.8160	5.0120	5.1020	5.1940	5.3470	5.4160	5.4360
Maximum	7.5200	7.9810	7.9010	7.4590	7.3330	7.3120	7.3130	7.3130
Minimum	1.5800	1.7150	2.3130	3.3210	3.6510	3.9390	3.9230	3.9050
Std. Dev.	2.1322	2.0241	1.7826	1.3127	1.1845	1.0867	1.0335	1.0156
Skewness	0.0760	0.0579	0.0595	0.0736	0.0879	0.1069	0.1274	0.1303
Kurtosis	1.3200	1.3779	1.4249	1.4473	1.4444	1.4554	1.4639	1.4696
Jarque-Bera	231.79	215.43	203.25	198.14	199.65	198.06	197.50	196.31
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Post-crisis	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	1,892	1,892	1,892	1,892	1,892	1,892	1,892	1,892
Mean	3.0487	3.1412	3.2987	3.6920	3.9296	4.1850	4.3782	4.3664
Median	2.5100	2.6950	2.9450	3.5865	3.8830	4.2205	4.5285	4.5045
Maximum	7.9100	7.0090	6.9130	6.2810	6.0580	5.9480	5.8220	5.7550
Minimum	1.6110	1.7010	1.7050	2.0450	2.3030	2.6010	2.8800	2.9270
Std. Dev.	1.5935	1.4997	1.3793	1.0785	0.9619	0.8567	0.7884	0.7721
Skewness	1.2839	1.3633	1.1205	0.5573	0.3705	0.2004	-0.0076	-0.0319
Kurtosis	3.2012	3.4427	3.1340	2.3543	2.1273	1.9236	1.6662	1.6177
Jarque-Bera	523.01	601.53	397.32	130.81	103.32	104.00	140.25	150.96
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Full Sample	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	3,847	3,847	3,847	3,847	3,847	3,847	3,847	3,847
Mean	3.7847	3.9940	4.1836	4.5429	4.7270	4.9164	5.0410	5.0460
Median	2.7700	3.0160	3.4750	4.2070	4.4740	4.7040	4.9040	4.9160
Maximum	7.9100	7.9810	7.9010	7.4590	7.3330	7.3120	7.3130	7.3130
Minimum	1.5800	1.7010	1.7050	2.0450	2.3030	2.6010	2.8800	2.9270
Std. Dev.	2.0205	1.9726	1.8188	1.4657	1.3354	1.2160	1.1285	1.1244
Skewness	0.6176	0.6266	0.5419	0.3532	0.3287	0.3347	0.3459	0.3438
Kurtosis	1.7348	1.7984	1.8672	2.0362	2.1114	2.1896	2.2688	2.2848
Jarque-Bera	501.15	483.19	393.96	228.89	195.83	177.10	162.41	157.78
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 4.3 Descriptive Statistics of daily yields at various maturities; SWEDEN: Pre-Crisis, Post-Crisis and Full Sample Period

Pre-crisis	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	1,955	1,955	1,955	1,955	1,955	1,955	1,955	1,955
Mean	3.2273	3.5200	3.9188	4.5645	4.7914	5.0005	4.9882	4.9693
Median	3.4300	3.6170	3.8910	4.4780	4.7610	5.0450	5.0050	4.9870
Maximum	4.5520	5.0620	5.8570	7.0720	6.6500	6.7830	6.7520	6.7360
Minimum	1.6350	1.6500	1.8760	2.6210	2.9510	3.2020	3.1850	3.1760
Std. Dev.	0.9094	0.9628	0.9870	0.9274	0.8955	0.8750	0.8573	0.8595
Skewness	-0.2138	-0.1112	-0.0046	0.0290	-0.0320	-0.1097	-0.0918	-0.0741
Kurtosis	1.5714	1.6924	1.9721	2.1901	2.1255	2.0413	2.0916	2.0645
Jarque-Bera	181.14	143.30	86.07	53.69	62.62	78.78	69.97	73.08
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Post-crisis	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	1,892	1,892	1,892	1,892	1,892	1,892	1,892	1,892
Mean	2.0649	2.3838	2.2766	2.8087	3.0560	3.2714	3.4477	3.4806
Median	1.4850	1.8360	1.7455	2.5285	2.8540	3.1170	3.3350	3.4565
Maximum	5.6000	5.7830	5.7020	5.6280	5.4660	5.3180	5.1870	5.0420
Minimum	0.4670	0.5260	0.4970	0.9530	1.2990	1.6710	2.0300	1.9780
Std. Dev.	1.4663	1.4406	1.2913	1.0978	1.0091	0.9340	0.8733	0.8394
Skewness	0.9368	0.8867	1.1142	0.7747	0.6067	0.5038	0.3504	0.1946
Kurtosis	2.6408	2.6039	2.9830	2.6274	2.3902	2.1885	1.9585	1.8604
Jarque-Bera	286.88	260.30	391.52	200.20	145.39	131.95	124.24	114.31
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Full Sample	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
Observations	3,847	3,847	3,847	3,847	3,847	3,847	3,847	3,847
Mean	2.6556	2.9612	3.1112	3.7010	3.9379	4.1501	4.2306	4.2371
Median	2.4700	2.7920	3.1020	3.7980	3.9880	4.1430	4.1920	4.1740
Maximum	5.6000	5.7830	5.8570	7.0720	6.6500	6.7830	6.7520	6.7360
Minimum	0.4670	0.5260	0.4970	0.9530	1.2990	1.6710	2.0300	1.9780
Std. Dev.	1.3472	1.3469	1.4103	1.3417	1.2888	1.2512	1.1583	1.1295
Skewness	0.0862	0.1100	0.1089	0.0162	-0.0038	0.0025	0.0247	0.0397
Kurtosis	1.8457	1.8708	1.7447	1.9507	1.9852	1.9692	2.0573	2.1440
Jarque-Bera	218.32	212.14	260.20	176.64	165.08	170.33	142.82	118.45
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

In general, the historical interest rates averages seem to increase with higher maturities while the opposite is true for the volatility in interest yield changes. While some of the time series are more symmetrical than others (for Sweden the skewness measure is close to zero) and some have fatter tails than others, overall none of the time-series analysed has a normal sample distribution. All full sample time-series involved are found to be autocorrelated (see Table 4.9), as autocorrelations coefficients at 1, 60 and 120-day lags decay rather slowly indicating a high level of persistence.

Table 4.4 Autocorelations for interest rates time-series of various maturities

	DENMARK							
Maturity	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
$\rho(1)$	1	0.999	0.999	0.999	0.996	0.998	0.998	0.997
$\rho(60)$	0.947	0.94	0.928	0.914	0.907	0.907	0.907	0.905
$\rho(200)$	0.718	0.707	0.708	0.714	0.715	0.724	0.729	0.733
	NORWAY							
Maturity	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
$\rho(1)$	0.999	0.999	0.999	0.999	0.999	0.998	0.998	0.998
$\rho(60)$	0.944	0.936	0.926	0.919	0.917	0.916	0.909	0.905
$\rho(200)$	0.684	0.681	0.696	0.738	0.751	0.758	0.748	0.743
	SWEDEN							
Maturity	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
$\rho(1)$	0.999	0.999	0.999	0.998	0.998	0.998	0.998	0.998
$\rho(60)$	0.909	0.89	0.889	0.888	0.893	0.898	0.891	0.892
$\rho(200)$	0.566	0.543	0.61	0.667	0.691	0.716	0.7	0.696

4.5 The Empirical Results

This section reports and examines the estimation results for the three versions of the Babbs and Nowman (1999) model. By adding new factors, the number of parameters to be estimated by the linear Kalman filter increases as follows: there are 12 parameters to be estimated for the one-factor specification, 16 parameters and 21 parameters for the two- and three-factor models, respectively. The time-series of the fitted interest rates are plotted against the actual data to observe where exactly the models don't fit the data and the standardised residuals are tested for desirable properties such as normality and other patterns required for the validation of model-specification.

4.5.1 The Estimation Results

The results for the one-factor model are presented in Table 4.5 for all the three countries. In general, most of the parameter estimates are highly significant over all three data-samples. Particularly all the estimates for the market price of risk are measured with high statistical significance. For Denmark, all twelve parameters are significant at 1% level, while for Norway only one parameter is statistically insignificant in the post-crisis period and for Sweden there are only two insignificant parameters, corresponding to the standard deviation of the measurement errors in the 2- and 7-year maturity interest rates.

The long-term mean parameter μ is always significant and increases during the crisis only for Denmark, from 4.48% to 5.26%. For Norway and Sweden, the crisis has a negative impact on the long-term mean of the short rate, lowering its value from 6.22% to 5.54% for Norway, and from 6.73% to 4.13% for Sweden, respectively. The market price of risk θ_1 associated with the single latent factor of the model is significant in all cases with different evolution as a result of the crisis. It increases after the crisis for Sweden and Norway from 0.1271 to 0.1664. The parameters that characterise the dynamics of the latent factor are also significant. The mean reversion parameter ξ_1 has the highest estimated value of 0.2021 before the crisis for Norway. This implies a mean half-life of the interest rate process of 3.43 ($-\ln(0.5)/\xi_1$) years, which is the time to return halfway to its long-term average value. The estimates of the diffusion parameter σ_1 increase for Norway and Sweden following the crisis, while the opposite happens for Denmark. Finally, the standard deviations of the measurement errors are estimated at satisfactorily low values for all eight maturities, ranging from the lowest value of 5.2 basis points for one-year Norwegian spot rate before the crisis to the highest value of almost 100 basis points in the case of the three-month Swedish government discount interest rate after the crisis.

The estimation results for the two-factor models are reported Table 4.6 for all the three countries. While most of the parameters are highly significant, there are cases (Denmark post crisis and Norway pre-crisis) when the long-term mean of the short interest rate and the market prices of risk parameters are difficult to estimate. Before the GFC the level of interest rates was higher and this is reflected in the values obtained for the long-term mean in the significant range of 4.29% - 4.68%. The crisis had a great impact on the estimates of this parameter with very low values such as 0.01% for Denmark and 0.06% for Norway. Regarding the two risk-premium parameters, the results are mixed and inconclusive, with values fluctuating between negative and positive values without any particular patterns. The two factors exhibit rather different processes with the mean reversion parameter for the first factor in the (0.42 – 1.88) range, and the second factor behaving like random walk as their mean reversion parameter are very low within the (0-0.05) range. The crisis also affects the correlation coefficient between the two factors which are strongly negatively correlated before the crisis ($\rho = -99\%$ for Denmark, $\rho = -84\%$ for Norway and $\rho = -88\%$ for Sweden) but they become positively correlated after the crisis ($\rho = 13\%$ for Norway and $\rho = 16\%$ for Sweden). The magnitude of the standard deviations of the measurement errors are considerably smaller than those in the one-factor models, indicating an improvement in the goodness-of-fit of the two-factor models over the less flexible models of one-factor models.

Turning to the three-factor models (see Table 4.7), most of the parameter-estimates are highly significant at 1% level. The values of the log-likelihood functions increase from the two-factor model levels, although for Sweden is not as much as from one- to two-factor specifications. Similarly, to the two-factor models, the estimates for the long-term mean parameter are systematically insignificant, with only one significant value of $\mu = 3.78\%$ for Denmark based on the pre-crisis data subsample. More positive results are obtained for the three risk-premium parameters compared to the results from the two-factor models. Moreover, an important pattern emerges: the market price of risk associated with the second factor is the highest in periods of turbulence. The latent factors seem to evolve rather differently, the first factor seem to have very short memory while the third factor behaves like a random walk (see Geyer and Pichler, 1999). More specifically, the mean reversion of the first factor is substantially higher compared to the other two factors, while for the third factor the processes possess very small speeds to revert to their zero mean and hence having much longer implied mean half-life. For example, for the first factor, the lowest and the highest mean reversion parameter are 0.75 (Sweden) and 2.57 (Denmark), implying a mean half-life of 0.92 and 0.26 years, respectively. Meanwhile, for the third factor the shortest mean half-life is 1.7 years, corresponding to a mean reversion speed of 0.39 which occurs for Norway after the crisis.

As in Geyer and Pichler (1999) the scale parameters for the conditional volatility indicate large spikes for the factor with no memory and less volatility for the third factor. The correlations among the factors diminish as a result of the crisis, changing from negative dependence before crisis to a positive relationship after the crisis. In general, the measurement errors are very small implying a very good fitting by the models. The largest measurement error of 113 basis points occurs in the post-crisis period for Denmark's three-month zero rates. In comparison with the two-factor models these errors seem smaller, suggesting an overall better fit to the data for the three-factor models.

As expected, the log-likelihood functions increase when more factors are included. In order to assess if the increments are statistically significant, the BIC (Bayesian Information Criterion) is calculated and reported in Table 4.8. Most of the J-factor models are rejected in favour of the $(J+1)$-factor models. For Sweden, however, the BIC criterion cannot reject the two-factor model against the three-factor model as the difference between their BIC values is smaller than 6 and hence not strongly significant. Otherwise, for all the other transitions to richer models, for Denmark and Norway, the three-factor model represents the superior model.

Table 4.5 Estimation Results for the Babbs and Nowman *one-factor* model for DENMARK, NORWAY and SWEDEN

	DENMARK			NORWAY			SWEDEN		
parameters	Pre-Crisis	Post-Crisis	Full Sample	Pre-Crisis	Post-Crisis	Full Sample	Pre-Crisis	Post-Crisis	Full Sample
μ	0.0448***	0.0526***	0.0611***	0.0622***	0.0554***	0.0588***	0.0673***	0.0413***	0.0313***
θ_1	0.2546***	0.1089***	0.1394***	0.0226***	0.0870***	0.0609***	0.1271***	0.1643***	0.2463***
ξ_1	0.0591***	0.0738***	0.0558***	0.2021***	0.0926***	0.1202***	0.0659***	0.0624***	0.0338***
σ_1	0.0200***	0.0107***	0.0104***	0.0073***	0.0108***	0.0099***	0.0112***	0.0137***	0.0111***
$\sqrt{h_1}$	0.0063***	0.0072***	0.0069***	0.0028***	0.0091***	0.0057***	0.0040***	0.0099***	0.0089***
$\sqrt{h_2}$	0.0060***	0.0051***	0.0053***	0.0005***	0.0073**	0.0026***	0.0025***	0.0091***	0.0088***
$\sqrt{h_3}$	0.0046***	0.0029***	0.0033***	0.0016***	0.0049***	0.0000***	0.0000***	0.0060***	0.0062***
$\sqrt{h_4}$	0.0014***	0.0000***	0.0000***	0.0031***	0.0012***	0.0032***	0.0034***	0.0014***	0.0017***
$\sqrt{h_5}$	0.0000***	0.0014***	0.0014***	0.0034***	0.0000	0.0038***	0.0041***	0.0000	0.0000
$\sqrt{h_6}$	0.0012***	0.0018***	0.0022***	0.0039***	0.0008***	0.0041***	0.0045***	0.0010***	0.0012***
$\sqrt{h_7}$	0.0020***	0.0018***	0.0026***	0.0048***	0.0017***	0.0043***	0.0040***	0.0021***	0.0016***
$\sqrt{h_8}$	0.0029***	0.0026***	0.0036***	0.0059***	0.0033***	0.0053***	0.0057***	0.0031***	0.0025***
LOGLF	**86,830.40**	**83,758.33**	**168,039.21**	**83,808.74**	**84,030.51**	**161,747.46**	**83,565.83**	**80,865.50**	**167,398.24**

Note: 1) Most parameter estimates are highly significant with their level of significance marked as following: 10% level of significance (*), 5% level of significance (**) and 1% level of significance (***). Estimates lower than 10^{-5} have been entered as zero.
2) The same conventions apply to Tables 4.6 and 4.7.

Table 4.6 Estimation Results for the Babbs and Nowman *two-factor* model for DENMARK, NORWAY and SWEDEN

2 -FACTOR	DENMARK			NORWAY			SWEDEN		
parameters	Pre-Crisis	Post-Crisis	Full Sample	Pre-Crisis	Post-Crisis	Full Sample	Pre-Crisis	Post-Crisis	Full Sample
μ	0.0429***	0.0560	0.0001***	0.04152	0.0006***	0.0136	0.04685***	0.00126	0.0386
θ_1	1.1248***	0.0001	0.0302***	-0.0000	0.4080***	0.2729***	-0.0016***	0.4158***	-0.0000
θ_2	-0.0211	0.1955***	0.5296***	0.1185***	-0.0199***	0.1628***	-0.0272***	-0.0235***	0.1271***
ξ_1	0.4595***	0.5975***	0.4172***	1.0882***	1.2853***	0.9849***	1.8842***	1.28736***	1.2575***
ξ_2	0.0517***	0.0354***	0.0350***	0.0000***	0.0000***	0.0000***	0.0019***	0.0001***	0.0000***
σ_1	0.0072***	0.0156	0.0112***	0.0101***	0.0088***	0.0123***	0.0101***	0.0083***	0.0124***
σ_2	0.0064***	0.01233***	0.0096***	0.0079***	0.0069***	0.0083***	0.0073***	0.0066***	0.0088***
ρ_{12}	-0.9999***	-0.8926***	-0.8318***	-0.8385***	0.1278***	-0.2726***	-0.8782***	0.1615***	-0.8540***
$\sqrt{h_1}$	0.0041***	0.0027***	0.0031***	0.0031***	0.0063***	0.0057***	0.0000***	0.0048***	0.0025***
$\sqrt{h_2}$	0.0013***	0.0006***	0.0004***	0.0004***	0.0024***	0.0021***	0.0023***	0.0037***	0.0001***
$\sqrt{h_3}$	0.0002***	0.0012***	0.0012***	0.0013***	0.0000***	0.0000***	0.0031***	0.0000***	0.0031***
$\sqrt{h_4}$	0.0026***	0.0011***	0.0013***	0.0024***	0.0015***	0.0019***	0.0021***	0.0016***	0.0029***
$\sqrt{h_5}$	0.0009***	0.0013***	0.0012***	0.0009***	0.0021***	0.0011***	0.0010***	0.0012***	0.0019***
$\sqrt{h_6}$	0.0002***	0.0013***	0.0010***	0.0007***	0.0009***	0.0001***	0.0009***	0.0003***	0.0013***
$\sqrt{h_7}$	0.0006***	0.0000	0.0002***	0.0002***	0.0008***	0.0010***	0.0002***	0.0014***	0.0000***
$\sqrt{h_8}$	0.0011***	0.0006***	0.0010***	0.0006***	0.0003***	0.0013***	0.0002***	0.0018***	0.0007***
LOGLF	**99,746.64**	**96,282.82**	**194,616.12**	**98,423.04**	**92,420.23**	**189,071.08**	**101,927.88**	**92,366.84**	**189,852.40**

Table 4.7 Estimation Results for the Babbs and Nowman *three-factor* model for DENMARK, NORWAY and SWEDEN

3-FACTOR	DENMARK			NORWAY			SWEDEN		
parameters	Pre-Crisis	Post-Crisis	Full Sample	Pre-Crisis	Post-Crisis	Full Sample	Pre-Crisis	Post-Crisis	Full Sample
μ	0.0378***	0.0018	0.0182	0.0199	0.0020	0.0519	0.0437	0.0135	0.0476
θ_1	-0.4144	0.1773***	-0.2431***	0.0029	0.1805	0.0029***	0.0598***	0.0407***	0.0609***
θ_2	-0.5947	0.3440***	0.1528***	-0.5100***	0.5778	-0.5124***	0.1553***	0.0669***	0.1555***
θ_3	-2.0132***	0.1303***	0.0077***	0.0366***	0.1004	0.0369***	0.0297***	0.0258***	0.0276***
ξ_1	2.5709***	1.1676***	1.6137***	0.9422***	1.6783***	0.9439***	0.7739***	0.7675***	0.7467***
ξ_2	0.0010***	0.0000***	0.4216***	0.6348***	0.0000***	0.6007***	0.1213***	0.2627***	0.1308***
ξ_3	0.3175***	0.3359***	0.0000***	0.0000***	0.3900***	0.0000***	0.0000***	0.0000***	0.0000***
σ_1	0.0214***	0.0180***	0.0190***	0.0488***	0.0254***	0.0484***	0.0098***	0.0289***	0.0179***
σ_2	0.0079***	0.0057***	0.0121***	0.0405***	0.0121***	0.0401***	0.0037***	0.0172***	0.0090***
σ_3	0.0090***	0.0241***	0.0059***	0.0070***	0.0115***	0.0072***	0.0047***	0.0062***	0.0075***
ρ_{12}	-0.8998***	0.0165***	-0.8844***	-0.9640***	-0.8731***	-0.9652***	-0.8436***	-0.9173***	-0.7918***
ρ_{13}	-0.8368***	0.0280***	-0.7060***	-0.6428***	-0.8186***	-0.6433***	-0.4896***	-0.6447***	-0.5664***
ρ_{23}	0.6617***	0.0137***	0.4209***	0.5399***	0.5834***	0.5484***	0.5413***	0.4632***	0.2480***

continued

Table 4.7	continued								
$\sqrt{h_1}$	0.0000***	0.0113***	0.0000***	0.0038***	0.0033***	0.0046***	0.0028***	0.0045***	0.0041***
$\sqrt{h_2}$	0.0017***	0.0001***	0.0010***	0.0003***	0.0004***	0.0003***	0.0001***	0.0000***	0.0000***
$\sqrt{h_3}$	0.0013***	0.0006***	0.0002***	0.0010***	0.0009***	0.0011***	0.0014***	0.0029***	0.0028***
$\sqrt{h_4}$	0.0004***	0.0002***	0.0010***	0.0004***	0.0002***	0.0003***	0.0030***	0.0006***	0.0017***
$\sqrt{h_5}$	0.0007***	0.0005***	0.0011***	0.0005***	0.0005***	0.0005***	0.0006***	0.0002***	0.0009***
$\sqrt{h_6}$	0.0010***	0.0008***	0.0009***	0.0007***	0.0006***	0.0007***	0.0005***	0.0004***	0.0001***
$\sqrt{h_7}$	0.0000***	0.0003***	0.0003***	0.0002***	0.0000***	0.0002***	0.0003***	0.0000***	0.0006***
$\sqrt{h_8}$	0.0013***	0.0013***	0.0011***	0.0005***	0.0006***	0.0006***	0.0001***	0.0007***	0.0001***
LOGLF	**105,657.76**	**106,801.70**	**206,787.00**	**104,057.63**	**99,920.24**	**203,857.51**	**102,515.71**	**101,171.04**	**201,966.78**

Note: There are 21 parameter estimates for the three-factor BN models; μ is the long-term mean, θ_j denotes the market price of risk parameter associated with factor X_j, ξ_j denote the reversion speed parameters, σ_j denote the diffusion (instantaneous volatility) parameters and ρ_{ij} denote the correlation parameters between the factors X_i and X_j.

The BIC (Bayesian Information Criterion) is used instead of the LR (Likelihood Ratio) test because the models are not nested and also the BIC test penalizes for any additional parameters. According to BIC the model with the lowest BIC value is preferred and it can be observed from the Table 4.8 that the most flexible model, that is the three-factor model is considered best in terms of goodness of fit for all the countries and across all data samples.

Table 4.8 The results for the Bayesian Information Criterion

BIC		DENMARK	
	Pre-crisis	Post-crisis	Full Sample
one-factor	-173,569.86	-167,426.12	-335,979.36
two-factor	-199,372.03	-192,444.91	-389,100.16
three-factor	-211,148.80	-213,437.40	-413,392.39
BIC		NORWAY	
	Pre-crisis	Post-crisis	Full Sample
one-factor	-167,526.54	-167,970.48	-323,395.86
two-factor	-196,724.83	-184,719.73	-378,010.08
three-factor	-207,948.54	-199,674.48	-407,533.41
BIC		SWEDEN	
	Pre-crisis	Post-crisis	Full Sample
one-factor	-167,040.72	-161,640.46	-334,697.42
two-factor	-203,734.51	-184,612.95	-379,572.72
three-factor	-204,864.70	-202,176.08	-403,751.95

4.5.2 The Time Series of the Fitted Interest Rates

Comparatively, in terms of in-sample fit the BIC information Criterion confirms that the three-factor model produces the best explanation of the observed yield curve. The same conclusion is generally valid when the model simulated interest rates are plotted against the actual interest time series across the whole sample period for each country. Visually the graphs are consistent with the estimation results and with the BIC criterion.

For Denmark, the fitted values implied by the one-, two- and three factor models are compared for each maturity in Figures 4.7- 4.9. When moving from one-factor to two-factor models, the fit improves across the eight maturities, while from the two- to three-factor model it is the three-month and the 20-year rates that show a better fit to the data. For Norway, the fitted values implied by the one-, two- and three-factor models are compared for each maturity in Figures 4.10 - 4.12, and for Sweden in Figures 4.13- 4.15 respectively.

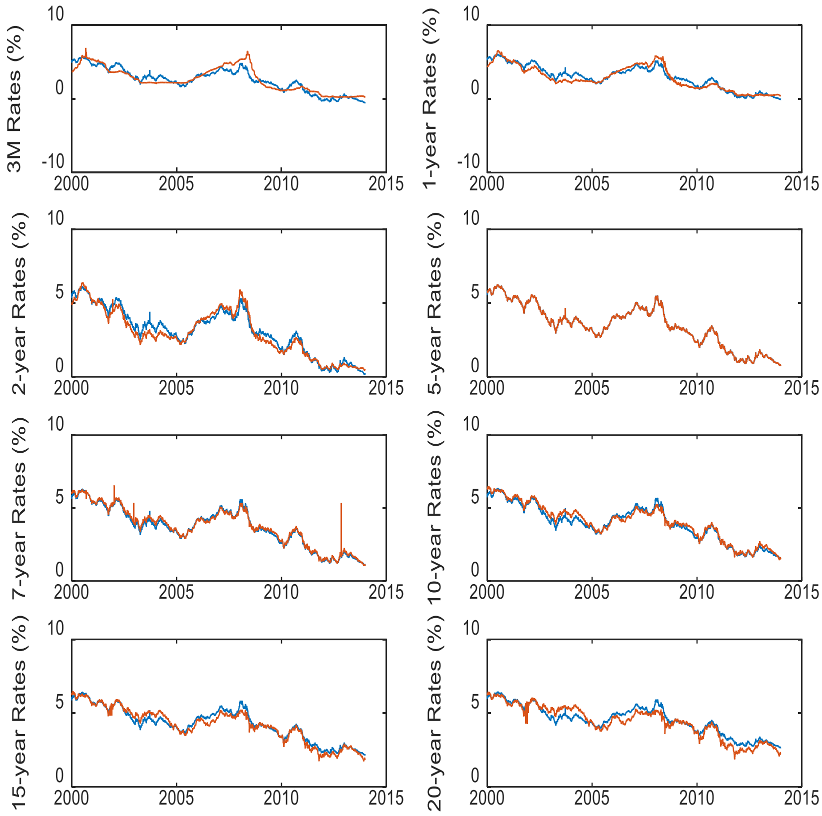


Figure 4.7 In-sample fitted values and actual interest rate time series for *Denmark* over the whole period 2000-2014, based on the *one-factor* BN term-structure model.

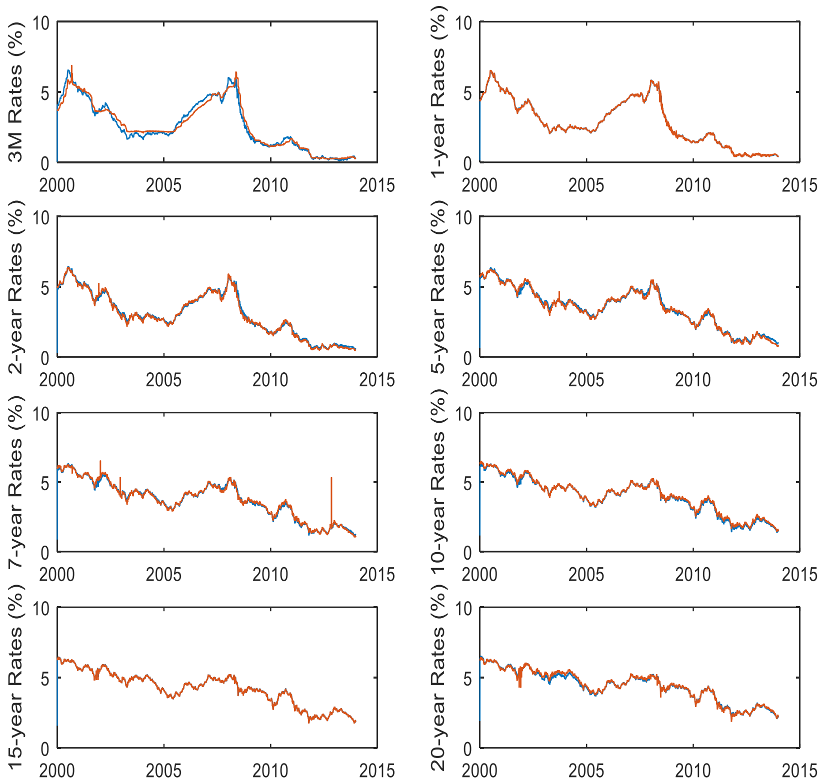


Figure 4.8 In-sample fitted values and actual interest rate time series for *Denmark* over the whole period 2000-2014, based on the *two-factor* BN term-structure model.

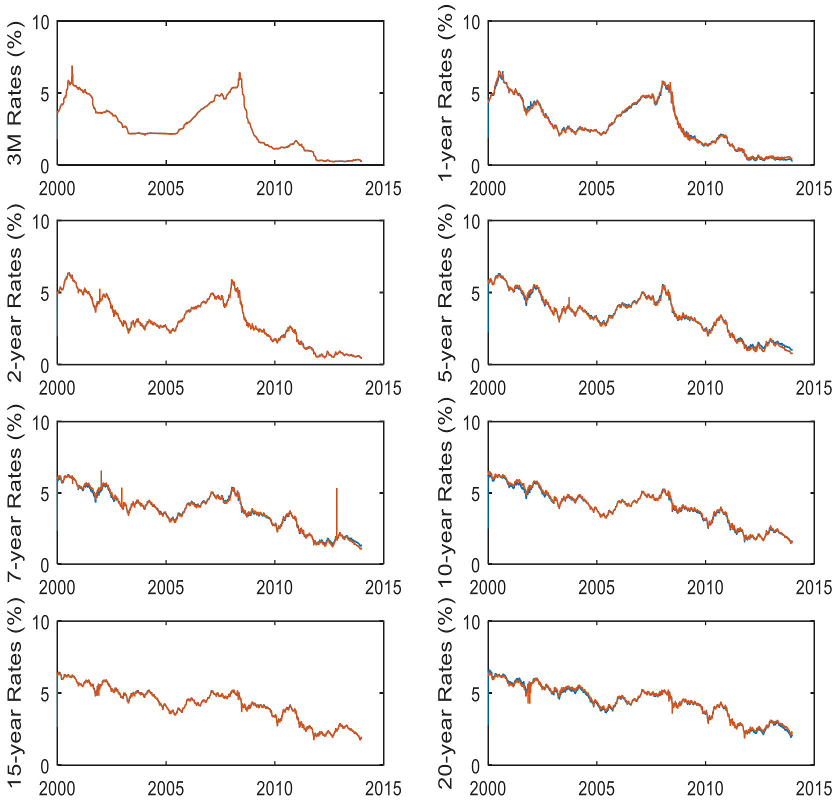


Figure 4.9. In-sample fitted values and actual interest rate time series for *Denmark* over the whole period 2000-2014, based on the *three-factor* BN term structure model.

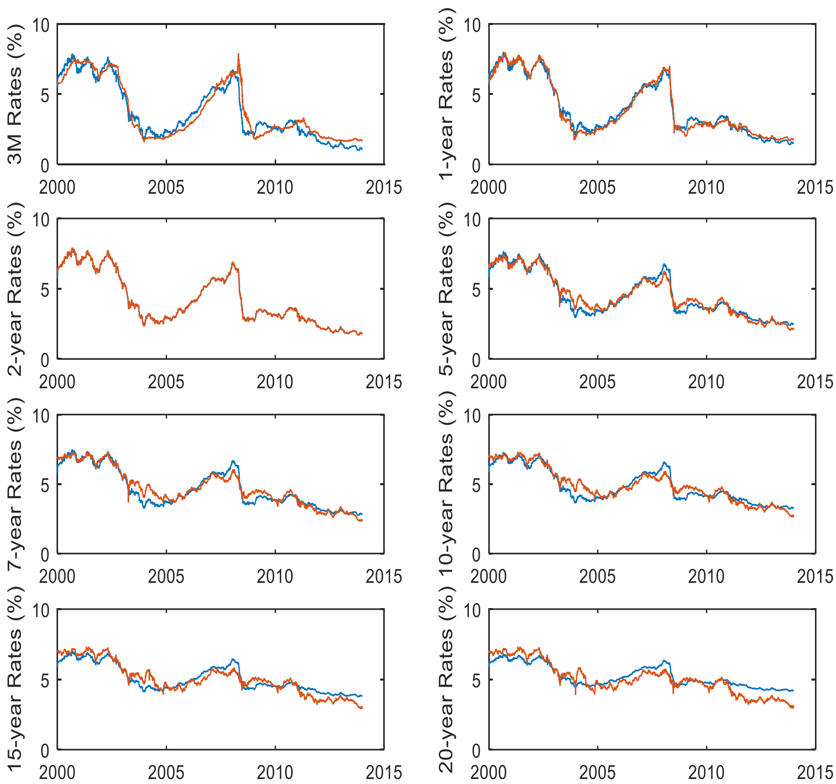


Figure 4.10. In-sample fitted values and actual interest rate time series for *Norway* over the whole period 2000-2014, based on the *one-factor* BN term structure model.

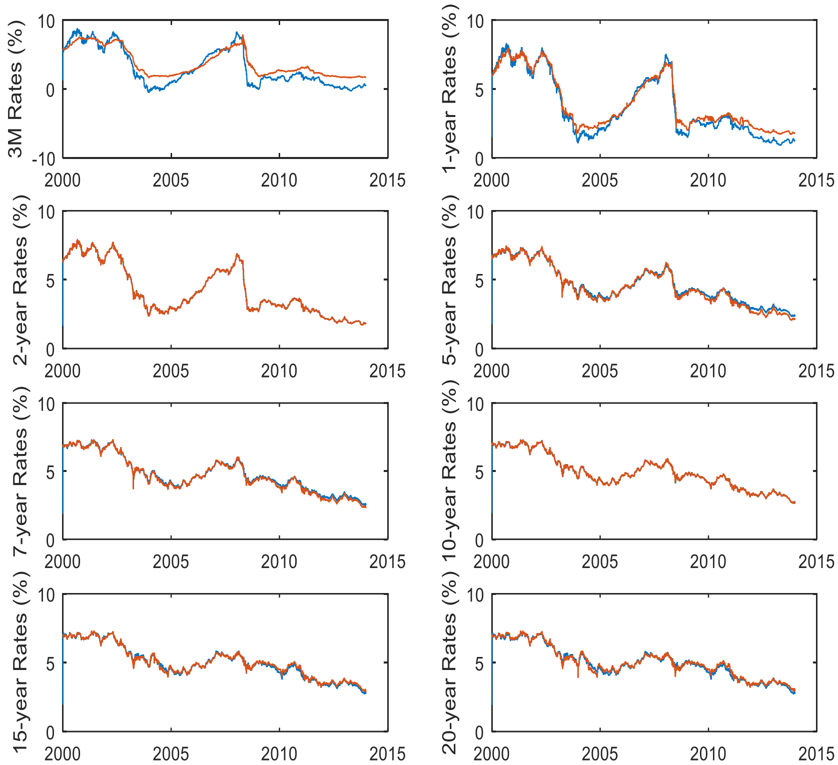


Figure 4.11. In-sample fitted values and actual interest rate time series for *Norway* over the whole period 2000-2014, based on the *two-factor* BN term structure model.

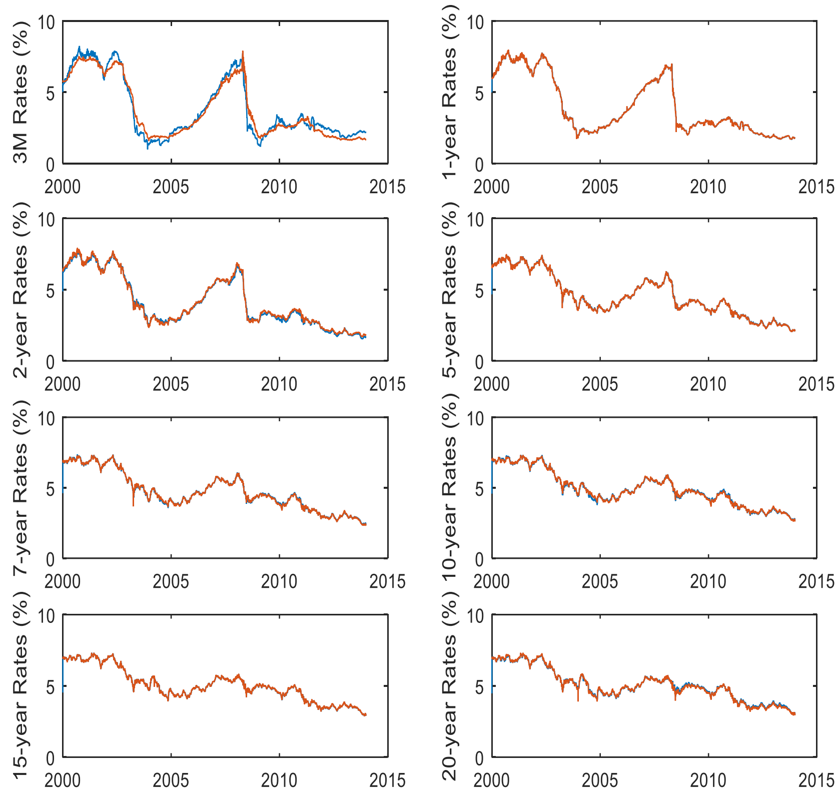


Figure 4.12. In-sample fitted values and actual interest rate time series for *Norway* over the whole period 2000 -2014, based on the *three-factor* BN term structure model.

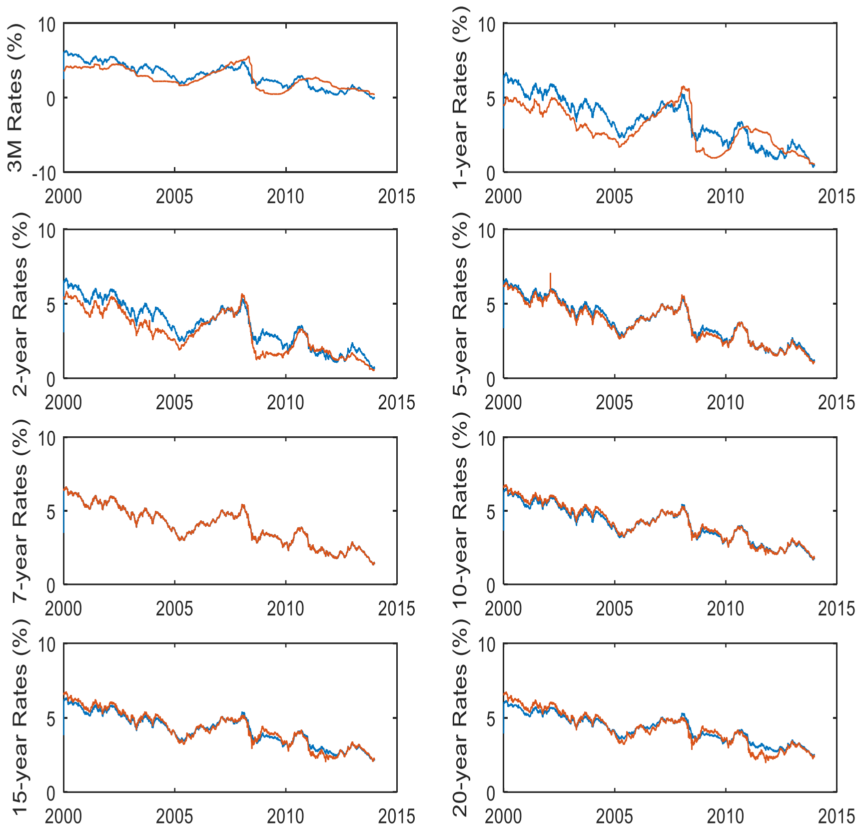


Figure 4.13. In-sample fitted values and actual interest rate time series for *Sweden* over the whole period 2000 - 2014, based on the *one-factor* BN term structure model.

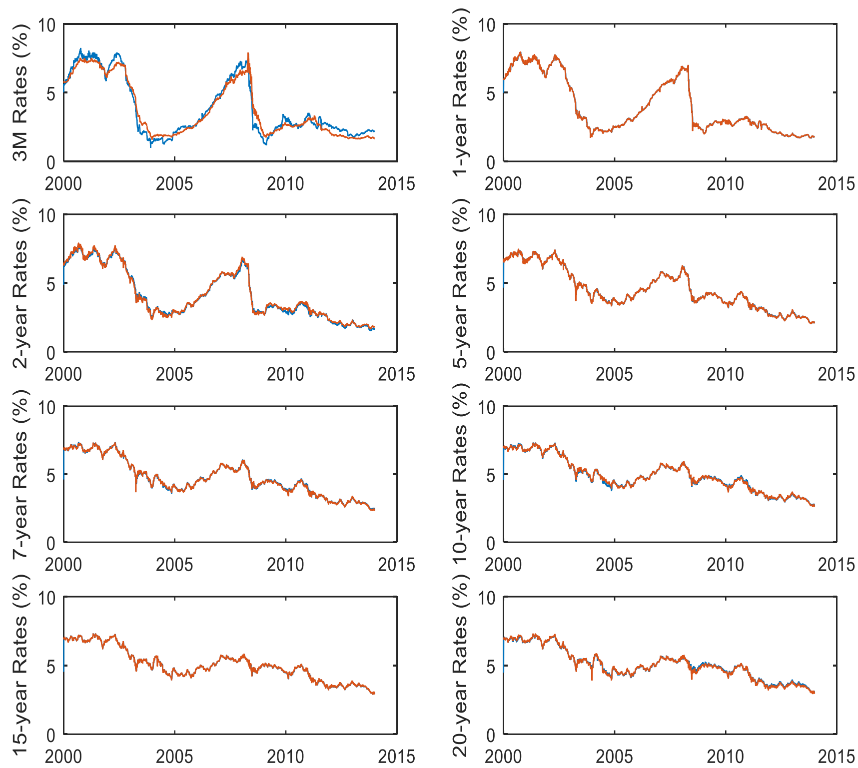


Figure 4.14. In-sample fitted values and actual interest rate time series for *Sweden* over the whole period 2000-2014, based on the *two-factor* BN term structure model.

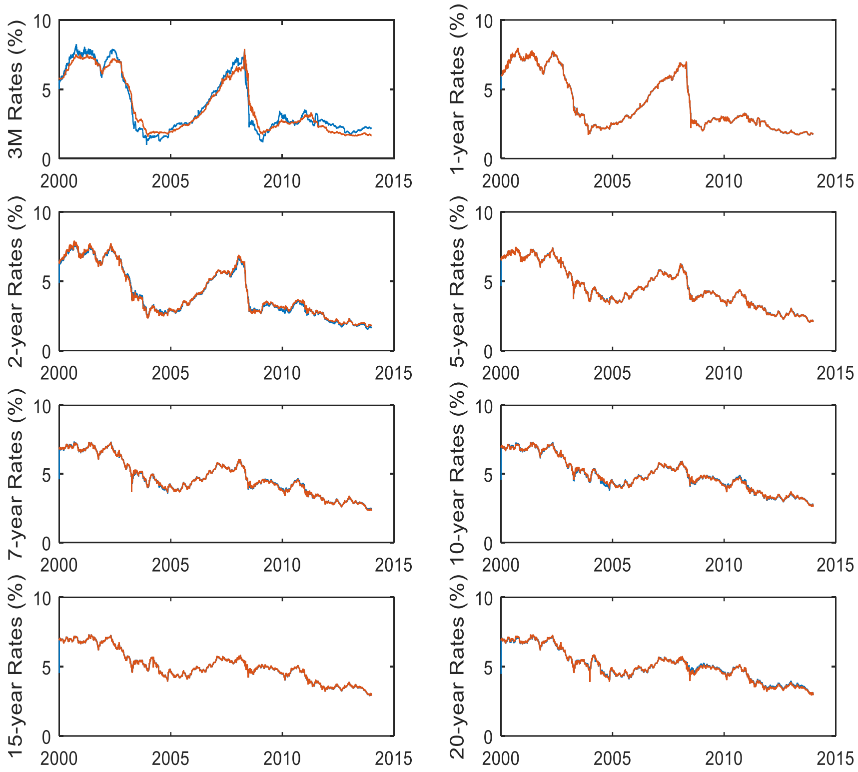


Figure 4.15. In-sample fitted values and actual interest rate time series for *Sweden* over the whole period 2000 - 2014, based on the *three-factor* BN term structure model.

After a comparative inspection of all the fitted time series of interest rates implied by the models, it is clear that by adding extra factors the fitting performance of the model improves, with the three-factor model supporting best the full period historical data, at least for Denmark and Norway. For Sweden, while it is clear that the two-factor model explains the data much better than the one-factor model especially for the shorter term interest rates, there is little difference in fitting the data when turning from the two- to three-factor model.

4.5.3 The Residuals Analysis

Another way of comparing the models is to analyse the measurement errors or the residuals for the different maturity yields. According to Geyer and Pichler, (1999), the properties of the standardised errors provide important insights about the measurement errors and therefore about the economic evaluation of the model The prediction errors $\nu_k = Y_k - \hat{Y}_k$ calculated before updating the conditional estimates for the state variables, are standardised using the diagonal elements of the covariance matrix F_k (see Harvey 1989, p.256) as following:

$$\tilde{v}_k = v_k / \sqrt{diag(F_k)}$$

For the model to be well-specified, it is required that each component of the error vector is a normally distributed white-noise time-series. This is a requirement rarely satisfied by empirical studies. In this investigation, there are only few maturities for which the residual time-series approach a normal distribution, however statistically the hypothesis for normality is rejected for all of them based on the JB test. Additionally, the time average of these errors, calculated over 3,847 observations, should not exhibit any maturity related patterns, and this desirable property is satisfied. The statistical means of the in-the-sample standardised errors are reported in Table 4.9 for each country, for the one-, two- and three-factor versions of Babbs and Nowman (1999) model.

For Denmark, the one-factor model produces a mixture of positively and negatively biased errors among different maturities. Some maturities like two-, five- and fifteen-years, seem to underestimate (on average) the actual interest rates across all specifications. The estimates from the two- and three-factor models are positively biased and there are no clear patterns of maturity dependence of these biases across maturities. Increasing the number of factors is associated with a decrease in the residual average for most maturities, with five out of eight maturities selecting the three-factor model as the best one.

The results for Norway, when the residual means are analysed, indicate the three-factor model as the best in fitting and explaining the data, while the one-factor model outperforms the richer models for only two maturities (three-months and two-years). However, there are no signs of the error averages dependency on maturity for any of the model versions.

In the case of Sweden, it is also the three-factor model that on average has the smallest biases for six out of eight maturities. For four out of eight maturities, all the models are negatively biased and no relationship between the magnitude of the residual-means and maturity can be found.

A positive general result is that the three-factor model produces small residual-means for all the countries, across all the maturities with the extreme values being realised for the Swedish interest rates: the smallest residual-mean of -0.017 and the largest mean of -0.5873 applying to the 20-year and 15-year interest rates, respectively.

Table 4.9 The means of the standardised estimation errors

DENMARK									
Maturity	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y	AVG
1F model	0.1266	-0.2134	-0.3336	-0.0308	0.3806	0.4069	-0.2477	-0.33	-0.1017
2F model	0.0546	-0.0007	-0.3542	-0.2637	0.109	0.6536	-0.2192	0.3217	0.1881
3F model	-0.0474	0.1405	-0.0528	-0.2609	0.0273	0.3826	-0.1837	0.3509	0.1518
Best model	3F	2F	3F	1F	3F	3F	3F	2F	1F
NORWAY									
Maturity	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y	AVG
1F model	0.0775	-0.1538	-0.0342	0.1506	0.1998	0.1939	-0.0458	-0.3161	-0.1193
2F model	1.147	1.1733	-0.0662	-0.4794	-0.5377	0.0069	0.251	0.256	0.7015
3F model	-0.1885	-0.0593	0.3363	-0.1189	-0.1083	0.0614	0.0427	-0.2566	-0.2226
Best model	1F	3F	1F	3F	3F	3F	3F	3F	1F
SWEDEN									
Maturity	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y	AVG
1F model	0.2169	-0.3867	-0.6535	-0.6027	-0.006	0.4214	-0.1136	-0.38	-0.0815
2F model	1.3893	-0.2533	-1.0541	-0.807	-0.6733	-0.1712	-0.0327	0.46	0.9246
3F model	-0.0821	-0.0831	-0.4337	-0.2829	-0.1848	-0.0245	-0.5873	-0.0169	-0.2119
Best model	3F	3F	3F	3F	1F	3F	2F	3F	1F

Following Geyer and Pichler (1999) the last column in Table 4.9 above reports the average of the means across maturities for each specification. However, this calculation seems to select the one-factor model as the best-fitting model, despite clear visual evidence of performance improvement when turning to multi-factor models. This is not of surprise as this criterion provides us with information about the estimation bias of the model and it is rather misleading as it cancels out the large positive and negative residuals, resulting in an erroneous small error measure and hence selecting the wrong model in terms of the error magnitude. For this reason, it is suggested here to consider a different comparison criterion based on a qualitative averaging instead of a numerical one. Based on this last criterion the three-factor model is selected as the best model for 6 out of 8 maturities. It is natural to conclude as a whole yield curve result that the three-factor specification outperforms the other model-specifications, a conclusion that is consistent with the visual observation of the fitted time series and the BIC test.

4.6 Factor Loadings Analysis

By using the Kalman filter technique one is able to extract the latent factors time series and their loadings which are of great importance in risk management where consistent revaluation is possible because the factor simulations play the role of the parameters used in the valuation process (Geyer and Pichler, 1999).

The factor loadings are defined as the reaction coefficients in a linear regression where the dependent variables are the bond yields and the explanatory variables are independent state variables. In the BN model, the assumption of independence is relaxed as the dynamics of the latent factors are driven by correlated Brownian motions, W_1, W_2, W_3 . Starting with the deterministic part of the measurement equation in the state-space form (eq. 4.4), and applying the continuous differential operator one obtains:

$$dR_m(\tau) = \sum_{j=1}^{J} \xi_j c_{mj} X_j dt - \sum_{j=1}^{J} \sigma_j c_{mj} dW_j \qquad (4.17)$$

Following Babbs and Nowman (1999) the innovations dW_j can be expressed as a particular linear combination of three independent (orthogonal) Brownian motions denoted by dZ_j , so the equation 4.17 above can be transformed as follows:

$$dR(\tau) = \sum_{j=1}^{J} H_j \xi_j X_j dt + \gamma_1(\tau) dZ_1 + \gamma_2(\tau) dZ_2 + \gamma_3(\tau) dZ_3 \qquad (4.18)$$

where the coefficients $\gamma_j (j = 1, 2, 3)$ are the factor loadings as functions of maturity and $H_j = H(\xi_j \tau) = \frac{1 - e^{-\xi_j \tau}}{\xi_j \tau}$. The curve for each factor loading represents the change in the spot interest rates due to a one standard deviation shock from the corresponding factor.

For the two-factor model the equivalent Gaussian processes using the independent Brownian motions are (see Babbs and Nowman 1997):

$$\begin{aligned} dX_1 &= -\xi_1 X_1 dt + k_{11} dZ_1 \\ dX_2 &= -\xi_2 X_2 dt + k_{21} dZ_1 + k_{22} dZ_2 \end{aligned} \qquad (4.19)$$

where $k_{11} = \sigma_1$, $k_{21} = \rho \sigma_2$ and $k_{22} = \sigma_2 \sqrt{1 - \rho^2}$.

Similarly, the linear combinations for the three-factor model are given by:

$$
\begin{aligned}
dX_1 &= -\xi_1 X_1 dt + k_{11} dZ_1 \\
dX_2 &= -\xi_2 X_2 dt + k_{21} dZ_1 + k_{22} dZ_2 \\
dX_3 &= -\xi_3 X_3 dt + k_{31} dZ_1 + k_{32} dZ_2 + k_{33} dZ_3
\end{aligned} \tag{4.20}
$$

where $k_{31} = \rho_{13}\sigma_3$, $k_{32} = \alpha\sigma_3$, $k_{33} = \sigma_3\sqrt{1-\rho_{13}^2-\alpha^2}$ and $\alpha = (\rho_{23}-\rho_{12}\rho_{13})/\sqrt{1-\rho_{12}^2}$.

Based on a classic unrestricted factor analysis, Litterman and Scheinkman (1991) identified three common (systematic risk) factors that explain most of the variation in bond returns and called them the *level, slope* and *curvature* factors. In their study Litterman and Scheinkman (1991) ordered the factors in terms of their power to explain the variance in the yields, with the first factor having the maximum impact. In the BN model, the unobservable factors are nominated in an arbitrary fashion and therefore there is no clear correspondence between the independent factors Z_1, Z_2, Z_3 defined here and the level, slope and curvature factors. The factor loadings extracted from eq. (4.18) are plotted in Figures 4.16 – 4.21 in order to determine the nature of the independent latent factors Z_1, Z_2 and Z_3. A change in the level factor should impact the yield curve in a similar way across all the maturities. The slope factor should have the greatest impact on the short-term segment of the yield curve, while the curvature-factor is related to the medium-term maturity segment. The factor loadings for Denmark are plotted in figures 4.16 and 4.17 for the two- and three-factor models respectively.

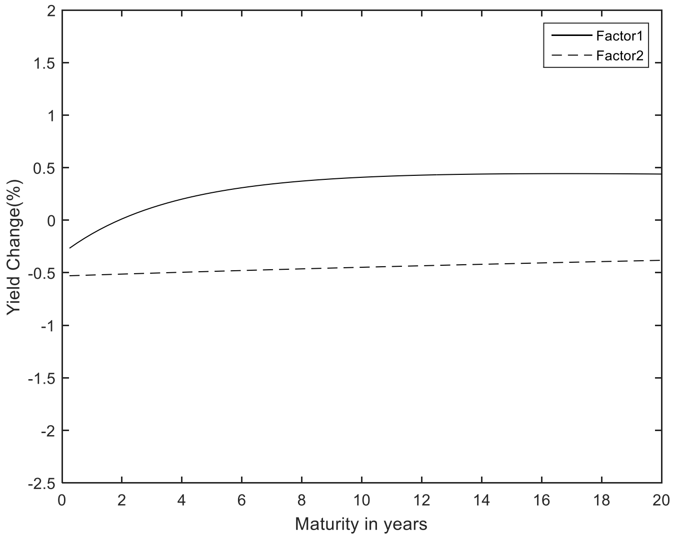


Figure 4.16 Denmark, the factor loadings for the two-factor BN model

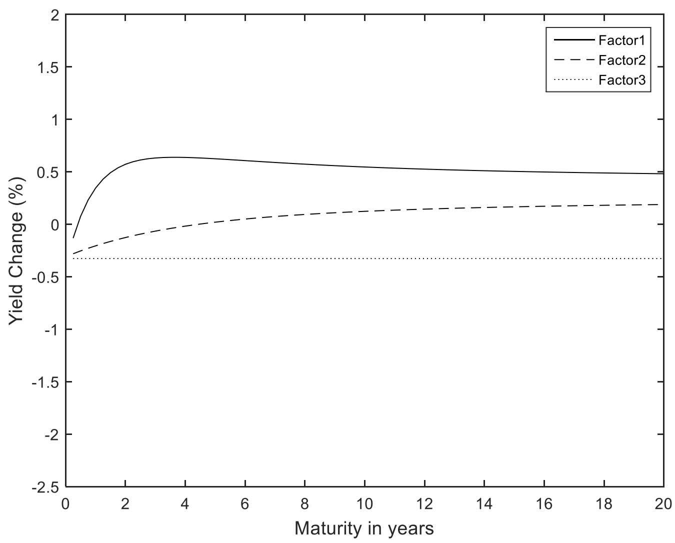


Figure 4.17 Denmark, the factor loadings for the three-factor BN model.

The first factor in the two-factor model (Figure 4.16) has a negative impact on the yield changes up to 2-year maturity after which it becomes positive slightly increasing the interest rate levels. For the second (level) factor the loading implies a negative effect on yield changes of approximately 0.5%.

Turning to the three-factor model in Figure 4.17, the first factor has an increasing positive impact on the change in the zero coupon yields and can be interpreted as the curvature factor since its strongest effect (larger than 0.5%) is across the medium-term maturity segment (between 2 and 6 years); after that the impact lowers to a constant positive level of 0.5%. The second factor possesses a negative loading lowering the interest rates up to 5-year maturity where its impact disappears and then becomes positive. Its strongest effect is exercised on the short-term interest rates of maturities under one year, hence the second factor can be interpreted as the slope factor. The loading of the third factor is negative across the whole maturity spectrum, with an equal effect across all maturities and therefore, can be interpreted as the level factor.

For Norway, the factors are identified in the same order as for Denmark however their loadings have slightly different magnitude (see Figure 4.18). The first factor (slope) has a decreasing negative impact on shorter maturities up to six years, and a negligible constant positive effect on yield of maturities longer than 6 years. The second factor, interpreted as the level factor is stronger than in the case of Denmark, lowering all the yields by approximately 0.8%. Turning to the three-factor model (Figure 4.19), the first factor has an increasing positive effect on yields change up to four years with the largest impact on the medium-term interest rates (2 to 6 years); for yields of longer maturities the curvature factor has a slowly decreasing positive influence. The second factor lowers the interest rates up to 10 years, after which its effect becomes small but positive. The third factor has the same negative impact of 0.5% across the whole yield curve, and therefore is interpreted as the level factor which is consistent with the results from the data-based factor analysis.

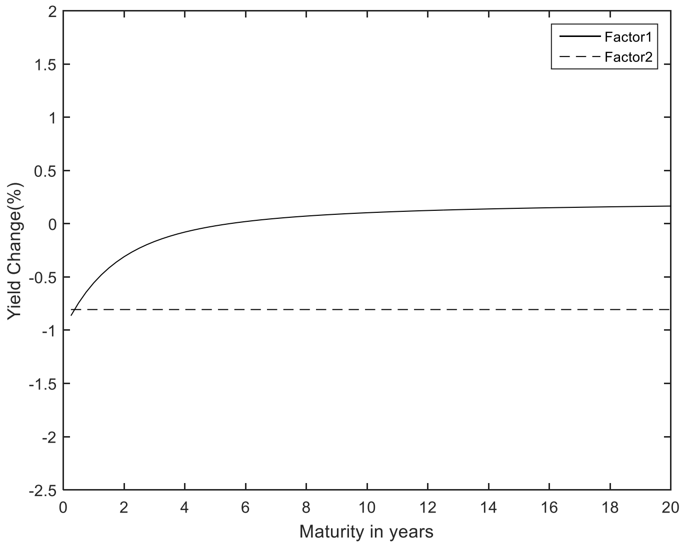


Figure 4.18 Norway, the factor loadings for the two-factor BN model

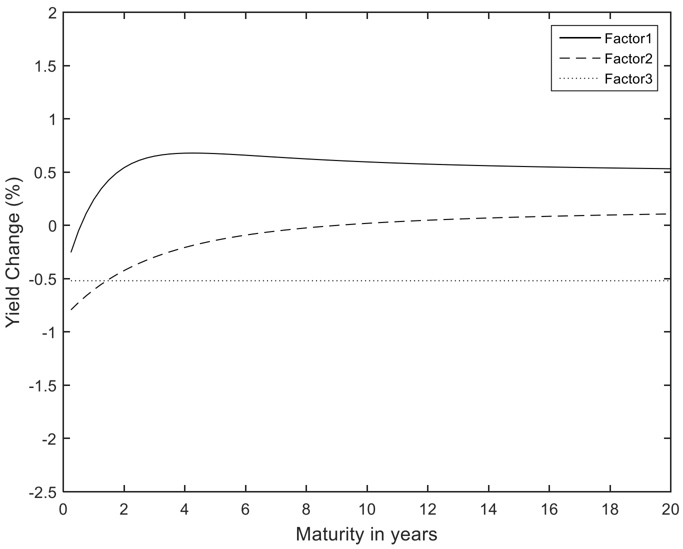


Figure 4.19 Norway, the factor loadings for the three-factor BN model

The factor analysis for Sweden portrays a slightly inconclusive situation when the three-factor specification is considered, a possible reflection of the failure to reject the two-factor model against the three-factor model in terms of goodness-of-fit. The factor loadings for the two- and three-factor models are plotted in the Figures 4.23 and 4.24, respectively. The first factor in the two-factor model has an increasing positive influence on the yield changes up to 8-year maturity. For yields of longer maturities, the impact remains at the same level of 0.7%. The second factor seems to act like a parallel shift by lowering all interest rates with 0.5%. While for the two-factor model the interpretation of the first factor as the slope and of the second factor as the level is highly supported by the data-based factor analysis, it is difficult to interpret the first and second factors in the three-factor model. The third factor can be easily interpreted as the level factor given its constant negative impact on the entire yield curve (see Figure 4.24). The first factor impacts negatively in a decreasing manner the short-term interest rates up to one-year maturity; after this point its influence becomes positive and increases substantially up to 5-year maturity, followed by a more stable impact of approximately 0.6%. The loading of the second factor is mostly negative and approaching zero as the maturity increases. The third factor lowers all the interest rates by just over 0.5% implying a downward parallel shift in the yield curve due to a shock of one standard deviation in this factor.

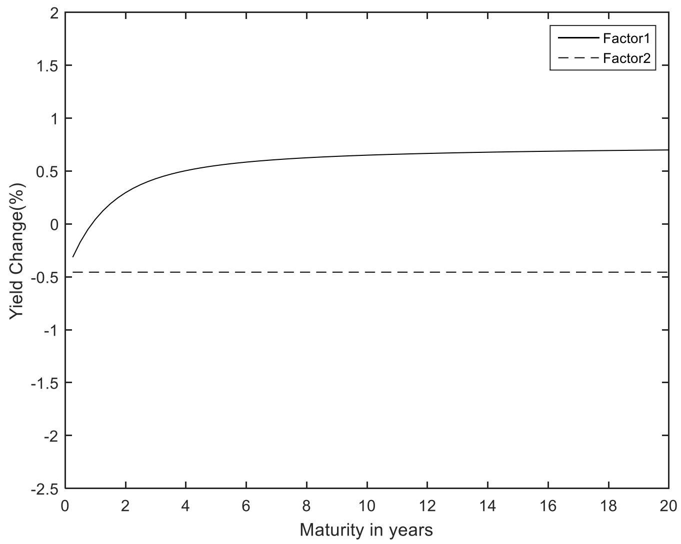


Figure 4.20 Sweden, the factor loadings for the two-factor BN model

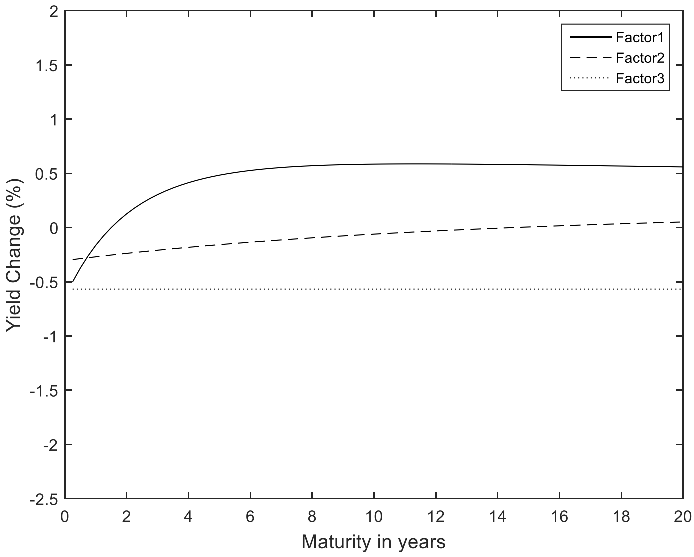


Figure 4.21 Sweden, the factor loadings for the three-factor BN model

Following Diebold and Li (2006), we compare the theoretical factors (X_1, X_2, X_3) implied by the Kalman filter with the empirical (data-based) factors and interpret the unobservable variables in terms of level, slope and curvature as suggested by Litterman and Scheinkman (1991).

It is important to see if this association is consistent with the factor loadings analysis.

The data-based factors are defined as follows: the long-term (level) factor is defined as the 20-year yield, the slope (short-term) factor as the difference between the 20-year and 3-month yields, and the curvature as twice the 5-year yield minus the sum of the 3-month and 20-year yields. The time-series of the factors extracted from the Kalman filter method are plotted against the unrestricted factors based on the information available on the yield curve and if the correlation between these time-series is considerably high then the factors are associated correspondingly.

For Denmark the data-based factor analysis is presented for the two- and three-factor models in Figure 4.24 (a,b) and Figure 4.25 (a,b,c), respectively.

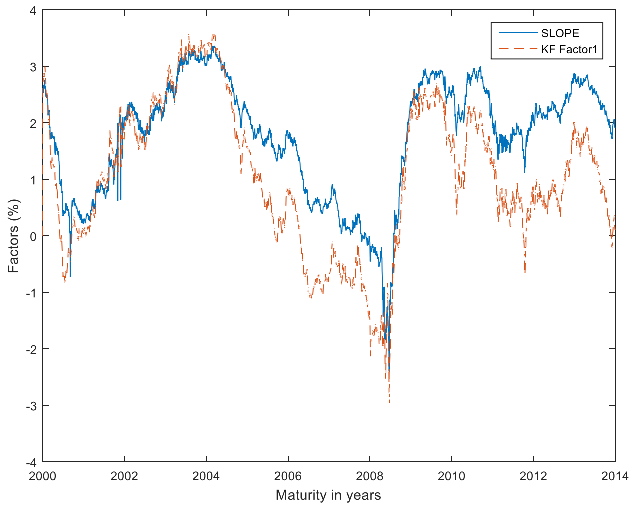


Figure 4.22a) Denmark, two-factor model; First factor (KF) and SLOPE (data-based)

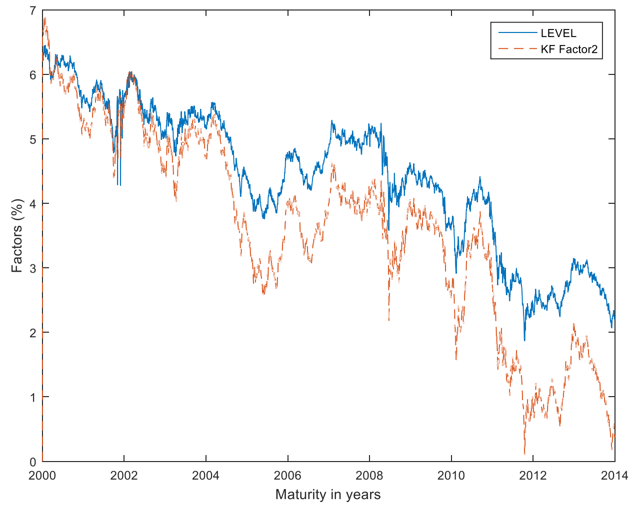


Figure 4.22b) Denmark, two-factor model; Second factor (KF) and LEVEL (data-based)

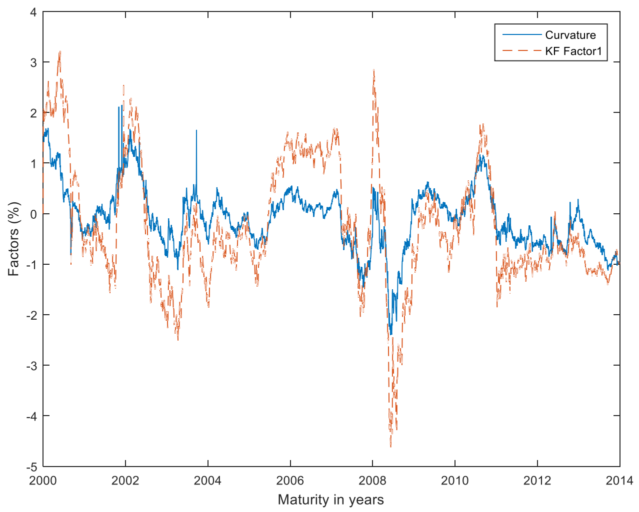


Figure 4.23a) Denmark, three-factor model; First factor (KF) and CURVATURE (data-based)

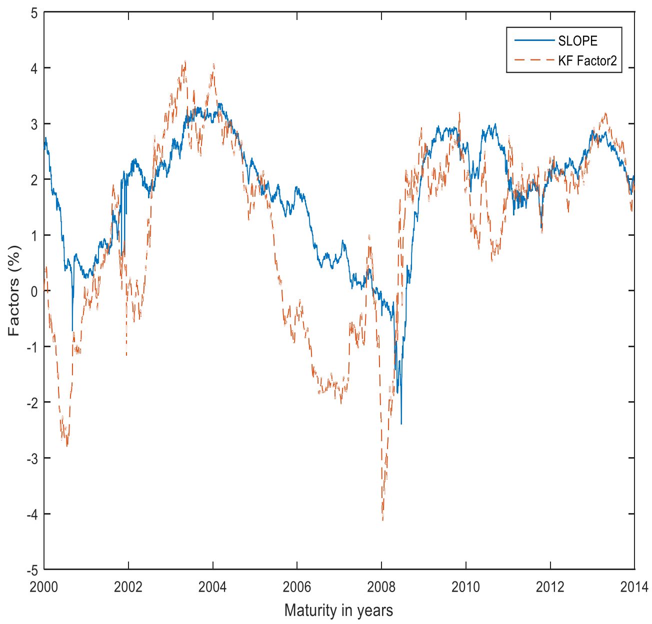


Figure 4.23b) Denmark, three-factor model; Second factor (KF) and SLOPE (data-based)

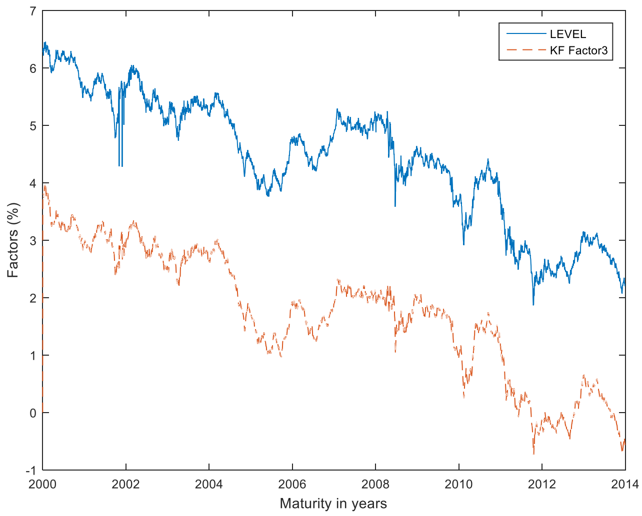


Figure 4.23c) Denmark, three-factor model; Third factor (KF) and LEVEL (data-based)

As suggested by Figure 4.22, for Denmark the first KF factor X1 in the two-factor model is highly correlated with the slope data-implied time series and the second factor X_2 with the level time series, while for the three-factor model (see Figure 4.23 above) the first factor is associated with the curvature, second factor with the slope and the third factor with the level data-implied factor. This correspondence is consistent with the conclusions from the factor analysis. While similar results apply to Norway, for Sweden the dynamics of the KF-implied factors $X_i(t)$ do not resemble any of the data defined factors. This can be possible as the factors $X_i(t)$ are not independent and they are in fact linear combinations of the independent Brownian motions $Z_i(t)$, used in the factor loadings analysis.

4.7 Forecasting Analysis

The Kalman filter technique can be applied to the estimation of various advanced TSIR models such as BN model to obtain multivariate times series of optimal estimate of the state vector. The forecasting procedure involves the recursive application of the space-state form given by the equations 4.6 and 4.7 over different length horizons.

The out-of-sample performance of the models is measured using RMSE (root-mean squared-errors) forecasting accuracy measure over one-, two- and three-month horizons. The one-month horizon ($H_1 = 23$ days) corresponds to the period 01 October 2014 – 31 October 2014, while the two-month horizon ($H_2 = 44$ days) covers the period 01 October

2014 – 28 November 2014 and the three-month horizon ($H_3 = 66$ days) covers the period 01 October 2014 - 31 December 2014.

The next section describes the forecasting algorithm using the state-space form suggested by Babbs and Nowman (1999). We retain the estimates for the matrix parameter B, C and the vector-parameter d , from the estimation part and the last filtered estimate of the latent vector, $X_{T|T}$ and use all this information to calculate one step-ahead optimal forecast for the latent vector $X(t)$ and one step-ahead prediction for $Y(t)$ conditional to the information up to time T :

$$\hat{X}_{T+1|T} = B\hat{X}_{T|T} \tag{4.21}$$

$$\hat{Y}_{T+1|T} = C\hat{X}_{T+1|T} + d \tag{4.22}$$

For future observations at $T+2, T+3$ and so on, the transition equation is used repeatedly to calculate the two-, three-step and so on ahead predictions, respectively. Therefore, for any positive integer p , the optimal forecast for the future value of the interest rates at time $T+p$ is given (see Harvey, 1989) by:

$$FY(T+p) \equiv \hat{Y}_{T+p|T} = C\hat{X}_{T+p|T} + d = CB^p \hat{X}_{T|T} + d \tag{4.23}$$

In order to assess the prediction power of these forecasts, the forecasting errors $FE(T+p) = Y(T+p) - FY(T+p)$ are calculated and implemented accordingly in the RMSE accuracy measure for all three horizons $(k = 23, 44, 66)$:

$$RMSE(k) = \sqrt{\frac{1}{k}\sum_{j=1}^{k} FE_{T+j}^2} \tag{4.24}$$

The RMSE measure provide us with a typical size of the forecasting errors over a certain period in the future. The RMSE calculations across the three horizons and the selection of the best model in terms of forecasting performance are presented bellow in Tables 4.10 – 4.12 for each country.

Table 4.10 DENMARK: The forecasting accuracy measure RMSE in percentages

horizon	model	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
	1F	0.1652	0.1062	0.0619	0.0092	0.0245	0.0353	0.0948	0.1236
H1	2F	0.0030	0.0070	0.0139	0.0404	0.0310	0.0091	0.0233	0.0157
	3F	0.0106	0.0324	0.0071	0.0587	0.0593	0.0346	0.0250	0.0124
	best model	2F	2F	3F	1F	1F	3F	2F	1F
	1F	0.1174	0.0728	0.0383	0.0192	0.0390	0.0523	0.0993	0.1252
H2	2F	0.0010	0.0043	0.0132	0.0386	0.0407	0.0306	0.0451	0.0449
	3F	0.0134	0.0234	0.0008	0.0535	0.0626	0.0501	0.0471	0.0252
	best model	2F	2F	3F	2F	1F	3F	2F	3F
	1F	0.0899	0.0505	0.0211	0.0221	0.0399	0.0551	0.0991	0.1265
H3	2F	0.0037	0.0023	0.0180	0.0351	0.0385	0.0348	0.0526	0.0589
	3F	0.0111	0.0132	0.0080	0.0489	0.0578	0.0518	0.0550	0.0434
	best model	2F	2F	3F	1F	2F	2F	2F	3F

Table 4.11 NORWAY: The forecasting accuracy measure RMSE in percentages

horizon	model	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
	1F	0.0920	0.0028	0.0669	0.1527	0.1759	0.1941	0.2422	0.2973
H1	2F	0.3754	0.4859	0.5703	0.5954	0.5671	0.5123	0.4571	0.4245
	3F	0.0946	0.0276	0.0194	0.0658	0.0775	0.0663	0.0502	0.0426
	best model	1F	2F	3F	3F	3F	3F	3F	3F
	1F	0.0679	0.0072	0.0602	0.1282	0.1484	0.1680	0.2028	0.2468
H2	2F	0.2827	0.3672	0.4265	0.4471	0.4296	0.3961	0.3564	0.3373
	3F	0.0429	0.0161	0.0198	0.0639	0.0758	0.0740	0.0624	0.0614
	best model	3F	1F	3F	3F	3F	3F	3F	3F
	1F	0.0311	0.0435	0.0890	0.1448	0.1591	0.1727	0.2025	0.2374
H3	2F	0.2654	0.3425	0.3897	0.4037	0.3868	0.3570	0.3262	0.3098
	3F	0.0398	0.0405	0.0516	0.0912	0.0988	0.0947	0.0866	0.0848
	best model	1F	3F	3F	3F	3F	3F	3F	3F

Table 4.12 SWEDEN: The forecasting accuracy measure RMSE in percentages

horizon	model	3M	1Y	2Y	5Y	7Y	10Y	15Y	20Y
	1F	0.1214	0.0437	0.0955	0.0887	0.0574	0.0250	0.0096	0.0166
H1	2F	0.0696	0.0806	0.1971	0.2282	0.1869	0.1269	0.0599	0.0148
	3F	0.0525	0.0438	0.0643	0.0736	0.0454	0.0601	0.0621	0.0579
	best model	3F	1F	3F	3F	3F	1F	1F	2F
	1F	0.0921	0.0276	0.0700	0.0657	0.0577	0.0391	0.0225	0.0206
H2	2F	0.0219	0.0766	0.1584	0.1840	0.1570	0.1172	0.0623	0.0220
	3F	0.0337	0.0296	0.0510	0.0699	0.0685	0.0693	0.0646	0.0541
	best model	2F	1F	3F	1F, 3F	1F	1F	1F	1F
	1F	0.0763	0.0234	0.0604	0.0731	0.0523	0.0403	0.0314	0.0329
H3	2F	0.0888	0.0824	0.1454	0.1579	0.1384	0.1080	0.0668	0.0366
	3F	0.0271	0.0274	0.0485	0.0631	0.0661	0.0696	0.0698	0.0637
	best model	3F	1F	3F	3F	1F	1F	1F	1F

For Denmark, the forecasting results are not clear cut. Overall, one could conclude that the two-factor model is superior to the other models in terms of prediction power. However, the models seem to forecast very well over all three sets of future observations, as their forecasting accuracy remains almost unchanged when the horizon increases. The forecasting results for Norway strongly suggest the three-factor model as the best model, while the predictions slightly deteriorate as the horizon is further into the future. Nevertheless, even for three-month horizon, the accuracy remains extremely high with values across all the maturities under 0.1%. Turning to Sweden, there is a clear pattern indicating that the three-factor model is most powerful in predicted the evolution of the short end of the yield curve where most undulations in shape occur, while for forecasting longer term interest rates the simplest one-factor model is selected as the best model. The forecasting results portray different conclusions for each country, however the magnitude of the forecasting errors is very small across all models as the numbers reported represent percentages.

To test statistically the efficiency of the forecasts produced by all three specifications (one-, two- and three-factor models) we employ the approach developed by Clark and West (2007) for nested models. To compare a simpler (nested) model A to a more general model, the authors consider the null hypothesis the additional parameters of the larger model B do not increase its predictive power, which is equivalent with an efficiency loss. Under their null hypothesis the model B produces an inflated RMSE. To adjusts for this bias Clark and West (2007) calculate the following f_t series:

$$f_t = FE_B^2 - FE_A^2 + (FE_B - FE_A)^2$$

where FE_A and FE_B denote the forecast-errors produced by model A and model B, respectively. The Clark and West (2007) test is a t-test; its test value is given by the t-statistic of the regression of f_t on a constant. Being a one-sided test, the alternative hypothesis states that model B is superior as it produces smaller RMSE metrics.

Table 4.13 The Results of the Clark-West Forecasting Errors Test

	DENMARK							
Clark-West test	M1	M2	M3	M4	M5	M6	M7	M8
1F v. 2F	149.12	73.25	40.81	-16.22	-8.37	13.78	26.40	29.35
1F v. 3F	84.24	58.97	42.02	-14.57	-20.11	7.21	27.50	30.55
2F v. 3F	7.58	-8.49	41.62	-33.02	-21.98	-8.29	-8.65	8.87
Best Model	3F	2F	3F	1F	1F	2F	2F	3F
	NORWAY							
Clark-West test	M1	M2	M3	M4	M5	M6	M7	M8
1F v. 2F	-23.85	-1.30	-8.96	23.70	28.77	34.50	43.41	58.64
1F v. 3F	22.27	5.53	10.97	25.23	29.83	34.77	43.35	58.47
2F v. 3F	23.90	6.96	10.88	21.44	20.60	-12.51	-5.48	-2.99
Best Model	3F	3F	3F	3F	3F	2F	2F	2F
	SWEDEN							
Clark-West test	M1	M2	M3	M4	M5	M6	M7	M8
1F v. 2F	14.34	-12.50	-26.50	-25.68	-17.55	-9.55	-4.51	-6.13
1F v. 3F	75.96	-6.95	64.40	17.42	-10.69	-9.41	-4.59	-5.83
2F v. 3F	7.52	-10.25	31.88	51.78	46.39	35.09	-11.01	-4.88
Best Model	3F	1F	3F	3F	1F	1F	1F	1F

For each country, the time series of the error forecasts for the three-month horizon are considered to compute the respective test statistics that are compared to the critical value of 1.645 at the 95% confidence level. The results of the Clark and West tests are reported in the Table 4.13 and they can be interpreted as following: any positive value larger than the critical value implies that the forecasts generated by the more general model are more reliable than those of the nested model. For negative values, the decision rule is that the the null cannot be rejected, therefore the predictive performance of the nested model is as good as the larger model. Using the transitivity law, the final conclusion on the best forecasting model is also reported. The verdict for the model with the best forecasts is compared to the results from the RMSE analysis. Despite few differences, the results are very similar for all countries under consideration. The consistency of the results may suggest that the inflation bias mentioned above is rather minimal and hence in our cases the sample dependent RMSE forecasting accuracy analysis still produces reliable outcomes.

4.8 Summary and Conclusions

In this empirical investigation, the term structure of interest rates is analysed for three Scandinavian countries (Denmark, Norway and Sweden) over the period 2000-2014 which includes the last global financial crisis of 2007-2009. One-, two- and three-factor versions of the general BN model are estimated using the linear exact Kalman filter technique in combination with a maximum likelihood estimator. This estimation method allows for the realistic feature of measurement errors in the zero nominal rates across eight maturities and also produces efficient ML parameter estimates. A high proportion of the parameter estimates are highly significant including some of the market price of risk parameters.

Based on formal statistical tests and residual analysis, the empirical results indicate that the three-factor specification explains best the changes over time in the shape of the yield curve for Denmark and Norway. For Sweden, the BIC test does not reject the two-factor model against the three-factor formulation, suggesting that the term structure of Swedish interest rates has simpler dynamics for which two factors are sufficient.

There is evidence of a structural break during the third quarter of 2007 as the estimation results for the pre-crisis data-sample differ considerably from those from the post-crisis period. Additionally, the loadings (sensitivity) of the yield curve on each factor are extracted and analysed in order to determine the nature of their associated factor. Moreover, the time series of the unobservable factors are extracted using the Kalman filter and compared to the level, slope and curvature factors defined using the data. For Denmark and Norway, the interpretation of the factors is very similar and straightforward, whether for Sweden the paths of the extracted factors are not matching the dynamics of any of the data-implied factors.

The estimation results are used to compute optimal daily forecasts for the last three months in 2014 and compare all the models in terms of prediction power. In terms of forecasting performance there is a clear winning model only for Norway where the three-factor model performs best, while for Denmark the best model is the two-factor specification. For Sweden, the one-factor and three-factor have comparable performance. Overall, the BN models achieve very good quality forecasts across all maturities and given their tractability these models can be very useful in hedging strategies and pricing interest rates derivatives in the current negative interest rates environment.

Chapter 5
The Impact of the Global Financial Crisis on the Return and Volatility Spillovers Empirical Evidence from Interest Rate and Equity Markets

5.1 Introduction

International financial markets have shown historically a significant level of interaction and interdependence with periods of intensified transmission of information through multiple channels, especially during/after periods of negative shocks. Despite the regularity of the financial crises[1] and their similarities, it is still difficult to control the propagation of a crisis and to contain its consequences to the market where the shock has originated. Post-examination of such events can always bring important insights about the dynamic evolution of a crisis with great implications for policy makers and regulators on one hand, and international investors and portfolio managers on the other hand.

In the aftermath of the most recent global financial crisis of 2007-2009 (GFC), the research on information spillovers has been revived with studies exploring new transmission channels such as liquidity and risk premium channels and developing new methods to model the dynamics of a crisis (Longstaff, 2010; Vayanos, 2004). Moreover, given one important facet of the GFC - the 2009 sovereign debt crisis in the Eurozone, there is an increasing interest on examining the inter-linkages involving international bond markets. Most of the spillovers empirical studies keep the domestic and the international transmission channels in isolation with only few studies (Christiansen, 2010;

[1] Examples of such crises include the critical events in the U.S. (1987), Mexico (1994), Asia (1997), Russia (1998).

Ehrmann et al., 2011) combining simultaneously equity and bond returns in a discrete time econometric framework.

According to Ehrmann et al. (2011, p 948) to better understand "the complexity of the financial transmission process across various assets—domestically as well as within and across asset classes—requires the simultaneous modeling of the various transmission channels in a single, comprehensive empirical framework". Following Ehrmann et al. (2011), this study employs a complex network of information transmission channels to comparatively investigate how the last financial crisis of 2007-2009 has spread from the U.S. (the country where the financial crisis started) to other major economies such as the U.K., Eurozone, Japan and Canada. This pair-wise analysis is conducted by implementing a discrete-time multivariate generalised autoregressive conditional heteroscedasticity (MGARCH) framework that is appropriate for investigating the return and volatility spillover channels between the U.S. and each other country across various asset classes. This modelling framework considers four asset prices, two from each country. Hence, the information can flow via six bidirectional routes (two direct domestic routes – same country different asset classes, two international *direct* routes - same asset classes, different country; and two international *indirect* routes – different asset classes and different country). It is the last type of spillovers – international indirect – that has received very little attention in the spillovers literature and this study aims to bring new evidence of its significance among the financial markets during the last global financial crisis of 2007-2009.

The two main asset classes considered are the equity markets on one side and the interest rate markets on the other side. Following Ehrmann et al. (2011) we differentiate between the two segments of the interest rate markets, the money market segment and the long-term segment, respectively. Apart from being the most important asset classes in an individual financial system, these three types of markets interact with each other at macroeconomic level. Indeed, any shock on the price of one of these assets will result in movements across all the asset classes. Previous empirical evidence (see Rigobon and Sack, 2004) tells us that a rise in the long-term interest rates should lower the equity prices, while an increase on the short-term interest rates will also result in declining equity prices through the new discounting of the future dividends. Conversely, changes on the equity prices will affect the equilibrium between the aggregate demand and supply and ultimately the expected monetary policy reflected in the market interest rates. However, the work of Rigobon and Sack (2004) is conducted at the domestic only.

Therefore, is of great interest to see how these asset classes interact with each other in an international context during turbulent economic and financial times.

By comparing the results, we aim to identify which country out of the four major economies has been mostly interacting with the U.S. especially after the last financial crisis. Despite the high degree of integration between the U.S. and each of these economies, their relationship with the U.S. can still be country-specific due to multiple factors such as differences in the structure of their financial systems, in the state of their economies and in the monetary policies implemented during the GFC. Findings of significant difference in their financial communication with the U.S. may have great implications for the new course of action that each of the four economies should take in order to contain a future crisis originating in the U.S.

The implementation of a four-dimensional model will allow to investigate possible answers to several important questions in relation to the mechanism of information transmission between different types of markets of any two countries. With both internal (domestic) and external (international) channels, one could identify which are the busiest *routes* that the information flows through. By considering the short- and long-term bond markets separately, one could determine if the information is transmitted in a specific way between the stock markets and between different maturity segments of the fixed income markets. The models employed are estimated over two periods - before the crisis and during the crisis – in order to observe any significant changes in the structural parameters and to assess the impact of the last financial crisis on the return and volatility spillover effects between the markets considered.

The severity of GFC and its snow-ball effect due to increased economic and financial integration prompted many central banks to act in a similar manner (by lowering the short-term rates) in order to ameliorate the impact of the crisis. By modelling simultaneously, the stock and the money markets we can investigate whether the monetary policy influences the stock markets via short-term rates[2] and we can observe any degree of convergence of those monetary policies across major economies.

The rest of this chapter is organised as follows. Section 2 briefly reviews the empirical literature on return and volatility spillover effects. Section 3 describes the modelling framework underlying our analysis. Section 4 presents the data sets and the results of the preliminary statistical analysis of the data. Section 5 provides the empirical results and their interpretation. Finally, section 6 concludes.

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[2] Treasury interest rates are recognised as one possible monetary policy tool among others through which a central bank can intervene in order to achieve its objectives (see BoE website)

5.2 Literature Review

5.2.1 The Information Transmission Mechanism between Financial Markets

The analysis of information spillovers among economic and financial markets is an important area of financial management, with the earliest empirical evidence being dated back only to the 1990s (Engle, Ito et al., 1990; Hamao et al., 1990). There is an extensive empirical literature on this topic, that stems from the necessity to understand better how financial markets coexist and respond to each other on a global platform when one or a group of markets are subject to either a positive or a negative "surprise".

The collapse of Bretton Woods system, the creation of European monetary union (EMU) and the merger of several stock markets along with important advances in computer technology and information processing have contributed to higher integration and liberalization of capital flows between national markets, creating a "terrain" that facilitates the propagation of a shock from a single financial market to many other markets around the world, therefore leaving markets more vulnerable during turbulent times (Kearney and Patton, 2000).

The way information spills from one market to another is a multifaceted process, with the cross-market linkage being facilitated through multiple channels. The relevant literature documents four main channels through which information is transmitted: 1) the price discovery or the *return* channel (King and Wadhwani, 1990; Kiyotaki and Moore, 2002); 2) the *volatility* channel (Fleming et al., 1998; Campbell and Taksler, 2003; Connolly et al., 2005); 3) the *liquidity* channel (Longstaff, 2010; Ding and Pu, 2012); and 4) the *risk premium* channel (Vayanos, 2004; Acharya and Pedersen, 2005). The most common modelling frameworks employed in the information spillover research are in discrete time and belong to the GARCH or VAR family. The first two channels are easily simultaneously accommodated by these typical spillover models, giving course to the development of a main strand in the literature – return and volatility spillovers, that can be tracked back to Engle, Ito et al. (1990) and Engle, Ng et al. (1990).

Advanced stock markets have been thoroughly analysed under the hypothesis of spillover effects. While the existence of return and volatility spillovers is widely acclaimed, a common feature of the findings points to the dominant role of the U.S. stock market as the source of the spillovers. For example, Hamao et al. (1990), Koutmos and Booth (1995) and Bae et al. (2002) found significant evidence of linkages between various developed stock markets with a clear global direction, i.e. the U.S. stock market being the main exporter of volatility. Using univariate GARCH models Base and Karolyi (1994) and Karolyi and Stulz (1996) also confirmed the influence of the U.S. as a "world" market on individual markets. The dynamics of these causal relationships seem to intensify especially after negative shocks mostly associated with a financial crisis.

For example, the global stock market crash of 1987 produced higher levels of both, contemporaneous and dynamic linkages across international stock markets, with the US exercising a stronger influence on French, German and UK stock markets after the crisis (see Arshanapalli and Doukas, 1993). Japanese and the U.S. stock markets are found to affect each other at a significant level through the volatility channel between 1986-1993 (Lin et al. 1994), while the volatility of the emerging stock markets is globally influenced by changes in the volatility of the U.S. stock market (Bekaert and Harvey (1997)). Similarly, Kim (2005) and Wang and Lee (2009) found that post - Asian crisis of 1997 the return and volatility spillovers from the U.S. to countries from the Asia-Pacific region have significantly increased.

Distinguishing between different types of shocks, Ng (2000) analysed separately the *world* (global) shock from U.S. and the *regional* shock from Japan on a group of Asian markets based on weekly data from January 1975 to December 1996. While there is significant evidence of volatility spillovers from both external developed markets, the shock from U.S. dominates the regional influence from Japan[3]. However, the strong bidirectional connection between Japan and Asian markets cannot be ignored, especially after the Asian crisis in 1997 when Japan's monetary expansion within Asian basin has created a higher interdependence among the portfolio of assets from both regions (see Fornari and Levy, 2000). This relationship has been explored further by Miyakoshi (2003) within a different econometric setting of a bivariate EGARCH model for Japan and an Asian market with an exogeneous influence from U.S. The daily data covering a shorter period (January 1998 - April 2000) had produced empirical results in contrast with those from Ng (2000)[4] as the endogeneous regional volatility shock from Japan is estimated to be stronger than the exogeneous world U.S. shock.

The volatility spillover effects are found to respond asymmetrically to the quality of the news, in the sense that they intensify greater following a negative shock than a positive shock. Earlier signs of such possible effects were found by Black (1976) and

[3] In the case of the GFC, Li and Giles (2015) found the opposite result with no influence from the US on to the Asian emerging markets.
[4]Additional to the difference of the econometric models, the data sets used cover also different period with different frequency.

Christie (1982) who provided evidence of the so-called *leverage* effect[5] in the stock market returns. The asymmetries of the volatility transmission process between New York, London and Tokyo stock markets is explicitly modelled by Koutmos and Booth (1995) in a multivariate EGARCH framework over a period including the 1987 stock crash in the U.S. Their findings provided support for the different reaction of the stock markets to different types of news, with markets being more sensitive to negative shocks (sudden decrease in the market). Hamao et al. (1990) employed a range of GARCH-M specification and found evidence that the Japanese stock market 'responded back' towards the U.S. and the UK stock markets in the sense that after the 1987 crash the information is transmitted in a reciprocal manner, implying therefore stronger interdependencies among these three stock markets.

The way a shock is transmitted from one market to another is a complex process that is not fully understood, as it is difficult to measure separately the effects of a crisis due to high levels of cross-market interdependence or investors's behaviour after a shock. There are various theories that try to explain how crises propagate based on different assumptions. Some theories assume that shocks spread via real linkages which exist and are stable before the shock, while other theories consider that investors change their trading strategies after the crisis (Forbes and Rigobon, 2002). In this investigation, we embrace the first type of theory as there is clear evidence of both domestic and international linkages across the markets.

According to Danielsson and Love (2006) the price of stocks can be affected with immediate effect by traders under speculative pressure (order flow) but not vice-versa. When aggregated over time the two will interact with impact on each other, a phenomenon called feedback trading. The price to price feedback theory is one of the oldest theories about financial markets (Shiller, 2003) but somehow is less known because it was mostly presented in non-academic papers. The main mechanism is that when speculative prices go up, the success of some investors leads to public attention that generates enthusiasm which in turn will increase expectations for further price increases. This process generally leads to an increase in investor demand and thus generates another round of price increases. If the feedback continues uninterrupted for many cycles, it may suddenly give rise to a speculative bubble. If the prices are high only because of expectations of further price increases, then they are not sustainable, and therefore the prices will start falling. The feedback mechanism could also generate a negative bubble,

[5] A reduction in stock prices leads to an increased debt to equity ratio and hence to a higher volatility level.

with decreasing price movements propelling further lower prices, until the market reaches an unsustainably low level.

Cutler et al. (1990) have advocated that serial correlation across many assets could be explained by models with feedback traders. This is in contradiction with the conclusion in Shiller (2003) who argued that simple feedback models do not imply strong serial correlation and that serial correlation in returns may be caused by many other reasons that are unrelated to the feedback traders model.

There is a large theoretical literature on feedback effects such as Leland (1992), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Boot and Thakor (1997), and Subrahmanyam and Titman (1999). These studies provide an insight on how financial markets may impact firms' investment and capital allocation decisions when there is feedback. Goldstein and Guembel (2008) argue that feedback effects may leave companies vulnerable to possible market manipulation and there is empirical evidence supporting this hypothesis in papers by Durnev, Morck and Yeung (2004), Luo (2005) and Chen, Goldstein and Jiang (2007). Hirshleifer, Subrahmanyam, and Titman (2006) describe a possible mechanism of how irrational traders can produce significant gains based on a feedback effect from asset prices to cash flows. Furthermore, Khanna and Sonti (2002) highlight that it is even possible to produce a "bubble-like" price evolution.
There has been a great interest in studying the implications of the feedback effect from prices to real value. Khanna and Sonti (2004) reveal that it is possible to explain herding based on the feedback from prices to asset value, as advocated also in Avery and Zemsky (1998). In a seminal paper, Sentana and Wadhwani (1992) presented empirical evidence suggesting that when volatility is low, stock returns at short horizons exhibit positive serial correlation, while when volatility is high the returns display negative autocorrelation. The authors show also that these stylised features are consistent with a model where some traders follow feedback strategies. The idea is that as volatility increases the positive feedback traders impose a greater influence on the asset price which then in turn causes greater negative serial correlation in returns. It also seems that there is greater feedback trading on a path of declining prices.

The spillovers literature refers to the controversial[6] concept of contagion, which may occur during a financial crisis when markets seem to move together in a much closer manner when compared to an otherwise calm period. Under different forms of overreaction, noise trading and/or speculation, contagion may explain why changes in

[6] The concept of contagion is theoretically controversial, while the empirical literature offers mixed evidence regarding its presence and magnitude (see Forbes and Rigobon, 2002)

stock prices in one market spread across world markets beyond the level of propagation implied by economic fundamentals (Lin et al., 1994). Forbes and Rigobon (2002) challenged the accuracy of the tests for the presence of contagion based on correlation coefficients that are influenced by market volatility arguing that the estimates were biased due to heteroschedasticity. Therefore, the previous association of a financial crisis with the phenomenon of contagion became questionable. When a correction for this bias was applied, the results changed and no longer evidenced the contagion effect. Financial contagion was investigated in an extensive study by Kenourgios et al. (2011) who examined the four emerging stock markets BRIC (Brazil, Russia, India and China) and two developed (U.S. and U.K.) over five financial crises inside the 1995-2006 period.

Contagion is recognised when there is evidence of a correlation break, i.e. only if the continuity of a high level of correlation is disturbed by a significant change, otherwise the shock is spread between the markets due to the ample interdependence present before the shock. Employing two different estimation techniques, Kenourgios et al. (2011) could identify what is the main reason for the contagion effects, either they are of behavioural or macroeconomic nature. While their findings support the presence of jumps in the correlation pattern with at least two regimes existent during each of the five crises, the industry-specific crises like the technology bubble (2000-2001) seem to propagate more powerfully than country-specific ones. They also conclude that policy makers have limited power to isolate the financial crises, because the evolution of the markets during a crisis has a strong behavioural component, hence the change in economic fundamentals is of secondary importance. Boyer et al. (2006) used two estimation methods, a regime-switching model and extreme value theory, to investigate spillovers among various stock markets and concluded that the well diversified portfolios held by international investors could be responsible for the so called “domino effect” observed in the global markets during a financial crisis.

Given their rapid economic growth, the group of the BRIC countries (Brazil, Russia, India and China) was the subject of numerous empirical studies that aimed to detect and measure the magnitude of various types of spillovers. It is important that the empirical results concerning spillover effects are compared over similar periods that include major events in the markets considered. For example, using a MGARCH modelling framework Li (2007) found no direct spillovers between China and U.S. stock markets during the interval 2000-2005. Also, Lin et al. (2009) reported no substantial changes in the correlation patterns over the period 1993-2006. However, in 2005 the Chinese stock market has been subject to structural reform, becoming more transparent and less regulated. Once the time horizon takes into account this event together with its potential lagged effects, the results are rather different. Using symmetric and asymmetric GARCH models Moon and Yu (2010) explored the interaction between China and U.S. stock markets between 1999 and 2007 and found evidence of a structural break in December 2005 in the Chinese stock market (SSE) Shangai Stock Exchange and of both, symmetric and asymmetric volatility spillovers after 2005.

Apart from shocks that impact negatively the financial markets, other major events such as the creation of European Monetary Union (EMU) had been analysed in terms of spillover effects. Using 3, 4 and 5 variables multivariate GARCH models Kearney and Patton (2000) examined the volatility transmission process across key currencies within the European Monetary System (EMS) between 1979 and 1997, prior to the European monetary unification. Their study extended the Bollerslev's (1990) approach using the BEKK parameterization and relaxing the assumption of constant conditional correlations. The empirical results generated by Kearney and Patton were inconclusive emphasizing the relativeness of the model specification, however some consistent features could be observed suggesting the dominance of the German mark. Additionally, the ECU currency worked as the "n-th" currency in the system by transmitting volatility rather indirectly (via its covariance) than through direct channels (its variance). Christiansen (2010) found that the introduction of the euro has caused a structural break in the volatility spillover effects, with a change in the regional and local influences. After the introduction of the euro the regional (aggregate European) spillovers gained intensity whereas local spillovers lost intensity diminishing the potential benefits of investment diversification.

Financial institutions are subject to frequent changes influenced by deregulation, technological change and financial innovation with the intra-industry relationships being more dynamic, therefore they require re-assessment more frequently (Allen and Gale, 2000; Allen and Santomero, 1997). In this regard, Elyasiany et al. (2007) tested the degree of convergence and competition[7] across the most prominent components of the financial industry in the U.S. namely, commercial banks, brokerage firms and life insurance companies (LIC). Extending on the work of Brewer and Jackson (2002), Elyasiany et al. (2007) included the brokers as a third financial sector and estimated separately the return and volatility spillover effects for small and large FIs within a more complex multivariate GARCH framework. This separation has empirically identified that the size of the FI or the degree of consolidation it is an important factor that influences the spillover patterns which is consistent with previous findings by Demsetz and Strahan

[7] Stronger spillover effects indicate both higher convergence and more intensive competition.

(1997) and Stiroh (2004). More specifically, large FI seem to exhibit stronger volatility spillover effects in contrast with small FI for which the volatility spillovers are limited and returns transmission is more pronounced. These conclusions have important implications for policy makers and regulators in the sense that they should expect dissimilarities in the impact of new regulation on financial services providers of different type and size.

In another intra-industry study Carson, Elyasiani and Mansur (2008) employed the System-GARCH modelling framework to investigate interdependences in returns and volatility across three segments from the U.S. insurance industry, namely accident and health (A&H), life (Life) and property and casualty (P&C) insurers. The multivariate model included macroeconomic factors as the market return and interest rate in the return equations. To test for the impact of the Financial Services Modernization Act (GLBA 1999) the volatility equations were augmented by including spillover factors across the industry segments together with a binary variable. The empirical results suggested that the Life insurers were mostly sensitive to changes in the long-term interest rate, while the A&H and P&C insurers were mostly exposed to market risk. In contrast with the banking industry, the channels by which information is transmitted in the insurance industry play different roles, as the volatility transmission among the industry segments was found to be weak, while the return spillovers were much stronger. The regulators of the insurance industry are interested in the assessment of the degree of connection among its sectors, as a high level of correlation typically increases the contagion effects that ultimately may lead to the collapse of the entire industry. The similar structure of the portfolios (predominantly of highly correlated bonds) of insurance companies is one of the multiple facets of the indirect interdependence that exists among the insurers. Being exposed to similar risks and subject to generally the same capital-requirements, increases the chances of a direct (positive) comovement, as each sector will be impacted and will respond in a similar manner.

5.2.2 Empirical Evidence for Spillover Effects in Bond Markets

The hypothesis of spillovers and contagion effects have been studied mostly in the context of equity and currency markets, with substantially less focus on bond markets. One possible reason might be the historical stability of bond markets during financial crises when compared to the more volatile equity and foreign exchange markets. Also, emerging bond markets data time series lack consistency or are not easily available. Nevertheless, several studies examined the mechanism of shock transmission across national bond markets over different periods including important events such as the multiple crises in the 1990s, the introduction of the euro in 2000 and the technology bubble spanning from 2000 to 2002. Early studies of international bond market spillovers during volatile periods include Borio and McCauley (1996) and Domanski and Kremer (2000).

In contrast with the popular conditional correlation analysis, Hartman et al. (2004) apply a nonparametric measure to evaluate the extreme linkages between stock and government bond markets for the G-5 countries. Based on weekly data over the 1987-1999 period they conclude that the extreme losses are generally much higher in the stock markets relative to government bond markets and find evidence for the flight-to-quality phenomenon. Both, emerging and developed bond markets were analysed by Dungey et al. (2006) who applied a latent factor model to bond spread data for twelve countries in order to test for a contagion channel in the transmission of two historical surprises, the Russian default and the recapitalisation announcement of the American hedge fund LTCM, during the summer of 1998. Their empirical findings suggest that the Russian crisis was spread through a contagion linkage whereas the LTCM shock the contagion effects were not particularly significant. Another important insight was that despite increased volatility was experienced more considerably by the emerging markets under study, the contagion effects were proportionately of similar magnitude for both, emerging and developed markets.

Ehrmann and Fratzscher (2005) found significant international bond market linkages between the USA and the euro area. Andersen et al. (2007) used high-frequency data over a relatively short period (1998-2002) to analyse the domestic contemporaneous relationship between equity and bond markets and the euro-dollar exchange rate. Connolly et al. (2007) investigated how the stock-bond returns relationship in the U.S., the UK and Germany varies with the changes in implied volatility (IV) from the U.S. stock index option market. On days with large changes (positive or negative) in the IV the stock-bond correlations were found to be negative. Bond and equity volatilities have been analysed simultaneously by Christiansen (2010) who decomposed individual European bond/equity variances into global, regional and local bond/equity effects in order to measure the impact of the introduction of the euro in 2000 on the spillover effects. However, this particular combination of variables has not been used before towards analysing the dynamics of information spillovers over a period that includes the 2007-2009 financial crisis.

The idea of including as many as possible transmission channels was followed and empirically investigated by Ehrmann et al. (2011) who applied the most comprehensive modelling framework to analyse simultaneously the linkages of three asset prices both domestically and internationally for USA and the euro area. Their structural multifactor model includes seven asset prices as the endogeneous variables: short-term interest rates, bond yields and equity markets returns in both economies and the exchange rate. Following the methodology presented in Rigobon (2003) of identification through heteroskedasticity (IH), their results show that the strongest shock transmission takes place within asset classes. Another key result suggests that the direct transmission of shocks in the bond markets is substantially increased by indirect transmission channels through other asset classes. For example, the equity markets in the U.S. also influence the other two markets in the euro area, the short-term interest rates and bond yields, respectively.

More recently, Claeys and Vasicek (2014) used factor-augmented VAR models (FAVAR) to measure the linkages between the government bond spreads of 16 EU countries, during the 2000 - 2012 period. They also tested for the contagion phenomenon and found evidence of its existence only during the period defined by the IMF and EU bailout-interventions for Greece and Ireland between 2010 and 2011. However, they conclude that contagion is a rather rare phenomenon and the strongest linkages are the result of a larger shocks rather than of contagion.

5.2.3 The Impact of the 2007-2009 Global Financial Crisis on Return and Volatility Spillover Effects – Empirical Evidence

The literature on spillover effects is vast given the multitude of financial crises that occurred over the last four decades. However, the GFC is still an open area for exploration within the spillover literature. Several authors have examined its impact via the return and volatility spillover effects between various types of financial markets, trying as well to identify possible factors that may explain how volatility is transmitted.

Cheung et al. (2010) introduces the term of *fear* spillovers for which the best indicator is the TED spreads. The credit risk becomes another channel of transmission of information that seems to have changed the correlation among international markets during the recent global financial crisis. The authors use a trivariate VAR model and a Granger causality test to examine the return and volatility spillovers between the TED spread, the returns in the S&P500 stock price index and other global stock markets such as the UK, Hong Kong, Japan, Australia, Russia and China. While the dominant role of the U.S. enhanced during the crisis, the TED spread also was found to Granger cause the S&P500 and provide spillover effects into the other global stock markets.

Ding and Pu (2012) examined closely connected financial sectors in the U.S. under different economic conditions. Stock, corporate bond and credit derivatives markets are considered for exploring both the static and dynamic structure of information spillovers and identifying factors that may influence any linkage across these markets. Based on a VAR estimation model and daily data over the period 2004-2009, the empirical results indicate a more intensified and timely information-transmission among these markets during the 2007-2009 crisis. While both volatility and liquidity factors influence separately the linkage between the markets, during the crisis when both factors are exogeneously included in the model, it is only the volatility channel that is significant in all the three sub-periods analysed and has a strong impact on all three financial markets. Furthermore, Ding and Pu (2012) found that the credit derivatives and the stock markets swap the leading role in sending shock signals once a systemic crisis exists. Before the crisis the surprise information is absorbed first by the credit derivatives market that quickly affects the stock market. However, during the crisis the stock market becomes more independent and plays a dominant role with investors taking more into account the signals from the equity markets rather than from the bond and the credit derivative markets. Similar evidence was provided by Diebold and Yilmaz (2012) who developed a spillover index to measure the magnitude of the volatility spillovers (total and directional) across the U.S. stock, bond, foreign exchange and commodities markets within a generalized VAR framework over the period 1999-2010. While the bond markets react rather slowly, the U.S. equity market plays the most important role during all seven phases of the crisis with net positive volatility spillovers from the stock market to the other markets exceeding 6% immediately after the collapse of Lehman Brothers in September 2008.

Using a multivariate BEKK GARCH- in-the-mean approach, Gilenko and Federova (2014) examined return and volatility spillovers within the group (internal) and from various developed and emerging equity markets (external). Moreover, their model included an interaction term of the external factors that permitted for the analysis of the dynamics of the external influences before, during and after the financial crisis of 2007-2009. The patterns of the internal volatility spillovers change over time from strong interconnections in the pre-crisis period to no sign of transmission during the crisis and recovery periods. In the case of internal mean-to-mean spillovers the Brasilian market seems to keep its dominant influence within the group before and after the crisis, despite that during the crisis these channels of propagation disappear with only one direction being significant, from the Indian to the Brazilian stock market. Nikkinen et al. (2012b) also studied the BRIC (Brasil, Rusia, India and China) countries within a sectorial context. The financial and industrial sectors of each market were tested using eight bivariate models for the presence of spillovers.

In a recent paper, Choudry and Jayasekera (2014) empirically investigate the effect of the GFC on the return and volatility spillovers across European banking industries. In a subperiod analysis they examined three aspects of the information transmission between major economies (ME) (Germany, U.K. and U.S.) and stressed economies (SE) (Italy, Ireland, Greece, Spain and Portugal). Employing a MGARCH-GJR framework their results indicate evidence of asymmetric spillover effects in the pre-crisis period 2002-2007. Moreover, the spillover effects have intensified during the crisis period (2007-2014) with significant transmission also from the SEs to the MEs. The transition between the two subperiods revealed presence of contagion between ME and the larger SE and signs of flight to perceived quality from smaller SEs such as Greece and Ireland, confirming that stronger linkages reduce domestic market insulation from global news. There is also evidence that smaller markets with weaker linkages have a delayed reaction when shocks occur in larger markets, an effect that is also present across other asset classes (Harris and Pisedtasalasai (2006) and McQueen, Pinegar and Thorely (1996)).

Jung and Maderitsch (2014) is another study based on intra-daily data on the main stock indices in Europe, U.S. and Asia covering the period 2000-2011. The five-minute frequency data used in their work allowed for the computation of the realised volatility series and hence making possible an investigation for a singular structural break in the linear patterns of observed volatility. Based on the HAR-DL (Heterogeneous Autoregressive Distributed Lag) model proposed by Corsi (2009), the null hypothesis of no structural break is rejected, indicating instability in the dynamics of volatility transmission. To investigate further for contagion effects, the authors created a series of spillover and found that they display a sudden and significant upward change in spillover patterns for all three markets around the inception of the financial crisis in 2007. Moreover, Jung and Maderitsch (2014) argued that the structural break present in the realized volatilities did not affect significantly the spillover dynamics, it is the conditional heteroskedasticity in the realised volatility that is accountable for the sudden upward shift in the spillovers patterns. Consequently, the presence of contagion effects was rejected for a high level of interdependence

Following Diebold and Yilmaz's (2009) generalized VAR approach (FAVAR - factor augmented VAR), Claeys and Vasicek (2014) measured the bilateral linkages between 16 EU bond markets and also proposed a test for the presence of contagion and detection of sudden changes in the information transmission process and direction of contagion. Based on daily bond spreads their empirical results showed spillover effects as a common feature in the pre-crisis period, however with a substantial increase in the market interdependences once the financial crisis has started. Some evidence of contagion was found on three occasions when Greece, Ireland and Portugal requested a fiscal bailout creating a sense of uncertainty that to a certain extent led to an increase in market co-movements.

In a recent study of volatility spillovers, Li and Giles (2015) employ an asymmetric MGARCH full BEKK (1,1) model to analyse shock and volatility transmissions across two developed stock markets (the U.S. and Japan) and a group[8] of Asian emerging stock markets over a period of twenty years (1993-2012) covering two important financial crises - the 1997 Asian crisis and the 2007 U.S. subprime crisis. Based on daily data, they found that on the long run, there is some evidence of shock spillovers between the markets above and volatility spillovers only from the Japanese market to the Asian developing. However, on the short run, the results during both crises differ from the results provided by the full sample. Moreover, the two crises portray a rather contradictory situation in terms of volatility spillovers. During the 1997 Asian financial crisis, there is evidence of volatility spillovers between the U.S. and the rest of the markets, confirming the leading role of the U.S. as a global factor. Neverthelesss, during the 2007 financial crisis it is Japan, the geographical factor, that connects bidirectionally with the Asian emerging markets, whereas the U.S. stock market communicates only with the Japanese market.

[8] The developing Asian stock markets considered are China, India, Indonesia, Malaysia, the Philippines and Thailand

5.3 Methodology

For the discrete-time modelling of return and spillovers the relevant literature suggests that the MGARCH framework is mostly appropriate. The multivariate GARCH approach has multiple advantages as it takes into account two channels of information transmission, one in the mean equation and one in the volatility equation. Also, it allows for the investigation of the linear relationships between the parameters both, within and across the equations in the system.

The multivariate GARCH framework considers a multitude of specifications[9] from a range of direct generalizations (VEC, BEKK and factor models) to nonlinear combinations (copula-GARCH) of the univariate GARCH model of Bollerslev (1986). In this investigation, the four variables BEKK[10] representation of Engle and Kroner (1995) is used to estimate the return and the direct and indirect volatility spillovers effects. Inside the BEKK methodology both, the mean and the variance equations are estimated simultaneously as in Li and Giles (2015).

The return and the volatility spillovers between the U.S. and another country are modelled in a comparative setting using the full BEKK(1,1) four-dimensional specification, to allow for a more complex information transmission network. For a four-variable BEKK model the information flows via six routes (two direct domestic routes between different asset classes, two international *direct* routes between the same asset classes and international *indirect* routes between different asset classes). Hence, for each country-pair we are interested in twelve parameters inside the mean system which represent the return spillover effects and another twelve parameters in the variance equations measuring the volatility spillover effects.

5.3.1 The Discrete Time Method: The MGARCH Model

The four-dimensional stochastic vector of returns $\{R(t)\}_t$ encompasses both stock returns and bond returns. The vector of returns is modelled using its conditional mean $\mu_t(\theta)$ that includes all possible feedbacks as follows:

$$R_t = \mu_t(\theta) + \varepsilon_t \tag{5.1}$$

[9] A comprehensive survey of MGARCH models is provided by Bauwens et al. (2006).
[10] The abbreviation comes from the Baba, Engle, Kraft and Kroner whose previous extensive studies on GARCH models resulted in the model presented by Engle and Kroner (1995).

where $\mu_t = \alpha + \beta R_{t-1}$ for $t = 1,...T$ and T is the sample size. The elements off the diagonal of the feedback matrix $\beta_{4\times4}$ are interpreted as the return spillover effects from one market to another. They are the reaction coefficients of the

For example, for the first U.S. - U.K. pair $R_t = (R_{1t}, R_{2t}, R_{3t}, R_{4t}) = (s_t^{US}, s_t^{UK}, r_t^{US}, r_t^{UK})$. The first two components are the log returns of two major international stock indices (s_t) and the last two components are the daily changes in the respective government bond/Treasury yields (r_t).

The scalar mean equations are given by:

$$\begin{aligned} R_{1,t} &= \alpha_1 + \beta_{11}R_{1,t-1} + \beta_{12}R_{2,t-1} + \beta_{13}R_{3,t-1} + \beta_{14}R_{4,t-1} \\ R_{2,t} &= \alpha_2 + \beta_{21}R_{1,t-1} + \beta_{22}R_{2,t-1} + \beta_{23}R_{3,t-1} + \beta_{24}R_{4,t-1} \\ R_{1,t} &= \alpha_3 + \beta_{31}R_{1,t-1} + \beta_{32}R_{2,t-1} + \beta_{33}R_{3,t-1} + \beta_{34}R_{4,t-1} \\ R_{1,t} &= \alpha_1 + \beta_{41}R_{1,t-1} + \beta_{42}R_{2,t-1} + \beta_{43}R_{3,t-1} + \beta_{44}R_{4,t-1} \end{aligned} \tag{5.2}$$

To recognize the fluctuation in the volatility of asset returns the innovations are modelled as $\varepsilon_t = \sqrt{H_t(\theta)} \cdot z_t$ where the random vector z_t is assumed to have a zero mean and unity variance, i.e. $E(z_t) = 0$ and $Var(z_t) = I_4$. The matrix $H_t(\theta)$ is the conditional variance matrix of R_t and hence it has to be positive definite.

The advantage of the BEKK formulation is that it ensures the positive definiteness of $H_t(\theta)$, avoiding heavy restrictions on the parameters. The simplest BEKK structure[11] for the conditional variance matrix is given by:

$$H_t = C'C + A'\varepsilon_{t-1}\varepsilon'_{t-1}A + G'H_{t-1}G \tag{5.3}$$

where C, A and G are 4x4 matrices, with C being a lower triangular matrix.

It is well known that the direct generalizations of the univariate GARCH models increase rapidly the number of parameters; for example, in the four dimensional setting the BEKK(1,1) model has 42 parameters. To lower the econometric burden, despite losing generality, the number of parameters can be reduced to 18 by imposing that the matrices A and G are diagonal. The constrained matrix coefficients are parameterised as following:

[11]For this simple structure Engle and Kroner (1995) showed that all the elements of the matrix coefficient have to be positive for the model to be identified.

$$C = \begin{pmatrix} \omega_{11} & 0 & 0 & 0 \\ \omega_{12} & \omega_{22} & 0 & 0 \\ \omega_{13} & \omega_{23} & \omega_{33} & 0 \\ \omega_{14} & \omega_{24} & \omega_{34} & \omega_{44} \end{pmatrix}, A = \begin{pmatrix} \lambda_{11} & 0 & 0 & 0 \\ 0 & \lambda_{22} & 0 & 0 \\ 0 & 0 & \lambda_{33} & 0 \\ 0 & 0 & 0 & \lambda_{44} \end{pmatrix} \text{ and } G = \begin{pmatrix} \gamma_{11} & 0 & 0 & 0 \\ 0 & \gamma_{22} & 0 & 0 \\ 0 & 0 & \gamma_{33} & 0 \\ 0 & 0 & 0 & \gamma_{44} \end{pmatrix} \quad (5.4)$$

Therefore, the system of volatility equations for the diagonal BEKK model can be written in the scalar form as:

$$\begin{aligned} h_{11,t+1} &= \omega_{11}^2 + \lambda_{11}^2 \varepsilon_{1,t}^2 + \gamma_{11}^2 h_{11,t} \\ h_{22,t+1} &= \omega_{22}^2 + \lambda_{22}^2 \varepsilon_{2,t}^2 + \gamma_{22}^2 h_{22,t} \\ h_{33,t+1} &= \omega_{33}^2 + \lambda_{33}^2 \varepsilon_{3,t}^2 + \gamma_{33}^2 h_{33,t} \\ h_{44,t+1} &= \omega_{44}^2 + \lambda_{44}^2 \varepsilon_{4,t}^2 + \gamma_{44}^2 h_{44,t} \end{aligned} \quad (5.5)$$

Nevertheless, given the diagonal form of the matrix G, this specification of the conditional variances fails to measure the volatility transmission across the markets considered. In order to measure the volatility spillover effects it is necessary to use the full BEKK specification, instead of the diagonal BEKK model.

In the case of full BEKK representation the 4x4 matrices, $A = (\lambda_{ij})_{1 \le i,j \le 4}$ and $G = (\gamma_{ij})_{1 \le i,j \le 4}$ are unrestricted, hence the total number of variance-covariance parameters to be estimated increases to 42. So, when the conditional mean is simultaneously estimated, another independent vector of 20 parameters (4 intercepts α_i and 16 feedback parameters β_{ij}) will be introduced. The more complex estimation of the full BEKK(1,1) model was conducted in RATS using the BFGS (Broyden, 1970; Fletcher, 1970; Goldfarb, 1970; and Shanno, 1970) algorithm to obtain the parameter estimates and their asymptotic standard errors.

The conditional variance equations in the full BEKK specification are more complex, containing also information about both, shock and volatility spillovers. The conditional variance comprises three parts: the intercept that depends on the elements of the matrix C, the ARCH-shock component involving elements of matrix A and the GARCH-volatility component involving elements of matrix G. For each of the four series in the model the conditional variance equation can be written as follows:

$$h_{ii,t+1} = Intercept_i + ARCH_{i,t} + GARCH_{i,t} \qquad \text{for all } i = 1,...,4 \text{, where}$$

$$Intercept_i = \sum_{j \ge i} \omega_{ji}^2$$

$$ARCH_{i,t+1} = \sum_{j=1}^{4} \lambda_{ji}^2 \varepsilon_{j,t}^2 + 2 \sum_{\substack{j,k=1 \\ j<k}}^{4} \lambda_{ji} \lambda_{ki} \varepsilon_{j,t} \varepsilon_{k,t}$$

$$GARCH_{i,t+1} = \sum_{j=1}^{4} \gamma_{ji}^2 h_{jj,t}^2 + 2 \sum_{\substack{j,k=1 \\ j<k}}^{4} \gamma_{ji} \gamma_{ki} h_{jk,t}$$

Therefore, the conditional variances can be computed using the following general formula:

$$h_{ii,t+1} = \sum_{j \geq i} \omega_{ji}^2 + \sum_{j=1}^{4} \lambda_{ji}^2 \varepsilon_{j,t}^2 + \sum_{j=1}^{4} \gamma_{ji}^2 h_{jj,t}^2 + 2 \sum_{\substack{j,k=1 \\ j<k}}^{4} \lambda_{ji} \lambda_{ki} \varepsilon_{j,t} \varepsilon_{k,t} + 2 \sum_{\substack{j,k=1 \\ j<k}}^{4} \gamma_{ji} \gamma_{ki} h_{jk,t} \qquad (5.6)$$

From the variance equation (5.6), the extent of the shock and volatility spillovers from market j to market i can be quantified.

The elements of matrix A measure the impact of different sources of news on market i : there are news from a single market j, and their effect is measured by λ_{ji}^2, and there are combined news from markets j and k with their effect measured by $\lambda_{ji}\lambda_{ki}$. Similarly, the elements of matrix G can be interpreted as the single effect of the current conditional variances (γ_{ji}^2) and the combined effect of the conditional covariances $(\gamma_{ji}\gamma_{ki})$ on the future level of the conditional variance. Hence, a shock and a volatility spillovers matrix can be created just as for the return spillovers and they are calculated as the transpose of the matrix formed by the squared elements of matrix A and G, respectively.

In both cases, diagonal and full BEKK volatility models, the parameter estimates are the solution of the maximization of a non-linear sample likelihood function conditional to some initial values $\mu_t(\theta_0)$ and $H_t(\theta_0)$:

$$\begin{aligned} LF(\theta) &= -T \ln(2\pi) - \sum_{t=1}^{T} \log |H_t(\theta)| - \sum_{t=1}^{T} \varepsilon_t' H_t^{-1}(\theta) \varepsilon_t \\ &= -T \ln(2\pi) - \sum_{t=1}^{T} \log |H_t(\theta)| - \sum_{t=1}^{T} (R_t - \mu_t)' H_t^{-1}(\theta)(R_t - \mu_t) \end{aligned} \qquad (5.7)$$

5.4 Data

5.4.1 The Data Sets

Given that the major information shock (the financial crisis of 2007-2009) originated in the U.S. it is important to detect and measure any return and volatility transmission between the U.S. and a second major economy over a period including the recent financial crisis. Four country-pairs namely U.S.-U.K., U.S.-Japan, U.S.-Germany and U.S.-Canada will be examined and two combinations of asset classes are considered for each country-pair. Hence, in one model two countries will be represented by two asset classes, allowing for a four-dimensional network in which information flows at both, domestic and international level. First, the stock and bond markets are analysed simultaneously within each country pair, and second the stock and money markets are investigated.

The equity markets are represented by the most diversified daily stock price indices of the five countries considered. The sampled indices are the Standard and Poor's 500 (S&P500) Composite Index for the U.S., Financial Times-Stock Exchange 100 Share (FTSE100) for the U.K., Nikkei 500 for Japan, DAX30 for Germany and S&P/TSX for Canada, respectively. All daily closing prices for the five stock indices are extracted from Datastream and the daily close-to-close returns are the continuously compounded returns, computed as $R(t) = \ln(p_t / p_{t-1})$, where p_t is the market total return index (dividend included) at time t.

The short-term interest rates are represented by one-month yields of the Government securities, provided by the Treasury bills for the U.S., UK and Canada, while for Japan and Germany[12] the one-month interbank rates are used, the FIBOR and Gensaki one-month rates, respectively. The long-term interest rates are represented by the 10- year yields of Government benchmark bonds. Following Dontis-Charitos et al. (2013), the analysis is refined by distinguishing between the pre-crisis and post-crisis period, delimited by July 2007- the third quarter of 2007, which is recognized as the starting point[13] of the recent global financial crisis.

[12] The data on one-month Treasury bill of Germany and Japan were unavailable, so other one-month rates were used; the interbank rates for Germany and the Gensaki rates for Japan, respectively.

[13] In July 2007, the first substantial signs of financial distress were observed in the U.S. subprime market.

5.4.2 Statistical Analysis of the Data

For a preliminary examination of the raw data, the levels of the equity indices, the one-month rates and the 10-year yield of Government bonds, respectively are plotted (Figures 5.1, 5.2 and 5.3) over the full sample period of 2nd July 2001 to 31st July 2014. The time-series of all countries are plotted in multiple graphs in Figure 5.4. It is observed that the S&P500 and the Nikkei500 have moved closely together, whereas the remaining indices follow a more particular path. The daily stock returns and the first differences of the short and long rates are plotted for an indication of the daily volatility patterns in Figures 5.5, 5.6 and 5.7. The period after summer 2007 seems more volatile than the pre-crisis period, also persisting over quite a long interval.

The stock markets considered in this empirical investigation (see Figure 5.1) exhibit the highest degree of co-movement in comparison with the other asset classes analysed, namely the 10-year Government bonds and one-month Government securities, especially before the GFC. Despite the gravity of this crisis, the stock markets follow a global trend of steady recovery reaching historical record high levels at the end of the period. Only the Nikkei500 time series follow a more particular path towards its recovery still to overpass the pre-crisis levels. Three major negative shocks seem to have impacted the paths of all stock price indices. First, the cumulative effect of two events, the burst of the dot.com bubble and the terrorist attack in 2001 in the U.S., reached maximum effects in 2002. Second, the spread of the 2007 subprime mortgage crisis in the U.S. culminated with the announcement of the collapse of Lehman Brothers in September 2008. The third negative shock is represented by the European sovereign debt crisis in 2011 that transferred also into the stock markets, however with less impact relative to the 2008 financial crisis in the U.S.

The graphs of five time-series for the 10-year Government bond yields show the particularities of each national bond market, with the Canadian and Japanese long-term bond markets evolving along more individual paths. Two common features can be observed across all time-series analysed. The stock market downturn in 2002 have impacted almost instantly all the other major bond markets around the world and the clear event of the GFC in September 2008 when again all the bond markets responded together with a sharp decline in long term interest rates. Another common reaction of the world bond markets, with the exception of a delayed response in the case of Japan is observed in 2011 as a result of the European sovereign bond crisis. After 2012 only the U.S., the U.K. and the Canadian bond markets show signs of recovering with the 10-year bond yields remaining stable above 2%.

The one-month interest rates time series exhibit the most diversity in their evolution. Their differences and the similarities in their paths over the sample period August 2001-July 2014 are clearly illustrated in Figure 3.3. In the pre-crisis period, the short-time interest rates for the U.S. and Germany evolved along a similar path, however with the German one-month FIBOR rates slightly lagged behind with a prolonged downturn in 2002 and a delayed start of the recovery in 2006 following the real-estate bubble in the U.S. The rest of the one-month interest rates have a slightly more particular evolution, with the U.K. and Canadian rates showing recovering as early as 2003 and 2004, respectively, while the Japanese rates reacted in jumps with un unprecedented increase in 2006. The ignition of the crisis in the summer of 2007 in the sub-prime mortgage market in the U.S. can be seen with immediate effect in the U.S. one-month Treasury Bills, followed by the Canadian one-month securities. The other short-term interest rates series respond initially with a period of high volatility until September 2008 when a major declined is observed. Beyond 2008, the money markets around the world evolved behave in a similar manner with the short-term rates just above the zero level, with only Canadian short-term rates close to 1%.

The time series of stock returns and the first-difference of the interest rates displayed are stationary around a zero mean and exhibit volatility clustering around the three turmoil periods that occurred in the sample period of 2001-2014. The most volatile markets are the stock markets followed by long-term bond markets, while the money markets appear very stable during clam periods with the most pronounced period of uncertainty being present in the U.S. one-month T-bills.

Summary Statistics

The standard statistical properties of the data considered for this investigation are presented for the pre-crisis and post-crisis periods in the Tables 5.1 and 5.2, respectively. The sample means of the stock returns and the first differenced interest rate series are not significantly different from zero, predominantly positive before the crisis and mostly negative during the crisis. Regarding the stock return series, the skewness and kurtosis measures reject the normality with the date being negatively skewed and leptokurtic over both subsamples. The skewness for both the short- and long-term rates changes between positive and negative values, and while both types of rates are leptokurtic, the short-term rates exhibit considerably much fatter tails than the normal distribution. The normality of all the time series over the two sub-periods is also rejected by the Jacque-Berra test as indicated by the zero p-values in the tables below.

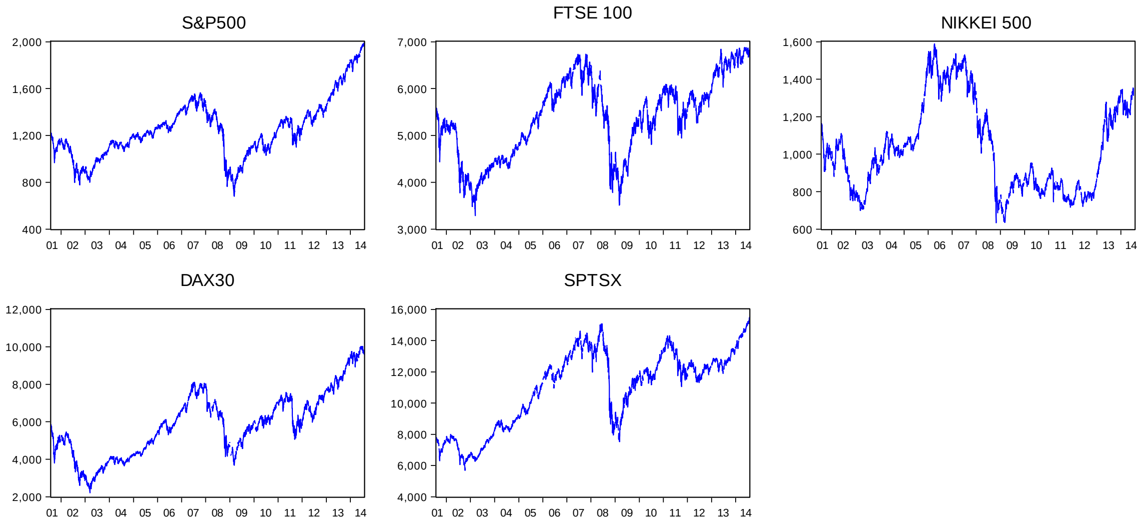

Figure 5.1. Daily Stock Price Indices: Levels: 2001- 2014

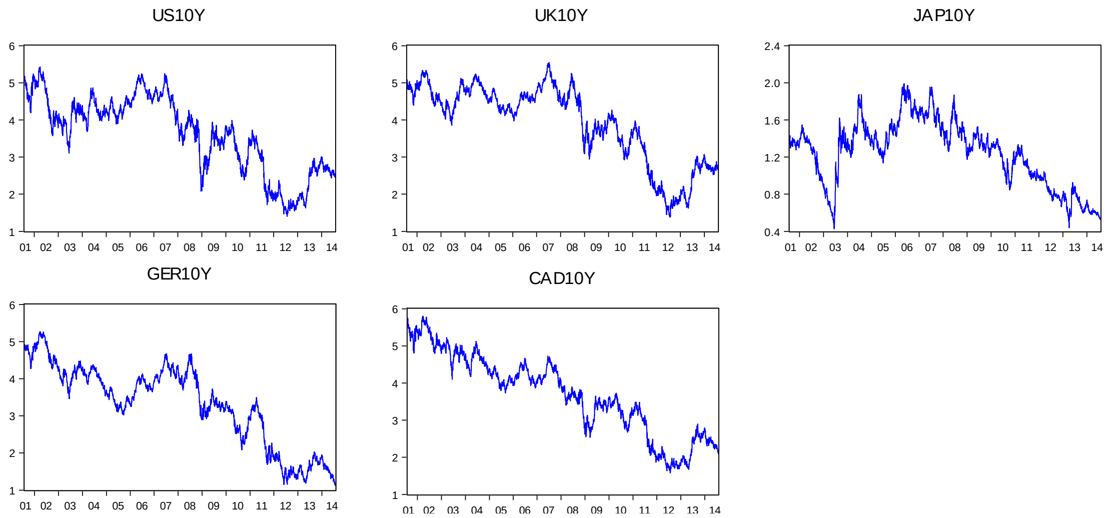

Figure 5.2 Daily Long -Term (10 years) Nominal Interest Rates: Levels: 2001-2014

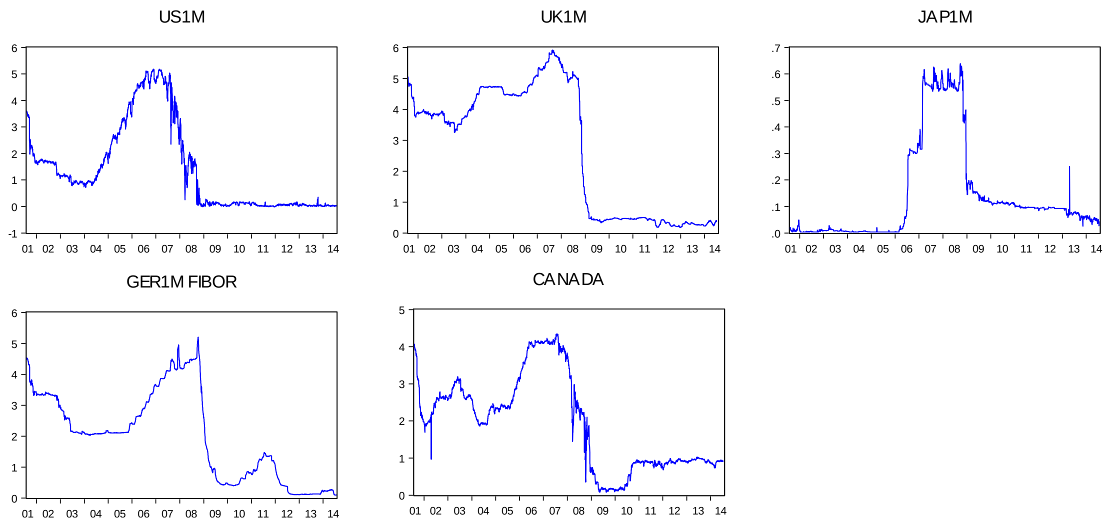

Figure 5.3 Daily Short-Term (one month) Treasury Bills Rates: Levels: 2001 -2014

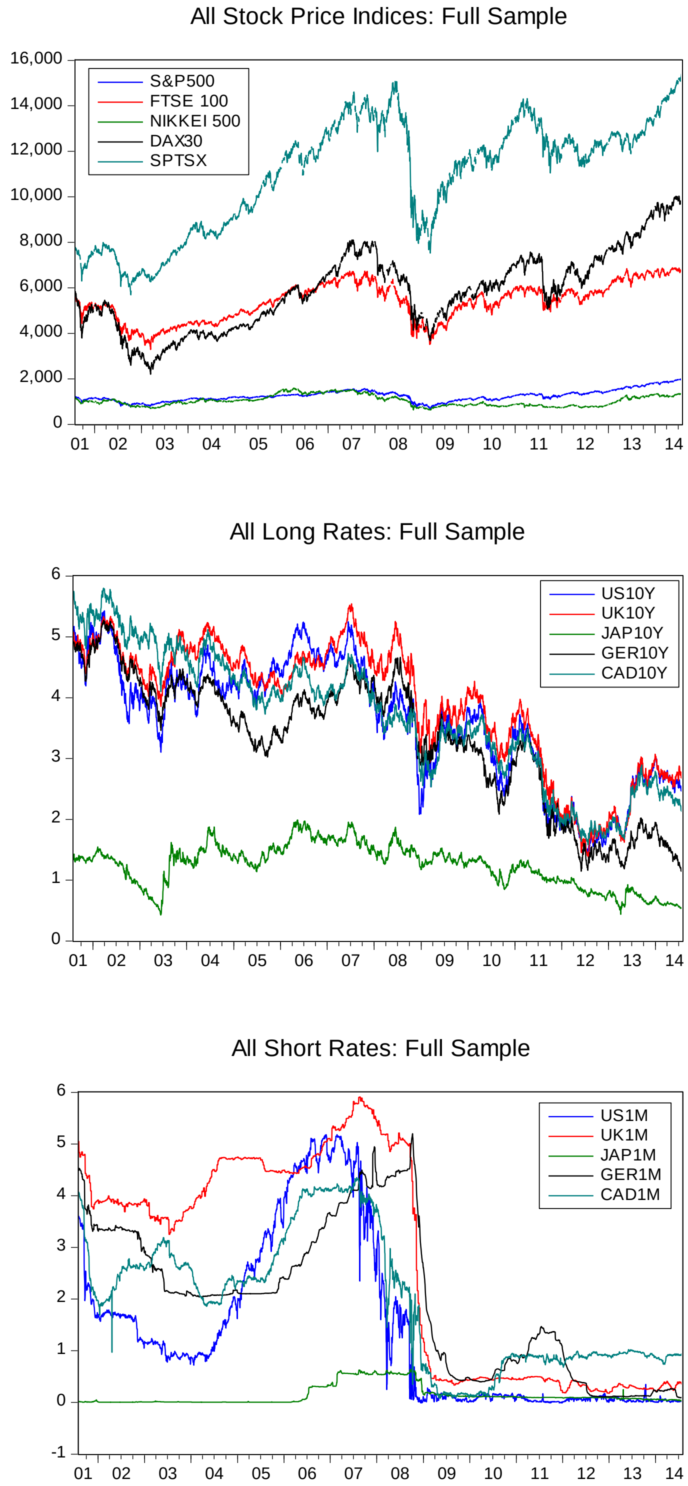

Figure 5.4 The time-series of all countries during 2001-2014:

Stock Price Indices, Long-Term Interest rates and Short-Term Interest Rates

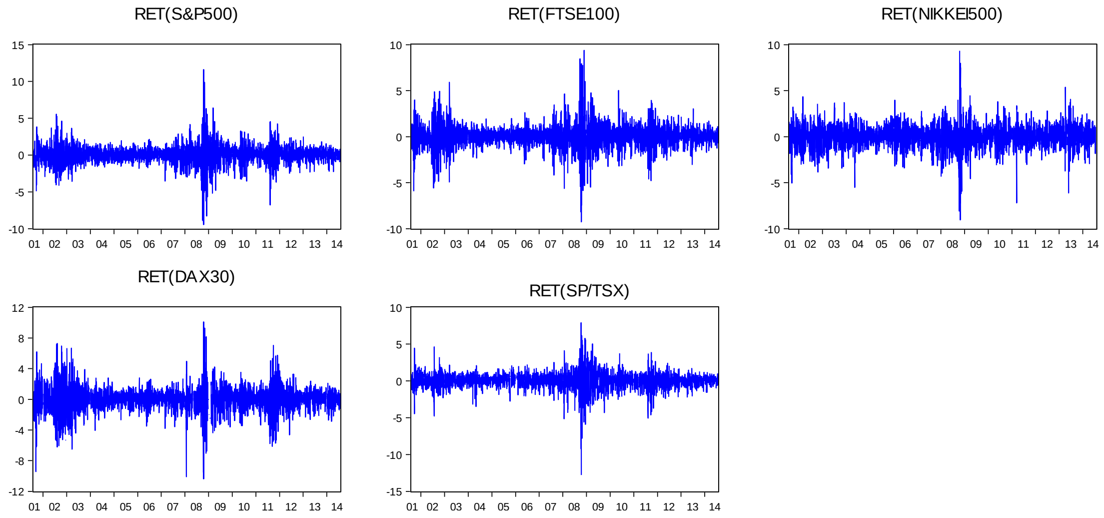

Figure 5.5 Stock Price Indices: Daily Returns: 2001-2014

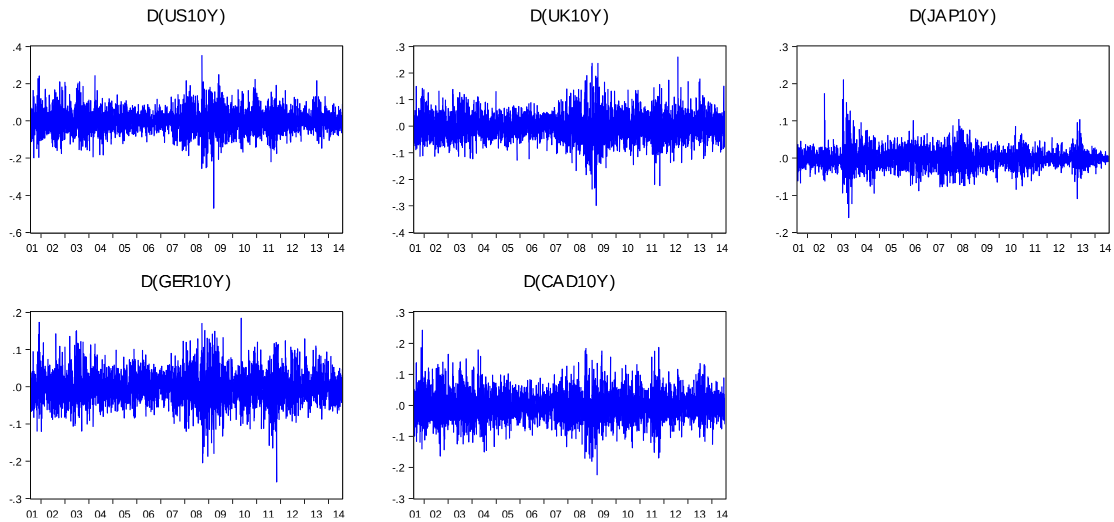

Figure 5.6 Long -Term Rates: Daily Changes: 2001-2014

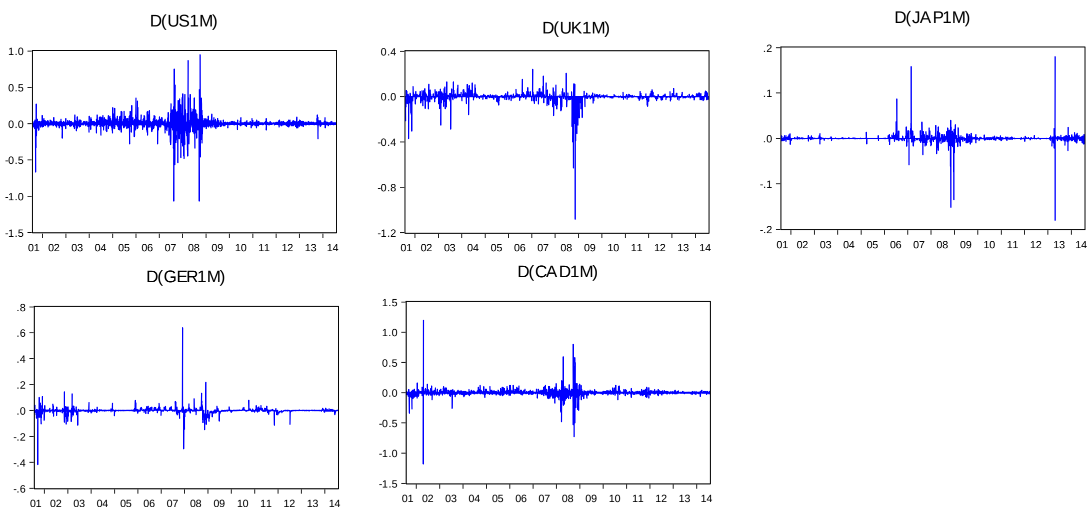

Figure 5.7 Short -Term Rates: Daily Changes: 2001-2014

Table 5.1 Descriptive Statistics for Stock Price Indices and their Returns: Pre-crisis period August 2001- June 2007

Stock Price Index	LEVELS					LOG RETURNS				
pre-crisis	S&P500	FTSE100	NIKKEI500	DAX30	SP/TSX	S&P500	FTSE100	NIKKEI500	DAX30	SP/TSX
Observations	1485	1494	1451	1500	1441	1484	1493	1450	1499	1440
Mean	1,149.3814	4,998.7599	1,108.5260	4,648.3152	9,135.4577	0.0001	0.0001	0.0002	0.0002	0.0004
Median	1,144.9400	4,982.2500	1,046.1800	4,387.5700	8,527.1300	0.0006	0.0005	0.0003	0.0008	0.0010
Maximum	1,539.1200	6,732.4000	1,588.5300	8,066.1800	14,161.0000	0.0557	0.0590	0.0434	0.0727	0.0464
Minimum	776.7600	3,287.0000	698.4900	2,203.9700	5,689.4300	-0.0492	-0.0589	-0.0551	-0.0943	-0.0478
Std. Dev.	165.4116	787.2527	237.6475	1,222.0813	2,214.0206	0.0100	0.0110	0.0114	0.0158	0.0082
Skewness	0.0681	0.2295	0.4025	0.4989	0.5848	0.0681	-0.2359	-0.2513	-0.1531	-0.3866
Kurtosis	2.6435	2.0800	2.0299	2.7541	2.1325	6.3695	7.5290	3.9317	6.8553	6.9423
Jarque-Bera	9.0128	65.8098	96.0636	66.0017	127.3159	703.1672	1289.8229	67.7151	934.1716	968.3944
Probability	0.011	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

This table reports the summary statistics for the five Equity Indices over the pre-crisis period. Both, daily levels and returns, respectively are examined. The central tendency of the samples is assessed using the mean, median, maximum and minimum values, while the variability is measured by the standard deviation. The normality of the time series is statistically assessed based on the skewness and kurtosis measures and tested using the JB test.
** denotes that the respective time series are insignificantly different from a normal distribution at 5% level.

Table 5.2 Descriptive Statistics for Stock Price Indices and their Returns: Post-crisis period: July 2007-July 2014

Stock Price Index	LEVELS					LOG RETURNS				
post-crisis	S&P500	FTSE100	NIKKEI500	DAX30	SP/TSX	S&P500	FTSE100	NIKKEI500	DAX30	SP/TSX
Observations	1784	1791	1736	1767	1729	1783	1790	1735	1766	1728
Mean	1,332.6019	5,707.2160	969.1330	6,897.5815	12,418.5882	0.0001	0.0000	-0.0001	0.0001	0.0001
Median	1,325.1350	5,783.6900	864.3400	6,842.3900	12,487.2500	0.0009	0.0003	0.0003	0.0009	0.0007
Maximum	1,988.0700	6,878.4900	1,530.4200	10,028.7100	15,524.8200	0.1161	0.0938	0.0933	0.1010	0.0790
Minimum	679.2800	3,512.0900	633.4800	3,677.0700	7,527.4400	-0.0946	-0.0926	-0.0904	-0.1038	-0.1279
Std. Dev.	281.6609	746.6390	219.3420	1,374.5365	1,511.8801	0.0141	0.0137	0.0139	0.0154	0.0131
Skewness	0.2422	-0.6520	0.6697	0.2535	-0.7844	-0.2844	-0.1054	-0.3248	-0.4691	-0.8819
Kurtosis	2.6711	2.9502	2.0910	2.7060	3.7213	12.4662	9.9845	9.0679	10.2257	13.5389
Jarque-Bera	25.4800	127.0652	189.5251	25.2841	214.7921	6,681.2000	3,641.7700	2,692.2414	3,906.5791	8,220.9394
Probability	0.000	0.000	0.000	0.000	0.000	0.0000	0.0000	0.0000	0.0000	0.0000

This table reports the summary statistics for the five equity indices during the crisis period. Both, daily levels and returns, respectively are examined. The central tendency of the samples is assessed using the mean, median, maximum and minimum values, while the variability is measured by the standard deviation. The normality of the time series is statistically assessed based on the skewness and kurtosis measures and tested using the JB test.

Table 5.3 Descriptive Statistics: Long-term Interest Rates for U.S., U.K., Japan, Germany and Canada; Pre-crisis period: August 2001- June 2007.

10Y Gov. Bonds Yields	LEVELS					FIRST DIFFERENCES				
pre-crisis	US10Y	UK10Y	JAP10Y	GER10Y	CAD10Y	US10Y	UK10Y	JAP10Y	GER10Y	CAD10Y
Observations	1543	1543	1543	1543	1543	1542	1542	1542	1542	1542
Mean	4.4446	4.6701	4.0795	1.3937	4.6321	0.0000	0.0003	0.0003	-0.0002	-0.0007
Median	4.4376	4.6639	4.0629	1.3998	4.5671	-0.0002	-0.0002	0.0000	-0.0004	-0.0002
Maximum	5.4298	5.5189	5.2732	1.9907	5.8014	0.2424	0.1497	0.2106	0.1737	0.2430
Minimum	3.1036	3.8561	3.0230	0.4261	3.7302	-0.1987	-0.1278	-0.1598	-0.1202	-0.1635
Std. Dev.	0.4357	0.3249	0.5143	0.3104	0.5050	0.0558	0.0395	0.0284	0.0375	0.0457
Skewness	-0.1093	0.0441	0.2425	-0.7235	0.3737	0.3242	0.2256	0.7153	0.4021	0.3031
Kurtosis	2.6379	2.3894	2.5683	3.5790	2.1066	4.5502	3.8124	8.8184	4.2079	4.3785
Jarque-Bera	11.5121	24.4848	27.1181	156.2726	87.2905	181.5206	55.5207	2,308.0709	135.3722	145.8040
Probability	0.0032	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

This table reports the summary statistics for the five 10- year Government bond yields, before the crisis. Both, daily levels and first differences, respectively are examined. The central tendency of the samples is assessed using the mean, median, maximum and minimum values, while the variability is measured by the standard deviation. The normality of the time series is statistically assessed based on the skewness and kurtosis measures and tested using the JB test.

Table 5.4 Descriptive Statistics: Long-term Interest Rates for U.S., U.K., Japan, Germany and Canada; During the crisis period: July 2007-July 2014.

10Y Gov Bonds Yields	LEVELS					FIRST DIFFERENCES				
post-crisis	US1M	UK1M	JAP1M	GER1M	CAD1M	US1M	UK1M	JAP1M	GER1M	CAD1M
Observations	1849	1849	1849	1849	1849	1848	1848	1848	1848	1848
Mean	2.9318	3.2564	1.1076	2.6619	2.8940	-0.0013	-0.0014	-0.0007	-0.0018	-0.0013
Median	2.8920	3.2970	1.1300	2.7117	2.9160	-0.0010	-0.0014	0.0000	-0.0013	-0.0004
Maximum	5.1941	5.5437	1.9556	4.6709	4.7096	0.3519	0.2600	0.1039	0.1848	0.1870
Minimum	1.4040	1.3820	0.4393	1.1187	1.5780	-0.4702	-0.2979	-0.1089	-0.2560	-0.2236
Std. Dev.	0.8314	1.0120	0.3382	1.0160	0.7516	0.0645	0.0558	0.0220	0.0459	0.0480
Skewness	0.1600	0.1242	0.0609	0.2144	0.1290	-0.0880	0.0371	0.1957	-0.0687	0.0519
Kurtosis	2.2619	2.0731	2.0284	1.7594	2.1033	5.8551	5.1215	5.5934	4.4873	4.2807
Jarque-Bera	49.8668	70.9358	73.8679	132.7424	67.0719	630.074 4	346.9839	529.6790	171.7726	127.1265
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

This table reports the summary statistics for the five 10-year Government bond yields, during the crisis. Both, daily levels and first differences, respectively are examined. The central tendency of the sample data is assessed using the mean, median, maximum and minimum values, while the variability is measured by the standard deviation. The normality of the time series is statistically assessed based on the skewness and kurtosis measures and tested using the JB test.

Table 5.5 Descriptive Statistics: Short-term Interest Rates for U.S., U.K., Japan, Germany and Canada; Pre-crisis period of August 2001- June 2007

1M Short Rates	LEVELS					FIRST DIFFERENCES				
pre-crisis	US1M	UK1M	JAP1M*	GER1M*	CAD1M	US1M	UK1M	JAP1M*	GER1M*	CAD1M
Observations	1543	1543	1543	1543	1543	1542	1542	1542	1542	1542
Mean	2.5584	4.3243	0.0743	2.7591	2.8719	0.0004	0.0005	0.0003	-0.0003	0.0001
Median	1.8750	4.4480	0.0065	2.6010	2.6300	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	5.2700	5.8236	0.6160	4.5290	4.2400	0.3400	0.2409	0.1580	0.1430	1.2000
Minimum	0.7400	3.2500	0.0030	2.0160	0.9700	-0.6300	-0.3700	-0.0585	-0.4180	-1.1800
Std. Dev.	1.5132	0.5504	0.1556	0.6777	0.7500	0.0493	0.0262	0.0056	0.0174	0.0505
Skewness	0.5438	0.1804	2.2229	0.5304	0.5855	-1.2629	-3.8553	16.8588	-8.8450	0.1492
Kurtosis	1.7516	2.2117	6.6773	2.0389	2.0192	27.6687	69.2018	463.2814	226.6594	403.1936
Jarque-Bera	1.75E+02	4.83E+01	2.14E+03	1.32E+02	1.50E+02	3.95E+04	2.85E+05	1.37E+07	3.23E+06	1.03E+07
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

This table reports the summary statistics for the five one-month short term rates, during the pre-crisis period. Both, daily levels and first differences, respectively are examined. The central tendency of the sample data is assessed using the mean, median, maximum and minimum values, while the variability is measured by the standard deviation. The normality of the time series is statistically assessed based on the skewness and kurtosis measures and tested using the JB test.

*For Japan and Germany, the one-month interbank interest rates have been used, as the data on the Government securities of one-month maturity were not available

Table 5.6 Descriptive Statistics: Short-term Interest Rates for U.S., U.K., Japan, Germany and Canada; Post-crisis period: July 2007-July 2014

1M Short Rates	LEVELS					FIRST DIFFERENCES				
post-crisis	US1M	UK1M	JAP1M*	GER1M*	CAD1M	US1M	UK1M	JAP1M*	GER1M*	CAD1M
Observations	1849	1849	1849	1849	1849	1848	1848	1848	1848	1848
Mean	0.5117	1.3420	0.1935	1.3642	1.1504	-0.0024	-0.0030	-0.0003	-0.0022	-0.0018
Median	0.0700	0.4378	0.1030	0.6200	0.9000	0.0000	0.0000	0.0000	0.0000	0.0000
Maximum	5.1300	5.9101	0.6380	5.1970	4.3500	0.9500	0.2047	0.1800	0.6400	0.8000
Minimum	-0.0000	0.1795	0.0260	0.0910	0.0800	-1.0500	-1.0795	-0.1800	-0.2960	-0.7300
Std. Dev.	1.0882	1.9192	0.1876	1.5855	1.0245	0.0887	0.0382	0.0091	0.0232	0.0571
Skewness	2.5461	1.5701	1.3967	1.2305	1.7401	-0.5618	-16.6944	-4.5792	10.2779	0.5608
Kurtosis	8.5934	3.5892	3.1291	2.8459	5.2285	43.6774	400.1652	235.0387	343.3530	70.2959
Jarque-Bera	4.40E+03	7.86E+02	6.02E+02	4.68E+02	1.32E+03	1.12E+05	1.22E+07	4.15E+06	8.95E+06	3.49E+05
Probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

This table reports the summary statistics for the five one-month short term rates, post-crisis. Both, daily levels and first differences are examined. The central tendency of the sample data is assessed using the mean, median, maximum and minimum values, while the variability is measured by the standard deviation. The normality of the time series is statistically assessed based on the skewness and kurtosis measures and tested using the JB test.

*For Japan and Germany, the one-month interbank interest rates have been used, as the data on the Government securities of one-month maturity were not available.

After a short examination of the summary statistics reported in the tables above several patterns seem to emerge. The dynamics of the sample means of the three time-series change between the two periods considered. While, on average, the interest rates have decreased significantly during the crisis, the stock indices seem to have recovered rather quickly, with an upward trend during the crisis. The DAX30 index recorded the highest percentage increase (48%) in the mean from 4,648.62 to 6,897.58. In contrast, the Japanese stock index, Nikkey500 average level decreased by 12% from 1,108.53 to 969.1. As a result, the correlation between the stock markets on one side and the short term and long-term bond markets on the other side are expected to change sign once the crisis has started. In term of the unconditional standard deviations, the S&P500 and DAX30 become more volatile during the crisis, while the rest of stock indices show lower uncertainty. Concerning the long-term interest rates all bond markets, except for Japan, present higher volatility during the crisis. For the short rates, surprisingly, the one-month U.S. T-bills yields become more stable, in contradiction with all the other one-month rates for which the volatility increases dramatically during the crisis (for example the sample standard deviation of the one-month UK treasury bills increased by 248%).

The normality of the distributions of all level time- series involved is rejected by all three standard measures, skewness and kurtosis and the Jarque-Berra test. In general, the raw data series are positively skewed, while the return series show negative skewness. The stocks and long rates present similar kurtosis patterns, with negative kurtosis in level series and moderate positive kurtosis for the returns and first difference series, respectively. However, the short-term rates series report a substantial excess kurtosis when first differenced, while generally platikurtic in levels. The Jarque-Bera test rejects the normality of all the time- series, apart from the S&P500 whose normality cannot be rejected at 1%.

Unit Root Tests Results

Typically, most financial time series are known to be nonstationary. However, it is often possible to reduce them to stationary series by first differencing. The property of stationarity is formally investigated using two statistical unit-root tests, namely, the Augmented Dickey-Fuller (1979) (ADF) and Phillips and Peron (1988) (PP) tests, and the stationarity Kwiatkowski-Phillips-Schmidt-Shin (1992) (KPSS) test. The statistical results indicate nonstationarity for most of the raw data, while the returns and the first-difference series become stationary when first differenced.

Table 5.7 The unit root tests: Stock Indices; Pre-crisis period of August 2001- June 2007

Unit Root Tests		**ADF**		**PP**		**KPSS**	
pre-crisis		t-Stat.	Prob.*	Adj. t-Stat	Prob.*	LM-Stat.	Crit.Val.**
S&P500	Log Level	-0.6481	0.8572	-2.9526	0.1463	0.4941	0.216
	Returns	-39.2780	0.0000	-39.3957	0.0000	0.0715	0.216
FTSE100	Log Level	-2.5719	0.2935	-2.4730	0.3417	0.8506	0.216
	Returns	-41.5803	0.0000	-42.2313	0.0000	0.0729	0.216
NIKKEI500	Log Level	-2.8354	0.1846	-2.7173	0.2296	0.6230	0.216
	Returns	-34.0031	0.0000	-33.9191	0.0000	0.0818	0.216
DAX30	Log Level	-2.4170	0.3704	-2.3962	0.3813	0.7928	0.216
	Returns	-39.5067	0.0000	-39.5437	0.0000	0.0698	0.216
SP/TSX	Log Level	-2.6316	0.2661	-2.6433	0.2609	0.6848	0.216
	Returns	-36.5066	0.0000	-36.4834	0.0000	0.0669	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; and also the sample test statistic and the critical values of the KPS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 5.8 The unit root tests: Stock Indices; Post-crisis period: July 2007-July 2014

Unit Root Tests		**ADF**		**PP**		**KPSS**	
post-crisis		t-Stat.	Prob.*	Adj. t-Stat.	Prob.*	LM-Stat.	Crit.Val.**
S&P500	Log Level	-2.1120	0.5382	-2.0592	0.5677	0.7743	0.216
	Returns	-33.592	0.0000	-46.2508	0.0000	0.0651	0.216
FTSE100	Log Level	-2.8742	0.1712	-2.6766	0.2465	0.5232	0.216
	Returns	-21.479	0.0000	-44.6861	0.0000	0.0561	0.216
NIKKEI500	Log Level	-1.9467	0.6293	-1.8260	0.6919	0.9566	0.216
	Returns	-37.528	0.0000	-37.3270	0.0000	0.0417	0.216
DAX30	Log Level	-2.4834	0.3364	-2.3987	0.3801	0.6394	0.216
	Returns	-41.663	0.0000	-41.7860	0.0000	0.0518	0.216
SP/TSX	Log Level	-2.1196	0.5339	-2.0248	0.5868	0.3389	0.216
	Returns	-24.8243	0.0000	-45.6365	0.0000	0.0620	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; and also the sample test statistic and the critical values of the KPS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 5.9 The unit root tests: Long-term Interest Rates;

Pre-crisis period: August 2001- June 2007

Unit Root Tests		**ADF**		**PP**		**KPSS**	
pre-crisis		t-Stat.	Prob.*	Adj. t-Stat	Prob.*	LM-Stat.	Crit.Val.**
US10Y	Level	-2.6949	0.2388	-2.7400	0.2204	0.6022	0.216
	First Diff.	-38.1707	0.0000	-38.1548	0.0000	0.0254	0.216
UK10Y	Level	-1.7361	0.7348	-1.7951	0.7069	0.2709	0.216
	First Diff.	-38.2425	0.0000	-38.2312	0.0000	0.0603	0.216
JAP10Y	Level	-2.3398	0.4115	-2.3991	0.3798	0.3222	0.216
	First Diff.	-38.2451	0.0000	-38.2616	0.0000	0.0411	0.216
GER10Y	Level	-0.9201	0.9521	-0.9369	0.9502	0.6092	0.216
	First Diff.	-38.2451	0.0000	-38.8395	0.0000	0.0702	0.216
CAD10Y	Level	-3.0641	0.1154	-3.1400	0.0974	0.4066	0.216
	First Diff.	-38.2508	0.0000	-38.2383	0.0000	0.0235	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; and also the sample test statistic and the critical values of the KPS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 5.10 The unit root tests: Long-term Interest Rates;

Post-crisis period: July 2007-July 2014

Unit Root Tests		**ADF**		**PP**		**KPSS**	
post-crisis		t-Stat.	Prob.*	Adj. t-Stat.	Prob.*	LM-Stat.	Crit.Val.**
US10Y	Level	-2.6578	0.2546	-2.4830	0.3366	0.4066	0.216
	First Diff.	-43.8958	0.0000	-44.0799	0.0000	0.0422	0.216
UK10Y	Level	-2.0116	0.5940	-1.9239	0.6415	0.4929	0.216
	First Diff.	-41.6571	0.0000	-41.7000	0.0000	0.0479	0.216
JAP10Y	Level	-4.7349	0.0006	-4.7332	0.0006	0.0757	0.216
	First Diff.	-44.9643	0.0000	-44.9834	0.0000	0.0215	0.216
GER10Y	Level	-3.0058	0.1308	-2.7748	0.2069	0.2651	0.216
	First Diff.	-38.4544	0.0000	-38.2240	0.0000	0.0315	0.216
CAD10Y	Level	-2.5667	0.2959	-2.3769	0.3916	0.3806	0.216
	First Diff.	-44.9977	0.0000	-45.2422	0.0000	0.0469	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; and also the sample test statistic and the critical values of the KPS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 5.11 The unit root tests: Short-term Interest Rates;

Pre-crisis period: August 2001- June 2007

Unit Root Tests		**ADF**		**PP**		**KPSS**	
pre-crisis		t-Stat.	Prob.*	Adj. t-Stat.	Prob.*	LM-Stat.	Crit.Val.**
US1M	Level	-3.6348	0.0272	-3.5867	0.0312	1.0912	0.216
	First Diff.	-15.0577	0.0000	-35.9307	0.0000	0.3241	0.216
UK1M	Level	-3.3390	0.0604	-3.3401	0.0602	0.4594	0.216
	First Diff.	-38.9851	0.0000	-38.9993	0.0000	0.2135	0.216
JAP1M	Level	1.4920	0.9993	1.2344	0.9984	0.8186	0.216
	First Diff.	-21.0035	0.0000	-39.4955	0.0000	0.0855	0.216
GER1M	Level	-1.4251	0.5713	-1.6256	0.4692	1.1545	0.216
	First Diff.	-7.7477	0.0000	-35.8555	0.0000	0.0470	0.216
CAD1M	Level	- 0.5927	0.8697	-0.7268	0.8380	0.7861	0.216
	First Diff.	-35.5228	0.0000	-57.3968	0.0001	0.1990	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; and also the sample test statistic and the critical values of the KPS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

Table 5.12 The unit root tests: Short-term Interest Rates;

Post-crisis period: July 2007-July 2014

Unit Root Tests		**ADF**		**PP**		**KPSS**	
Short Rates post-crisis		t-Stat.	Prob.*	Adj. t-Stat.	Prob.*	LM-Stat.	Crit.Val.**
US1M	Level	-3.8243	0.0155	-3.2955	0.0673	0.8499	0.216
	First Diff.	-15.9247	0.0000	-40.1889	0.0000	0.1093	0.216
UK1M	Level	-1.7682	0.7199	-1.1854	0.9123	1.0074	0.216
	First Diff.	-4.1491	0.0054	-44.4946	0.0000	0.1817	0.216
JAP1M	Level	-1.4893	0.8333	-1.5179	0.8235	0.9855	0.216
	First Diff.	-34.6956	0.0000	-49.8181	0.0000	0.0478	0.216
GER1M	Level	-1.0407	0.9366	-1.0480	0.9356	0.7982	0.216
	First Diff.	-10.1353	0.0000	-38.8175	0.0000	0.1269	0.216
CAD1M	Level	-2.7044	0.2348	-2.3776	0.3912	0.9468	0.216
	First Diff.	-12.6441	0.0000	-44.6301	0.0000	0.1017	0.216

This table presents the sample test-statistics and the probabilities for ADF and PP unit root tests; and also the sample test statistic and the critical values of the KPS test, computed using EViews.
*MacKinnon (1996) one-sided p-values
**Kwiatkowski-Phillips-Schmidt-Shin (1992, Table1)

5.5 Empirical Results: The Full BEKK Model

The estimation results for the discrete time MGARCH – full BEKK models are are organised pairwise, for example for the U.S.-U.K. pair we look first at the individual channels (return and volatility) for each combination of the equity markets with short-term interest rates markets one-month T-bills and then the equity markets with Government bond markets. The results for each segment of the yield curve are presented separately, with Tables 5.13 - 5.16 containing the empirical findings for the equity and the money market segment, and Tables 5.17 -5.20 reporting the estimation results for the equity and bond markets, respectively. Each table is organised in two panels: panel A contains the estimates for the twenty parameters of the mean equation (four intercepts, four autoregressive coefficients and twelve return spillover parameters), while panel B reports the estimates for the thirty-six parameters of the variance equation, including the shock and volatility spillovers matrices.

5.5.1 The Estimation Results for the Full BEKK model: Stock and Money Markets

Return Spillovers – The Mean Equation

U.S. – U.K.

In the case of U.S.-U.K. pair for stock and money markets, the results regarding the mean equation (see Table 3.13, panel A)show very little evidence of return spillovers prior to the crisis with only two significant estimates at 1% level of significance, $\beta_{21} = 0.3849$ measuring the feedback from the S&P500 to FTSE100 (direct international) and $\beta_{34} = 0.1171$ from the U.K. money market to U.S. money market. There is also a reciprocal negative weak feedback in the returns from the U.S. to the U.K. During the crisis, both U.S. markets have a leading role, with S&P500 and U.S. money market affecting the U.K. equity market ($\beta_{21} = 0.466$ and $\beta_{23} = 1.4234$). The unidirectional return spillovers from S&P500 to FTSE100 intensify as a result of the crisis from 0.38 to 0.44. Also, the U.S. money market transmits information to both U.S and U.K equity markets. In conclusion, for the U.S. – U.K. pair, there is evidence of international indirect spillovers which are not present before crisis.

U.S. – Japan

Turning to U.S.-Japan stock and money market results (Table3.14, Panel A), there is evidence of increasing unidirectional direct international return spillover effects from S&P500 to NIKKEI500 from $\beta_{21} = 0.0683$ before the crisis to $\beta_{21} = 0.1097$ post-crisis. The strongest linkages exist before the crisis from the U.S money market to both U.S. and Japanese equity markets (domestic and indirect international, respectively). However, the information transmission via these two routes slightly weakens during the crisis. Interestingly, domestic return spillovers in Japan are highly negative during the crisis suggesting that the two asset classes evolve in opposite directions in terms of returns. In addition, there is some evidence of decoupling effects as the U.S. money market transmits information to its Japanese counterparty with increased negative effect $\beta_{43} = -0.0063$ during the crisis. In general, during the crisis the transmission of shocks via return channel is stronger between the equity markets and weaker between the money markets of the U.S and Japan.

U.S. – Germany

The return effects for the U.S. and German stock and money markets are reported in Table 3.15, Panel A. There is some unidirectional (from U.S. to Germany) interaction between the two money markets has increased from 0.006 to 0.010. Regarding the equity markets, the impact of the S&P500 on DAX30 is decreasing from 0.202 to 0.154 whereas the influence from DAX30 on S&P500 more than doubled from 0.08 to 0.174. Similar to the U.S.-U.K. analysis, the position of the U.S. money market in relation with the equity markets changes dramatically, from no impact before the crisis to significant linkages to S&P500 ($\beta_{13} = 0.995$) and to DAX30 ($\beta_{23} = -0.843$).

U.S. – Canada

The parameter estimates of the mean equation for U.S.-Canada are presented in Table 3.16 (Panel A) provides some evidence for the presence of return spillover effects over both periods. There is only a unidirectional (from SPTSX to S&P500) flow of return information between the two equity markets that intensifies during the crisis from 0.187 to 0.290. On the money markets side, there is no sign of interaction during the crisis, however some markets connect indirectly with substantial return spillovers from the Canadian money market on S&P500 ($\beta_{14} = 2.812$). Hence, the Canadian markets are exporting information shock via return channel to the U.S and not the other way as expected given the crisis originated in the U.S.

The transmission of information via return channel inside this complex route-network, between the stock and money markets can be summarised as follows. Regarding the domestic route, the busiest information flows exist mostly in one direction from the U.S. money market to the equity market. The other economies do not provide evidence of interaction between their equity and money markets via the return channel (apart from Japan where the money market has a negative feedback effect on the equity market). For the direct international route, the U.S. equity markets have the leading role of exporting information, while the money markets seem to exchange very little information or with negative effects (U.S.- Japan). The leading role of the U.S. exporting information is also observed via the indirect international transmission route with the busiest direction from the U.S. money markets to the equity markets of the other economies.

Volatility Spillovers – The Volatility Equation

U.S. – U.K.

Analysing the estimates for the volatility equation parameters (see Table 3.13, Panel B), one could observe that while before the crisis the volatility information flows in most directions, during the crisis many routes disappear as several estimates are statistically insignificant. However, the crisis has increased the volatility spillover effects in both directions between the S&P and FTSE100. While the communication consolidates between the equity markets it weakens between U.S. and U.K. money markets, where the relationship becomes very weak and unidirectional ($\gamma_{34}^{2} = 0.0002$). Domestically, only inside the U.K the money-market exports volatility to FTSE100. There is evidence of highly significant indirect volatility spillovers from the UK money market to the U.S. equity market ($\gamma_{41}^{2} = 56.00$). Therefore, the direct volatility channel is active only between the equity markets.

U.S. – Japan

There are linkages in terms of volatility shocks between S&P5000 and NIKKEI500 before the crisis which disappear in the crisis period. (see Table 3.14, Panel B). The direct relationships between money markets of the two countries on one side and the equity markets on the other side become almost inexistent. Also, the domestic routes inside each country are less busy during the crisis. The only indirect volatility transmission that persists over both periods is that between the U.S. money market and NIKKEI500 $(\gamma_{32}^{2} = 0.2966)$.

U.S. – Germany

In terms of volatility impact, the transmission channels during the crisis change differently when compared to the return linkages. The interaction between the U.S. money market and the equity markets dissipates (like in the U.S.-Japan case). The crisis seems not to affect the way in which the equity markets exchange volatility, while the domestic routes are very busy only for Germany ($\gamma_{42}^2 = 10.177$). There is evidence of increased insirect international volatility spillovers from the German money market to S&P500 ($\gamma_{41}^2 = 45.407$).

U.S. – Canada

The transmission of the volatility information between the U.S and Canada is subject to some changes during the crisis (see Table 3.16, Panel B), as the equity markets are slightly more interconnected in the sense that their relationship becomes bidirectional. The inverse change is observed for the money markets where there is no more influence from the Canadian money market during the crisis. Similarly, to the UK and Germany, the Canadian money market exports volatility to the U.S. markets indirectly through the equity market ($\gamma_{41}^2 = 12.107$) however with substantially greater intensity.

The information transmission via volatility channel with its six bidirectional routes can be summarised in general terms as follows. For all the analysed countries, there are clear similar patterns with very few differences. The domestic routes suggest that in all cases the U.S. markets interact less, while for the other economies the equity and the money market are communicating at a much higher intensity during the crisis. Regarding the direct (between the same asset classes) international routes there is a general pattern: the equity markets become closely interconnected (except for Japanese equity markets), while between the money markets there is barely any volatility spillover effects. There is evidence of substantial indirect international volatility transmission from the money markets of the countries analysed to S&P500 (except for Japan).

5.5.2 The Estimation Results for the Full BEKK model: Stock and Bond Markets

Return Spillovers – The Mean Equation

U.S. – U.K.

First, the analysis of the U.S.-U.K. pair (see Table 5.17, Panel A) finds that there is evidence of some return spillover effects during both sub-periods. At the domestic level, the information flows in one direction only, from the bond markets to the equity markets in both countries. The U.S. long-term bond market seems to be most impactful, with its feedbacks increasing from $\beta_{13} = 1.057$ before the crisis to $\beta_{13} = 6.493$ during the crisis. and Similar changes can be observed in the influence of the U.K. long-term bond market on the equity market ($\beta_{23} = 1.645$ before the crisis and $\beta_{23} = 4.566$ during the crisis).

The direct international route is dominated by the U.S. markets with information being transmitted unidirectionally from S&P500 towards FTSE100 that consolidates during the crisis and from U.S. bond market to U.K bond market. There is evidence of indirect international return spillovers from from the U.S. bond market to FTSE100 and from the UK bond market to S&P500, respectively.

U.S. – Japan

For the U.S.-Japan pair the estimation results are presented in Table 5.18 (Panel A). The feedback effects in the mean equations are barely present before the crisis. Internally there is only one direction flow of information from the bond markets to the equity markets. Internationally, the direct linkages between same asset classes are dominated by the U.S. markets. Concerning the equity markets there are strong unidirectional feedbacks from S&P500 to NIKKEI500. There is some evidence of indirect return transmission as a result of the crisis, from the U.S. bond market to the NIKKEI500 and from S&P500 to Japanese bond market. Hence, the U.S. markets become the main exporters of information via the return channel, both directly and indirectly.

U.S. - Germany

The estimation results for the U.S.-Germany (Table 5.19, Panel A) show important changes in the mechanism of price discovery transmission as the result of the crisis. As for the previous pairs, inside each economy the bond markets have a great impact on the respective equity markets ($\beta_{13} = 6.2286$ and $\beta_{24} = 6.0799$). The direct international linkages are more intense between the equity markets with the DAX30 index being dominant over S&P500 during the crisis. The bond markets are weakly linked with feedbacks from the U.S. bond market ($\beta_{43} = 0.1916$) being stronger before the crisis. From one asset class to another, the event of the crisis makes the equity markets more sensitive to what happens within the bond markets. During the crisis, the indirect external routes are extremely busy with significant information transmission from each bond market to the other equity market ($\beta_{23} = 4.9189$ and $\beta_{14} = 3.8735$).

U.S. - Canada

The return spillovers results for the mean equation for U.S. - Canada show different results from the other three pairs (see Table 5.20, Panel A). During the crisis, the feedbacks from U.S. bond market to both equity markets increased few times fold, while within each asset class there is influence only from the Canadian market, as in the money market context. An important change in the return transmission is the new indirect impact of the two stock indices, S&P500 and SPTSX on the bond markets.

In conclusion, while the other three economies seem to be dominated by the U.S. equity and bond markets, for Canada the situation reverses as there are strong bidirectional return spillovers especially from both equity and bond Canadian markets to their U.S. counterparty markets.

Volatility Spillovers – The Volatility Equations Equation

U.S. – U. K.

In terms of volatility channel, there are more significant estimates compared to the return parameters, with less pre-crisis presence of volatility spillover effects than during the crisis. The equity markets communicate uni-directionally from the S&P 5000 to FTSE100 ($\gamma_{12}^2 = 0.0154$), while the internal volatility flows are extremely high from each bond market to its respective equity sector ($\gamma_{31}^2 = 186.9651$ and $\gamma_{42}^2 = 17.1581$). Overall, the bond markets seem to play a rather dominant role especially during the crisis along both direct and indirect routes.

U.S. - Japan

Turning to the volatility channel for the U.S. – Japan analysis, before the crisis there is significant evidence of volatility spillovers between the equity and bond markets of the two the economies in all directions. However, during the crisis the flow of information through volatility channel intensifies only along some directions. The estimates for the domestic routes show that the bond markets are transferring volatility to the equity markets; are The U.S. equity market becomes the main source of volatility spillovers across all the other markets when compared with its Japanese counterparty, while the Japanese bond market seems to export more volatility than the U.S. bond market. The volatility of the U.S. bond market does not play the leading role anymore which is in contrast with the return spillovers results. The Japanese bond market affects to a great extent the two equity markets, S&P500 ($\gamma_{41}^2 = 186.068$) and NIKKEI500 ($\gamma_{32}^2 = 32.974$).

U.S. - Germany

Looking at the Table 5.19 (Panel B), there is clearly more evidence of the volatility spillover effects between the U.S and Germany, as all sixteen parameters in the matrix of volatility spillovers are highly significant in both sub-periods considered. However, the impact of the crisis is unclear as the value of several coefficients decrease during the crisis. The dominance of the U.S. markets present before the crisis seems to diminish while the influence of the German bond and equity markets on their U.S. counterparties increases during the crisis.

U.S. - Canada

There is weak evidence of volatility spillover effects during both periods with most estimates slightly increasing during the crisis. The U.S. is the main volatility exporter in the equity markets ($\gamma_{12}^2 = 0.055$ compared to $\gamma_{21}^2 = 0.041$) and Canada in the bond markets, respectively ($\gamma_{43}^2 = 0.013$ compared to $\gamma_{34}^2 = 0.003$). The indirect connections are weak, with only the volatility of the Canadian bond market affecting significantly the volatility of the S&P500 ($\gamma_{41}^2 = 1.340$).

The volatility channel seems to facilitate mostly the transmission of information via indirect external routes and via the domestic route in one direction only, from the bond markets to the equity markets. The results from both return and volatility spillovers analyses suggest that Canada exchanges information with the U.S. in a different way. During the crisis, for many routes the direction of the information flows changes such that the Canadian markets are dominant over the U.S. markets.

One of the aims of this study is to investigate how the last global financial crisis has spread from the U.S. (the country where the crisis originated) to other major economies. To assess the role of the U.S markets as the most important source of information shocks we have estimated the full BEKK four-variable model for all the possible combinations of pairing any two countries. A total of other 24 models have been estimated corresponding to six country-pairs over pre-crisis and post-crisis periods, keeping the same asset class combinations across the two segments of the yield curve. The estimation results for the additional combinations are presented in the Appendix at the end of the chapter. The findings from the analysis of the pairs that do not contain the U.S. markets reveal weaker spillover effects especially through the volatility channel as the parameter estimates measuring the intensity of the information flow are substantially lower. Moreover, where the U.S. markets are modelled there is significant evidence of volatility spillovers effects along the indirect international routes, whereas when the U.S. markets are excluded these linkages are substantially diminished.

Table 5.13 U.S.-U.K., Stock and Money Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	FTSE100	US TB	UK 1M	S&P500	FTSE100	US 1M	UK 1M
intercept	0.025296	0.035852	-0.001348	0.000897	0.090339***	0.027369	-0.000461	-0.000060
AR(1)	-0.054885**	-0.261938***	-0.037685	0.003815	-0.088172***	-0.301711***	0.022004	-0.011939
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from FTSE100	from US 1M	from UK 1M	from S&P500	from FTSE100	from US 1M	from UK 1M
to S&P500		-0.025839	0.490693	-0.846969		-0.005459	0.864732***	-1.318798
to FTSE100	0.388486***		-0.083479	-1.199790	0.445833***		1.423429***	-0.997557
to US 1M	-0.000133	-0.001537*		0.117130***	0.000012	-0.000419		-0.010318
to UK 1M	-0.000876	0.000348	-0.022914*		0.000483***	-0.000212	0.009237	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	S&P500	FTSE100	US 1M	UK 1M	S&P500	FTSE100	US 1M	UK 1M
Intercept matrix	-0.127376***				-0.067956			
C	0.014975	0.091260***			-0.037689	0.110118***		
i=1,…,4	-0.010649	0.014608***	0.000000		0.003475***	-0.000302	0.000000	
	-0.004701*	-0.003225	0.000000	0.000000	0.000029	-0.000490***	0.000000	0.000000
ARCH effect matrix	0.201644***	-0.112957***	-0.003029**	0.006358***	0.015602	0.167818***	-0.000292	-0.000725***
A	0.061418**	0.266175***	0.016550***	-0.002294***	0.149183***	0.103032***	-0.000769	0.000052
i=1,…,4	0.378500	-0.882946**	0.719879***	0.069817***	-0.720056***	-0.224335	0.623558***	0.019871***
	0.131130	1.821763***	-0.052835	-0.051453	-0.706872	-2.079203***	-0.057734**	0.191632***
GARCH effect matrix	-1.018739***	-0.404689***	0.012832***	0.008284***	0.136733	-0.847421***	0.000229	-0.000286
G	0.627083***	1.079446***	-0.008693***	0.000589	-1.097538***	-0.130892	0.000543	0.000196
i=1,…,4	-3.313100***	1.251530*	0.473664***	-0.361381***	0.352114	-0.057005	0.849714***	-0.014450***
	16.243943***	5.531147***	0.755907***	0.527481***	7.483250***	7.039526***	0.007864	0.978931***

Shock Spillovers	from S&P500	from FTSE100	from US 1M	from UK 1M	from S&P500	from FTSE100	from US 1M	from UK 1M
to S&P500	0.040660***	0.003772**	0.143262	0.017195	0.000243	0.022256***	0.518481***	0.499668
to FTSE100	0.012759***	0.070849***	0.779593**	3.318820***	0.028163***	0.010616***	0.050326	4.323086***
to US 1M	0.000009**	0.000274***	0.518225***	0.002792	0.000000	0.000001	0.388824***	0.003333**
to UK 1M	0.000040***	0.000005***	0.004874***	0.002647	0.000001***	0.000000	0.000395***	0.036723***
Volatility Spillovers	from S&P500	from FTSE100	from US 1M	from UK 1M	from S&P500	from FTSE100	from US 1M	from UK 1M
to S&P500	1.037828***	0.393233***	10.976633***	263.865684***	0.018696	1.204591***	0.123984	55.999024***
to FTSE100	0.163773***	1.165204***	1.566328*	30.593588***	0.718123***	0.017133	0.003250	49.554928***
to US 1M	0.000165***	0.000076***	0.224357***	0.571395***	0.000000	0.000000	0.722014***	0.000062
to UK 1M	0.000069***	0.000000	0.130596***	0.278236***	0.000000	0.000000	0.000209***	0.958306***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively

Table 5.14 U.S. – JAPAN Stock and Money Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	NIKKEI500	from US 1M	JAP 1M	S&P500	NIKKEI500	US 1M	JAP 1M
intercept	0.056131***	0.069043***	-0.002086**	-0.000001	0.065854***	0.019098	0.000138	0.000012
AR(1)	-0.094721***	0.099057***	0.144364***	0.137531***	-0.030599	-0.011690	-0.004563	0.202389***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from NIKKEI500	from US 1M	from JAP 1M	from S&P500	from NIKKEI500	from US 1M	from JAP 1M
to S&P500		0.021382	0.993042***	-2.861105		-0.008863	0.766808***	-2.737455
to NIKKEI500	0.068281**		1.056396**	-4.872645	0.109678***		0.982511***	-11.959657***
to US 1M	0.007026***	-0.002080**		0.062720	-0.000749**	0.000297		-0.009560
to JAP 1M	0.000073***	-0.000073***	0.000563**		0.000038	-0.000056	-0.006276***	

Panel B	Pre-crisis				Post-crisis			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.100043***				0.125558***			
C	0.726525***	0.000001			0.135768***	-0.182554***		
i=1,…,4	0.024560***	0.000000	0.000000		0.000591	0.000787	-0.004538***	
	0.000060***	0.000000	0.000000	0.000000	0.000101***	-0.000109***	0.000036	0.000000
ARCH effect matrix	0.123417***	-0.092027***	-0.022678***	0.000131***	0.221187***	-0.018190	-0.001910***	-0.000067***
A	0.024638	0.206863***	-0.002186	-0.000136***	0.006953	0.194363***	0.001574***	0.000144***
i=1,…,4	0.563099	2.285312**	0.888880***	0.000878***	0.404400	1.000598***	0.646242***	-0.003971***
	1.203542	-1.502579	-0.070373	0.746446***	-0.267110	-3.712818	-0.050244	0.554472***
GARCH effect matrix	-1.097223***	-0.665604***	0.004416***	0.000037	0.970334***	0.006307	0.000325***	0.000018***
G	0.305272***	0.843902***	-0.018676***	-0.000022*	-0.000701	0.959833***	-0.000322*	-0.000057***
i=1,…,4	0.650073	0.516289	-0.190602***	0.001637***	-0.180146***	-0.544623***	0.840624***	0.001048***
	-8.133501	-4.165638	0.984142***	0.886889***	0.284055	1.258251**	0.016694	0.923871***
Shock Spillovers	from S&P500	from NIKKEI500	from US 1M	from JAP 1M	from S&P500	from NIKKEI500	from US 1M	from JAP 1M
to S&P500	0.015232***	0.000607	0.317081	1.448514	0.048924***	0.000048	0.163540	0.071348
to NIKKEI500	0.008469***	0.042792***	5.222650**	2.257743	0.000331	0.037777***	1.001196***	13.785018
to US 1M	0.000514***	0.000005	0.790107***	0.004952	0.000004***	0.000002***	0.417629***	0.002524
to JAP 1M	0.000000***	0.000000***	0.000001***	0.557182***	0.000000***	0.000000***	0.000016***	0.307439***
Volatility Spillovers	from S&P500	from NIKKEI500	from US 1M	from JAP 1M	from S&P500	from NIKKEI500	from US 1M	from JAP 1M
to S&P500	1.203898***	0.093191***	0.422596	66.153835	0.941549***	0.000000	0.032453***	0.080687
to NIKKEI500	0.443029***	0.712170***	0.266554	17.352538	0.000040	0.921279***	0.296614***	1.583195**
to US 1M	0.000020***	0.000349***	0.036329***	0.968536***	0.000000***	0.000000***	0.706648***	0.000279
to JAP 1M	0.000000	0.000000*	0.000003***	0.786572***	0.000000***	0.000000***	0.000001***	0.853538***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively

Table 5.15 U.S. – Germany, Stock and Money Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	DAX30	US 1M	GER 1M	S&P500	DAX30	US 1M	GER 1M
intercept	0.054877***	0.115261***	0.002131**	-0.000009	0.084436***	0.070638***	-0.000561	-0.000107*
AR(1)	-0.129968***	-0.129122***	0.032071	0.379127***	-0.153461***	-0.086617	0.041216***	0.970145***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from DAX30	from US 1M	from GER 1M	from S&P500	from DAX30	from US 1M	from GER 1M
to S&P500		0.080260***	-0.218816	1.282842		0.173600***	0.955313***	4.366920***
to DAX30	0.202328***		-0.465932	-4.319040**	0.154321***		0.843047***	2.533530
to US 1M	-0.000473	-0.001272*		-0.045627	-0.000306	-0.000087		-0.039709
to GER 1M	-0.000272*	0.000431***	0.005760***		-0.001066***	0.000178***	0.009716***	
Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.076832***				0.127876***			
C	0.116586***	0.022818			0.210413***	0.039138		
i=1,…,4	-0.002470	0.014643***	-0.000001		-0.000064	-0.001005***	0.000000	
	0.000001	-0.000182***	0.000000	0.000000	-0.001257***	-0.000167	0.000000	0.000000
ARCH effect matrix	0.193398***	0.320415***	-0.002299*	0.000113	0.144674***	-0.060268**	-0.000558*	-0.004170***
A	-0.105175***	-0.000116	0.006538***	-0.000093	0.100708***	0.161372***	0.000012	0.001340***
i=1,…,4	0.851034**	1.486965***	0.641630***	0.004254*	1.159865***	0.377138	0.269349***	0.165652***
	-2.090648*	-4.909012***	-0.014902	0.589312***	14.441266***	8.105497***	-0.081592	1.974711***
GARCH effect matrix	0.885180***	-0.198984***	0.001319***	-0.000049	1.037518***	0.131143***	0.000142	0.000584***
G	0.095776***	1.058327***	-0.000932***	0.000035	-0.094879***	0.891752***	0.000029	-0.000127
i=1,…,4	-0.414615***	-0.622621***	0.787018***	0.000776	-0.020744	0.053339	0.970299***	-0.010743***
	0.870251***	0.429943	-0.015794	0.924036***	-6.738536***	-3.190080***	0.039633	0.248095***

Shock Spillovers	from S&P500	from DAX30	from US 1M	from GER 1M	**from S&P500**	from DAX30	from US 1M	from GER 1M
to S&P500	0.037403***	0.011062***	0.724259**	4.370809*	0.020931***	0.010142***	1.345287***	208.550156***
to DAX30	0.102666***	0.000000	2.211064***	24.098399***	0.003632**	0.026041***	0.142233	65.699081***
to US 1M	0.000005*	0.000043***	0.411689***	0.000222	0.000000*	0.000000	0.072549***	0.006657
to GER 1M	0.000000	0.000000	0.000018*	0.347288***	0.000017***	0.000002***	0.027441***	3.899483***
Volatility Spillovers	from S&P500	from DAX30	from US 1M	from GER 1M	from S&P500	from DAX30	from US 1M	from GER 1M
to S&P500	0.783543***	0.009173***	0.171906***	0.757337***	1.076443***	0.009002***	0.000430	45.407864***
to DAX30	0.039595***	1.120056***	0.387657***	0.184851	0.017199***	0.795221***	0.002845	10.176609***
to US 1M	0.000002***	0.000001***	0.619398***	0.000249	0.000000	0.000000	0.941481***	0.001571
to GER 1M	0.000000	0.000000	0.000001	0.853843***	0.000000***	0.000000	0.000115***	0.061551***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively

Table RATS 5.16 U.S. – CANADA Stock and Money Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	SPTSX	US 1M	CAD 1M	S&P500	SPTSX	US 1M	CAD 1M
intercept	0.043281**	0.076261***	-0.000199	0.000775*	0.059812***	0.036209*	-0.000061	-0.000412
AR(1)	-0.203001***	0.002361	0.209623***	0.202101***	-0.230687***	-0.054882**	-0.006196	-0.122254***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from SPTSX	from US 1M	from CAD 1M	from S&P500	from SPTSX	from US 1M	from CAD 1M
to S&P500		0.186882***	0.090056	-0.371459		0.290353***	0.892475***	2.811756***
to SPTSX	-0.009304		-0.525441	-0.509030	0.035043		0.107829	0.543477
to US 1M	0.000506	0.004321***		0.188716***	-0.000807**	0.001285***		0.002800
to CAD 1M	-0.001648*	0.000767	-0.000927		-0.000037	0.000462	-0.014918	

Panel B	Pre-crisis				Post-crisis			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.061269***				0.121509***			
C	-0.190261***	-0.150115***			0.059913***	0.045894***		
i=1,…,4	-0.005356	-0.001002	0.030075***		-0.002145	-0.003032*	0.000006	
	-0.009213***	-0.005316	-0.001713	0.000000	-0.000850**	0.000163	0.000001	0.000000
ARCH effect matrix	0.039465**	-0.071785***	-0.020195***	-0.017670***	0.201473***	0.078428***	0.000244	0.000001
A	-0.086773***	-0.216288***	0.003653	0.009891***	-0.108312***	0.135453***	-0.001220***	-0.002676***
i=1,…,4	-0.115432	-0.095499	0.772505***	-0.050012*	-0.325253	0.128580	0.600153***	0.061269***
	-0.176640	-0.169373	0.105129***	0.704024***	-0.927384***	-0.782737***	0.012595	0.295599***
GARCH effect matrix	1.000708***	0.040352***	-0.000856	0.002762***	-0.955691***	-0.036555***	0.832501	3.479543
G	-0.013944	0.888097***	-0.000554	-0.006746***	0.029895***	-0.996617***	2.439321	4.742052
i=1,…,4	-0.288196	-0.945182	0.352325***	0.091326***	0.000425	0.000711***	0.856810***	-0.006144***
	0.573539***	0.195392	-0.127834***	0.726470***	0.000818***	-0.002044***	-0.026183	0.956238***
Shock Spillovers	from S&P500	from SPTSX	from US 1M	from CAD 1M	from S&P500	from SPTSX	from US 1M	from CAD 1M
to S&P500	0.001557**	0.007529***	0.013325	0.031202	0.040592***	0.011732***	0.105790	0.860041***
to SPTSX	0.005153***	0.046780***	0.009120	0.028687	0.006151***	0.018347***	0.016533	0.612677***
to US 1M	0.000408***	0.000013	0.596764***	0.011052***	0.000000	0.000001***	0.360184***	0.000159
to CAD 1M	0.000312***	0.000098***	0.002501*	0.495650***	0.000000	0.000007***	0.003754***	0.087379***
Volatility Spillovers	from S&P500	from SPTSX	from US 1M	from CAD 1M	from S&P500	from SPTSX	from US 1M	from CAD 1M
to S&P500	1.001417***	0.000194	0.083057	0.328948***	0.913346***	0.001336***	0.693058	12.107222***
to SPTSX	0.001628***	0.788716***	0.893369	0.038178	0.000894***	0.993246***	5.950285***	22.487053***
to US 1M	0.000001	0.000000	0.124133***	0.016342***	0.000000	0.000001	0.734123***	0.000038
to CAD 1M	0.000008***	0.000046***	0.008341***	0.527759***	0.000001	0.000004	0.000686***	0.914390***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively

Table 5.17 U.S. - U.K. Stock and Bond Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	FTSE100	US 10Y	UK 10Y	S&P500	FTSE100	US 10Y	UK 10Y
intercept	0.054295***	0.054067***	0.000394	0.001126	0.069432***	0.026146	-0.000198	-0.00086
AR(1)	-0.089378***	-0.225684***	-0.030975	-0.117975***	-0.043080*	-0.28250***	-0.048370*	-0.109000***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from FTSE100	from US 10Y	from UK 10Y	from S&P500	from FTSE100	from US 10Y	from UK 10Y
to S&P500		0.030640	1.056586***	1.134996**		0.001744	6.492774***	2.271236***
to FTSE100	0.339259***		1.644884***	2.054306***	0.431665***		4.566273***	4.293195***
to US 10Y	-0.000821*	0.000872		0.090644***	-0.000772	-0.001141		0.042179
to UK 10Y	0.000438	0.000505	0.165452***		-0.000904	0.000179	0.218115***	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.084895***				0.001171			
C	0.039413	0.093000***			-0.205557**	0.150410		
i=1,…,4	0.002334***	0.001393*	0.000000		0.003969	-0.002914	0.000001	
	0.006243***	0.004761***	0.000000	0.000001	0.007487**	-0.005496	0.000001	0.000000
ARCH effect matrix	-0.221384***	0.086551***	0.002252***	0.001809	0.193490***	-0.205298***	-0.006362***	0.003515***
A	0.182835***	0.269696***	0.001325	-0.000916	-0.189846***	0.150497***	-0.009331***	-0.007001***
i=1,…,4	0.237864	-0.796658**	0.038590*	-0.092809***	-3.884820***	-1.104321***	-0.030253	0.060438***
	-0.941924	0.854587*	-0.004277	0.144843***	2.667708***	-0.657617	0.057376***	0.040079*
GARCH effect matrix	0.966610***	0.048721***	0.001362***	0.001183***	0.691614***	-0.124196***	0.038047***	0.024243***
G	0.005582	0.916042***	-0.001663***	-0.000759*	0.081894	0.843316***	-0.001925	-0.023370***
i=1,…,4	-0.179762	0.279603***	1.003224***	0.018594***	-13.673521***	-10.375747***	0.747183***	0.102816

	0.532199**	-0.911371***	-0.020582***	0.951411***	10.772947***	4.142230***	-1.048534***	-0.990592***
Shock Spillovers	from S&P500	from FTSE100	from US TB	from UK TB	from S&P500	from FTSE100	from US Bond	from UK bond
to S&P500	0.049011***	0.033429***	0.056579	0.887220	0.037438***	0.036042***	15.091827***	7.116666***
to FTSE100	0.007491***	0.072736***	0.634664**	0.730318*	0.042147***	0.022649***	1.219524***	0.432460
to US 10Y	0.000005***	0.000002	0.001489*	0.000018	0.000040***	0.000087***	0.000915	0.003292***
to UK 10Y	0.000003	0.000001	0.008613***	0.020980***	0.000012***	0.000049***	0.003653***	0.001606*
Volatility Spillovers	from S&P500	from FTSE100	from US TB	from UK TB	from S&P500	from FTSE100	from US Bond	from UK bond
to S&P500	0.934335***	0.000031	0.032314	0.283235**	0.478330***	0.006707	186.965166***	116.056393***
to FTSE100	0.002374***	0.839133***	0.078178***	0.830597***	0.015425***	0.711182***	107.656116***	17.158067***
to US 10Y	0.000002***	0.000003***	1.006458***	0.000424***	0.001448***	0.000004	0.558282***	1.099423***
to UK 10Y	0.000001***	0.000001*	0.000346***	0.905182***	0.000588***	0.000546***	0.010571	0.981272***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively.**L**

Table 5.18 U.S. – Japan, Stock and Bond Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	NIKKEI500	US 10Y	JAP 10Y	S&P500	NIKKEI500	US 10Y	JAP 10Y
intercept	0.045200***	0.044777*	0.000512	-0.000261	0.069013***	0.046515***	0.000057	-0.000928**
AR(1)	-0.086098***	0.062718***	0.034683	-0.036071	-0.050845**	0.019932	-0.062397***	-0.028946
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from NIKKEI500	from US 10Y	from JAP 10Y	from S&P500	from NIKKEI500	from US 10Y	from JAP 10Y
to S&P500		0.034843*	-0.496176	0.743916		0.024465	6.704387***	0.098445
to NIKKEI500	0.042043		0.116648	7.187995***	0.042188*		5.996694***	6.762224***
to US 10Y	0.002839*	-0.000978		-0.082546	-0.000970	0.000501		0.095242
to JAP 10Y	0.000778	0.000377	0.060773***		-0.001197***	0.001115***	0.114669***	

Panel B	Pre-crisis				Post-crisis			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.058902**				0.114558***			
C	0.302745**	0.305349***			0.164581**	-0.268067***		
i=1,…,4	-0.001145	-0.000560	-0.000002		-0.003634*	-0.005610***	-0.000002	
	-0.004935***	-0.001087	0.000004	0.000228	-0.001063**	-0.000231	-0.000002	0.000000
ARCH effect matrix	-0.180557***	0.090650***	-0.003488***	-0.000768	0.253322***	-0.164406***	0.004322***	0.000134
A	0.129374***	0.343448***	0.000630	-0.001311***	-0.001532	0.382244***	0.001782	-0.000090
i=1,…,4	-0.828209***	-1.605463***	0.016082	-0.025283***	2.242576***	1.687185***	-0.103583***	-0.005471
	-0.537251	-2.920203***	0.023937	-0.223112***	-4.185626***	-8.989765***	-0.068811	-0.214000***
GARCH effect matrix	1.006113***	0.127698***	0.000291	-0.000934***	0.939326***	0.045129***	-0.002705***	-0.001231
G	-0.053814***	0.767290***	-0.001856***	0.001959***	0.002476	0.865595***	-0.001295*	0.000001
i=1,…,4	-0.890979***	-1.981806***	-0.997966***	-0.015282***	1.122146***	0.552415***	0.998771***	0.042340**
	0.846092***	2.854006***	0.296192***	0.941878***	-13.640667***	-5.742270*	-0.579707***	-0.981502***
Shock Spillovers	from S&P500	from NIKKEI500	from US 10Y	from JAP 10Y	from S&P500	from NIKKEI500	from US 10Y	from JAP 10Y
to S&P500	0.032601***	0.016738***	0.685930***	0.288639	0.064172***	0.000002	5.029146***	17.519468***
to NIKKEI500	0.008217***	0.117957***	2.577512***	8.527586***	0.027029***	0.146111***	2.846593***	80.815869***
to US 10Y	0.000012***	0.000000	0.000259	0.000573	0.000019***	0.000003	0.010729***	0.004735
to JAP 10Y	0.000001	0.000002***	0.000639***	0.049779***	0.000000	0.000000	0.000030	0.045796***
Volatility Spillovers	from S&P500	from NIKKEI500	from US 10Y	from JAP 10Y	from S&P500	from NIKKEI500	from US 10Y	from JAP 10Y
to S&P500	1.012263***	0.002896***	0.793844***	0.715871***	0.882333***	0.000006	1.259212***	186.067794***
to NIKKEI500	0.016307***	0.588734***	3.927556***	8.145350***	0.002037***	0.749254***	0.305162***	32.973662*
to US 10Y	0.000000	0.000003***	0.995935***	0.087730***	0.000007***	0.000002*	0.997544***	0.336061***
to JAP 10Y	0.000001***	0.000004***	0.000234***	0.887135***	0.000002	0.000000	0.001793**	0.963347***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively.

Table 5.19 U.S. – Germany, Stock and Bond Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	DAX30	US 10Y	GER 10Y	S&P500	DAX30	US 10Y	GER 10Y
intercept	0.056605***	0.109746***	0.001452	0.000753	0.099182***	0.095280***	0.000913	-0.000906
AR(1)	-0.150844***	-0.072663***	0.051220**	-0.191196***	-0.202242***	-0.109867***	-0.054282**	-0.053741**
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from DAX30	from US 10Y	from GER 10Y	from S&P500	from DAX30	from US 10Y	from GER 10Y
to S&P500		0.094229***	1.845535***	-0.557286		0.120078***	6.228559***	3.873497***
to DAX30	0.129119***		3.428322***	-0.021693	0.102639***		4.941885***	6.079898***
to US 10Y	0.000156	-0.001131		-0.064974*	-0.000672	0.000608		0.011617
to GER 10Y	-0.001835*	0.000682	0.363440***		-0.001042	-0.001243	0.191554***	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.114491***				0.044738			
C	-0.137642***	0.000000			0.188741***	0.000025		
i=1,…,4	0.022109***	0.000000	0.000000		0.004004**	0.000003	-0.000001	
	0.022434***	0.000000	0.000000	0.000000	0.012492***	0.000003	-0.000001	0.000000
ARCH effect matrix	0.135708***	0.098573**	0.006630***	0.002260	0.182058***	0.042031	0.012279***	0.001479
A	-0.093010***	0.183203***	0.003261***	0.005201***	0.041644*	0.214488***	-0.004998***	-0.006307***
i=1,…,4	1.212548***	1.850986***	0.001338	0.045588*	2.748556***	1.427812***	-0.008980	-0.093877***
	0.630704	3.166946***	-0.009108	-0.058627	0.561579	2.953336***	0.055876*	0.239269***
GARCH effect matrix	0.661904***	-0.200451***	-0.036500***	0.007995***	0.956980***	0.076861*	-0.030318***	-0.006574***
G	0.071928***	0.982807***	0.005118***	-0.003445***	-0.079797***	0.875325***	0.018602***	0.011216***
i=1,…,4	11.171125***	8.244266***	0.807513***	0.115445***	6.698426***	3.148348***	0.965210***	0.105227***
	-8.860482***	-4.057351***	-0.405219***	0.555093***	-7.126246***	-8.681736***	-0.128908***	0.790813***

Shock Spillovers	from S&P500	from DAX30	from US 10Y	from GER 10Y	from S&P500	from DAX30	from US 10Y	from GER 10Y
to S&P500	0.018417***	0.008651***	1.470272***	0.397788	0.033145***	0.001734*	7.554561***	0.315371
to DAX30	0.009717**	0.033563***	3.426148***	10.029549***	0.001767	0.046005***	2.038646***	8.722194***
to US 10Y	0.000044***	0.000011***	0.000002	0.000083	0.000151***	0.000025***	0.000081	0.003122*
to GER 10Y	0.000005	0.000027***	0.002078*	0.003437	0.000002	0.000040***	0.008813***	0.057250***
Volatility Spillovers	from S&P500	from DAX30	from US 10Y	from GER 10Y	from S&P500	from DAX30	from US 10Y	from GER 10Y
to S&P500	0.438116***	0.005174***	124.794042***	78.508141***	0.915810***	0.006368***	44.868914***	50.783388***
to DAX30	0.040180***	0.965909***	67.967915***	16.462097***	0.005908*	0.766194***	9.912097***	75.372546***
to US 10Y	0.001332***	0.000026***	0.652077***	0.164202***	0.000919***	0.000346***	0.931630***	0.016617***
to GER 10Y	0.000064***	0.000012***	0.013327***	0.308128***	0.000043***	0.000126***	0.011073***	0.625385***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively

Table 5.20 U.S. – Canada, Stock and Bond Markets; The estimation results for the Variance Equation in the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	S&P500	SPTSX	US 10Y	CAD 10Y	S&P500	SPTSX	US 10Y	CAD 10Y
intercept	0.061043***	0.094276***	0.000141	-0.000703	0.071375***	0.062035***	-0.000064	-0.000925
AR(1)	-0.145458***	-0.033483	-0.098493	-0.098493	-0.194434***	-0.046581	-0.176814***	-0.023708
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	from S&P500	from SPTSX	from US 10Y	from CAD 10Y	from S&P500	from SPTSX	from US 10Y	from CAD 10Y
to S&P500	-0.145458	0.146036***	0.637649	1.270641*	-0.194434***	0.229226***	3.702519***	4.747858***
to SPTSX	0.038973*	-0.033483	0.592392	-0.544040	0.019598	-0.046581	1.585542***	1.556437
to US 10Y	0.001900	-0.002060	-0.098493***	0.161330***	-0.003673***	0.006595***	-0.176814***	0.212745***
to CAD 10Y	0.000336	-0.001007	-0.038054	0.050377	-0.002077***	0.002875***	-0.022300	-0.023708

Panel B	Pre-crisis				Post-crisis			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.056737**				0.077148***			
C	0.408662***	-0.123604			0.026723	-0.018275		
i=1,…,4	0.000419	-0.001227	0.000000		-0.009037**	-0.007454	0.000000	
	0.000857	0.003031***	0.000000	0.000000	-0.011596***	-0.006972	0.000000	0.000000
ARCH effect matrix	0.167332***	0.166353***	0.003968***	-0.002653***	0.246621***	0.098074***	0.001462	0.000491
A	0.007727	0.347958***	0.000304	0.003111***	-0.095436***	0.207653***	0.003921***	0.001521
i=1,…,4	-1.776918***	-0.103037	0.127612***	-0.000297	-3.649553***	-2.675723***	-0.032628	0.121376***
	1.297067***	-0.887555	-0.232549***	-0.145274***	-0.647361	-0.022209	0.197751***	0.067741
GARCH effect matrix	0.973010***	0.120487***	0.000232	0.002295***	0.797494***	-0.235162***	-0.000417	0.000175
G	0.011697	0.624927***	-0.002041	-0.003075**	0.203377***	1.072551***	-0.001768***	-0.000602
i=1,…,4	0.860309***	-0.759251*	0.944203***	-0.023238***	-0.318091	0.202946	1.042737***	0.057592***
	-1.067437***	1.448311***	0.070352***	1.011602***	1.157601***	0.128027	-0.113999***	0.873327***
Shock Spillovers	from S&P500	from SPTSX	from US 10Y	from CAD 10Y	from S&P500	from SPTSX	from US 10Y	from CAD 10Y
to S&P500	0.028000***	0.000060	3.157438***	1.682383***	0.060822***	0.009108***	13.319234***	0.419076
to SPTSX	0.027673***	0.121075***	0.010617	0.787754	0.009618***	0.043120***	7.159494***	0.000493
to US 10Y	0.000016***	0.000000	0.016285***	0.054079***	0.000002	0.000015***	0.001065	0.039106***
to CAD 10Y	0.000007***	0.000010***	0.000000	0.021105***	0.000000	0.000002	0.014732***	0.004589
Volatility Spillovers	from S&P500	from SPTSX	from US 10Y	from CAD 10Y	from S&P500	from SPTSX	from US 10Y	from CAD 10Y
to S&P500	0.946748***	0.000137	0.740132***	1.139422***	0.635997***	0.041362***	0.101182	1.340041***
to SPTSX	0.014517***	0.390534***	0.576462*	2.097605***	0.055301***	1.150365***	0.041187	0.016391
to US 10Y	0.000000	0.000004	0.891519***	0.004949***	0.000000	0.000003***	1.087300***	0.012996***
to CAD 10Y	0.000005***	0.000009**	0.000540***	1.023339***	0.000000	0.000000	0.003317***	0.762699***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively

Table 5.21 Summary Results: The busiest routes in the post-crisis period

		MONEY MARKETS			BOND MARKETS		
Pairs	Channel	Domestic routes	Direct external route	Indirect external routes	Domestic routes	Direct external route	Indirect external routes
U.S. - U.K.	return channel	US : MM to EM 0.86	EM : US to UK 0.45	US (MM) to UK (EM) 1.42	US : BM to EM 6.47 UK: BM TO EM 4.29	EM : US to UK 0.43	US (BM) to UK (EM) 4.57 UK (MM) to US (EM) 2.27
	volatility channel	UK : MM to EM 49.55	EM: UK to US 1.21 MM : US to UK 0.72	UK (MM) to US (EM) 55.99	US : BM to EM 186.97 UK : BM to EM 17.16	MM: UK to US 0.98	UK (BM) to US (EM) 116.06 US (BM) to UK (EM) 107.66
US - JAP	return channel	US : MM to EM 0.76 JAP : MM to EM -11.96		US (MM) to JAP (EM) 0.98 JAP (MM) to US (EM) -2.74	JAP : BM to EM 6.76 US : BM to EM 6.70		US (BM) to JAP (EM) 5.99
	volatility channel	JAP : MM to EM 1.58	MM : US to Japan 0.71	US (MM) to JAP (EM) 0.3	JAP : BM to EM 32.97 US : BM to EM 1.26		JAP (BM) to US (EM) 186.07 US (BM) to JAP (EM) 2.85
US - GER	return channel	US : MM to EM 0.95		GER (MM) to US (EM) 4.37	US : BM to EM 6.23 GER : BM to EM 6.07		US (BM) to GER (EM) 4.94 GER (BM) to US (EM) 3.87
	volatility channel	GER : MM to EM 10.18	MM : US to GER 0.94	GER (MM) to US (EM) 45.41	GER : BM to EM 75.37 US : BM to EM 44.87		GER (BM) to US (EM) 50.78 US (BM) to GER (EM) 9.91

US - CAD	return channel	US : MM to EM 0.89		CAD (MM) to US (EM) 2.82	US : BM to EM 3.7		CAD (BM) to US (EM) 4.74 US (BM) to CAD (EM) 1.58
	volatility channel	CAD: MM to EM 22.49		CAD (MM) to US (EM) 12.11 US (MM) to CAD (EM) 5.95			CAD (BM) to US (EM) 1.34

The comparative analysis of the summary results reported in the Table 5.21 concludes that out of the three types of routes of information transmission, the most active route is the indirect external route followed by the domestic one. This result is valid for both return and volatility channels. Along these routes, the information flows unidirectionally from the interest rate markets to the equity markets and not vice-versa, implying that the interest rate markets dominate the equity markets in transmission of information. When comparing the results of the two segments of the yield curve it is found that the return and volatility spillover effects are much stronger when the equity markets are modelled in combination with the long-term markets than with the money markets. Among the countries considered, the results for Canada are rather different as the Canadian markets seem to influence indirectly the U.S. markets.

5.5.3 Model Implied Conditional variances and covariances

The time series of daily conditional variances and covariances implied by the full BEKK(1,1) model are presented for the long term money markets in Figures 5.8 to 5.15 and for the bond markets in Figures 5.16 to 5.23. Each figure includes part **a** and part **b**, for the two periods studied, before and during the crisis, respectively.

For the one-month interest rates, the graphs of the conditional variances and covariances show more diversity with specific dynamics to each country. For each of these markets the graphs clearly indicate the turbulent periods in the two sample periods analysed. First, in the pre-crisis periods the most noticeable signs of instability are during important events such as the introduction of the euro in 2001 when all the money markets seem to be affected; the technology bubble in 2002-2003[14] mostly affecting Canada and Germany for a very short period. Second, during the financial crisis the signals are clearly present much earlier in the U.S. from 2007 to 2009, while for the other markets the conditional variances and covariances become highly unstable during the interval 2008 - 2009. Two other periods of uncertainty can be observed during 2011 and 2013 respectively, corresponding to different episodes of the sovereign crises in the Eurozone.

When stock returns and long interest rates are combined, a common pattern is observed in the evolution of the conditional variances of the stock returns. Across three out four stock markets, we distinguish several periods: one of medium volatility from 2002 to 2003, followed by a relative calm interval over the period 2003 – 2007, and then a period of great uncertainty marked by the financial crisis of 2007-2009. A totally different pattern emerges in the daily conditional variance time series of the NIKKEI500, where prior to the global financial crisis it follows a stationary process while during the crisis the Japanese stock market moves very close together with the U.S. stock market. Shifting the focus on the long-term bond markets the figures indicate similar dynamics but of significantly less magnitude for the conditional variances, confirming empirically that the long-term bond markets are in general less volatile than the stock markets.

The conditional covariances graphs suggest that inside each asset class, over both periods there is always a positive relationship between the two countries under study (except for UK-Japan). The highest degree of co-movement is present in 2002 and 2008 - the two turbulent periods corresponding to the technology bubble burst and the latest

[14] The dot.com bubble in 2002 impacted significantly only the economic sectors of all countries analysed, the money markets have not been impacted by this event. However, the last financial crisis has spread across different markets from fixed-income initially to the equity and sovereign bond markets.

global financial crisis, respectively. For the rest of the combinations, the conditional covariance time series oscillate around zero with the most amplitude around the same time-points of 2002 and 2008, reflecting the great instability that characterizes these markets during a crisis.

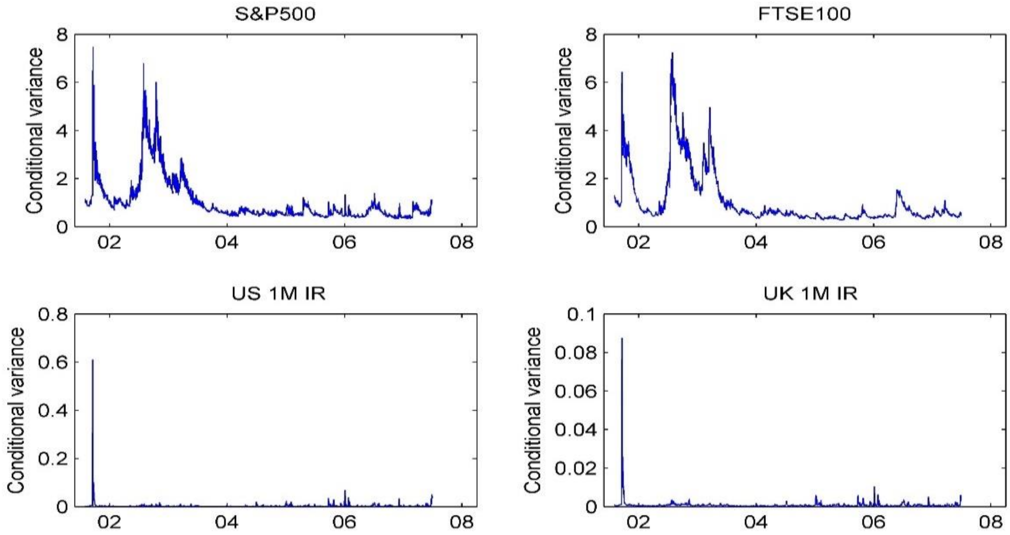

Figure 5.8a Conditional Variances: U.S. – U.K. Equity and Money Markets; Before Crisis.

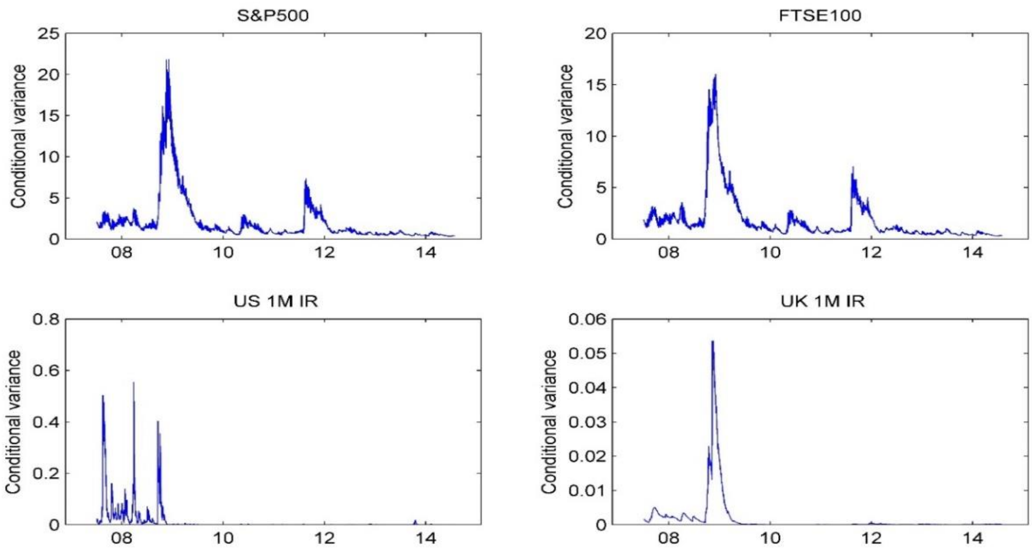

Figure 5.8b Conditional Variances: U.S. – U.K. Equity and Money Markets; Post-Crisis.

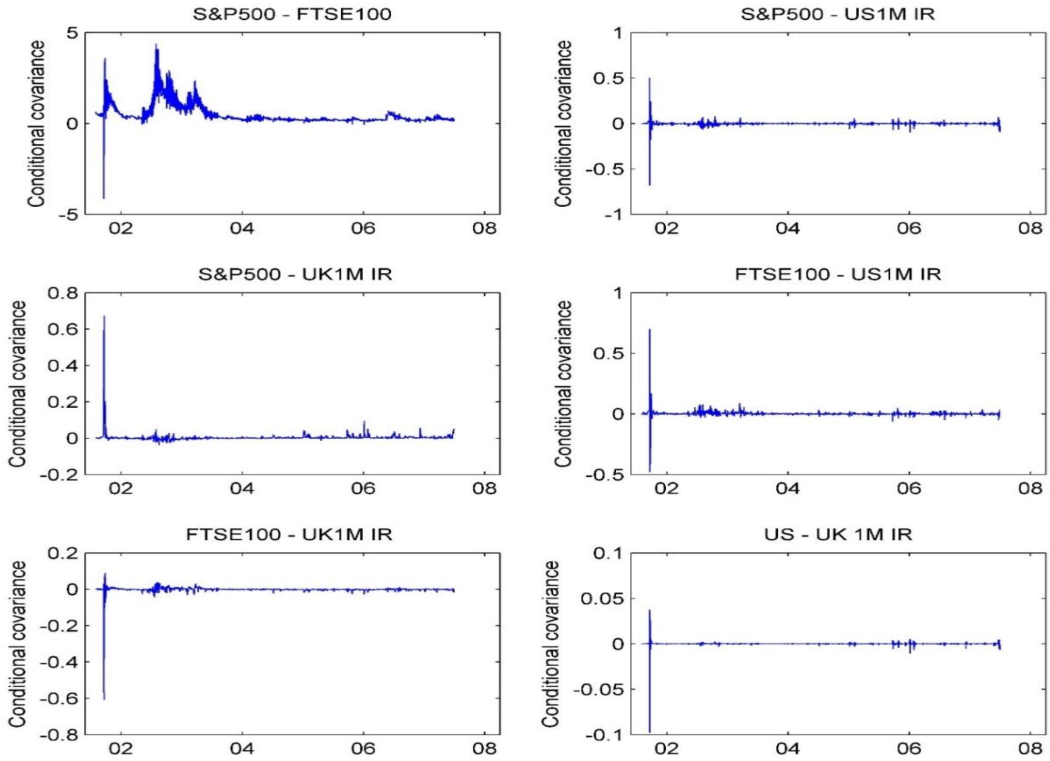

Figure 5.9a Conditional Covariance: U.S. – U.K. Equity and Money Markets Before the Crisis.

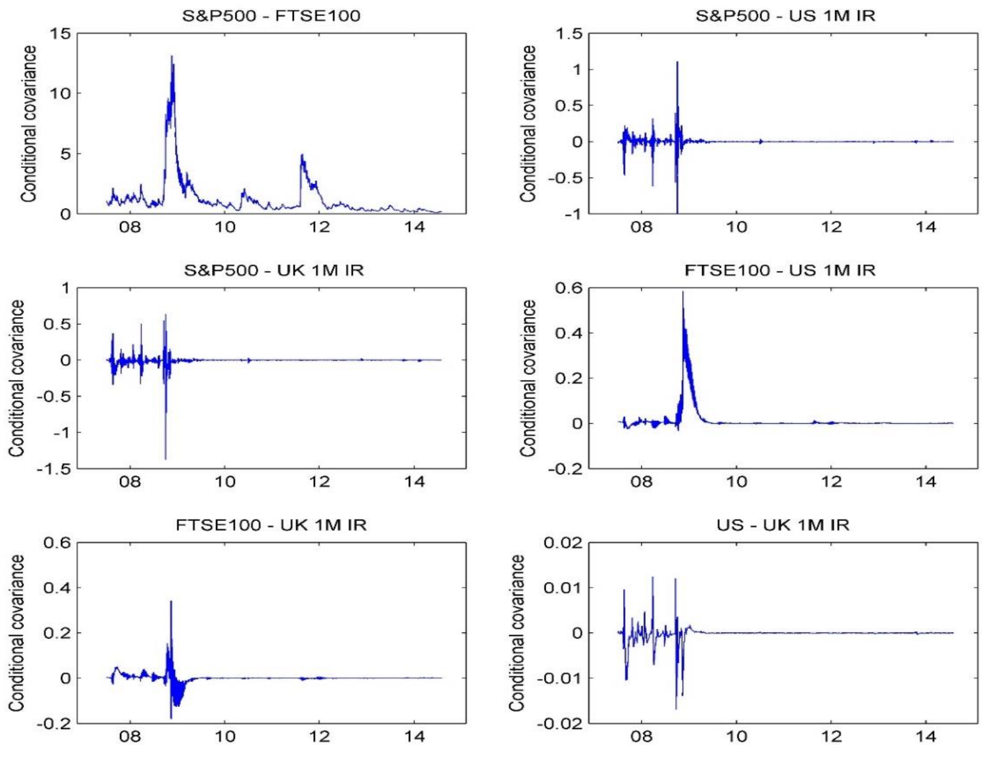

Figure 5.9b Conditional Covariance: U.S. – U.K. Equity and Money Markets Post-Crisis.

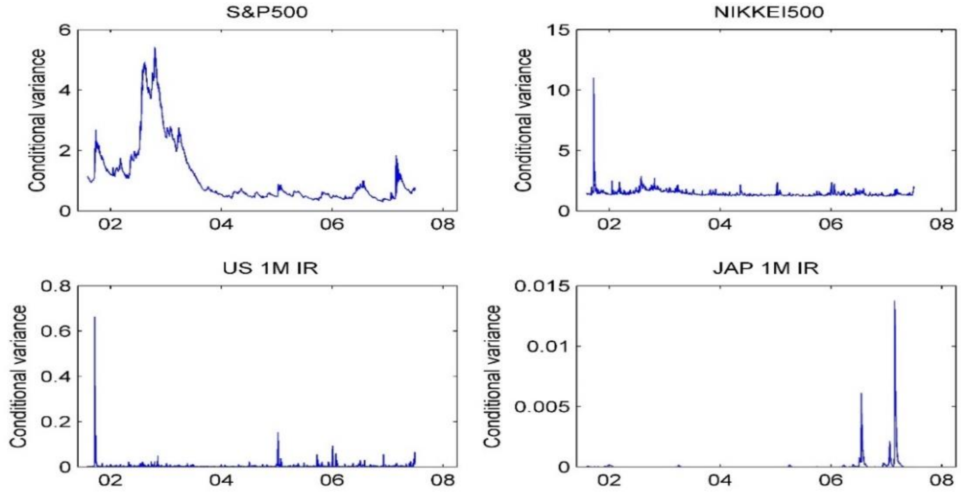

Figure 5.10a Conditional Variances: U.S. – Japan Equity and Money Markets; Before the Crisis.

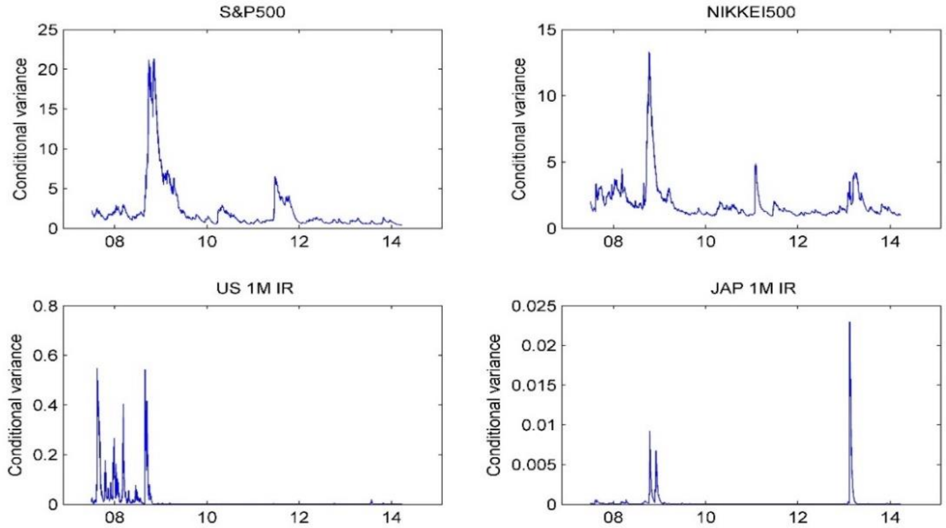

Figure 5.10b Conditional Variances: U.S. – Japan Equity and Money Markets; Post-Crisis.

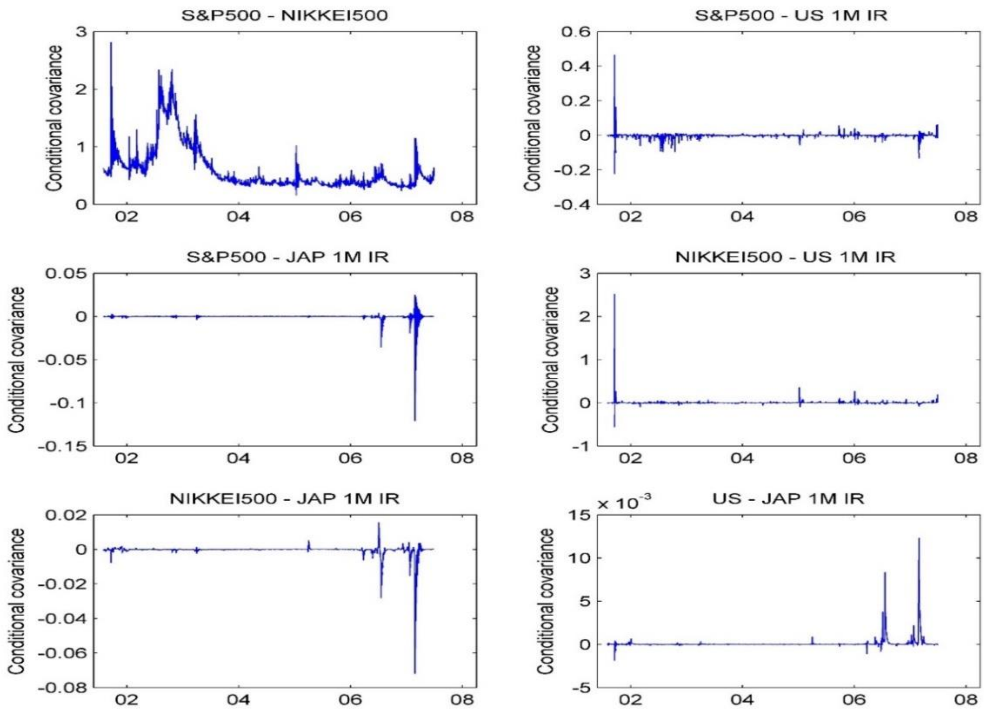

Figure 5.11a Conditional Covariance: U.S. – Japan Equity and Money Markets; Before the Crisis.

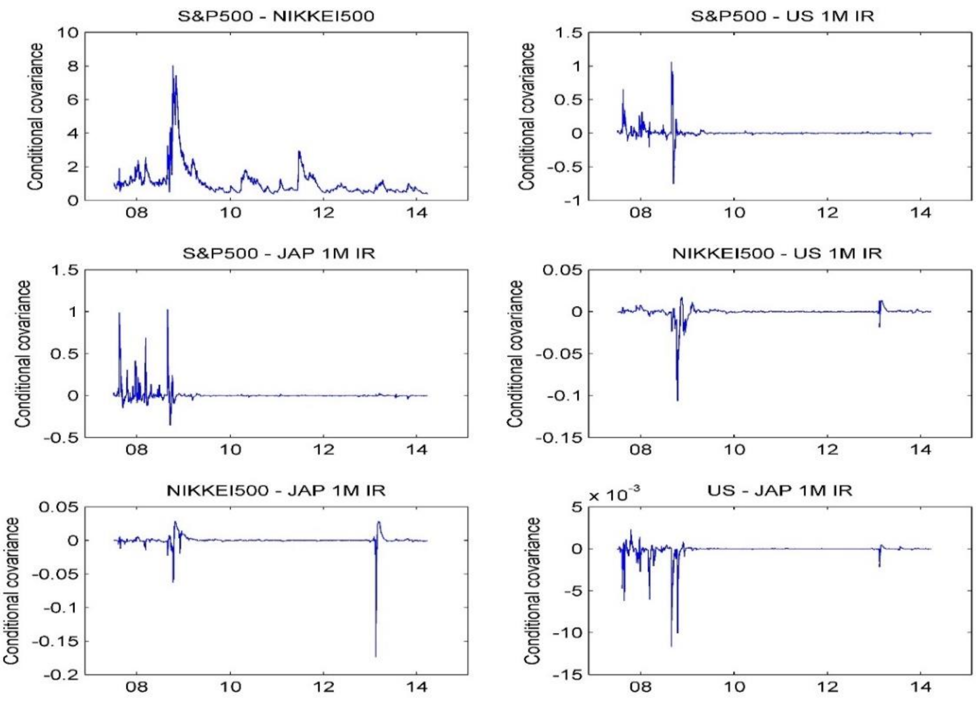

Figure 5.11b Conditional Covariance: U.S. – Japan Equity and Money Markets; Post-Crisis.

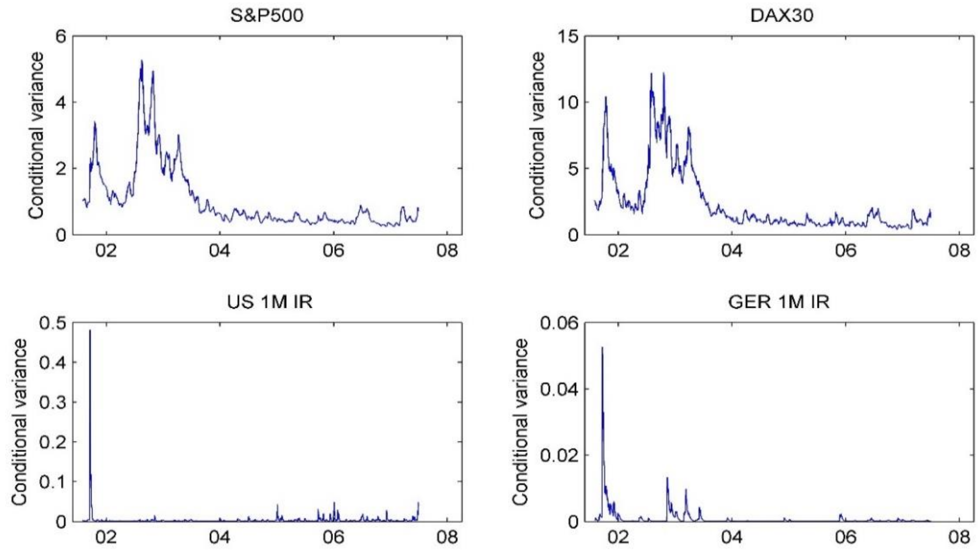

Figure 5.12a Conditional Variances: U.S. – Germany Equity and Money Markets; Before the Crisis.

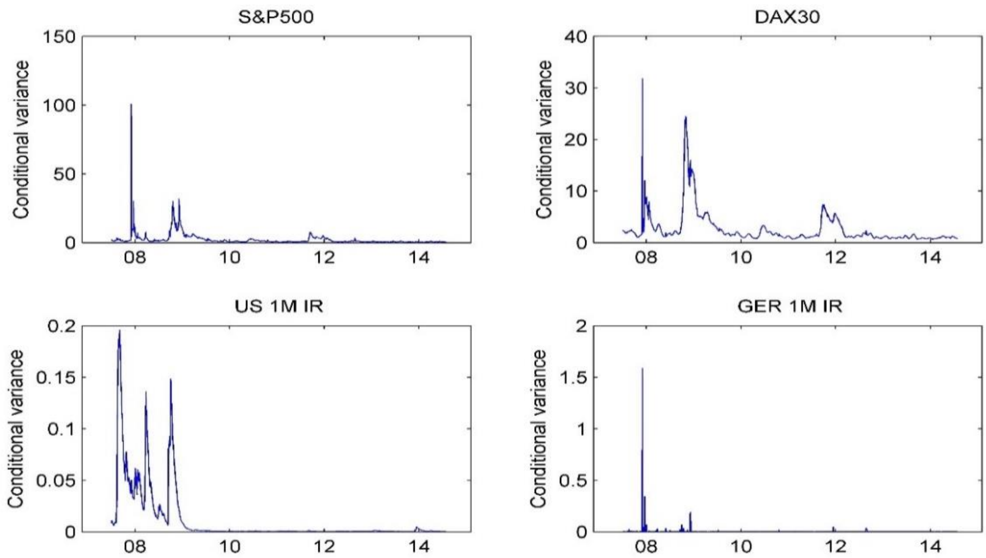

Figure 5.12b Conditional Variances: U.S.– Germany Equity and Money Markets; Post-Crisis.

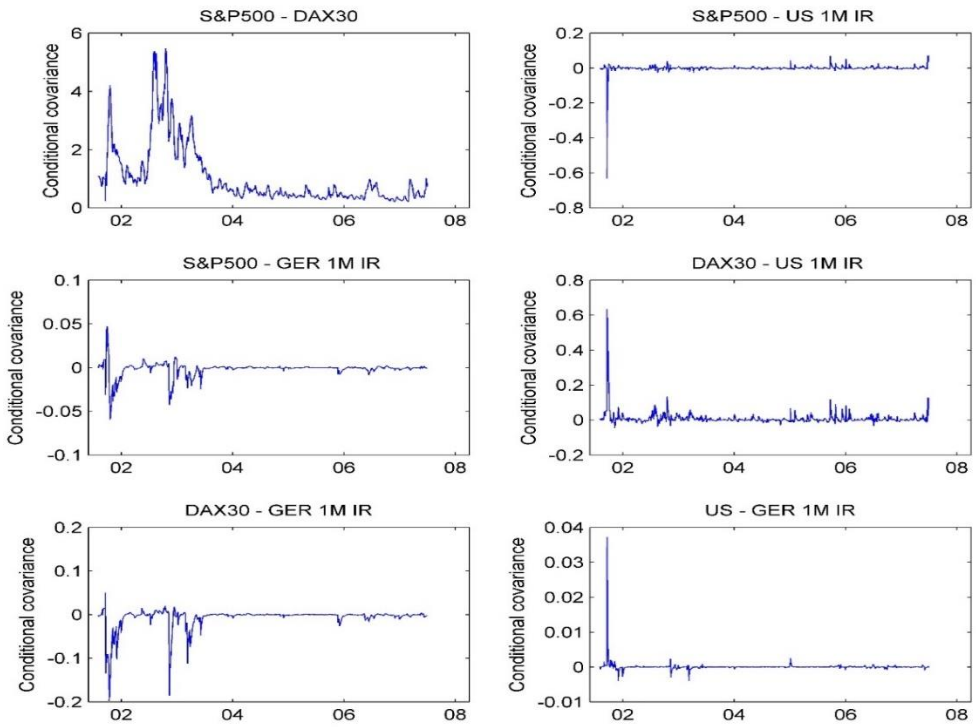

Figure 5.13a Conditional Covariance: U.S. – Germany Equity and Money Markets; Before the Crisis.

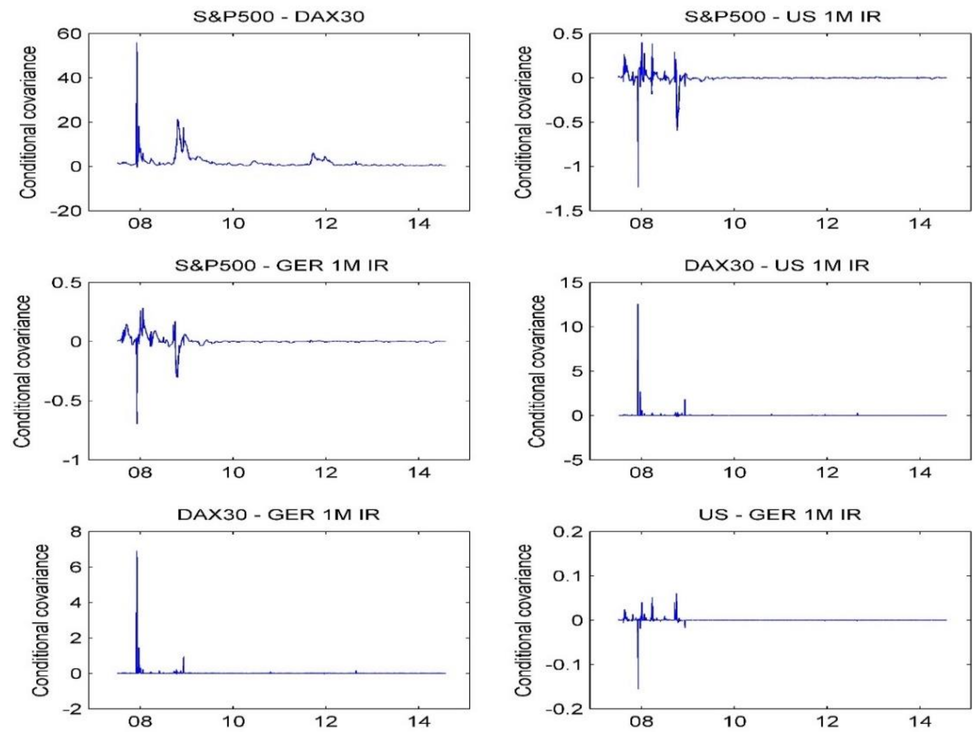

Figure 5.13b Conditional Covariance: U.S. – Germany Equity and Money Markets; Post-Crisis.

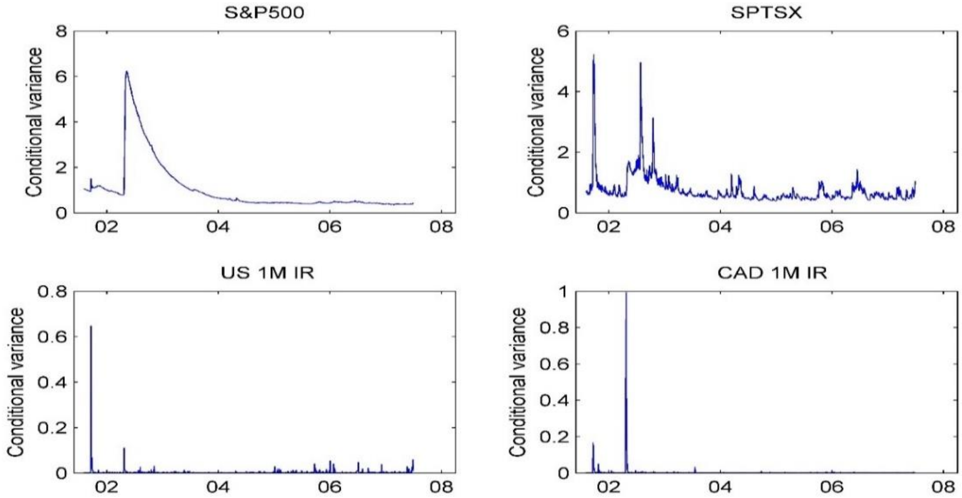

Figure 5.14a Conditional Variances: U.S. – Canada Equity and Money Markets; Before the Crisis.

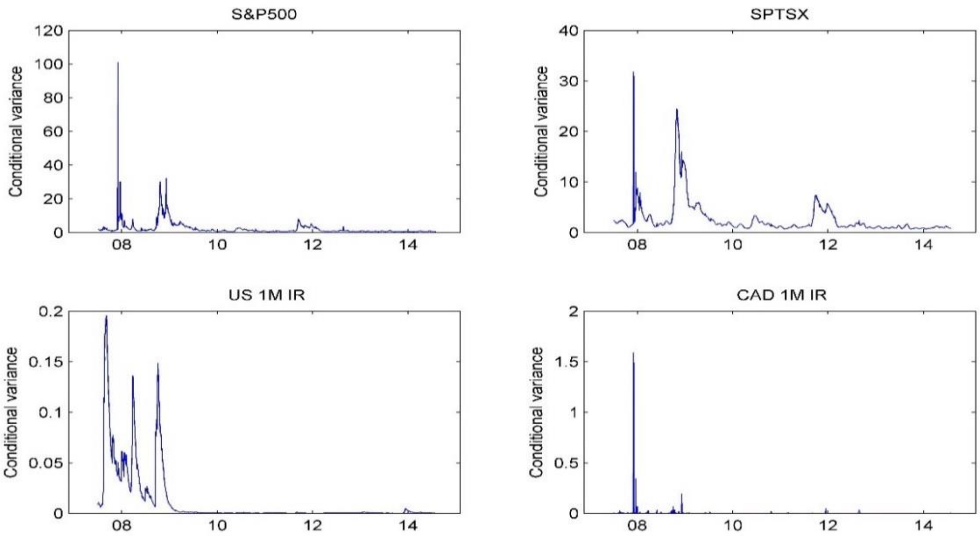

Figure 5.14b Conditional Variances: U.S. – Canada Equity and Money Markets; Post-Crisis.

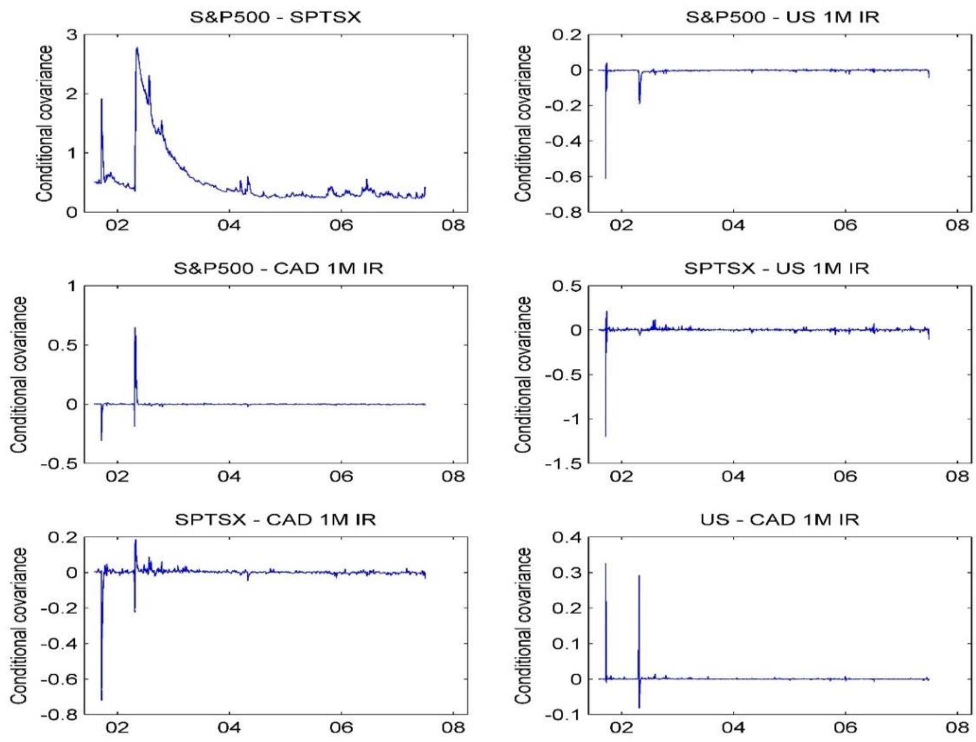

Figure 5.15a Conditional Covariance: U.S. –Canada Equity and Money Markets; Before the Crisis

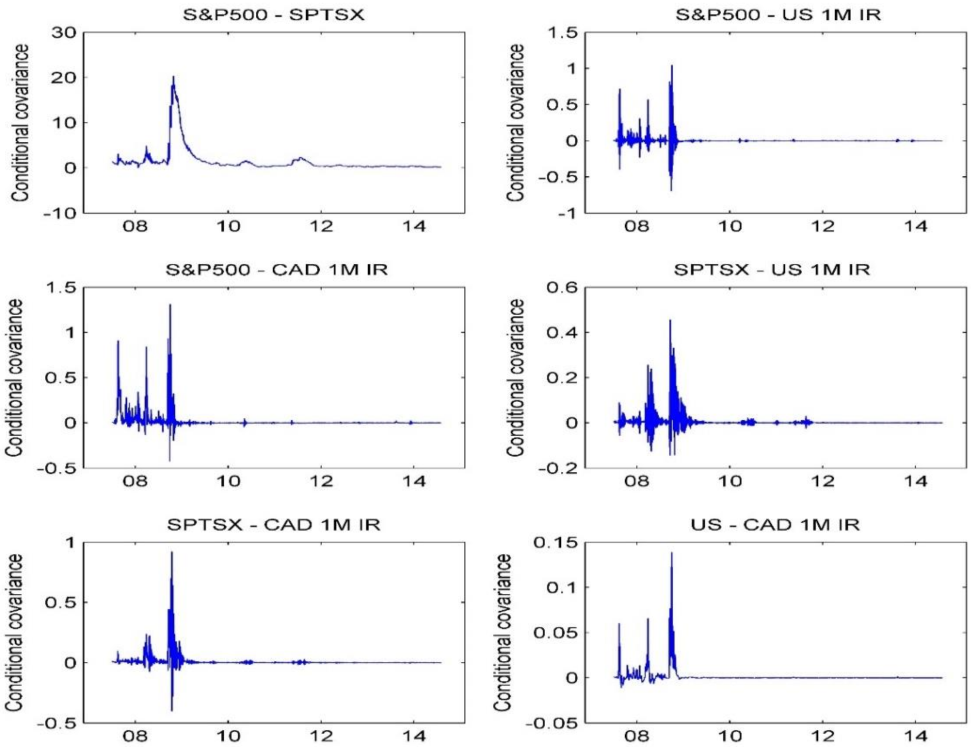

Figure 5.15b Conditional Covariance: U.S. – Canada Equity and Money Markets; Post-Crisis.

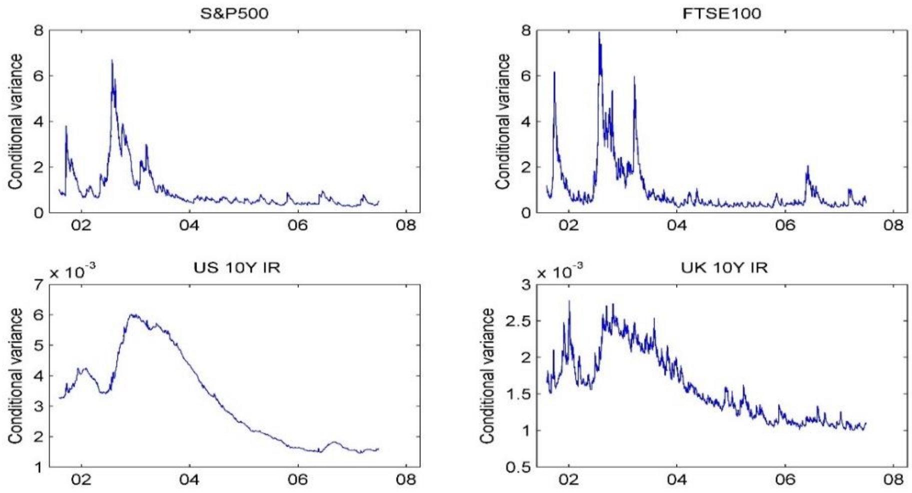

Figure 5.16a Conditional Variance: U.S. – U.K. Equity and Long-Term Bond Markets; Before the Crisis.

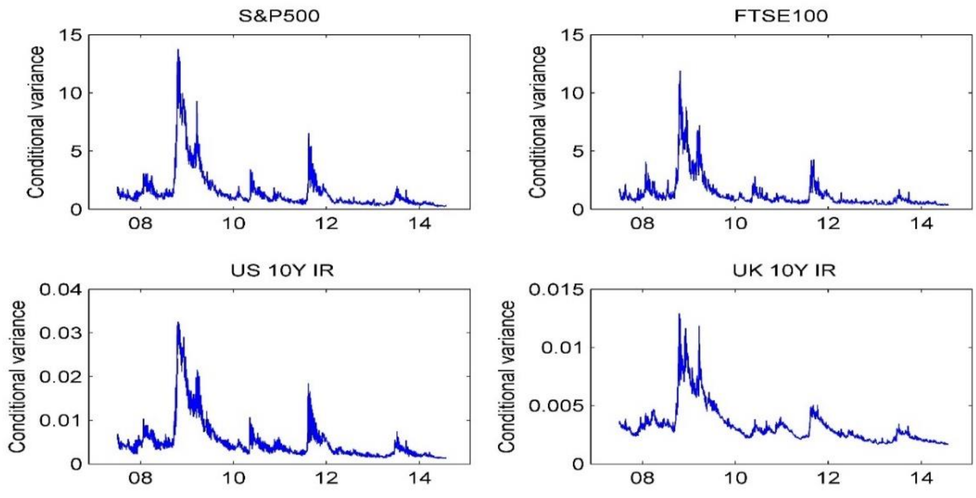

Figure 5.16b Conditional Variances: U.S.-U.K. Equity and Long-Term Bond Markets; Post-Crisis

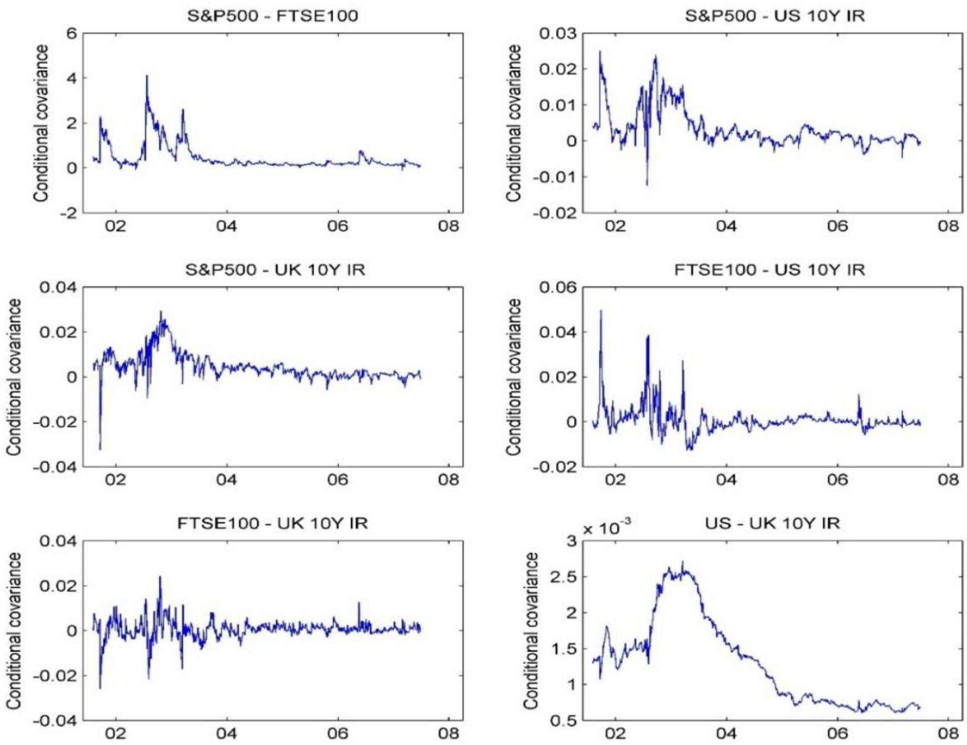

Figure 5.17a Conditional Covariances: U.S. - U.K. Equity and Long-Term Bond Markets; Before the Crisis.

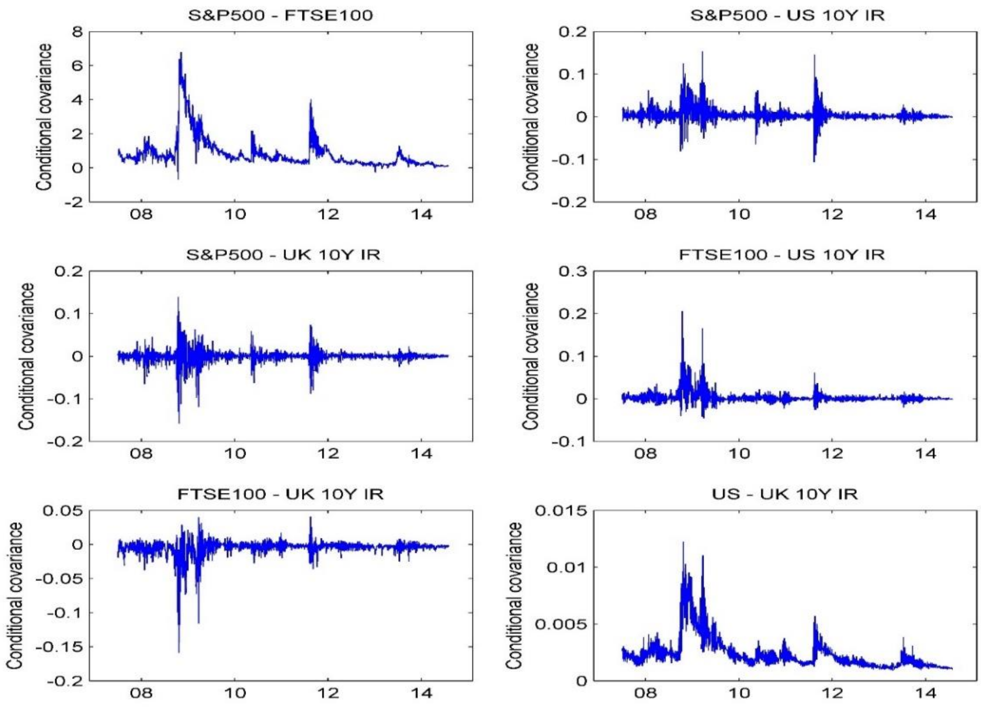

Figure 5.17b Conditional Covariance: U.S. - U.K. Equity and Long-Term Bond Markets; Post-Crisis.

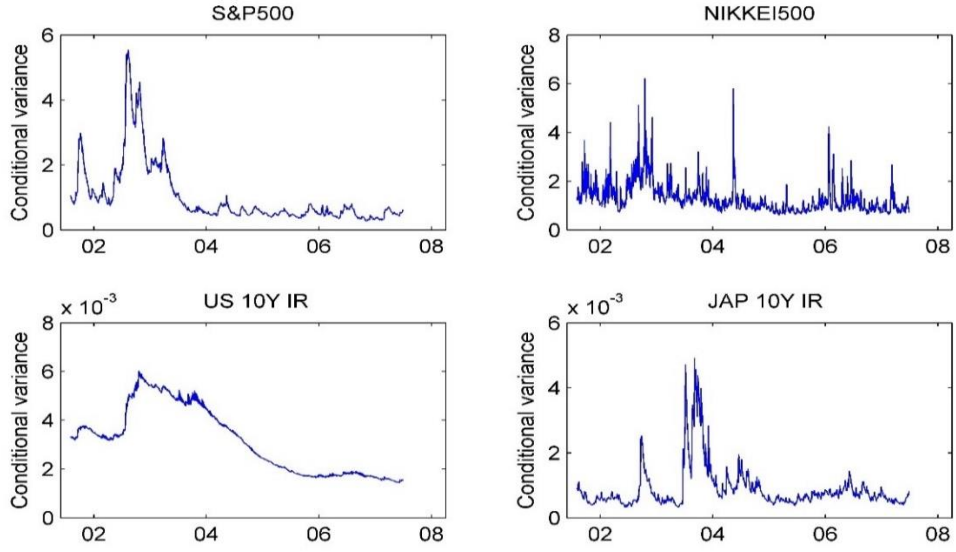

Figure 5.18a Conditional Variances: U.S.-Japan Equity and Long-Term Bond Markets; Before the Crisis

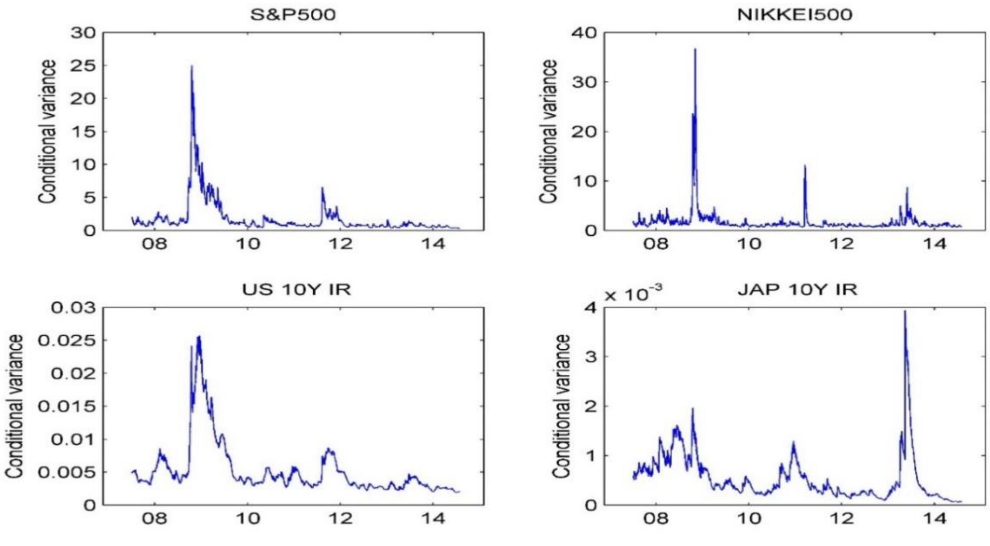

Figure 5.18b Conditional Variances: U.S.-Japan Equity and Long-Term Bond Markets; Post-Crisis

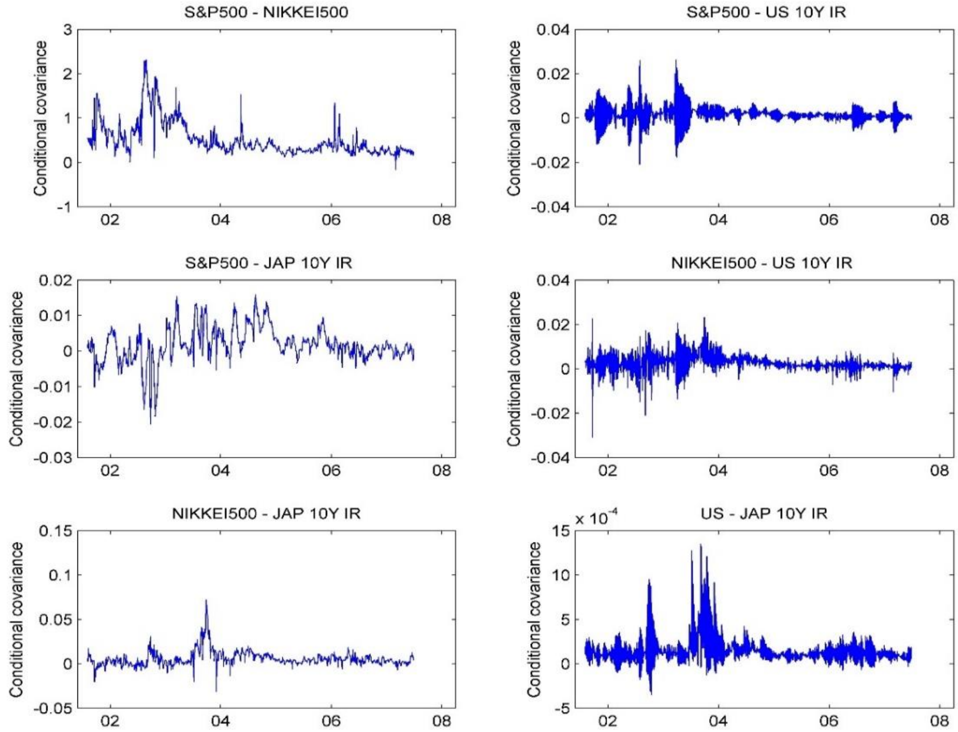

Figure 5.19a Conditional Covariance: U.S. – Japan Equity and Long-Term Bond Markets; Before the Crisis.

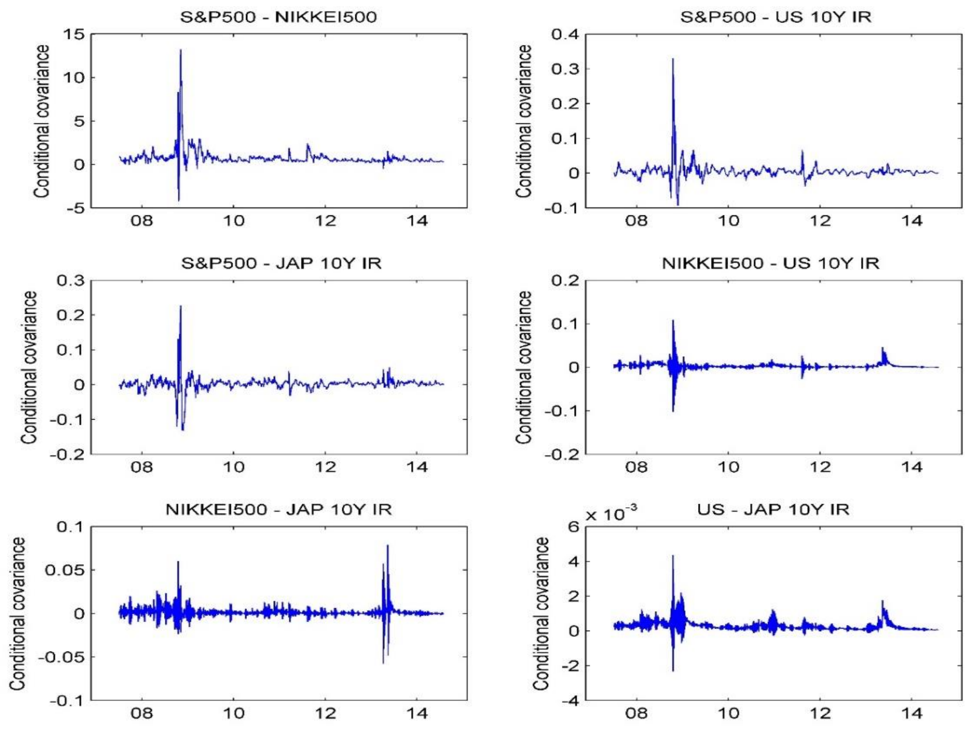

Figure 5.19b Conditional Covariance: U.S. – Japan Equity and Long-Term Bond Markets; Post-Crisis.

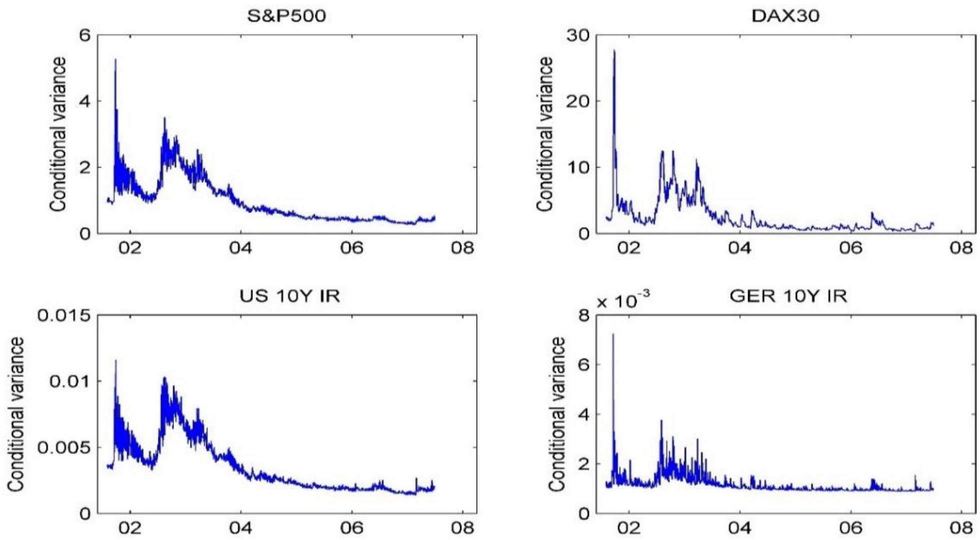

Figure 5.20a Conditional Variance: U.S. – Germany Equity and Long-Term Bond Markets; Before the Crisis

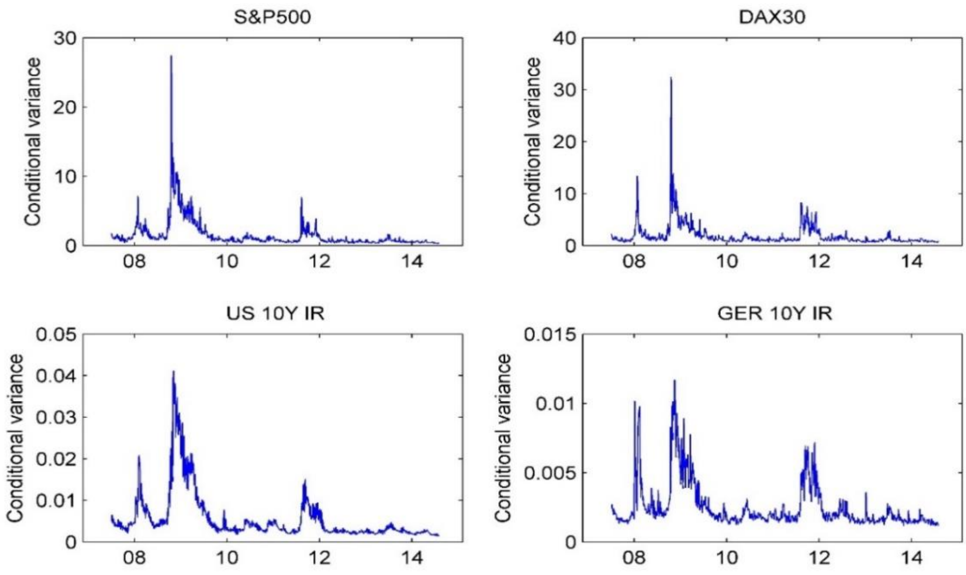

Figure 5.20b Conditional Variances: U.S. – Germany Equity and Long-Term Bond Markets; Post-Crisis

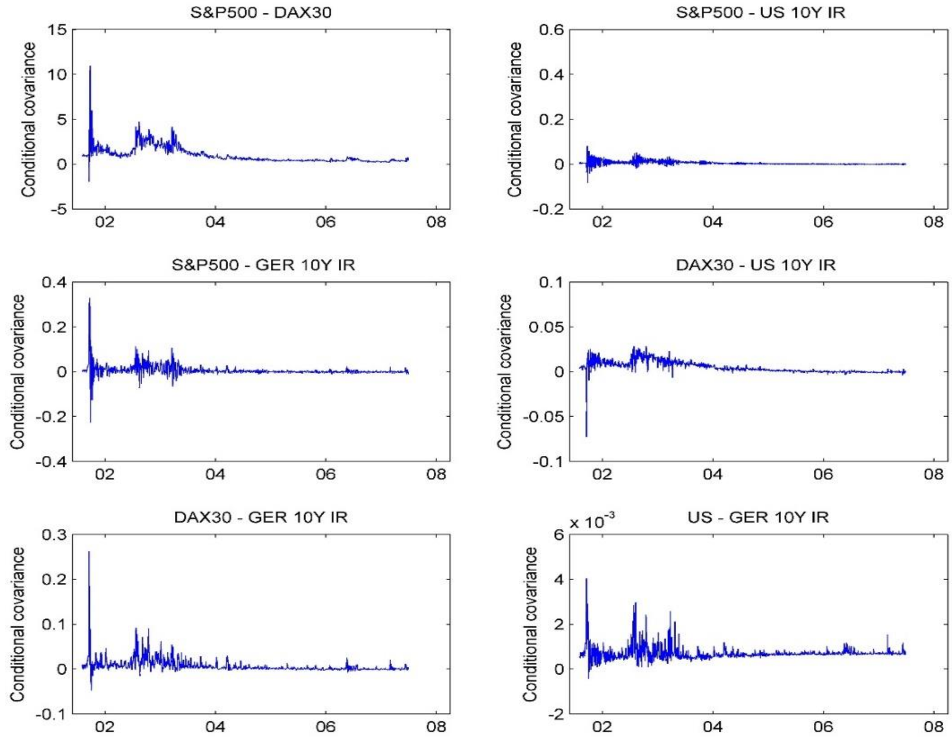

Figure 5.21a Conditional Covariance: U.S. – Germany Equity and Long-Term Bond Markets; Before the Crisis

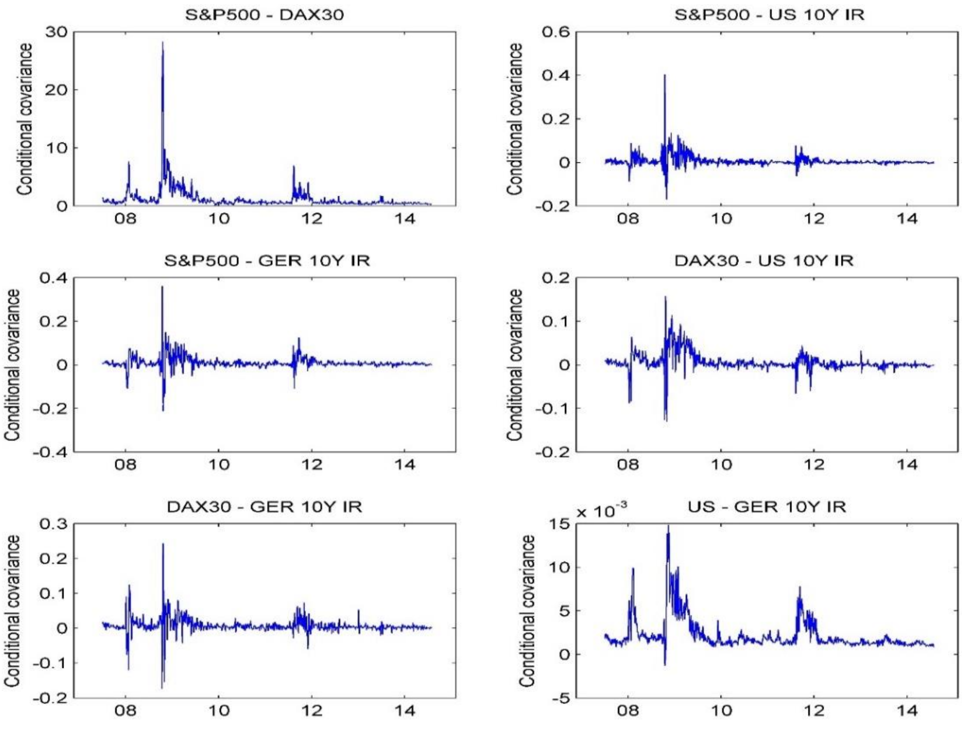

Figure 5.21b Conditional Covariances: U.S. – Germany Equity and Long-Term Bond Markets; Post-Crisis

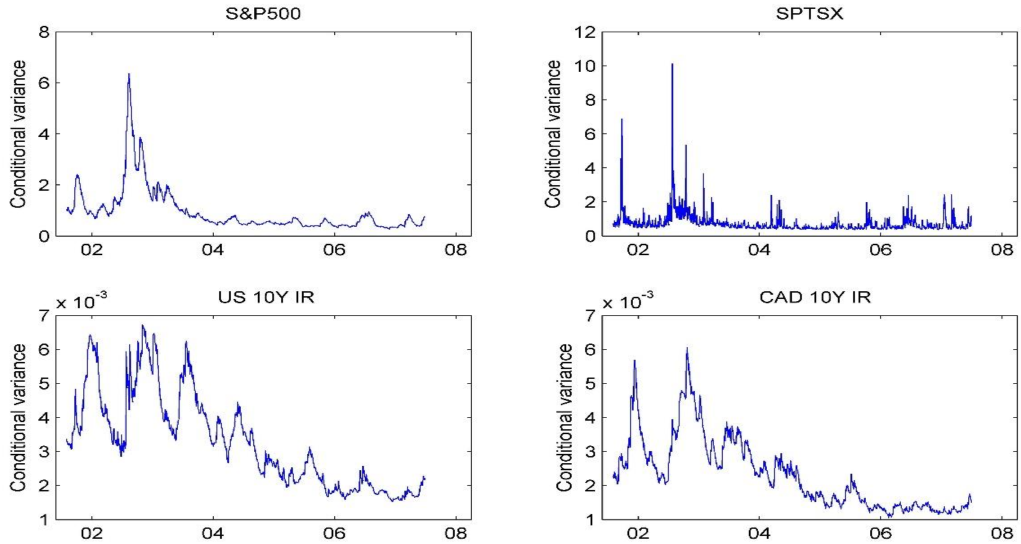

Figure 5.22a Conditional Variances: U.S. – Canada Equity and Long-Term Bond Markets; Before the Crisis

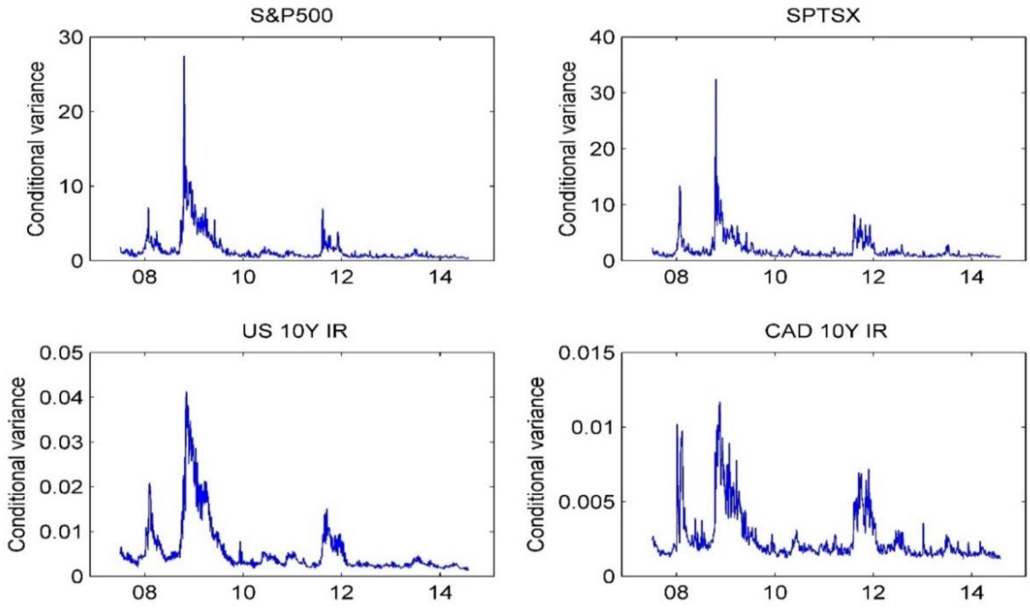

Figure 5.22b Conditional Variances: U.S. – Canada Equity and Long-Term Bond Markets; Post-Crisis

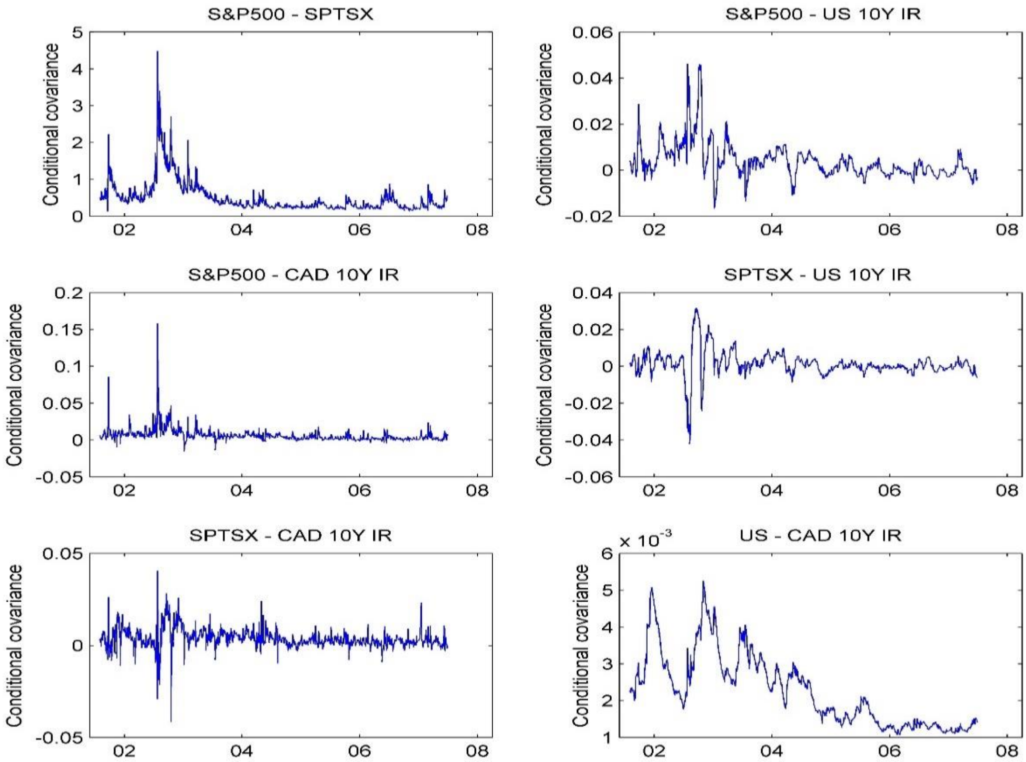

Figure 5.23a Conditional Covariance: U.S. – Canada Equity and Long-Term Bond Markets; Before the Crisis

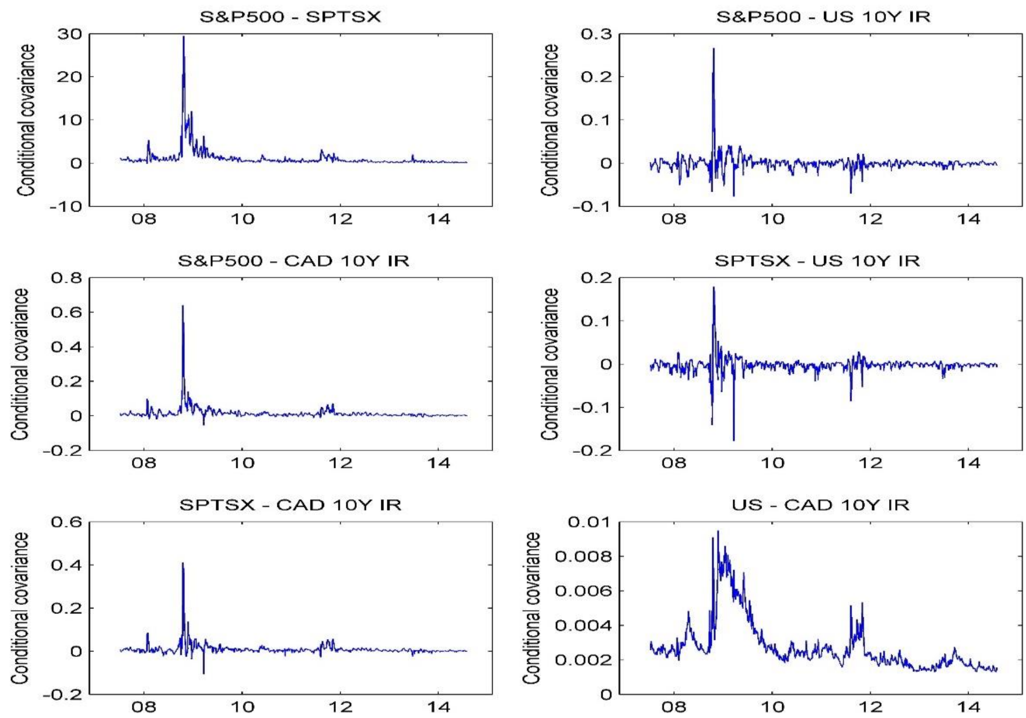

Figure 5.23b Conditional Covariance: U.S. – Canada Equity and Long-Term Bond Markets; Post-Crisis

5.7 Summary and Conclusions

This chapter investigates the impact of the global financial crisis of 2007-2009 (GFC) on the return and volatility spillovers dynamic effects among different types of markets, more specifically equity, long-term bonds and money markets. Given the high degree of financial integration shocks are likely to spread simultaneously at both, domestic and international level. We examine how the GFC has spread from the U.S. to four major economies, namely the U.K., Japan, Germany and Canada. We employ the discrete time MGARCH technique – the full BEKK(1,1) model with four variables. The empirical results are organised pairwise, for example for the U.S.-U.K. pair we look first at the individual channels (return and volatility) for each combination of the equity markets with short-term interest rates markets on one side, and the equity markets with Government bond markets on the other side.

The empirical results provide evidence that the GFC had impacted the relationships between major economic and financial markets. Despite the fact, that for each country analysed the information transmission mechanism relatively to the U.S., has its own particularities before the crisis, this mechanism is subject to similar changes during the crisis, with the exception of Canada.

When the equity markets are simultaneously modelled with the short-term interest rates markets, the individual analysis of each country-pair yields the following results. The transmission of information via return channel takes place in the domestic markets mostly in one direction from the U.S. money market to the equity market. The other economies do not provide evidence of interaction between their equity and money markets via the return channel (apart from Japan where the money market has a negative feedback effect on the equity market). For the direct international route, the U.S. equity markets have the leading role of exporting information, while the money markets seem to exchange very little information or with negative effects (U.S.- Japan). The leading role of the U.S. exporting information is also observed via the indirect international transmission route with the busiest direction from the U.S. money markets to the equity markets of the other economies.

The information transmission via volatility channel has similar patterns across countries. The domestic routes suggest that in all cases the U.S. markets interact less, while for the other economies the equity and the money market are communicating at a much higher intensity during the crisis. Regarding the direct (between the same asset classes) external routes there is a general pattern: the equity markets become closely interconnected (except for Japanese equity markets), while between the money markets there is barely any volatility spillover effects. There is evidence of substantial indirect international volatility transmission from the money markets of the countries analysed to S&P500 (except for Japan). In general, the values of the parameter estimates measuring the spillover effects are rather smaller. However, the intensity of the information flows between Canadian and U.S. markets is much greater than between the U.S. markets and the other economies. There can be two lines of reasoning for these findings, one being the geographic closeness between the two countries and second one - the different structure of the Canadian financial and baking markets proved most efficient in managing the consequences of the last global financial crisis of 2007-2009.

When the long-term segment of the yield curve is modelled in conjunction with the equity markets, the U.K., Japan and Germany seem to be dominated by the U.S. equity and bond markets. The situation for Canada reverses as there are strong bidirectional return spillovers especially from both equity and bond Canadian markets to their U.S. counterparty markets. The volatility channel seems to facilitate mostly the transmission of information via indirect external routes and via the domestic route in one direction only, from the bond markets to the equity markets. The results from the analysis of both return and volatility channels suggest that Canada exchanges information with the U.S. in a different way. During the crisis, for many routes the direction of the information flows changes such that the Canadian markets are transmitting information to the U.S. markets.

To assess the role of the U.S markets as the most important source of information shocks we have estimated the full BEKK four-variable model for all the possible combinations of pairing any two countries. A total of other 24 models have been estimated corresponding to six country-pairs over pre-crisis and post-crisis periods, keeping the same asset class combinations across the two segments of the yield curve. The estimation results for the additional combinations are presented in the Appendix at the end of the chapter. The findings from the analysis of the new pairs that do not contain the U.S. markets reveal weaker spillover effects especially through the volatility channel as the parameter estimates measuring the intensity of the information flow are substantially lower. Therefore, when the U.S. markets are modelled there is significant evidence of volatility spillovers effects along the indirect international routes, whereas when the U.S. markets are excluded these linkages are substantially diminished. This can be interpreted as additional evidence for the impact of the GFC on the return and volatility communication channels between the major global economies.

In more general terms, the comparative analysis of the summary results reported in the Table 5.21 concludes that out of the three types of routes of information transmission, the most active routes are the indirect external route followed by the domestic one. These results are valid for both return and volatility channels and it emphasises the importance of considering this type of routes, ignored previously in the spillovers literature. Along these routes, the information flows unidirectionally from the interest rate markets to the equity markets and not vice-versa, implying that the interest rate markets dominate the equity markets in transmission of information. When comparing the results of the two segments of the yield curve it is found that the return and volatility spillover effects are much stronger when the equity markets are modelled in combination with the long-term markets than with the money markets. Among the countries considered, the results for Canada are rather different as the Canadian markets seem to influence indirectly the U.S. markets, reflecting the relative stability that Canadian markets sustained during the crisis.

APPENDIX

Table A.1 U.K. – Japan, Stock and Money Markets; The estimation results for the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	FTSE100	NIKKEI500	UK 1M	JAP 1M	FTSE100	NIKKEI500	UK 1M	JAP 1M
intercept	0.06316***	0.00552	0.0010*	0.00002**	0.04117	0.02685	0.00007	-0.00001
AR(1)	-0.06938**	0.13186***	-0.03156	0.15354**	-0.01497	0.09154***	0.00978	0.11816***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromFTSE100	fromNIKKEI500	fromUK 1M	fromJAP 1M	fromFTSE100	fromNIKKEI500	fromUK 1M	fromJAP 1M
to FTSE100		0.02379	-0.26599	-1.98479		0.02052	-0.08845	-5.90688*
to NIKKEI500	0.02445		2.68493**	-4.78553	0.09962		2.54618***	-13.0478***
to UK 1M	0.00078	0.00032		0.00554	0.00023	0.00032	0.00978	
to JAP 1M	-0.00006***	-0.00009***	0.00219***		0.00002	-0.00003	-0.00386	
Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.03354				0.13340***			
C	0.12945***	-0.00000			-0.45215***	0.42476**		
i=1,…,4	0.00112**	-0.00000	-0.00000		-0.00084	-0.00154***	0.00000	
	0.00003	-0.00000	-0.00000	-0.00000	0.00007	0.00014*	0.00000	0.00000
ARCH effect matrix	0.12390***	-0.13182***	0.00248***	-0.00022***	0.18290***	0.15919***	-0.00054***	-0.00001
A	0.03942**	-0.09823***	-0.00347***	-0.00024 ***	-0.00754	0.36610***	-0.00093***	0.00003
i=1,…,4	0.55575	-4.38211***	0.06122***	0.00047	0.39480	4.36790***	-0.04221**	-0.05896***
	-0.52127	-20.09670***	0.06662	1.06070***	-3.59985	-11.52234***	0.50170***	0.60340***

GARCH effect matrix G i=1,…,4	0.99061***	0.03915***	-0.00158***	0.00004***	0.97372***	-0.00537	-0.00003	0.00001
	-0.00764**	0.97708***	-0.00142***	-0.00003	0.04113**	0.78389***	0.00127***	-0.00009***
	2.76855	3.42125***	0.96508***	-0.00117	-0.61547*	0.05068	0.87881***	0.08417***
	-0.18738	7.13702***	0.01019	0.80521***	0.69697	3.73540*	-1.59020***	0.82748***
Shock Spillovers	fromFTSE100	fromNIKKEI500	fromUK 1M	fromJAP 1M	fromFTSE100	fromNIKKEI500	fromUK 1M	fromJAP 1M
to FTSE100	0.01535***	0.00155***	0.30886	0.27173	0.03345***	0.00006	0.15587	12.95890
to NIKKEI500	0.01738***	0.00965***	19.20297***	403.877***	0.02534***	0.13403***	19.9853***	133.456***
to UK 1M	0.00001***	0.00001***	0.00375***	0.00444	0.00000***	0.00000***	0.00178**	0.25170***
to JAP 1M	0.00000***	0.00000***	0.00000	1.1250***	0.00000	0.00000	0.00348***	0.36409***
Volatility Spillovers	fromFTSE100	fromNIKKEI500	fromUK 1M	fromJAP 1M	fromFTSE100	fromNIKKEI500	fromUK 1M	fromJAP 1M
to FTSE100	0.98133***	0.00006***	7.66490	0.03511	0.94813***	0.00169***	0.37880*	0.48577
to NIKKEI500	0.00153***	0.95469***	11.70499***	50.93719***	0.00003	0.61448***	0.00257	13.953*
to UK 1M	0.00000***	0.00000***	0.93139***	0.00010	0.00000	0.00000***	0.77231***	2.52874***
to JAP 1M	0.00000***	0.00000	0.00000	0.64838***	0.00000	0.00000***	0.00708***	0.68472***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: The price discovery information is barely transmitted between the U.K. and Japanese markets, both before and after the crisis. The money markets seem to transmit some information to equity markets domestically and indirectly with negative effects though. However, the volatility channel is more active with dominance from the Japanese equity market (direct international transmission to FTSE100, 0.61448) and money market (direct international transmission to the U.K. money market, 0.68472).

Table A.2 U.K. – Japan, Stock and Bond Markets; The estimation results for the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	FTSE100	NIKKEI500	UK 10Y	JAP 10Y	FTSE100	NIKKEI500	UK 10Y	JAP 10Y
intercept	0.04413**	0.04158	-0.00016	-0.00002	0.03388**	0.06976	-0.00091	-0.00067
AR(1)	-0.12623***	0.03909*	0.04064	-0.04679	-0.10914***	0.02703*	0.00336	-0.04762
The Return Spillovers	**Pre-crisis**				**Post-crisis**			
Matrix	from FTSE100	fromNIKKEI500	fromUK 10Y	fromJAP 10Y	fromFTSE100	fromNIKKEI500	fromUK 10Y	fromJAP 10Y
to FTSE100		0.05962***	3.68213***	1.53502**		0.15197***	5.99571***	2.21252**
to NIKKEI500	0.06052**		2.92427***	7.45934***	-0.02386**		4.99501***	8.78173***
to UK 10Y	0.00077	-0.00018		0.00851	-0.00154	0.00205		0.18633
to JAP 10Y	0.00000	0.00044	0.09719***		-0.00119	0.00045	0.08796***	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.14350***				0.06287***			
C	-0.01071	-0.03922*			-0.04644	0.22426*		
i=1,…,4	0.00314***	0.00203***	0.00130*		-0.03621***	-0.00773***	0.03549*	
	0.00047	0.00135	0.00451***	-0.00078	-0.00051	-0.00095	0.00048***	0.00000
ARCH effect matrix	0.33953***	0.01397	0.00178***	-0.00011	0.27770***	-0.04622	-0.01396***	-0.00012
A	-0.01653	0.12064***	-0.00025	-0.00005	-0.05001	0.29814***	0.00016	-0.00148
i=1,…,4	-0.78285	-2.51206***	-0.03095*	0.00157	0.38267	0.14143***	0.19012*	-0.01342
	0.42819	1.40912**	0.05072**	0.24905***	1.25919	9.42070**	-0.21169***	0.16942***

GARCH effect matrix G i=1,…,4	0.92569***	-0.00273	-0.00083***	0.00006	0.95243***	0.01152	0.00312***	-0.00008
	0.01386**	0.98832***	0.00024**	0.00008	0.02455**	0.92613***	0.00515**	0.00085
	-0.21094	0.03201	0.99380***	-0.00054	-1.48128*	-1.87137	0.00711***	-0.00600
	-0.09985	-0.11994	-0.01148**	0.95453***	0.18295	-0.61524	0.42724**	0.97898***
Shock Spillovers	fromFTSE100	fromNIKKEI500	fromUK 10Y	fromJAP 10Y	fromFTSE100	fromNIKKEI500	fromUK 10Y	fromJAP 10Y
to FTSE100	0.11528***	0.00027	0.61285	0.18335	0.07712***	0.00250	0.14643***	1.58557
to NIKKEI500	0.00020	0.01455***	6.31047***	1.98563**	0.00214	0.08889***	0.02000***	88.74957**
to UK 10Y	0.00000***	0.00000	0.00096	0.00257**	0.00019***	0.00000	0.03615*	0.04481***
to JAP 10Y	0.00000	0.00000	0.00000	0.06202***	0.00000	0.00000	0.00018	0.02870***
Volatility Spillovers	fromFTSE100	fromNIKKEI500	fromUK 10Y	fromJAP 10Y	fromFTSE100	fromNIKKEI500	fromUK 10Y	fromJAP 10Y
to FTSE100	0.85691***	0.00019**	0.04450	0.00997	0.90713***	0.00060**	2.19418*	0.03347
to NIKKEI500	0.00001	0.97678***	0.00102	0.01438	0.00013	0.85772***	3.50202	0.37852
to UK 10Y	0.00000***	0.00000**	0.98764***	0.00013**	0.00001***	0.00003**	0.00005***	0.18253**
to JAP 10Y	0.00000	0.00000	0.00000	0.91112***	0.00000	0.00000	0.00004	0.95840***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: When bond markets are modelled simultaneously with the equity markets the information via return channel flows on all six routes with intensified effects as a result of the crisis. The domestic and the indirect international flows from the bond markets to the equity markets double in the post-crisis period. The volatility spillovers effects are relatively small (two out of twelve estimates are larger), the strongest linkages being between the U.K. markets (domestic bond to equity effect) and between the Japanese and the U.K. bond markets (direct international transmission).

Table A.3 U.K. – Germany, Stock and Money Markets; The estimation results for the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	FTSE100	DAX30	UK 1M	GER 1M	FTSE100	DAX30	UK 1M	GER 1M
intercept	0.05524***	0.10301***	0.00105*	0.00009	0.00465**	0.00803***	-0.00011	-0.00041***
AR(1)	-0.30259***	0.05456*	-0.00321	0.26287***	-0.29292***	0.00524	-0.07088***	0.78179***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromFTSE100	fromDAX30	fromUK 1M	fromGER 1M	fromFTSE100	fromDAX30	fromUK 1M	fromGER 1M
to FTSE100		0.22987***	-0.56586	-0.64735		0.31932***	-0.07595	0.65290***
to DAX30	-0.15017***		-0.91912	-0.99591	-0.04774*		-0.00944	0.78127***
to UK 1M	0.00168**	-0.00134**		0.05025	-0.00203	0.00475***		0.09301***
to GER 1M	-0.00001	-0.00014*	0.01385***		0.00593***	0.00060***	-0.04027***	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.06166***				0.01163***			
	0.00345	-0.01731			0.01088***	0.01654***		
C	0.01724***	-0.0050***	0.00000		0.00009***	0.00006	0.00000	
i=1,…,4	-0.00016	-0.00024	0.00000	0.00000	0.00003	0.00000	0.00000	0.00000
ARCH effect matrix	0.26814***	0.15395***	0.00100***	-0.00188***	0.23001***	0.06424***	-0.00330***	-0.00145
	-0.06025***	0.09817***	0.00047	0.00132***	0.00222	0.23303***	0.00180**	-0.00639***
A	-0.82213	0.18377	-0.02753	-0.04262***	0.34878***	0.67346***	0.23204***	0.08414***
i=1,…,4	-11.61518***	-16.51926***	-0.29700***	1.37995***	2.57187***	2.69329***	0.16644***	2.22396***

GARCH effect matrix G i=1,…,4	0.89303***	-0.16357***	0.00154**	0.00012	0.97095***	-0.00597	0.00049***	-0.00041
	0.07047***	1.04935***	-0.00190***	0.00000	-0.00348***	0.95514***	-0.00038***	0.00031
	-3.79382***	-8.65907***	-0.61946***	0.01890***	-0.02168*	-0.06436***	0.97992***	0.00297
	4.47776***	6.40879***	0.46509***	0.69791***	-0.38578***	-0.37433***	-0.04938***	0.61098***
Shock Spillovers	fromFTSE100	fromDAX30	fromUK 1M	fromGER 1M	fromFTSE100	fromDAX30	fromUK 1M	fromGER 1M
to FTSE100	0.07190***	0.00363***	0.67591	134.91249***	0.05291***	0.00000	0.12165***	6.61452***
to DAX30	0.02370***	0.00964***	0.03377	272.88592***	0.00413***	0.05430***	0.45355***	7.25383***
to UK 1M	0.00000***	0.00000	0.00076	0.08821***	0.00001***	0.00000**	0.05384***	0.02770***
to GER 1M	0.00000***	0.00000***	0.00182***	1.90427***	0.00000	0.00004***	0.00708***	4.94599***
Volatility Spillovers	fromFTSE100	fromDAX30	fromUK 1M	fromGER 1M	fromFTSE100	fromDAX30	fromUK 1M	fromGER 1M
to FTSE100	0.79750***	0.00497***	14.39309***	20.05030***	0.94275***	0.00001***	0.00047*	0.14883***
to DAX30	0.02676***	1.10114***	74.97957***	41.07256***	0.00004	0.91229***	0.00414***	0.14012***
to UK 1M	0.00000**	0.00000***	0.38373***	0.21631***	0.00000***	0.00000***	0.96025***	0.00244***
to GER 1M	0.00000	0.00000	0.00036***	0.48708***	0.00000	0.00000	0.00001	0.37329***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: Clearly, U.K. communicates closer with Germany than with Japan via both return and volatility channels. The busiest return routes are from the German money market to both its equity market (domestic transmission) and the U.K. money market (direct international transmission). The volatility channel between these two countries is has most of its twelve flow estimates significant, however their values are rather small. The most impact via the volatility channel comes from the German money market towards all the other types of markets in the model.

Table A.4 U.K. – Germany, Stock and Bond Markets; The estimation results for the FULL BEKK Model

Panel A **The Mean Equation**	**Pre-crisis**				**Post-crisis**			
	FTSE100	DAX30	UK 10Y	GER 10Y	FTSE100	DAX30	UK 10Y	GER 10Y
intercept	0.00529**	0.01127***	0.00079	-0.00006	0.00484**	0.00801***	-0.00030	-0.00140
AR(1)	-0.30028***	0.05100	0.04426*	-0.01315	-0.22227***	-0.06378**	0.01408	0.03609
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromFTSE100	fromDAX30	fromUK 10Y	fromGER 10Y	fromFTSE100	fromDAX30	fromUK 10Y	fromGER 10Y
to FTSE100		0.18766***	0.44088***	-0.02711		0.24243***	0.15259**	0.85350***
to DAX30	-0.15070***		0.55072***	0.16480**	0.03471		0.11924	0.85895***
to UK 10Y	0.01605	-0.01120		-0.05718**	-0.00699	-0.00771		0.01397
to GER 10Y	-0.02023**	0.03515***	0.59209***		-0.00956	-0.01728*	0.05478*	

Panel B **The Variance Equation**	**Pre-crisis**				**Post-crisis**			
	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.02189***				0.01555***			
C	-0.01413***	-0.00539***			0.00672**	0.01080***		
i=1,…,4	-0.00212***	0.00241***	0.00000		0.00115	0.00416***	0.00000	
	-0.00090**	0.00023	0.00000	0.00000	0.01310***	0.01132***	0.00000	0.00000
ARCH effect matrix	0.30680***	0.01854	0.00155	-0.00514	0.35161***	0.25810***	0.01453*	0.07732***
A	0.05686*	0.25645***	0.00785*	0.01241***	-0.11570***	0.13751***	-0.00459	-0.07532***
i=1,…,4	0.03529	0.11144	0.00961**	0.01210	-0.18545***	-0.20975***	0.00491	-0.28117***
	-0.02437	-0.07962	0.04482	-0.01988	0.19580***	0.26771***	0.04715**	0.39732***

GARCH effect matrix G i=1,…,4	0.88605***	-0.06460***	-0.00748***	-0.00157	0.90997***	-0.09298***	-0.01275***	-0.04509***
	0.01894*	0.98907***	0.00035	-0.00234***	0.05236***	0.99443***	0.00798**	0.04207***
	-0.00610	0.01542*	0.99540***	0.00025	0.25596***	0.42280***	0.97082***	0.12327***
	-0.01931	-0.00726	-0.00338**	0.99686***	-0.39282***	-0.70007***	0.03251***	0.74710***
Shock Spillovers	fromFTSE100	fromDAX30	fromUK 10Y	fromGER 10Y	fromFTSE100	fromDAX30	fromUK 10Y	fromGER 10Y
to FTSE100	0.09412***	0.00323*	0.00125	0.00059	0.12363***	0.01339***	0.03439***	0.03834***
to DAX30	0.00034	0.06577***	0.01242	0.00634	0.06662***	0.01891***	0.04400***	0.07167***
to UK 10Y	0.00000	0.00006*	0.00009**	0.00201	0.00021*	0.00002	0.00002	0.00222**
to GER 10Y	0.00003	0.00015***	0.00015	0.00040	0.00598***	0.00567***	0.07906***	0.15786***
Volatility Spillovers	fromFTSE100	fromDAX30	fromUK 10Y	fromGER 10Y	fromFTSE100	fromDAX30	fromUK 10Y	fromGER 10Y
to FTSE100	0.78508***	0.00036*	0.00004	0.00037	0.82805***	0.00274***	0.06551***	0.15431***
to DAX30	0.00417***	0.97825***	0.00024*	0.00005	0.00865***	0.98889***	0.17876***	0.49010***
to UK 10Y	0.00006***	0.00000	0.99082***	0.00001**	0.00016***	0.00006**	0.94250***	0.00106***
to GER 10Y	0.00000	0.00001***	0.00000	0.99372***	0.00203***	0.00177***	0.01520***	0.55816***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: As in the case of money markets the information is transmitted in a very similar way as the return channel is concerned.; the German bond market improve its leading role both internally and internationally. The crisis had a great impact on the volatility spillovers effects, as after the crisis all the twelve flows in the network are significant. The busiest routes are the direct international route from the German to the U.K. bond market and the domestic route inside Germany from the bond to the equity market.

Table A.5 U.K. – Canada, Stock and Money Markets; The estimation results for the FULL BEKK Model

Panel A **The Mean Equation**	**Pre-crisis**				**Post-crisis**			
	FTSE100	SPTSX	UK 1M	CAD 1M	FTSE100	SPTSX	UK 1M	CAD 1M
intercept	0.00248	0.00712**	0.00131**	0.00287***	0.00094	0.00521**	0.00006	0.00004
AR(1)	-0.21592***	0.03135	-0.01041	0.20999***	-0.16534***	-0.02305	-0.03330	-0.12953***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromFTSE100	fromSPTSX	fromUK 1M	fromCAD 1M	fromFTSE100	fromSPTSX	fromUK 1M	fromCAD 1M
to FTSE100		0.44023***	-0.06545	0.06443		0.56523***	0.02875	0.38843***
to SPTSX	0.02248		-0.13701	-0.02306	0.02023		-0.01608	0.00639
to US 1M	-0.00357	-0.00358		-0.00500	-0.00013	0.00407		0.07196***
to CAD 1M	-0.00655	-0.00767	-0.03444*		-0.00317	0.00786**	0.07229**	

Panel B **The Variance Equation**	**Pre-crisis**				**Post-crisis**			
	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.00850***				0.01004***			
C	0.03326***	0.00341			0.00926***	0.00432***		
i=1,…,4	0.00293***	0.00005	0.00000		0.00046***	0.00070***	0.00000	
	0.00563**	-0.00608**	0.00000	0.00000	0.00166***	-0.00112***	0.00000	0.00000
ARCH effect matrix	0.24606***	0.28949***	-0.04463***	0.03916***	0.13890***	-0.11680***	-0.00626***	-0.01340***
A	-0.11438***	-0.02918	0.05431***	0.00130	0.06587***	0.19964***	0.00142	0.00119
i=1,…,4	0.82963***	0.58327***	-0.14074***	0.07810**	-0.04444	0.00330	0.23322***	0.12489***
	0.07894	0.02454	-0.01643	1.02937***	0.12936***	0.18861***	-0.01262***	0.41090***

GARCH effect matrix G i=1,…,4	0.72336***	-0.11377***	0.13592***	0.00931*	0.94425***	-0.08662***	0.00141***	0.00105
	0.30541***	0.87360***	-0.07368***	-0.03629***	0.09912	1.00462	-0.00188***	-0.00116*
	-2.07281***	0.79103***	0.76261***	0.02788	-0.01982	0.01827	0.97399***	0.00797***
	-0.00927*	-0.00659	-0.03038***	0.76508***	-0.09844****	-0.07342***	-0.00687***	0.92694***
Shock Spillovers	fromFTSE100	fromSPTSX	fromUK 1M	fromCAD 1M	fromFTSE100	fromSPTSX	fromUK 1M	fromCAD 1M
to FTSE100	0.06055***	0.01308***	0.68828***	0.00623	0.01929***	0.00434***	0.00197	0.01674***
to SPTSX	0.08380***	0.00085	0.34020***	0.00060	0.01364***	0.03986***	0.00001	0.03557***
to US 1M	0.00199***	0.00295***	0.01981***	0.00027	0.00004***	0.00000	0.05439***	0.00016***
to CAD 1M	0.00153***	0.00000	0.00610	1.05961***	0.00018***	0.00000	0.01560***	0.16884***
Volatility Spillovers	fromFTSE100	fromSPTSX	fromUK 1M	fromCAD 1M	fromFTSE100	fromSPTSX	fromUK 1M	fromCAD 1M
to FTSE100	0.52325***	0.09328***	4.29654***	0.00009*	0.89160***	0.00982	0.00039	0.00969***
to SPTSX	0.01294***	0.76318***	0.62573***	0.00004	0.00750	1.00926	0.00033	0.00539***
to US 1M	0.01847***	0.00543***	0.58157***	0.00092***	0.00000	0.00000***	0.94865***	0.00005***
to CAD 1M	0.00009*	0.00132***	0.00078	0.58535***	0.00000***	0.00000*	0.00006*	0.85923***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: The equity and money markets of the U.K and Canada seem to communicate more before the crisis. After the crisis, some routes of information transmission disappear, the bidirectional exchange of information become unidirectional, from the Canadian markets to the U.K. markets. Most information spillovers effects are severely reduced to minimal influences after the crisis. The return channel facilitates the highest flows when compared to the volatility channel, with the indirect international routes from the Canadian markets to the U.K markets being the busiest ones.

Table 5.6 U.K. – Canada, Stock and Bond Markets; The estimation results for the FULL BEKK Model

Panel A **The Mean Equation**	**Pre-crisis**				**Post-crisis**			
	FTSE100	SPTSX	UK 10Y	CAD 10Y	FTSE100	SPTSX	UK 10Y	CAD 10Y
intercept	0.00413*	0.00957***	0.00065	-0.00071	0.00347	0.00423**	-0.00058	-0.00115
AR(1)	-0.24311***	-0.01094	-0.03467	-0.06141**	-0.12143***	-0.09585***	-0.05483***	-0.06914***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromFTSE100	fromSPTSX	fromUK 10Y	fromCAD 10Y	fromFTSE100	fromSPTSX	fromUK 10Y	fromCAD 10Y
to FTSE100		0.39981***	0.29242***	0.15234***		0.39408***	0.55819***	0.39690***
to SPTSX	-0.00011		0.04618	0.09204*	0.01688		0.04675	0.29745***
to UK 10Y	0.00089	0.00504		0.16462***	-0.01420	0.00885		0.20072***
to CAD 10Y	0.00755	-0.01593	0.15026***		-0.01672**	0.01360	0.03331	

Panel B **The Variance Equation**	**Pre-crisis**				**Post-crisis**			
	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
	0.00480*				0.01989***			
intercept matrix	0.04576***	0.00000			0.00413	-0.00308		
C	-0.00052	0.00000	0.00000		0.00052	-0.00377***	0.00000	
i=1,…,4	-0.00549***	0.00000	0.00000	0.00000	0.00127	0.00325***	0.00000	0.00000
ARCH effect matrix	0.26220***	0.11368***	-0.00391	-0.00758	0.25369***	0.11665***	-0.03010***	-0.02200***
A	-0.01981	0.39380***	0.02029***	0.00371	-0.05114	0.28393***	0.03156***	0.03010***
i=1,…,4	-0.21157***	0.14931	-0.01143	0.03482*	-0.01773	-0.02543	0.03877**	-0.08168***
	0.01990	-0.11258	-0.01821**	-0.12081***	-0.09205*	-0.10306**	-0.14773***	-0.03228*

GARCH effect matrix G i=1,…,4								
	0.94315***	-0.00296	-0.00102	-0.00807***	0.94330***	-0.04198***	0.00784***	0.00722***
	0.06004***	0.73766***	-0.00170***	0.02621***	0.03514**	0.95869***	-0.00233	-0.00673***
	-0.01486*	0.02262	0.99435***	-0.01107***	-0.02347**	-0.01589	0.98595***	0.00124
	0.01395	0.08713***	0.00759***	0.98664***	-0.03157	0.02206	0.00887**	0.99209***
	fromFTSE100	fromSPTSX	fromUK 10Y	fromCAD 10Y	fromFTSE100	fromSPTSX	fromUK 10Y	fromCAD 10Y
to FTSE100	0.06875***	0.00039	0.04476***	0.00040	0.06436***	0.00261	0.00031	0.00847*
to SPTSX	0.01292***	0.15508***	0.02229	0.01267	0.01361***	0.08062***	0.00065	0.01062**
to UK 10Y	0.00002	0.00041***	0.00013	0.00033**	0.00091***	0.00100***	0.00150**	0.02182***
to CAD 10Y	0.00006	0.00001	0.00121*	0.01459***	0.00048***	0.00091***	0.00667***	0.00104*
	fromFTSE100	fromSPTSX	fromUK 10Y	fromCAD 10Y	fromFTSE100	fromSPTSX	fromUK 10Y	fromCAD 10Y
to FTSE100	0.88953***	0.00360***	0.00022*	0.00019	0.88982***	0.00124***	0.00055**	0.00100
to SPTSX	0.00001	0.54414***	0.00051	0.00759***	0.00176***	0.91908***	0.00025	0.00049
to UK 10Y	0.00000	0.00000***	0.98872***	0.00006***	0.00006***	0.00001	0.97210***	0.00008**
to CAD 10Y	0.00007***	0.00069***	0.00012***	0.97345***	0.00005***	0.00005***	0.00000	0.98424***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: The dynamics of the information spillovers effects between the U.K and Canada are similar across the two segments of the yield curve. The crisis had different effects on the return and volatility channels. The influence of Canadian markets has been consolidated after the crisis in terms of price discovery channel, while for the volatility channel the transmission of information has lost intensity, implying that the return channel facilitates better the communication between these two countries.

Table 5.7 Japan – Germany, Stock and Money Markets; The estimation results for the FULL BEKK Model

Panel A **The Mean Equation**	**Pre-crisis**				**Post-crisis**			
	NIKKEI500	DAX30	JAP 1M	GER 1M	FTSE100			
intercept	0.00492	0.01182***	0.00000	-0.00006	0.00450***	0.01583**	-0.00010**	-0.00089***
AR(1)	0.01958	-0.03304	0.05335	0.11247***	-0.17653***	0.11175***	0.23070***	0.81915***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromNIKKEI500	fromDAX30	fromJAP 1M	fromGER 1M	fromNIKKEI500	fromDAX30	fromJAP 1M	fromGER 1M
to NIKKEI500		0.16774***	-0.56310	-0.16605		0.00005	-0.02338***	-0.01017***
to DAX30	0.05164**		-0.18441	-0.03646	-2.95689**		-1.35121**	0.09400
to JAP 1M	-0.00011	0.00014*		-0.00907***	0.02504***	0.00010		0.00964***
to GER 1M	-0.00025	0.00225***	-0.01800		0.24591***	-0.00313***	0.00740	

Panel B **The Variance Equation**	**Pre-crisis**				**Post-crisis**			
	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix C i=1,…,4	0.14088***				0.00204***			
	0.03749***	0.00811***			0.01535***	0.00000		
	0.00012***	-0.00014***	0.00000		0.00001	0.00000	0.00000	
	-0.00065	0.00054	0.00000	0.00000	-0.00038***	0.00000	0.00000	0.00000
ARCH effect matrix A i=1,…,4	0.23290***	-0.00281	-0.00014	-0.00300**	-0.03570	1.75675*	0.04892***	0.63308***
	-0.00414	0.16817***	0.00019***	0.00762***	-0.00095***	0.16062***	0.00071***	-0.00654***
	0.52021	-0.44167**	0.77428***	0.09980**	-0.01801**	-0.83674***	0.60967	-0.00269***
	-1.12405***	-1.97329***	0.01736***	1.50643***	-0.02199***	0.61290***	0.02603***	1.72173***

GARCH effect matrix G i=1,…,4	0.45204***	-0.15203***	-0.00099***	0.00665***	0.61501***	-9.55115***	-0.01122	0.08029
	0.09685***	1.00708***	0.00018***	0.00145***	0.00031***	0.97267***	-0.00019***	0.00066***
	-0.13499	0.10525***	0.86373***	-0.01482*	0.00198	0.20722***	0.92117***	-0.00025
	0.11979	1.96873***	-0.00060	-0.66198***	0.00235***	-0.12069***	-0.00277***	0.68873***
	fromNIKKEI500	fromDAX30	fromJAP 1M	fromGER 1M	fromNIKKEI500	fromDAX30	fromJAP 1M	fromGER 1M
to NIKKEI500	0.05424***	0.00002	0.27062	1.26349***	0.00127	0.00000***	0.00032**	0.00048***
to DAX30	0.00001	0.02828***	0.19507**	3.89388***	3.08617*	0.02580***	0.70013***	0.37564***
to JAP 1M	0.00000	0.00000***	0.59950***	0.00030***	0.00239***	0.00000***	0.37170	0.00068***
to GER 1M	0.00001**	0.00006***	0.00996**	2.26933***	0.40079***	0.00004***	0.00001***	2.96436***
	fromNIKKEI500	fromDAX30	fromJAP 1M	fromGER 1M	fromNIKKEI500	fromDAX30	fromJAP 1M	fromGER 1M
to NIKKEI500	0.20434***	0.00938***	0.01822	0.01435	0.37824***	0.00000***	0.00000	0.00001***
to DAX30	0.02311***	1.01421***	0.01108***	3.87588***	91.22450***	0.94608***	0.04294***	0.01457***
to JAP 1M	0.00000***	0.00000***	0.74602***	0.00000	0.00013	0.00000***	0.84855***	0.00001***
to GER 1M	0.00004***	0.00000***	0.00022*	0.43821***	0.00645	0.00000***	0.00000	0.47434***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: The relationships between the Japanese and German equity and money markets are described by numerous negative effects, implying some evidence of divergence/decoupling effects. The results regarding the return channel suggest a certain level of divergence between the equity markets of the two countries and also domestically for Japan with a negative influence from the money market to its equity market. Volatility wise, the strongest route is between the equity markets especially after the crisis (91.2245).

Table 5.8 Japan – Germany, Stock and Bond Markets; The estimation results for the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation								
intercept	0.08633***	0.01404***	-0.00013***	0.00030	0.01007**	0.00927***	-0.00020	-0.00139
AR(1)	0.04910*	-0.01111	0.14060***	-0.00096	-0.05459**	-0.02937**	-0.05742	0.07356***
The Return Spillovers	**Pre-crisis**				**Post-crisis**			
Matrix	fromNIKKEI500	fromDAX30	fromJAP 10Y	fromGER 10Y	fromNIKKEI500	fromDAX30	fromJAP 10Y	fromGER 10Y
to NIKKEI500		0.82967***	-1.12261	4.47905***		0.21738***	0.52098**	0.93002***
to DAX30	0.00616*		-0.99135***	0.43218***	-0.02506		-0.36302***	0.91524***
to JAP 10Y	-0.00001	0.00042***		-0.00170***	0.00052**	-0.00710		0.13522***
to GER 10Y	-0.00001	-0.00017	-0.19181		-0.01378**	-0.02183***	0.10879**	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.91903***				-0.01646***			
C	0.03941***	-0.01255***			0.00645***	-0.01819***		
i=1,…,4	-0.00014***	0.00000	0.00000		-0.00116**	-0.00061	0.00000	
	0.00070	0.00382***	0.00000	0.00000	0.00196***	-0.00390***	0.00002	0.00001
ARCH effect matrix	-0.36976***	-0.01248***	-0.00003	-0.00079	0.08088***	0.05802***	0.00157	-0.00092
A	0.32416	0.23815***	0.00011	0.00387*	0.05392***	0.30550***	0.00224	-0.00572
i=1,…,4	1.02085	-0.67507	1.00789***	-0.20316	-1.91228***	-0.34756***	-0.14434***	-0.11540***
	-2.82991**	0.12555	0.00338***	-0.04555**	-0.02573	-0.00707	0.01693*	0.01731

GARCH effect matrix	0.49754***	-0.01851***	0.00021***	-0.00072*	0.92378***	-0.01258*	-0.03590***	-0.01593***
G	0.67832***	0.98683***	-0.00028***	0.00345***	-0.00376	0.93678***	0.01018***	0.01341***
i=1,…,4	2.04300	0.27347	0.82965***	0.04451*	2.10849***	0.46390***	0.95740***	0.04163*
	0.41397***	0.04127***	-0.00007	0.99255***	-0.08786***	-0.09142***	-0.00114	0.98718***
	fromNIKKEI500	fromDAX30	fromJAP 10Y	fromGER 10Y	fromNIKKEI500	fromDAX30	fromJAP 10Y	fromGER 10Y
to NIKKEI500	0.13672***	0.10508	1.04213	8.00842**	0.00654***	0.00291***	3.65681***	0.00066
to DAX30	0.00016***	0.05671***	0.45572	0.01576	0.00337***	0.09333***	0.12080***	0.00005
to JAP 10Y	0.00000	0.00000	1.01583***	0.00001***	0.00000	0.00001	0.02083***	0.00029*
to GER 10Y	0.00000	0.00001*	0.04127	0.00207**	0.00000	0.00003	0.01332***	0.00030
	fromNIKKEI500	fromDAX30	fromJAP 10Y	fromGER 10Y	fromNIKKEI500	fromDAX30	fromJAP 10Y	fromGER 10Y
to NIKKEI500	0.24755***	0.46011***	4.17385	0.17137***	0.85337***	0.00001	4.44571***	0.00772***
to DAX30	0.00034***	0.97383***	0.07479	0.00170***	0.00016*	0.87756***	0.21520***	0.00836***
to JAP 10Y	0.00000***	0.00000***	0.68832***	0.00000	0.00129***	0.00010***	0.91661***	0.00000
to GER 10Y	0.00000*	0.00001***	0.00198*	0.98515***	0.00025***	0.00018***	0.00173*	0.97453***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: When the long-term segment of the yield curve is combined with the equity markets for Japan – Germany pair, the linkages along the return channel got stronger as a result of the crisis with the indirect international route being the busiest one. The volatility channel, however, has been less active post crisis, the only route that allowed for increased information flow is the domestic route from the Japanese bond market to its equity market represented by the return on the NKKEI500 index.

Table 5.9 Japan – Canada, Stock and Money Markets; The estimation results for the FULL BEKK Model

Panel A **The Mean Equation**	**Pre-crisis**				**Post-crisis**			
	NIKKEI500	SPTSX	JAP 1M	CAD 1M	NIKKEI500	SPTSX	JAP 1M	CAD 1M
intercept	0.00265	0.00213	0.00001	0.00282***	0.00478	0.00365**	-0.00003	0.00005
AR(1)	0.06575***	0.05066**	0.18759***	0.34323***	-0.02457	-0.04042**	0.27219***	-0.11071***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromNIKKEI500	fromSPTSX	fromJAP 1M	fromCAD 1M	fromNIKKEI500	fromSPTSX	fromJAP 1M	fromCAD 1M
to NIKKEI500		0.38006***	-0.45151	0.39200***		0.38261***	1.54115***	0.42209***
to SPTSX	0.04522***		0.11574	0.39225***	0.01880*		0.21542	0.04136
to JAP 1M	-0.00010	0.00074***		0.00116***	0.00011	0.00121***		0.00960***
to CAD 1M	-0.00687*	0.00808**	-0.09941**		0.00026	0.00812**	-0.02306*	

Panel B **The Variance Equation**	**Pre-crisis**				**Post-crisis**			
	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.03123***				-0.04878***			
C	-0.07333***	0.00000			-0.01022***	-0.00340***		
i=1,…,4	0.00011***	0.00000	0.00000		0.00052***	-0.00001	0.00000	
	0.00435***	0.00000	0.00000	0.00000	-0.00013	0.00107***	0.00000	0.00000
ARCH effect matrix	0.09052***	-0.03936***	-0.00095***	-0.00568	-0.13650***	-0.01227***	0.00124***	0.00196***
A	0.14686***	0.33318***	0.00310***	-0.02541***	-0.04329***	0.19784***	0.00279***	-0.01497***
i=1,…,4	0.01538	0.04015	0.83006***	-0.02467	2.13632***	-0.13750***	0.62062***	0.01925***
	0.36102***	0.50545***	0.00094*	1.09893***	0.66345***	0.29384***	0.00168**	0.35640***

GARCH effect matrix G i=1,…,4	0.98148***	-0.02943***	0.00007**	-0.00077	0.86254***	-0.01021***	0.00752***	0.00044***
	-0.09522***	0.53647***	0.00104***	0.11449***	0.03511***	0.97889***	-0.00346***	0.00247***
	-0.04712	-0.39873**	0.85787***	0.12150**	-16.49322***	-0.57526***	0.81732***	0.01808***
	-0.15315***	-0.23281***	-0.00169***	0.69336***	-0.25764***	-0.07894***	0.00071***	0.94972***
	fromNIKKEI500	fromSPTSX	fromJAP 1M	fromCAD 1M	fromNIKKEI500	fromSPTSX	fromJAP 1M	fromCAD 1M
to NIKKEI500	0.00819***	0.02157***	0.00024	0.13034***	0.01863***	0.00187***	4.56385***	0.44017***
to SPTSX	0.00155***	0.11101***	0.00161	0.25548***	0.00015***	0.03914***	0.01891***	0.08634***
to JAP 1M	0.00000***	0.00001***	0.68900***	0.00000*	0.00000***	0.00001***	0.38517***	0.00000**
to CAD 1M	0.00003	0.00065***	0.00061	1.20765***	0.00000***	0.00022***	0.00037***	0.12702***
	fromNIKKEI500	fromSPTSX	fromJAP 1M	fromCAD 1M	fromNIKKEI500	fromSPTSX	fromJAP 1M	fromCAD 1M
to NIKKEI500	0.96330***	0.00907***	0.00222	0.02346***	0.74398***	0.00123***	272.02615***	0.06638***
to SPTSX	0.00087***	0.28780***	0.15898**	0.05420***	0.00010***	0.95823***	0.33093***	0.00623***
to JAP 1M	0.00000**	0.00000***	0.73594***	0.00000***	0.00006***	0.00001***	0.66802***	0.00000***
to CAD 1M	0.00000	0.01311***	0.01476**	0.48075***	0.00000***	0.00001***	0.00033***	0.90197***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: For Japan and Canada combination the stock and money markets have been complexly interconnected before the crisis. While some of the routes in the network lost some intensity, there are two particular routes through which the information transmission intensifies after the crisis in the context of both return and volatility channels. They are the indirect international routes from the Japanese and Canadian money markets to the equity markets of the other country. There is weak evidence of information flows from the other direction, more specifically from the equity markets to the bond markets. Internally, in Japan the money market greatly influences post-crisis the NIKKEI500 index via both return and volatility channels.

Table 5.10 Japan – Canada, Stock and Bond Markets; The estimation results for the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	NIKKEI500	SPTSX	JAP 10Y	CAD 10Y	NIKKEI500	SPTSX	JAP 10Y	CAD 10Y
intercept	0.00675***	0.00672***	0.00005***	-0.00067***	0.00644*	0.00469**	-0.00081*	-0.00206*
AR(1)	0.09577***	0.01412***	0.26982***	-0.00319	-0.02064	-0.03753*	-0.06138***	-0.04977*
The Return Spillovers	**Pre-crisis**				**Post-crisis**			
Matrix	fromNIKKEI500	fromSPTSX	fromJAP 10Y	fromCAD 10Y	fromNIKKEI500	fromSPTSX	fromJAP 10Y	fromCAD 10Y
to NIKKEI500		0.39451***	-1.11962***	1.91312***		0.31005***	0.98940***	1.02598***
to SPTSX	0.00676***		0.34398***	0.03986**	0.02275*		0.03590	0.29853***
to JAP 10Y	0.00002*	-0.00197***		0.00251***	0.00269	-0.00010		0.13409*
to CAD 10Y	0.00154***	0.00609***	-0.02986**		-0.00102	0.01455*	0.03057**	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	-0.00560***				0.09321***			
C	-0.00658***	0.00002			0.00457***	-0.00280***		
i=1,…,4	0.00001*	-0.00001	0.00001***		0.00007	0.00104***	0.00000	
	0.00034***	0.00062***	-0.00058***	0.00000	0.00218***	0.00503***	0.00066	0.00035
ARCH effect matrix	0.20349***	0.01019***	-0.00002***	0.00002	0.44523***	0.01623**	-0.00060	0.00030
A	-0.08174***	0.10843***	-0.00453***	0.00228***	-0.41739***	0.26622***	0.00456***	0.01677***
i=1,…,4	-0.70486***	0.00091	0.70840***	0.01394***	0.72429***	0.20409***	-0.19831***	-0.09012***
	-0.32674***	-0.02339***	0.00011***	0.23849***	-0.06458	-0.11509***	0.00688***	0.13668***

GARCH effect matrix G i=1,…,4	0.97949***	-0.00182***	0.00000***	-0.00002*	0.65095***	-0.00152	-0.00206***	-0.00766***
	0.01794***	0.98598***	0.00049***	0.00008	0.14495***	0.95864***	-0.00055***	-0.00304***
	0.04542***	-0.01891***	0.90854***	-0.00015	0.33205***	0.11083***	0.98188***	-0.00073
	0.08242***	0.00668***	0.00004***	0.97294***	-0.08188***	0.02825***	-0.00185***	0.98170***
	fromNIKKEI500	fromSPTSX	fromJAP 10Y	fromCAD 10Y	fromNIKKEI500	fromSPTSX	fromJAP 10Y	fromCAD 10Y
to NIKKEI500	0.04141***	0.00668***	0.49682***	0.10676***	0.19823***	0.17421***	0.52460***	0.00417
to SPTSX	0.00010***	0.01176***	0.00000	0.00055***	0.00026**	0.07087***	0.04165***	0.01324***
to JAP 10Y	0.00000***	0.00002***	0.50182***	0.00000***	0.00000	0.00002***	0.03933***	0.00005***
to CAD 10Y	0.00000	0.00001***	0.00019***	0.05688***	0.00000	0.00028***	0.00812***	0.01868***
	fromNIKKEI500	fromSPTSX	fromJAP 10Y	fromCAD 10Y	fromNIKKEI500	fromSPTSX	fromJAP 10Y	fromCAD 10Y
to NIKKEI500	0.95939***	0.00032***	0.00206***	0.00679***	0.42374***	0.02101***	0.11026***	0.00670***
to SPTSX	0.00000***	0.97216***	0.00036***	0.00004***	0.00000	0.91898***	0.01228***	0.00080***
to JAP 10Y	0.00000***	0.00000***	0.82545***	0.00000***	0.00000***	0.00000***	0.96408***	0.00000***
to CAD 10Y	0.00000*	0.00000	0.00000	0.94662***	0.00006***	0.00001***	0.00000	0.96373***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: The long segment of the yield curve modelled simultaneously with the stock markets between Japan and Canada do not change substantially from the analysis involving the short-term segment of the yield curve. Both channels (return and volatility) are active across most of the possible routes before and after the crisis. The magnitude of the estimates for the return channels are higher than those for the volatility channel. Also, the dynamics of information transmission process are affected in a similar way along the two channels. The domestic route in both countries bond-to-equity markets and the direct external route between the two bond markets becomes busier as a result of the crisis. The Canadian markets seem to have a leading role in exporting more information to the Japanese markets.

Table 5.11 Germany – Canada, Stock and Money Markets; The estimation results for the FULL BEKK Model

Panel A **The Mean Equation**	**Pre-crisis**				**Post-crisis**			
	DAX30	SPTSX	GER 1M	CAD 1M	DAX30	SPTSX	GER 1M	CAD 1M
intercept	0.01074***	0.00850***	0.00007**	0.00093***	0.00786**	0.00392	-0.00009**	0.00014
AR(1)	-0.05721***	-0.05952***	0.30168***	0.05145***	-0.18058***	-0.07700***	0.72524***	-0.14770***
The Return Spillovers Matrix	**Pre-crisis**				**Post-crisis**			
	fromDAX30	fromSPTSX	fromGER 1M	fromCAD 1M	fromDAX30	fromSPTSX	fromGER 1M	fromCAD 1M
to DAX30		0.27014***	-0.28100**	0.11638***		0.30402***	0.81204***	0.20509***
to SPTSX	0.04207***		-0.23133***	-0.00116	-0.01245		0.46647***	0.00071
to GER 1M	0.00181***	0.00116**		0.00132**	0.00341***	-0.00459***		0.02977***
to CAD 1M	-0.00006	-0.01446***	-0.06547**		-0.00361	0.00478	0.01575	

Panel B **The Variance Equation**	**Pre-crisis**				**Post-crisis**			
	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
intercept matrix	0.01045***				-0.02662***			
C	0.04573***	0.01075***			-0.00428***	-0.00226*		
i=1,…,4	0.00056***	-0.00092***	0.00000		0.00000	0.00014	0.00000	
	-0.00431***	-0.00209*	0.00000	0.00000	-0.00018	-0.00215***	0.00000	0.00000
ARCH effect matrix	0.12191***	0.02565***	0.01098***	-0.00758**	0.20460***	0.08791***	-0.01519***	0.00423***
A	0.09304***	0.27782***	-0.00121	-0.04290***	-0.02827***	0.17427***	0.01979***	-0.00368***
i=1,…,4	-2.93243***	-2.60824***	2.00236***	-0.97771***	2.40315***	1.01357***	2.04706***	-0.15265***
	0.17813***	0.10449***	0.00522***	0.64904***	-0.07157***	0.11095***	-0.03440***	0.38816***

GARCH effect matrix	0.98739***	0.02512***	0.00036	-0.01564***	0.91364**	-0.07571***	0.00095**	-0.00085***
G	-0.03026***	0.75485***	-0.00409***	0.09378***	0.11040***	1.01863***	-0.00098**	0.00003
i=1,…,4	0.73911***	0.62923***	0.60005***	0.28474***	-0.40861***	-0.03793***	0.66743***	0.02288***
	-0.15427***	-0.23094***	-0.00243	0.87256***	0.00609*	-0.02984***	0.01079***	0.94094***
	fromDAX30	fromSPTSX	fromGER 1M	fromCAD 1M	fromDAX30	fromSPTSX	fromGER 1M	fromCAD 1M
to DAX30	0.04186***	0.00087***	8.59914***	0.03173***	0.04186***	0.00080***	5.77514***	0.00512***
to SPTSX	0.00066***	0.07718***	6.80291***	0.01092***	0.00773***	0.03037***	1.02733***	0.01231***
to GER 1M	0.00012***	0.00000	4.00945***	0.00003***	0.00023***	0.00039***	4.19044***	0.00118***
to CAD 1M	0.00006**	0.00184***	0.95591***	0.42125***	0.00002***	0.00001***	0.02330***	0.15067***
	fromDAX30	fromSPTSX	fromGER 1M	fromCAD 1M	fromDAX30	fromSPTSX	fromGER 1M	fromCAD 1M
to DAX30	0.97494***	0.00092***	0.54629***	0.02380***	0.83474**	0.01219***	0.16696***	0.00004*
to SPTSX	0.00063***	0.56980***	0.39593**	0.05334***	0.00573***	1.03762***	0.00144***	0.00089***
to GER 1M	0.00000	0.00002***	0.36006***	0.00001	0.00000**	0.00000**	0.44546***	0.00012***
to CAD 1M	0.00024**	0.00879***	0.08108***	0.76137***	0.00000***	0.00000	0.00052***	0.88536***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: The Canadian markets have well- established connections with Germany before the crisis, some of which consolidate in the post-crisis period. Their equity markets communicate better from both directions via both channels however with greater influence fom SPTSX index to the DAX30 index, while the bond markets are also more interlinked with Canadian money market as the main exporter of information shocks through both return and volatility markets.   There is little evidence of substantial spillovers effect through the indirect international route.

Table 5.12 Germany – Canada, Stock and Bond Markets; The estimation results for the FULL BEKK Model

Panel A	**Pre-crisis**				**Post-crisis**			
The Mean Equation	DAX30	SPTSX	GER 10Y	CAD 10Y	DAX30	SPTSX	GER 10Y	CAD 10Y
intercept	0.00514	0.00639***	0.00028	-0.00105	0.00576**	0.00898***	-0.00064	-0.00048
AR(1)	-0.11320***	-0.00909	-0.13880***	0.01870	-0.14692***	-0.08080***	0.10850***	-0.17472***
The Return Spillovers	**Pre-crisis**				**Post-crisis**			
Matrix	fromDAX30	fromSPTSX	fromGER 10Y	fromCAD 10Y	fromDAX30	fromSPTSX	fromGER 10Y	fromCAD 10Y
to DAX30		0.27520***	0.09520	0.38080***		0.27453***	0.65640***	0.30349***
to SPTSX	0.01300		-0.02310	0.07420	-0.00353		0.11584**	0.17586***
to GER 10Y	-0.00287***	0.00024**		0.40300***	-0.02151***	0.02332**		0.10456***
to CAD 10Y	0.00039	-0.01050	0.0107		-0.00456	0.00731	0.42978***	

Panel B	**Pre-crisis**				**Post-crisis**			
The Variance Equation	j=1	j=2	j=3	j=4	j=1	j=2	j=3	j=4
	0.00822				0.02063***			
intercept matrix	0.05090**	0.00972			0.00149	-0.00379**		
C	0.00498	-0.01170	0.03040		0.00517***	-0.00323***	-0.00234***	
i=1,…,4	0.01140	-0.02950	0.01830	0.00000	0.00008	0.00183***	-0.00094*	0.00000
ARCH effect matrix	0.25540***	0.11290***	0.01320	0.01410	0.11327***	-0.12748***	-0.03299***	0.00114
A	-0.04540	0.24150***	-0.01170	-0.01960	0.27532***	0.30098***	0.00701***	-0.00036
i=1,…,4	-0.03140	0.07610	0.00111	0.18050**	0.16574***	0.09538***	0.15033***	0.01404*
	-0.13550	-0.12640	0.10600***	-0.32290***	-0.14874***	-0.00848	-0.01610	0.13145***

GARCH effect matrix G i=1,…,4	0.97020***	0.03440	0.00942	0.00192	0.96014***	0.02613***	0.01770***	-0.00267***
	0.01860	0.69490***	0.04740	0.02970	-0.03503***	0.95125***	-0.00999***	0.00092***
	-0.24780	-0.34600	0.40030	-0.35310	-0.16579***	-0.03662***	0.96668***	-0.01080***
	-0.05550	-0.13980	-0.16430	0.65900***	0.09706***	0.02384***	0.02096***	0.99382***
	fromDAX30	fromSPTSX	fromGER 10Y	fromCAD 10Y	fromDAX30	fromSPTSX	fromGER 10Y	fromCAD 10Y
to DAX30	0.06523***	0.00206	0.00099	0.01836	0.01283***	0.07580***	0.02747***	0.02212***
to SPTSX	0.01275***	0.05832***	0.00579	0.01598	0.01625***	0.09059***	0.00910***	0.00007
to GER 10Y	0.00017	0.00014	0.00000	0.01124***	0.00109***	0.00005***	0.02260***	0.00026
to CAD 10Y	0.00020	0.00038	0.03258**	0.10426***	0.00000	0.00000	0.00020*	0.01728***
	fromDAX30	fromSPTSX	fromGER 10Y	fromCAD 10Y	fromDAX30	fromSPTSX	fromGER 10Y	fromCAD 10Y
to DAX30	0.94129***	0.00035	0.06140	0.00308	0.92188***	0.00123***	0.02749***	0.00942***
to SPTSX	0.00118	0.48289***	0.11972	0.01954	0.00068***	0.90487***	0.00134***	0.00057***
to GER 10Y	0.00009	0.00225	0.16024	0.02699	0.00031***	0.00010***	0.93446***	0.00044***
to CAD 10Y	0.00000	0.00088	0.12468	0.43428***	0.00001***	0.00000***	0.00012***	0.98768***

Note: ***, ** and * indicate that the estimate is statistically significant at 1%, 5% and 10%, respectively;

The value 0.00000*** means that the parameter estimate is extremely small (non-zero) but still statistically significant.

Comments: These results provide evidence of the impact that the crisis had on the dynamics of the spillovers of information when the German and Canadian bond markets are simultaneously modelled in conjunction with equity markets. The pre-crisis period reveals a less degree of interconnection between the two countries, with more routes becoming busy after the crisis. The return channel is more active when compared with the volatility channel as the estimates measuring the intensity of information spillovers have larger values in Panel A of the table. There most active routes in the post-crisi period are the indirect external routes (unidirectional, only from the bond to the equity markets), the direct external link between the equity markets with greater influence from SPTSX index, and the direct external route between the two bond markets.

Chapter 6

Conclusions and Further Research

The three extensive empirical studies presented in this thesis have the general aim to dynamically estimate and forecast the term structure of interest rates within contexts that have not been considered before.

The first empirical investigation contributes not only empirically but also in the theoretical direction by extending the multivariate CKLS modelling framework to four- and five-factor specifications. There are very few empirical studies in the TSIR literature that test such highly dimensional yield curve models, one reason being the computational and econometric challenges[1] that arise when using such complex models.

Following Nowman's (2003, 2006) approach, four continuous time term structure models, the general CKLS model and three other classic models nested in the CKLS framework (Vasicek, CIR and BS) are employed to estimate the short- and long-term segments of the yield curve using the Gaussian methods of dynamic estimation developed by Bergstrom (1983,1984). The short-end of the yield curve is examined in an international context based on five major currency-LIBOR rates over the period 2000-2013, with a total of forty continuous time models to be estimated. For longer maturities the nominal U.K. yield curve is dynamically estimated using eight continuous time models.

The empirical results favour the five-factor models over the four-factor models as the addition of the fifth factor increases substantially the goodness of fit across all four specifications. For all five LIBOR currencies (GBP, USD, JPY, EUR and CAD) the restricted models are statistically rejected against the unrestricted CKLS specification. Interestingly however, in the case of the longer-term segment of the nominal U.K. curve the best fit from the restricted models has been provided by the Vasicek model, which narrowly failed the validity test against the CKLS model. This result could suggest that

[1] By adding extra variables, the optimization algorithm (the objective function) becomes more complex; it may also lead to over-fitting the data and to identification issues (see Hamilton and Wu, 2012).

in the current environment of very low interest rates models models that have been previously rejected based on their admittance of negative interest rates (such as Vasicek) should be reconsidered.

Another important finding is that the transition from four- to five-factor specification can be associated with a decrease in the value of the diffusion parameter measuring the sensitivity of the instantaneous volatility with respect to the level of the instantaneous interest rate. This may suggest that less flexible models introduce a specification bias by overestimating how elastic the volatility is.

In addition, the benefit of increasing the number of factors, is that one could observe the changes in the structure of the variance-covariance matrix between the two extensions. This allows the identification of which maturity yields have the strongest interconnections. For the short-term segment of the yield curve, it was found that the last three factors - the three-, six- and twelve-month LIBOR rates - move together very closely. Based on this evidence one may conclude that if any twists were to exist in the term structure of interest rates over the period 2000-2013 they should have occurred outside this three to twelve-month maturity segment. This feature of the analysis has important implications for the investment decision making process; investors who focus on certain segments of the term structure of interest rates could determine the regions where a twist/inversion may occur along the yield curve.

The forecasting performance of the TSIR models employed here is assessed and compared with the forecasting results from discrete time benchmark models such as VAR (1) and AR(1). Based on five, both statistical and economic, measures of forecasting accuracy it was found that for shorter maturity (up to six months) interest rates, the continuous time models nested in the CKLS framework outperform consistently the discrete time models. However, once the model involves interest rates of longer maturities, the situation reverses. This is an important conclusion, that adds new insights into the debate of parsimonious versus complex modelling, suggesting that complex models are necessary to capture well the dynamics of the short-end of the yield curve. These findings could have great implications for financial areas where the accuracy of interest rate forecasting is crucial. In conclusion, the forecasting results suggest that the availability of alternative forecasting methods should become an intrinsic feature of any forecasting analysis of the short end of the yield curve. If most of the richer models would produce this superior performance over parsimonious models, then the final averaged forecasts should give more weight to more the sophisticated models when this maturity segment is concerned.

For further research, the newly extended models in this chapter have a great appeal for applications to any asset class where there is a term structure such as futures contracts in general and commodity (oil, gold) futures, dividend, FX and real estate futures in particular.

In the second empirical investigation of this thesis, the dynamics of the yield curves for three Scandinavian countries (Denmark, Norway and Sweden) are explored for the first time in the literature by using one-, two- and three-factor versions of the general Babbs and Nowman (1999) model. The Kalman filter and maximum likelihood estimates of twenty-seven models are mostly highly significant, including several market prices of risk parameters which are difficult to estimate in general. Based on formal statistical tests and residual analysis, the empirical results indicate that the three-factor specification explains best the changes over time in the shape of the yield curve for Denmark and Norway. For Sweden, the BIC statistical test does not reject the two-factor model against the three-factor formulation. There is evidence of a structural break during the third quarter of 2007 as the estimation results for the pre-crisis data-sample differ considerably from those from the post-crisis period. Additionally, the loadings (sensitivity) of the yield curve on each factor are extracted and analysed in order to determine the nature of their associated factor. Moreover, the time series of the unobservable factors implied by the Kalman filter are compared to the data-based factors of level, slope and curvature. For Denmark and Norway, the interpretation of the factors is very similar and straightforward in terms of level, slope and curvature as there is a high level of correlation between the two types of factors. However, for Sweden the paths of the extracted factors are not consistent with the dynamics of the data-implied factors.

The estimation results are used to compute optimal daily forecasts for the last three months in 2014 and compare all the models in terms of prediction power. In terms of forecasting performance there is a clear winning model only for Norway where the three-factor model performs best, while for Denmark and Sweden the one-factor and three-factor have comparable performance. Overall, the BN models achieve very good quality forecasts across all maturities and given their tractability they can be very useful in hedging strategies and pricing interest rates derivatives.

The latent factor BN model can be further discussed in comparison with more recent extensions of the classic non-parametric Nelson-Siegel (1987) model, such as the dynamic version (DNS) and the arbitrage-free version (AFNS) presented in Diebold and Rudebusch (2013). Following Diebold et al. (2006), one could investigate which macroeconomic variables can relate to the theoretical factors implied by these models. In this way one could determine which model is most appropriate to extract continuous information regarding particular macroeconomic variables such inflation and monetary policy instruments.

The third empirical study in this thesis, investigates the impact of the financial crisis of 2007-2009 on the return and volatility spillovers dynamic effects among different types of markets, more specifically equity, long-term bonds and money markets. Given the high degree of financial integration, shocks in one market are likely to spread simultaneously at both domestic and international level. To include these two levels, the dynamic interaction between two asset classes (internal) across two different countries (external) is modelled simultaneously in the four-factor modelling framework the traditional discrete time multivariate M-GARCH model, the full BEKK variant. This framework allows to examine how the global financial crisis of 2007-2009 has spread from the U.S. to four major economies, namely U.K., Japan, Germany and Canada. The empirical results are organised pairwise, for example for the U.S.-U.K. pair we look first at the combination of stocks and one-month T-bills markets and then stock and 10-year government bonds. The empirical results resulted from the estimation of sixteen models contain evidence that the last financial crisis has definitely impacted the relationships between major economic and financial markets. The empirical results emphasize that, relative to the U.S., for each country analysed the information transmission mechanism has its own particularities even before the crisis. This mechanism is subject to changes during the crisis, with one consistent result that the U.S. bond markets become the dominant exporters of information either through the price discovery or volatility channel mostly to the equity markets of the other country. Out of the three types of routes of information transmission, the most active routes are the indirect external route followed by the domestic one. These results are valid for both return and volatility channels and it emphasises the importance of considering this type of routes, ignored previously in the spillovers literature. Along these routes, the information flows unidirectionally from the interest rate markets to the equity markets and not vice-versa, implying that the interest rate markets dominate the equity markets in transmission of information.

The return and volatility spillover effects are much stronger when the equity markets are modelled in combination with the long-term markets than with the money markets. Among the countries considered, the results for Canada are significantly different, as the Canadian markets seem to influence indirectly the U.S. markets, reflecting the relative stability that Canadian markets sustained during the crisis.

This line of research could be continued further by employing the five-factor extension of the CKLS framework, where the fifth factor would be the exchange rate between the U.S. and the respective country. It would be of interest to conduct a comparative investigation between this continuous time framework and the discrete time approach followed by Ehrmann et al. (2011).

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