Classroom design demonstrations for complex IIR filters.

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CLASSROOM DESIGN DEMONSTRATIONS FOR COMPLEX IIR FILTERS

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ABSTRACT

An environment for exceptionally fast design of FIR and IIR filter design by pushbutton and easy drag-and-drop manipulations is described. Filter gain and phase credentials are immediately available to the designer, along with the facility to simply sketch the desired filter characteristics. The design flow for a specimen complex FIR and IIR realization of an arbitrary combination of gain and group delay is presented in detail, and a subsequent phase compensation filter also shown. A varied selection of other design examples emphasizes the flexibility and abundance of choice afforded the User.

1. INTRODUCTION

DSP Learners gain a lot from quick hand calculations of a digital filter's coefficients carried out in front of their own eyes, especially if there is immediate, painless visual display of the resulting gain and z-plane pole-zero pattern which are subsequently pondered and discussed in real time. But the fascination quickly wears thin if lengthy, tedious calculations have to be endured, or even if tortuous syntax typing of design commands has to be passively witnessed in a classroom situation. What is needed is a snappy, high dazzle-factor way of **involving the audience** in a few readily-followed design manipulations!

We have developed a MATLAB-GUI-based Tool called *Sketch-a-Filt* which performs most things to do with digital filter design through sketches and pushbutton or drag-and-drop graphical operations. Where standard filter types (lowpass, bandpass, differentiators, etc.) are wanted, the User selects from a menu, setting a few key parameters in edit boxes. Where curiosities with arbitrary-shaped gain curves are desired, it's merely a matter of mouse-sketching what is ideally sought. An FIR approximation (using Frequency-Sampling) is delivered, even as the drawing is being done! Immediate display of impulse response, gain (or phase or group delay) and pole-zero pattern follows a single button-push.

Up to six simultaneous design styles (three FIR and three IIR) can be selected as standard for this near-real-time design response. In addition, a User-Defined mode allows Users to embed their own MATLAB design algorithm(s) in *Sketch-a-Filt* in order to employ its facilities as a test and comparison environment.

Even greater design power is unleashed when arbitrary combinations of gain and phase are being pursued. A separate GUI hosting a waveform calculator (our "Target Profile Calculator") gets launched, and elaborate manipulations of curve shapes can be initiated by button presses, or MATLAB instructions can be typed (or m-files invoked) in-place for a customized frequency-dependent shape. Once a target gain/phase combination is settled upon, the main GUI takes over and pumps out the design, as before.

2. SPECIMEN DESIGN FLOW

This environment makes even design of demanding complex-coefficient FIR and IIR filters a breezy and enjoyable affair. A typical problem will illustrate the procedure. In Figure 1, a piecewise linear gain target (the red "target profile") has been auto-drawn by typing in just 10 vertices of straight lines.

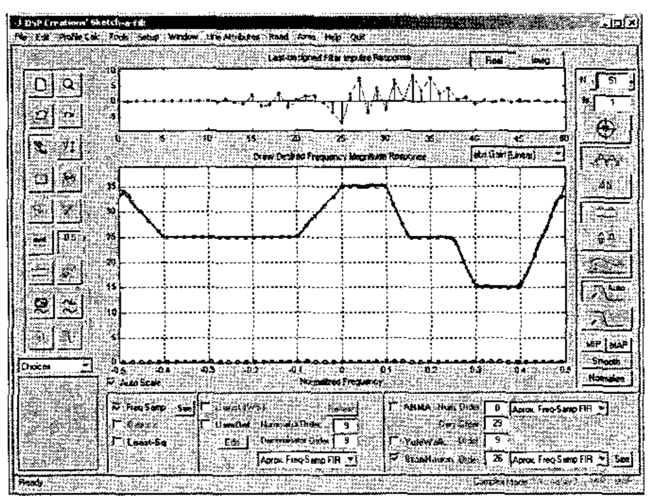

Fig. 1 Our GUI Showing the Gain Target for a Complex Filter (in red) [design results in black and green]

If the User is satisfied with a linear phase characteristic to accompany this gain, then the job is already finished: simply take the Frequency Sampling FIR result. But, to make things more interesting, a most unorthodox design goal is set: let's have a filter with precisely the **same group delay as its gain**! And, for good measure, let's have both complex IIR and FIR versions!

Figure 2 shows the Target Profile Calculator with the gain target (duplicated on three of its Displays) it has inherited from Figure 1. A mere push on the upper-left calculator button invokes the service utility command shown as an m-code line in the long edit box and we end up with the two-trace (real and imaginary) plot residing on Display C. Sending that back to the main GUI we see the tight hugging gain curves evident in Figure 1.

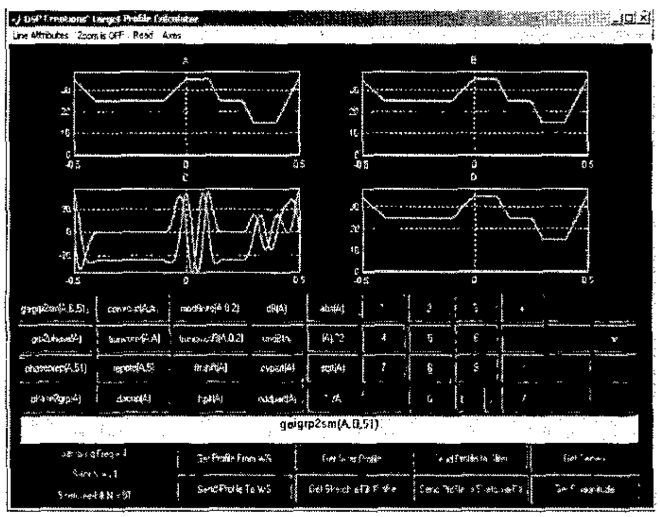

Fig. 2 A Target Profile Calculator Scene

One of these achieved gain plots has resulted from an FIR frequency-sampling design with 51 coefficients. The IIR design subsequently approximates that FIR approximant (not the ideal target!) By employing the phenomenally powerful (but too little-recognized) brandenstein-unbehauen algorithm [1]. Do we obtain the strange group delay we aspire to? Figure 3 shows how well (especially for so little effort!) We have done:

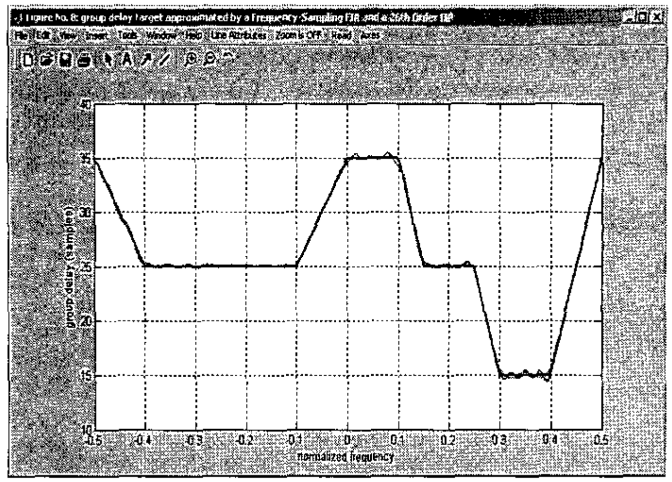

Fig. 3 Group Delay Curves- The Target With 51-Coefficient Fir & 26th Order IIR Designs

Now suppose we come upon both the filters designed thus far and decide we want to design follow-on compensator filters that will flatten the group delay to 50 samples for all frequencies, but yield gains which are the square of what we saw in Figure 1. All we need do in the Target Calculator is set 50 minus the old Display A into a new Display A and proceed exactly as before. Here are the gains we get:

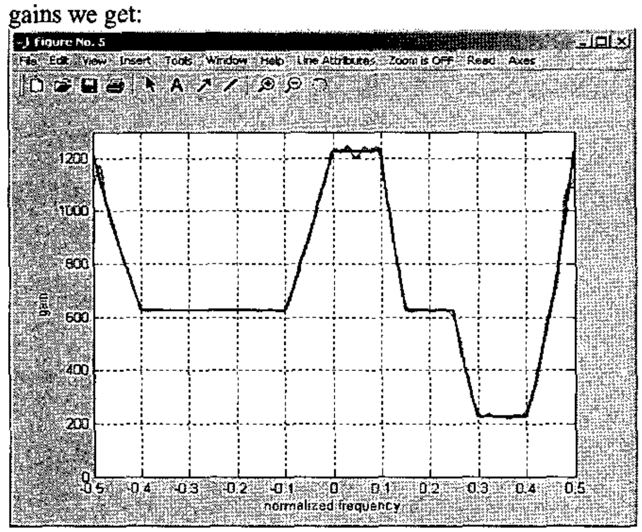

Fig. 4 Gains of Cascaded Filters (101-Coeff FIR and 50th Order IIR overall)

And we have our group delay compensated close to flatness - a thoroughly satisfactory outcome:

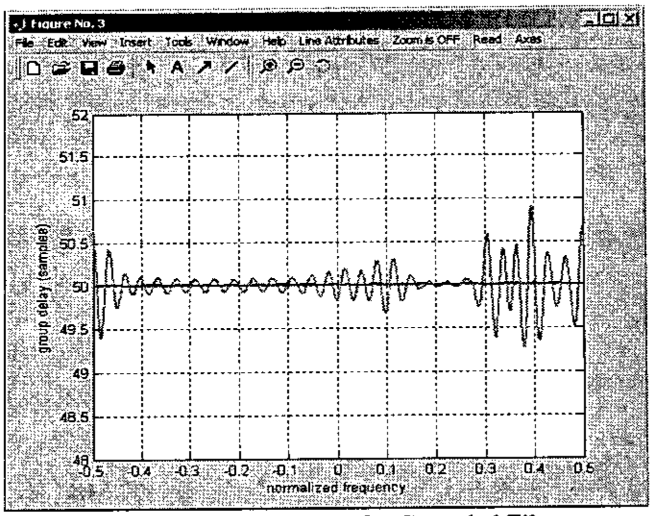

Fig. 5 Group Delay Curves for Cascaded Filters (101-Coeff FIR and 50^{th} Order IIR overall)

3. PHASE LINEARITY ACROSS PASSBANDS

Early work on Balanced Model Reduction [2] showed it to be a powerful way of achieving linear phase in IIR filters, at least over passbands. This has also been powerfully evident in the designs the Brandenstein-Unbehauen algorithm delivers. A simple example can be taken: a 51-coefficient FIR complex bandpass prototype obtained in MATLAB by invoking **cremez** and subsequently imported into *Sketch-a-Filt*; it is shown as the gray Guest filter in Figure 6, along with an Order 10 Brandenstein-Unbehauen approximant (in green).

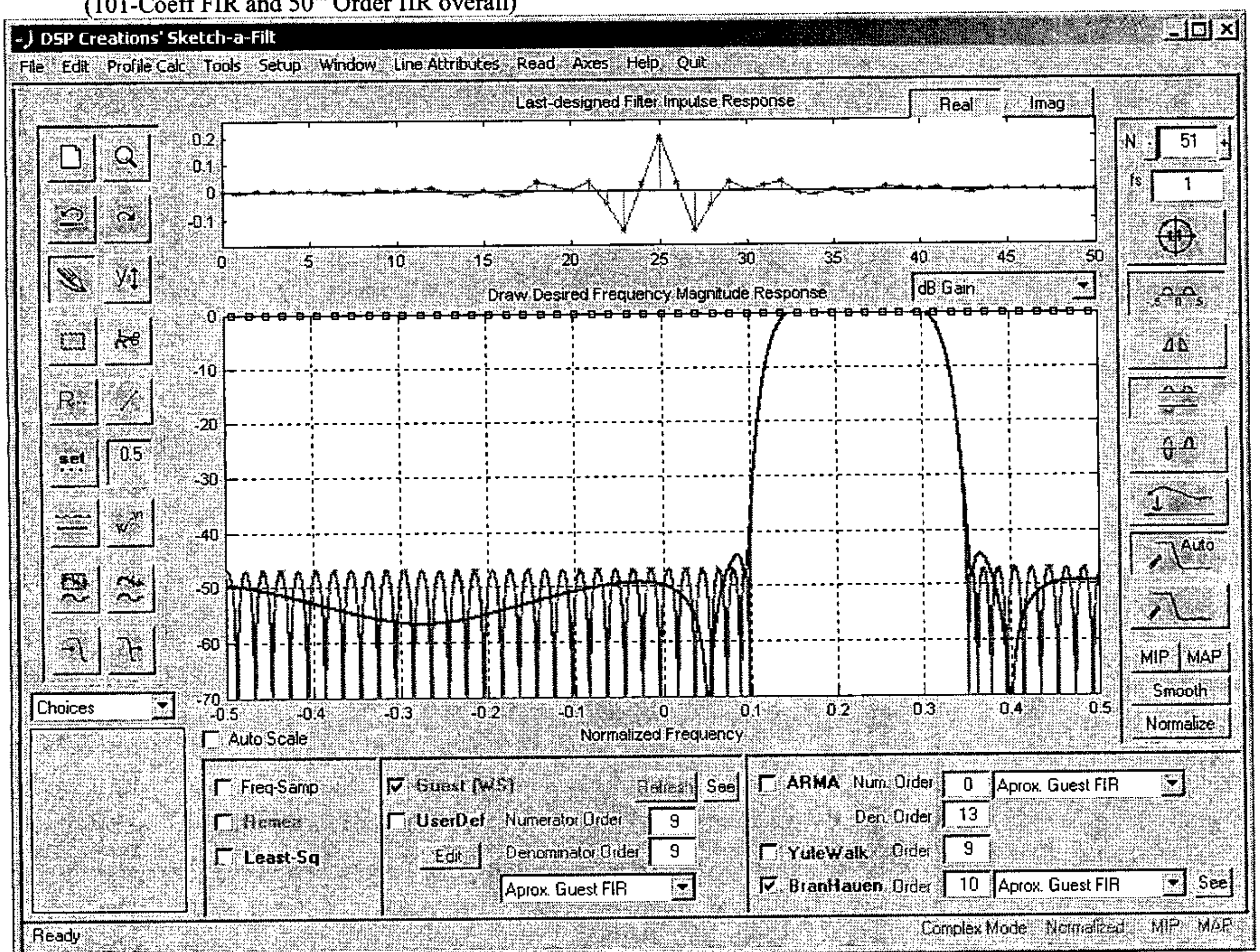

Fig. 6 A 51-Coefficient FIR Complex Bandpass Filter with a 10^{th}-Order IIR Approximant

It is clear that - though equiripple behaviour has been sacrificed - the stopband error structure (except for two small bulges right at the passband edges) resides comfortably below that of the prototype, making the approximation very good indeed.

Pushing *Sketch-a-Filt*'s "See" button for the filter in green we are able to view most of the important "filter credentials". This has been captured in Figure 7. The noteworthy thing is that the passband phase for this IIR filter is breathtakingly linear!

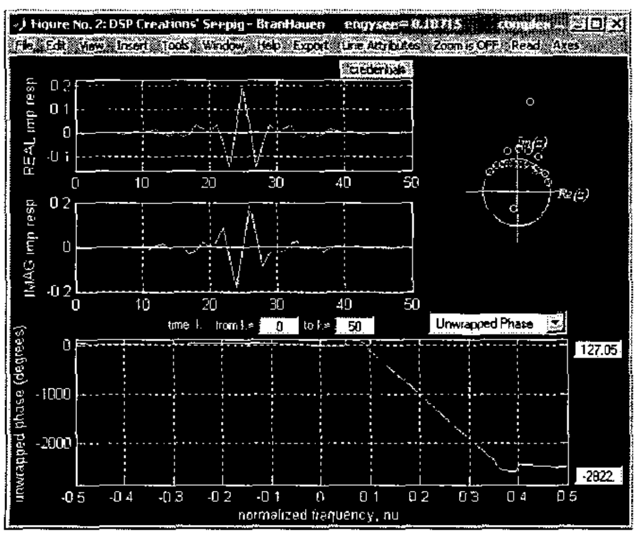

Fig. 7 Credentials of the 10th Order IIR Approximant to the FIR of Figure 6

4. THE ATTRACTIONS OF MINIMUM-PHASE PROTOTYPES

It has been observed before that the gratifying linear-phase approximation power we see for Brandenstein- Unbehauen-type designs become even more spectacular when minimum-phase FIRs are targeted [2]-[4]. This is borne out by a slight change to the previous design. We simply press the MIP (for MInimumPhase) button and again approximate the "MIPized" Guest, getting a successful approximation with a Brandenstein-Unbehauen filter of only Order 6!

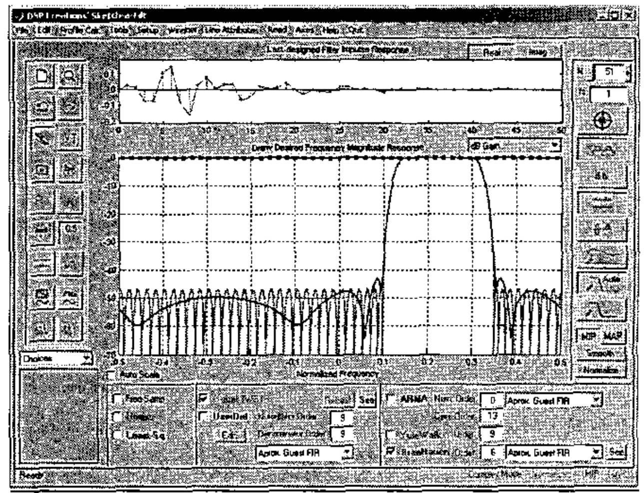

Fig. 8 A 6th Order IIR Approximant to the FIR of Figure 6 when MIPized

Now we can move on to easily undertake whimsical departures from what we've done. It only costs a button push or so! Suppose we want our IIR filter to be complex, minimum-phase, bandpass(ish) and to exhibit Nyquist behaviour, peaking at time sample 6 and having zero-valued samples every 5 samples (anchored about sample 6). We simply invoke Nyquist imposition in *Sketch-a-Filt* and get this curious result:

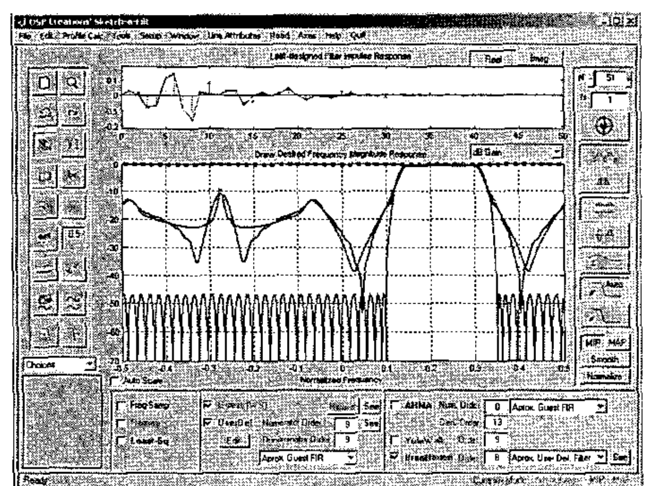

Fig. 9 An 8th Order IIR Approximant to the foregoing FIR, with periodic samples forced zero

We see the blue curve, which is the FIR, obeying (somewhat recklessly) our impulse response zero-impostion instruction faultlessly. We have gone back up to Order 8 to get the excellent Brandenstein-Unbehauen approximation (in green) seen in Figure 9.

5. A COMPLEX IIR DIFFERENTIATOR EXAMPLE

Preuss [5] has a nice example of a complex differentiator target specification which has its transfer function non-zero only over the normalized frequency interval (0.0375, 0.425). We can rapidly concoct such a bandlimited linear target profile by a calling up a standard differentiator through popup menu selection and opening the Target Profile Calculator to do our construction. Figure 10 displays how easy the necessary work is:

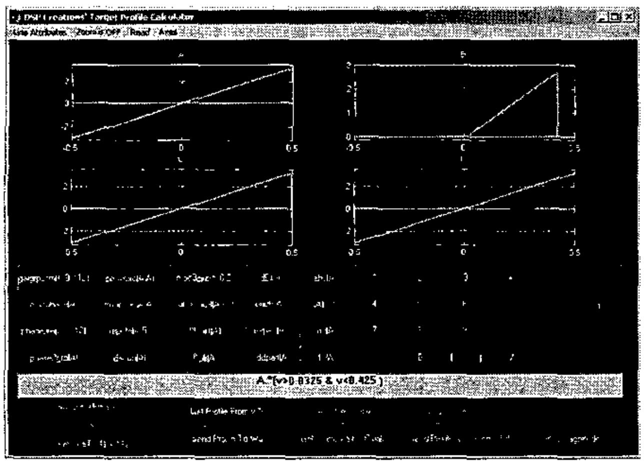

Fig.10 Making a Complex Differentiator Target Profile

Next we design a lavishly-dimensioned FIR by Frequency Sampling (investing no less than 112 coefficients for this stepping-stone!).Then we merely decree that two IIR designs are required. One is a Brandenstein- Unbehauen of Order 25 (green in Figure 11, and faithfully delivering linear phase) and an All-Pole design costing about the same in number of implementation multipliers (by clicking on ARMA, as shown in magenta in Figure 11). Such arbitrary dabblings are painless in this environment and are very conducive to free-wheeling exploration of alternatives.

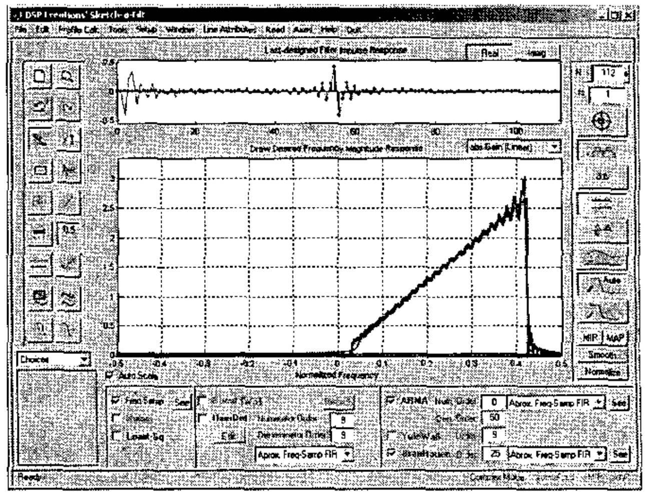

Fig.11 A 25th Order Near-Linear-Phase IIR Approximant and a 50th Order All-Pole IIR Approximant to a Complex Bandpass Differentiator

6. REMARKS

We have several tales of successful "serious" filter designs that have been done for researchers, quite apart from numerous in-class design occasions - all of which have evoked enthusiastic comments, especially when desirable transfer function features have been cloned and morphed or outlandish gain/phase hybrids have been "surgically joined". Some have been heard to say: "Why didn't somebody tell us that filter design was this easy?"

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