A Novel Optimization Algorithm for Notch Bandwidth in Lattice Based Adaptive Filter for the Tracking of Interference in GPS

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Abstract—**The weak signal levels experienced at the reception of the messages transmitted by navigation satellites, makes Global Positioning System (GPS) vulnerable to unintentional and intentional interference. This calls for appropriate modelling of GPS signal sources and jammers to assess the anti-jamming and interference mitigation capabilities of algorithms developed to be implemented for GPS receivers. Using a practical simulation model, this work presents an anti-jamming technique based on a novel algorithm. A fully adaptive lattice based notch filter is presented that provides better performance when compared to existing adaptive notch filter based techniques, chosen from the literature, in terms of convergence speed whilst delivering superior performance in the excision of the interference signal. To justify the superiority of the proposed technique, the noise and interference signal power is varied for in a wide dynamic range assessing jamming-to-noise density versus effective carrier-to-noise density performance at the output of the correlator.**

Keywords—Anti-Jamming, Adaptive Notch Filter, GPS receivers, Lattice-Based Notch Filter.

I. INTRODUCTION

For navigation systems, reliability and real-time positioning have become more important nowadays. The use of a navigation system is now a daily necessity to find the route to our destinations where we just simply take a look at our smartphones, or satnavs when driving, and it is an easy process to retrieve our position and get the directions to where we are heading. Global Positioning System (GPS) is the most popular Global Navigation Satellite System (GNSS) with a worldwide coverage making it the most favourable choice for the development of navigation receivers for the global satellite navigation market. The GNSS uses the Direct Sequence Spread Spectrum (DSSS) technique to modulate the navigation data, which inherently has some level of interference mitigation capability [1]. When the power of the interference increases to a certain level, the positioning and navigation functions of GPS will no longer work properly or even fail totally. That is why the GPS receivers should be accompanied with anti-jamming capabilities to tackle this problem. It is possible to mitigate jamming using spatial-filtering [2–4], adaptive frequency-domain filtering [5-6], methods applied in Time-Frequency Domain (TFD) [7-8] or adaptive time-domain filtering [9-15].However, the main disadvantage of these techniques is the necessity for complex mathematical transformations to be implemented in digital domain, which contributes highly to the hardware complexity of the GPS receivers despite the limited hardware resources and power available on-board. The common choice being the Fast Fourier Transform (FFT) followed by inverse FFT or sometimes wavelet transforms [16], the implementation of these techniques require larger amount of logic resources and memory accesses, increasing the power dissipation of the GPS receiver. In this paper, we present and analyse a low-complexity time-domain anti-jamming technique, which is constructed of Adaptive Notch Filter (ANF) modules. Each module is able to detect, estimate, and notch out one single-tone Continuous Wave Interference (CWI) with varying frequency. The ANF module is composed of a simple fully adaptive second-order Infinite Impulse Response (IIR) filter built on lattice structure.

This paper is divided in five sections. Section II describes the modelling of the transmitted and received signal. Section III discusses ANF algorithms based on lattice structure with brief literature review on the simultaneous adaptation of both notch bandwidth and notch centre frequency. Section IV provide insight into the proposed algorithm. Section V demonstrates a simulation based results and provided comparison with the existing and the proposed method and last section summarizes the conclusion.

II. SIGNAL DEFINITION

The ideal GPS L1 signal with CWI at receiver front-end can be expressed as in (1) where $s[n]$ is transmitted signal, $\eta[n]$ is the thermal noise level and $j_{int}[n]$ is the CWI interference.

$$r[n] = s[n] + \eta[n] + j_{int}[n] \tag{1}$$

$$s[n] = \sqrt{2P_i}D_s[n - \tau_0]C[n - \tau_0]\cos(2\pi f_{L1+d}nt + \theta) \tag{2}$$

In (2) the P_i is the signal power, D_s is the navigation data bit with a chip rate of 50 Hz, and C is coarse C/A code running at a chip rate of 1.023MHz and τ_0 is the code phase delay. Frequency parameter f_{L1+d} represents the L1 carrier frequency with a Doppler shift of f_d (where the sum of these two frequency components results in $f_{L1+d} = f_{L1} \pm f_d = 1575.42MHz \pm f_d$). The performance of the GPS acquisition systems are evaluated and processed based on the Cross Ambiguity Function (CAF), which in the discrete time domain, can be defined as (3) and the Signal-to-Noise Ratio (SNR) and Carrier to Noise Density in CAF can be calculated
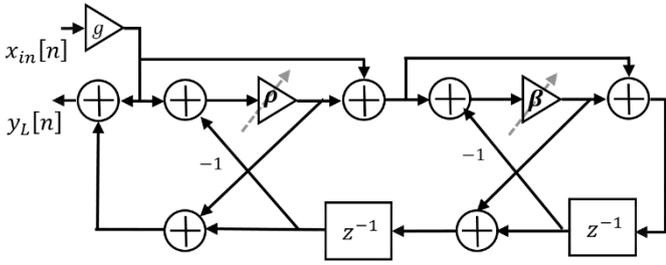
Figure 1. Lattice based ANF Structure

[17] by equation (4) and (5) respectively, where B is bandwidth of final stage filter.

$$S(\tau_0, f_d) = \left| \frac{1}{N} \sum_{n=0}^{N-1} r[n] c_L[n - \tau_0] e^{-j2\pi f_d} \right|^2 \quad (3)$$

where c_L is the local replica of the C/A code and N represents number of samples used for evaluation.

$$SNR(dB) = 10 log \left[\frac{\max[S(\tau_0, f_d)] - mean[S(\tau_0, f_d)]}{s.t.d[S(\tau_0, f_d)]} \right] \quad (4)$$

where SNR is simply evaluated by dividing the difference of maximum correlation peak and mean noise of the search space by standard deviation of the search space.

$$C/N_0 = 10 log 10(SNR \times B) \; [dB - Hz] \quad (5)$$

III. ADAPTIVE NOTCH FILTERS

There is an extensive literature on the detection and mitigation of CWI in GNSS. Second order ANF IIR notch filters provide a cost effective solution as they requires only two coefficients to be set up system and can perform as good as higher order FIR filter if carefully tuned and adapted for the given environment. There are two major ANF structures, the direct form second order ANF [11] and lattice based ANF. Direct form second order ANF has two main drawbacks. Firstly notch bandwidth parameter cannot be adapted optimally and secondly if the notch bandwidth parameter is very close to unity, then the convergence of direct form ANF to the target frequency is very slow. Whereas from the literature review [12-15] it is evident that lattice based adaptive notch filter is the preferred choice when it comes to adapting both parameters, the notch bandwidth and notch centre frequency. Hence in this research paper various different algorithms based on lattice structure are studied and compared with our proposed method in terms of convergence speed and excision of interference in GPS. In an adaptive IIR lattice structure the notch filter transfer function from input $x_{in}[n]$ to notch output $y_L[n]$ is expressed as.

$$H_L(z) = \frac{1 + \rho[n]}{2} \frac{1 - 2\beta[n]z^{-1} + z^{-2}}{1 - \beta[n](1 + \rho[n])z^{-1} + \rho[n]z^{-2}} \quad (6)$$

where β is the notch centre frequency parameter and ρ is notch bandwidth parameter of the lattice notch filter, hence from now on these parameters will be referred as β and ρ .

A. Single parameter adaptation

Adaptation of parameter β can be achieved by simply minimizing the cost function at the output of the ANF while keeping ρ fixed. A gradient signal is generated within the structure that helps to update value of β one sample at a time. How an update equation for β is implemented depends on the choice of the adaptive algorithm such as LMS, RLS, signed-LMS, fixed-step size or time-varying step-size. In [18] only β

is adapted in the lattice based structure via modified version of the signed NLMS algorithm shown below

$$\beta[n] = \beta[n - 1] - \mu[n]. y_L[n]. sign \left[\frac{g_1[n]}{1 + |g_1[n]|} \right] \quad (7)$$

$$g_1[n] = (1 + \rho[n])x[n - 1] \quad (8)$$

where $g_1[n]$ the gradient is signal utilize to update the parameter β $\mu[n]$ is the variable step-size and $y_L[n]$ is the output of the filter. The work by regalia [19] and [20] uses simplified gradient term to update the β. In [19] dynamic adaptation of ρ is proposed. Furthermore [19-20] uses same gradient function as derived in [13-14] referred as partial gradient and shown as follows

$$grad(J_\beta[n]) = (1 - \rho[n])x[n - 1] \quad (9)$$

which can further be evaluated [20] to update β using

$$\beta[n] = \beta[n - 1] - \mu[n]. y_L[n] grad(J_\beta[n]) \quad (10)$$

In (10) $\mu[n]$ is the time varying step-size formulated [20] by

$$\mu[n] = \mu_\beta / \phi_\beta[n] \quad (11)$$

$$\phi_\beta[n] = \gamma \phi_\beta[n - 1] + (1 - \gamma) grad(J_\beta[n])^2 \quad (12)$$

where $\phi_\beta[n]$, γ and μ_β are respectively instantaneous power estimation of the gradient signal, forgetting factor (valued between 0.9 and 0.99) and fixed step size respectively. In this research paper adaptation of β is accomplished as in [20] and to make fair comparison to the relevant work a variable step-size is utilised as it will provide faster convergence irrespective of any other parameters within the adaptive algorithm.

B. Simultaneously Adaptation of both β and ρ

There are only a few study to adaptation relevant to the adaptation of notch bandwidth parameter ρ for ANFs. The ANF were first introduce in 1975 [21], where the idea was retrieve a single-tone signal immerse in wideband noise. A constrained ANF [22] exponentially adapts the ρ toward unity (0.995) to attain sharp notch and idea was further improved in [23] and developed into a fully adaptive notch filter. Benefits of adapting ρ is clearly stated in [14-15] and [19] [22] which is to increases the convergence of β to the target frequency. In [13-14] the adaptation of ρ via partial gradient term g_ρ to update the value of ρ as in (13) using fixed step-size μ_ρ and somewhat similar update equation is also used by [10].

$$\rho[n] = \rho[n - 1] - \mu_\rho y_L[n] g_\rho[n] \quad (13)$$

In most of these research works authors are simply taking in account that single-tone or multi-tone sinusoidal interference present in desired signal, but does not contain information about how to adapt both parameters of ANF if the single-tone frequency is varying or hopping within desired signal or spectrum of GPS.

IV. PROPOSED METHOD

In this research work, the gradient signal to update ρ is derived using the output of the notch filter by the differentiation of (6) with respect to ρ which yields.

$$\frac{\partial}{\partial \rho}(H_L(z)) = \overbrace{\frac{1 + \rho[n]}{2} \frac{1 - 2\beta[n]z^{-1} + z^{-2}}{1 - \beta[n](1 + \rho[n])z^{-1} + \rho[n]z^{-2}}}^{H_L(z)} \times$$

$$G_f(z) - \left[\frac{1 - z^{-2}}{1 - \beta[n](1 + \rho[n])z^{-1} + \rho[n]z^{-2}} \times \frac{1}{1 + \rho[n]} \right] \quad (14)$$
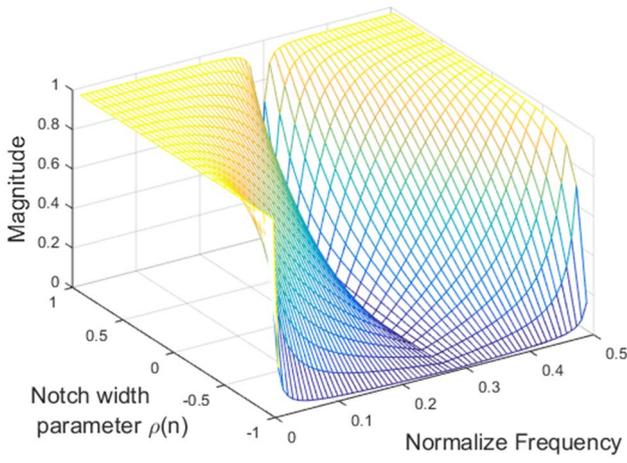
Figure 2. Frequency response of $H_L(z)$ as ρ changes from -1 to 1

which gives the full gradient expression $G_f(z)$, rather than a partial gradient to update ρ different from [10] and [13-14]. Following from (13), the update equation for ρ becomes

$$\rho[n] = \rho[n-1] + \mu_\rho \, y_L[n] g_f[n]/\phi_\beta[n] \qquad (15)$$

where y_L is the output of the filter, μ_ρ is the fixed-step size and g_f is the gradient signal corresponding to $G_f(z)$. Every time β tries to lock on to the new target frequency, the value ϕ_β tend to approach 0. When β locks on to the target frequency, the recursive calculation (12) of ϕ_β keep on growing until next target frequency, which controls how much the value of ρ in (15) has to increase. Initially ϕ_β is smaller therefore ρ converges at a fast rate, but as ϕ_β get larger in size, the ρ is updated at a lower rate.

Fig 2 and Fig 3 are respectively the 3D frequency response and 3D cost function for values of ρ ranging from -1 to 1and β ranging from -1 to1. In Fig.2, as ρ approaches towards -1, lattice based filter produces an all-stop kind response and notch widens to spread over complete frequency response. This testifies the necessity to adapt ρ by (15) along with a constraint set for ρ has to be constrained to avoid instability. Fig 3 illustrates frequency response of the cost function, from which it can be observed as the ρ gets smaller, the gradient signal is steeper providing fast convergence. On the other hand if ρ is closer to unity the gradient is flatter and it will take more time to reach a global minima. Our interest is to obtain fast convergence whilst tracking the frequency of the interference signal. $\rho = 1$ means pole is on the unit circle and the filter is unstable. Therefore, ρ is constrained close to 1, and for this study is set in between 0.70 and 0.95 being lower and upper limits respectively. The algorithm for the fully adaptive is given in Table I.

It should be noted that ρ should not converge before β does in order to benefit from having an adaptive ρ parameter. To ensure β always adapts before ρ, the adaption of ρ is initiated after a certain number of samples, M. Resetting ρ reset to the lower limit of 0.7, as shown in line 5 of the algorithm, allows β to converge to target frequency at a quicker pace before ρ converges to its upper limit 0.95 which can be observed in Fig.5. The simulation results shown in Fig 5 were produced setting μ_ρ =set to 0.03 and μ_β=0.018 respectively. The forgetting factor γ was fixed at 0.9 for the adaption of ρ and β. As per algorithm, ρ was initialized at 0.70, ρ adapts after 'M' number of samples so β starts to converging to target frequency at a faster rate. The proposed algorithm have three
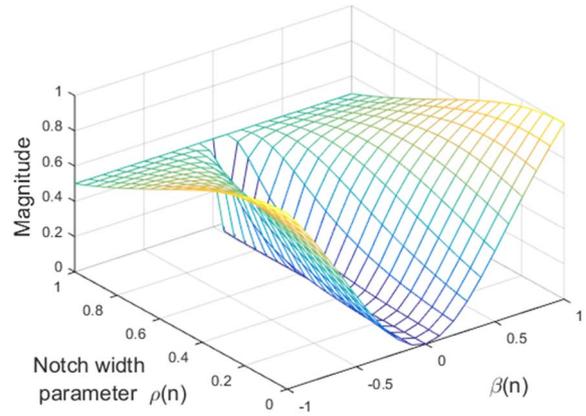

Figure 3. 3D illustration of the cost function $J(\beta,\rho)$ for lattice based

sets of conditions **A**, **B**, and **C** as highlighted in line 6, 10 and 13. Condition **A** of the algorithm ensures ρ only to update its value when ρ is between 0.70 and 0.95. Condition **B** acts as a stability upper limit (in this case 0.95), previous value of ρ

Table I. Proposed Algorithm to adapt ρ

Algorithm: *Adaptation of ρ with constraints*
Input: $x[n]$ & **Output** : $y_L[n]$
Initialization: $[n] = 0.70$; $\mu_\rho = 0.03; \gamma = 0.90, M = 120$
1:     **For** n =1:1:N_{sample}
2:     $y_L[n] = latticfilter(\beta[n], \rho[n], x[n])$
3:     $\psi[n] = \gamma\psi[n-1] + (1-\gamma)|y_L|^2$
4:     **Update**$\rightarrow \beta[n]$
5:     **If** $n > j + M$
6:   **A**   **If** $0.70 < \rho[n] < 0.95$
7:     $\rho[n] = \rho[n-1] + \mu_\rho y[n]g[n]/\phi_\beta$
8:     $\rho[n-1] = \rho[n]$
9:     **end**
10: **B**   **If** $\rho[n] < 0.70 \ or \ \rho[n] > 0.95$
11:     $\rho[n] = \rho[n-1]$
12:     **end**
13: **C**   **If** $\psi[n] > V_{threshold}$
14:     $\rho[n] = 0.70$
15:     $j = N_{sample}$
16:     **end**
17:    **end**
18: **end**

is retained in case ANF diverges. Finally, the condition **C** represents the prompt resetting of ρ to the value of 0.70, exactly when the frequency of the interferer changes. To ensure the fast convergence of β for the new subsequent frequency. The instantaneous power at the ouput of the ANF is monitored which provide prompt information when there is slight variation at the input of the ANF. This information can be used to reset ρ when the frequency of CWI changes. The instantaneous power at the output of ANF given by (16) and Fig.4 shows the $\psi[n]$ peaks as the CWI hop at 1000, 2000 and 3000 samples point.

$$\psi[n] = \gamma\psi[n-1] + (1-\gamma)|y_L|^2 \qquad (16)$$

ρ resets only when value of $\psi[n]$ is more than $V_{Threshold}$. It is the equivalent of twice the size of the expected value of the input signal (noisy signal) as show in (17). In this algorithm how ρ is adapted is show in Fig 5 by the black curve and blue curve shows the adaptation of ρ in [14] via proposed method.
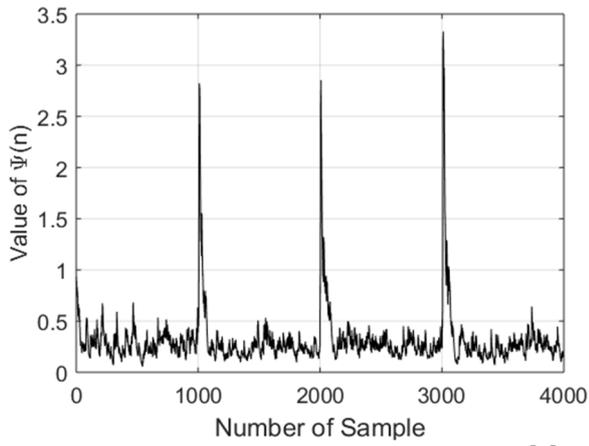
Figure 4. The instantaneous power at the output of ANF $\psi[n]$

$$V_{thershold} = 2E\{|[x[n]]|\} \qquad (17)$$

V. SIMULATION RESULTS

The red and blue curves in Fig.6 show the simulation results for the algorithm in [18] and [19]. As it can be observed from the results in Fig.6, the black curve shows superior convergence performance, which is due to the adaptation of notch bandwidth parameter. The black curve in Fig.5 represents the corresponding constraints on the adaptation of ρ via the algorithm described in the previous section. At the beginning of each new centre frequency, ρ rapidly converges towards unity from the initial value of 0.70 but as the time advances ϕ_β grows larger in size and the convergence rate of ρ slows down just before reaching the maximum limit of 0.95. These results clearly demonstrate benefits of adapting ρ for tracking CWI interference for any specific application. Furthermore, by simultaneously adapting both the coefficient in lattice based structure the performance of an ANF can be improved significantly. The performance of the lattice based ANF with the proposed full gradient adaptation of notch bandwidth parameter ρ with constraints is shown by the black curve in Fig.7.Its performance is far superior in comparison to the second order direct form [11] ANF and C/N_0 is significantly higher than [14] with the use of proposed adaptation algorithm presented in Table I. It is able to mitigate higher level of jamming signal levels. By simultaneously adapting both parameters, the transient endurance by an ANF at the beginning of each frequency change is reduced considerably.
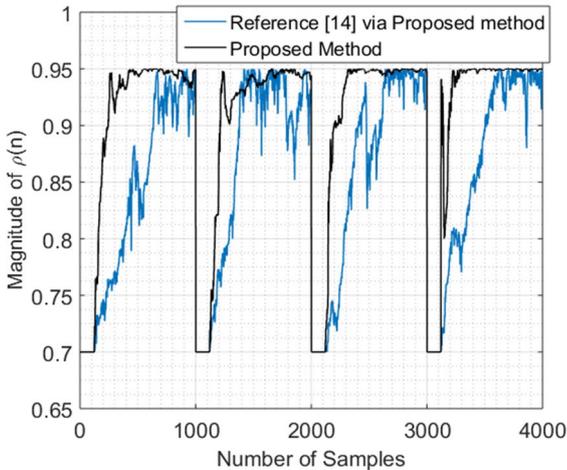

Figure 5. Adaptation of the parameter ρ for the proposed algorithm (in black) and reference [14] implemented via proposed algorithm (in blue).
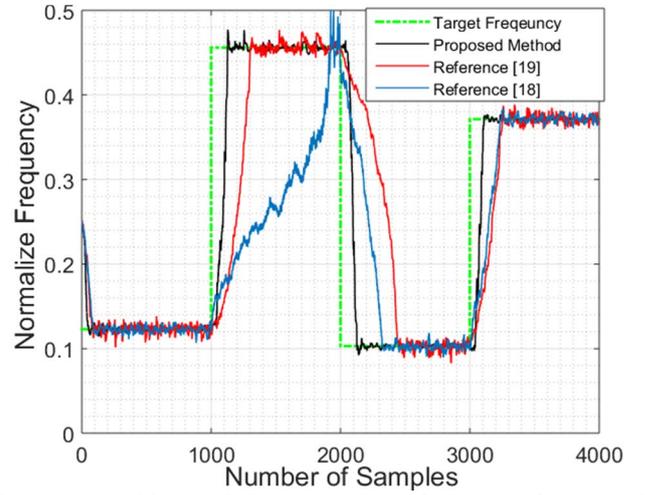

Figure 6. Tracking and Convergence performance of proposed method, reference [17] and [18], where $\mu_\beta = 0.02$, $\mu_\rho = 0.018$ and $\gamma = 0.9$
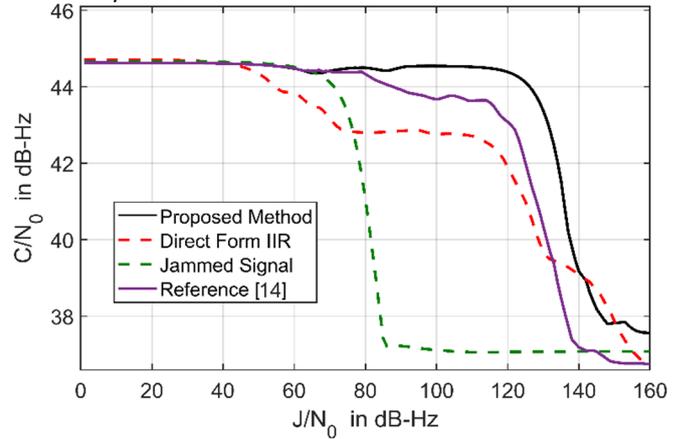

Figure 7. C/N_0 At output of the acquisition module. Red curve represents Direct form IIR [11], black curve represents lattice base ANF with proposed algorithm, blue curve shows when [14] implemented via proposed algorithm in Table I. Brown curve represented the jammed signal. Black curve shows higher C/N_0.

VI. CONCLUSION

A novel optimization algorithm for the lattice based adaptive IIR notch algorithm is presented, which can simultaneously adapt both its notch bandwidth and centre frequency parameters. The proposed algorithm shows favourable convergence properties for both of its adaptive parameters and resulting in improved tracking performance and higher C/N_0 at the output of the acquisition block, when compared with the relevant designs chosen from the open literature. In this research article different types of ANF algorithms are compared in terms of tracking rate, convergence speed, and output C/N_0 specific to GPS based applications. A constrained adaptation of the notch bandwidth parameter is developed, modelled and presented. It is shown in the simulation results that by simultaneously adapting both ρ and β, the performance of the lattice based adaptive notch filter can be enhanced significantly. Furthermore, the tracking ability and convergence speed of proposed algorithm is compared with the existing techniques to provide in-depth comparative analysis between existing methods. The existing and proposed methods are tested for frequency varying interference and the proposed method shown to be superior. The trade-off between quality of interference mitigation and optimization of computational complexity of system is something that is needed to been addressed in a future study.

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