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Design of Generalized Chebyshev Lumped Element Filters by Computer Optimisation

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Abstract
A numerical method for the optimization of a symmetrical lumped element lowpass and bandpass filters with Generalized Chebyshev response is considered. By exploiting the fact that a network based on generalized Chebyshev prototype has a prescribed number of turning points in the insertion loss and an identical number of independent parameters which can be assigned as variables to adjust their levels the method gives fast convergence.

Key-Words: Low-pass Filters, Bandpass Filters, LC Filters, Lumped Elements, Generalized Chebyshev Filters, Optimization

1. INTRODUCTION
When a common approach to the design of filters results in a design passband which differs considerably from that which is specified, optimization is required to tune the filter elements to achieve a design that meets certain requirements. Most RF and microwave filters have not yielded exact optimum synthesis. Taking into account parasitic effects, high frequency operation, frequency dependent elements, a narrow range of element values, and so on, a common approach to design provides, at best, only approximate answers. Not infrequently, a common approach may be used to great advantage in providing the initial points for optimization. In this paper, we introduce an optimization procedure based on equal ripple optimisation to optimise filters based upon Generalized Chebyshev function prototype.

This method searches for tuning points in the filter transfer function and forces the ripple levels at these points to have specified values. The method requires knowledge of the filter insertion or return loss at these points. The method will generate a set of equations which are solved to give a new set of parameter values. The cycle is then repeated, until the filter characteristic is within an arbitrarily close value to the desired specification. This technique requires less calculation of the electrical parameters of filter discontinuity than generalised optimization routines so far applied [1].

2 PROBLEM FORMULATION

The double terminated low-pass prototype network shown in Figure 1 satisfies a generalized Chebyshev insertion loss response.

![Figure 1. Generalised Chebyshev low pass prototype](image)

This characteristic in terms of insertion loss, L, is given by

\[
L = 1 + 2 \cos^2 \left( \frac{n-1}{2} \right) \cosh \left( \frac{\pi}{2} \eta - \frac{1}{2} \right)
\]

where the transmission zeros are of order \((n-1)\) at
w=±w0 and one at infinity. n is an odd number equal
to the degree of the network, E = \left[ 10^{\frac{L_i}{10}} - 1 \right]^{\frac{1}{2}}
and R.L. is the minimum return loss level (dB) in the
passband.

A typical insertion loss response is illustrated in Figure 1a, where \( w_m \) is the frequency of the minimum
insertion loss level in the stopband and \( w_i \) is the
bandedge frequency of the stopband.

In general, approximate methods based on the
synthesis of a generalized Chebyshev prototype to the
design a symmetrical filter will not meet the
specifications satisfied by (1). Assume that an nth
degree symmetrical low-pass filter has an insertion
loss response \( L_i \) of the form shown in Figure 1a. It
exhibits \( m-1 \) \((m=n-1)\) zeros and \( m-2 \) ripples, the
maxima of which occur at the frequencies \( f_2, f_3, ..., f_m \).
For a symmetrical low-pass filter all of these \( m-2 \) frequencie
lie within the specified passband \( f_i \approx f_0 \).
The deviation of a ripple maximum from the
maximum allowed insertion loss in the passband, \( L_{on} \),
is a function of the \( m=n+1 \) symmetrical filter parameter
values required to specify the low-pass filter. There are \( n-1 \) such functions for the symmetrical
case:

\[
E_i = L_i(f_i) - L_{on}, \quad i=2,3,n-3
\]  
(2)

\( E_c \) and \( E_m \) are defined by:

\[
E_c = L_1(f_c) - L_b
\]  
(3)

\[
E_m = L_1(f_m) - L_m
\]  
(4)

\( E_c, E_m \) are also functions of the \( m=n-1 \) parameter
values of the symmetrical filter.

The specifications

\[
L_i(f) \leq L_b, \quad 0 \leq f \leq f_c
\]  
(5)

\[
L_i(f) \geq L_m, \quad f_o \leq f \leq f_m
\]

are satisfied when

\[
E_i = 0, \quad i=1,2,3,...,m
\]  
(6)

This is a system of \( m=n-1 \) nonlinear equations in
\( m=n-1 \) variables for the symmetrical case. Solving (6)
gives the parameter values of a filter satisfying (5). The
\( E_i (i=1,...,m) \) can be regarded as the components
of an \( m \) dimensional error vector. Optimization is
carried out by equating each of these components to
zero (a vector process) rather than minimizing the
magnitude of the vector (a scalar process). Thus equal
ripple optimization can be regarded as a vector
procedure whereas general purpose optimization routines
are scalar procedures. Usually the
convergence criterion applied in general purpose
optimization routines is that the gradient, with respect
to the filter elements, of the magnitude of the error
vector is zero. However a zero gradient may
respond to a local minimum and the error may not
be truly minimized. The convergence criterion applied
in equal ripple optimization is that each component of
the error vector is zero. Thus on convergence the error
is reduced to zero. The problem of local minima does
not arise.

To apply an iterative nonlinear equation solver it is
necessary for a given set of filter parameter values to
know the insertion loss only at the bandedge
frequency, \( f_o \) (minimum) and at the ripple maxima.
However, the frequencies at which the ripple maxima
occur are unknown and are functions of the filter
parameter values.

The Newton-Raphson method [4] is a rapidly
convergent technique for the solution of a system of
nonlinear equations if a good initial approximation is
available. The number of times the function is
evaluated in the process of finding its root is the usual
measure of computational effort. This includes
function evaluations required to calculate derivatives
numerically.

The Newton-Raphson method has the general form [5]

\[
x^{k+1} = x^k - J^{-1}(x^k) E(x^k)
\]  
(7)

where \( k \) is the iteration number \((k=1,2,...)\) and \( J^{-1} \) is the
inverse of the \( m \times m \) Jacobian matrix evaluated at
\( x^k \). The above identifies the regions within the
passband which need to be sampled in order to
calculate \( E(x^k) \) (and \( J(x^k) \)). The response and errors
after each iteration are computed again with the new
corrected parameters, until the errors are judged to be
sufficiently small.
3 NUMERICAL RESULTS

In order to illustrate our approach, a fifth order lumped element low-pass and band-pass filters have been designed. The low-pass filter can be described by 4 parameters: inductors \( L_1=L_5, \ L_2=L_4, \ L_3 \) and capacitor \( C_2=C_4 \) as marked in Figure 2. We used equal ripple optimization with \( L_1, L_2, L_3 \) and \( C_2 \) as variables for filter shown in Figure 2. Figure 3a shows the calculated return loss (dashed line) and insertion loss (solid line) of filter before optimization. The return loss (dashed line) and insertion loss (solid line) calculated using the filter elements obtained on convergence are shown in Figure 3b. The band-pass filter can be described by 6 parameters: inductors \( L_1=L_5, \ L_2=L_4, \ L_3 \) and capacitors \( C_1=C_5, \ C_2=C_4, \ C_3 \) as marked in Figure 4. We used equal ripple optimization with \( L_1, L_2, L_3 \) and \( C_1, C_2, C_3 \) as variables for filter shown in Figure 4. Figure 5a shows the calculated return loss (solid line) and insertion loss (dashed line) of filter before optimization. The return loss (solid line) and insertion loss (dashed line) calculated using the filter elements obtained on convergence are shown in Figure 5b.

![Figure 2. Generalized Chebyshev low-pass filter](image)

![Figure 3a. Simulated Insertion and Return loss of generalized Chebyshev low pass filter before optimization](image)

![Figure 3b. Simulated Insertion and Return loss of generalized Chebyshev low pass filter after optimization](image)

![Figure 4. Generalized Chebyshev band-pass filter](image)
4 CONCLUSION

The method presented here offers a simple but reliable method for optimization of low-pass and band-pass filters with Generalized Chebyshev function prototype. The method provides fast convergence.

References