# Structured tone mitigation in 3rd and 4th order MASH Delta-Sigma Modulators-comparative study.

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# Structured Tone Mitigation in 3rd and 4th Order MASH Delta-Sigma Modulators-Comparative Study

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## Abstract

*A Delta-Sigma Modulator (DSM) can be thought as a nonlinear chaotic system that may exhibit tonal behaviour in its output spectrum. These tones are sometimes referred to as spurs and they are undesirable. To provide for the mitigation of structured tones, application of dithering, using chaotic modulators, loading irrational initial conditions and maintaining controllable maximum sequence lengths are commonly used and advised methods primarily in Multi-stAge noise SHaping (MASH) DSMs. Higher order MASH-DSMs are less problematic and are commonly used in many high speed and low noise frequency synthesiser circuits. As MASH is composed of cascaded first order digital DSM stages, it is unconditionally stable. In this paper, the tone mitigation techniques for MASH 1-1-1 and MASH 1-1-1-1 modulators are compared and their noise performances presented.*

## 1. Introduction

A common type of DSM deployed in Fractional-N (FN) frequency synthesizer is the MASH DSM that is composed of cascaded first order digital DSMs (DDSM) [1]. The Simulink models of MASH 1-1-1, MASH 1-1-1-1 ($3^{rd}$ and $4^{th}$ order) and a first order DDSM used in their stages are given in Figure 1, Figure 2, and Figure 3, respectively.

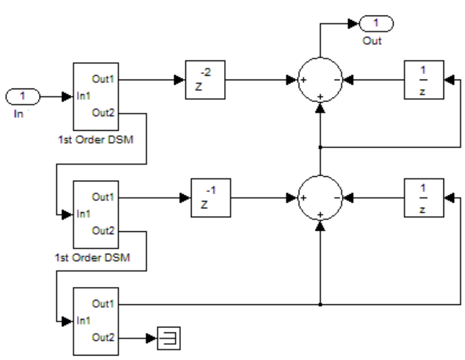


**Figure 1. Simulink Model of MASH 1-1-1 DDSM**

The advantage of MASH is that it is unconditionally stable since it is composed of $1^{st}$ order stages that are proven to be unconditionally stable themselves. For an $n^{th}$ order MASH, the quantization error of each stage is fed to the next stage, the outputs are then combined in a noise shaping block that cancels the noise from the first stages by producing multi bit outputs that exhibit $n^{th}$ order noise shaping characteristics. So, the total noise is shaped and pushed to higher frequencies. The major drawback of a MASH DDSM employing single bit quantizers with first order accumulators in all its stages is that its output is multi bit (n+ 1 bit in length with $2^n$ output levels). It is worth clarify that an $n^{th}$ order MASH would produce $2^n$ levels which would require (n+1) bits to represent the output levels in signed digital arithmetic [2]. For example, a $4^{th}$ order MASH would produce 16 output levels {-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8}. The time domain output of waveform of a MASH 1-1-1-1, and the histogram of its output levels are given in Figures 4 and 5, respectively.

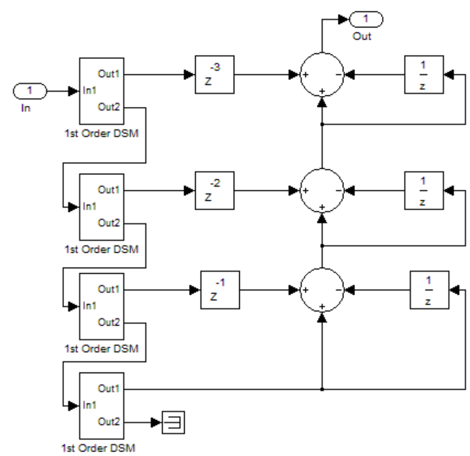


**Figure 2. Simulink Model of MASH 1-1-1-1 DDSM**

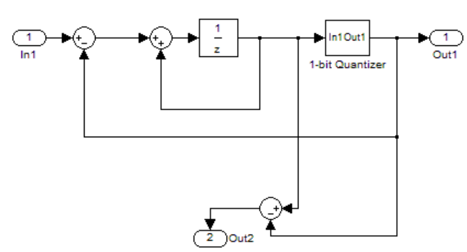


**Figure 3. Simulink Model of 1st Order DDSM used in MASH 1-1-1 and MASH 1-1-1-1 stages**

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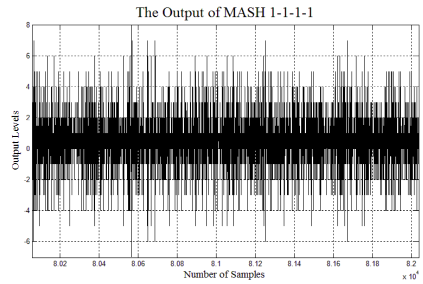


**Figure 4. MASH 1-1-1-1 Output Levels**

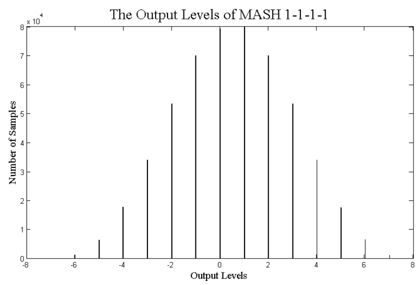


**Figure 5. Histogram of MASH 1-1-1-1 Output Levels**

## 2. Tone Mitigation Techniques

To reduce DSM tonal behaviour, some methods have been reported and evaluated in the open literature. One general method is to use dithering to randomize the behaviour of the modulator by introducing noise at any stages of the DSM [2]. Dithering breaks up short cycles, increasing the effective sequence length resulting in smoother noise shaped spectrum [3]. The major drawbacks of dithering are the need for additional hardware and increased noise floor [1], [4]-[5]. One-bit input dithering can be applied by adding one-bit random sequences (filtered (coloured) or un-filtered) to the input of the first stage, to the last stage, or to all stages of the modulator [1]. Moreover, by changing the Least Significant Bit (LSB) of the input of the modulator or by increasing the number of bits to represent the input by one bit (adding an additional one bit at the LSB), dithering can also be provided [6] -[7]. However, dithering the LSB at the input will result in frequency error also by decreasing the frequency resolution at the output of the FN-frequency synthesizer.

The second method for tone reduction in DSMs, is the use of chaotic modulators [3]-[9]. However, this method will not always guarantee that the DSM output will be tone free due to the inherent unpredictable characteristics of the chaotic systems. Moreover, chaos is sensitive to initial conditions.

The third method is to load pre-defined initial conditions to the modulator. It has been shown by [6] - [7] that the spectrum will be structured tone free if irrational initial conditions are applied to the first stage of the modulator. However, for a fixed point digital implementation, it is strictly not possible to realize irrational initial condition so it is advised and proved that an initial condition that is an odd number may also be used. It is suggested in [6] that if the internal digital word length of a fixed point DDSM is greater than 15 for 3$^{rd}$ or higher order MASH, irrational initial conditions can be approximated by odd numbers.

One other method to have tone free outputs is to design the modulator to maintain a very long controllable sequence lengths that results in the smooth distribution of the quantization noise [4]-[5], [8]. For DDSMs, if they are operating with rational input and rational initial conditions, they always produce finite length sequences since they are finite state machines [4]. In this method, instead of avoiding the periodic behaviour, this behaviour can be controlled to generate maximum length sequences to provide internal randomization. This method demonstrates that for DDSMs, if the sequence length is long enough there will be no tonal behaviour in the output even if there is no initial condition applied for DC inputs as well [4]-[5], [8].

In the literature, there is not much comparison given for these four structured tone mitigation techniques in terms of their noise and spurs performances. For example, according to [9], dithering may provide 9 dB better SNR than a chaotic modulator for the same order and amplitude range.

In this paper, other than the technique of maintaining very long controllable sequence lengths all the other techniques will be investigated together and their performances will be compared. For a DDSM FN-frequency synthesizer application, higher order DSMs such as, MASH 1-1-1 and MASH 1-1-1-1 have been chosen as the DSM topologies.

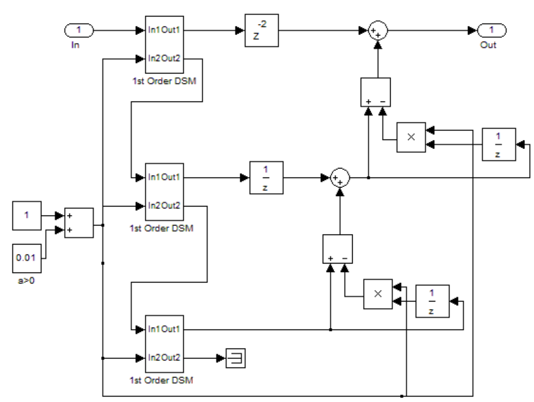


**Figure 6. Simulink Model of MASH 1-1-1 Chaotic Modulator**

The Simulink model of a MASH 1-1-1 chaotic modulator, a first order DDSM used in its stages and its output power spectrum are given in Figure 6, Figure 7 and Figure 8, respectively. Please note that for power spectrum estimations in the paper, 16 %50 overlapping periodgrams, each with 32K of length (specified by Hanning window), were averaged before the Fourier transform by using Welch's method. Please also note that the output of one bit quantizer is 0 if its input is <1 and 1 if its input is ≥ 1.

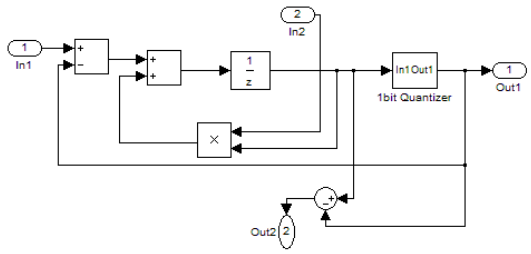


**Figure 7. Simulink Model of 1st Order DDSM used in Chaotic Modulator**

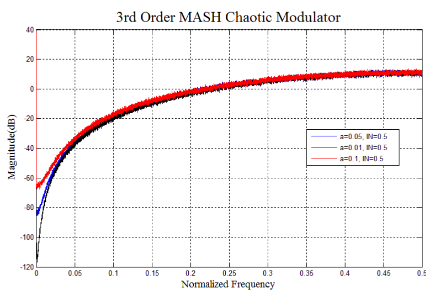


**Figure 8. Output Power Spectrum of MASH 1-1-1 Chaotic Modulator**

For making the modulator chaotic, the value "a+1" is inserted to the modulator providing the pole of the feedback filter is outside the unit circle [9]. As seen in Figure 7, the value "a" has an effect on low frequency values. At DC, for the same input value of 0.5, there is almost 60 dB difference in spectrum for the values a=0.1 and a=0.01. Numerous simulations results have also shown that for all "a" values such that $10^{-12} < a < 10^{-2}$, the spectrum is free of tones for all input values between 0 and 0.9 for a 3rd order chaotic modulator as far as it could be observed. However, exhaustive simulation results with automated tonality measures will be reported in a future paper.

Dithering is applied by adding two random sequences to the input of the single bit quantizer in the first stage of the DSMs given in Figures 1 and 2. The Simulink model of a first order DDSM with dithering is given in Figure 9. In dithered modulators, the mean and the variance values for dithering random sequences, are critical and important for noise shaping performance of the modulator.

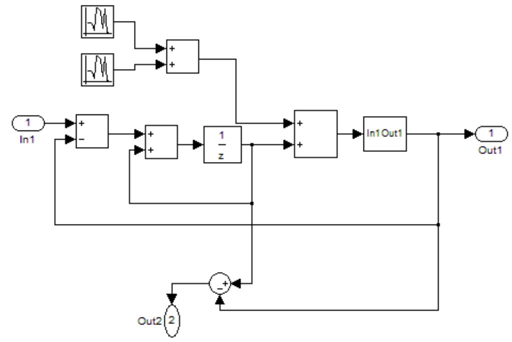


**Figure 9. Simulink Model of a 1st Order DDSM with Dither**

In Figure 10, the simulation results for 3rd order dithered MASH 1-1-1 modulators is given. Numerous simulation results have shown that the output power spectrum of the modulator depends on the chosen dithering values, random sequences, and the input value of the modulator. As seen in Figure 10, the power spectrum has no tones for IN=0.5 with random sequence (Mean=0.5 and Var=0.035). Moreover, for IN=0.5 with random sequence (Mean=0 and Var=0.035), and for IN=0.6 with random sequence (Mean=0 and Var=0.05), the output spectrum has two tones at 0.25, 0.5 and 0.2, 0.4 normalized frequencies, respectively.

To observe the effect of dithering in other stages on the output power spectrum, dithering is applied by adding two random sequences to the input of the single bit quantizer in the last stage and to the inputs of the single bit quantizers in all stages of DDSM. The output power spectrums of MASH 1-1-1 dithered modulator for these two cases are given in Figure 11. As it is clear from Figure 11 that there is a tone at normalized frequency 0.25 for the case where the dithering is only at last stage. However, for the case where all stages are dithered the output power spectrum is free of tones.

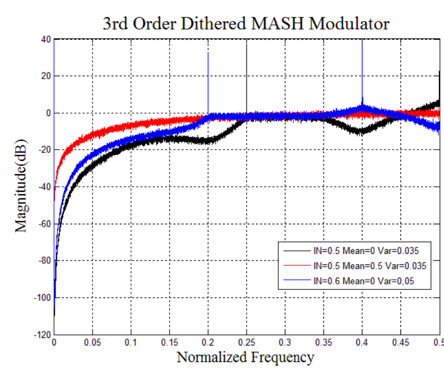


**Figure 10. Output Power Spectrum of MASH 1-1-1 Dithered (1st Stage) Modulator**

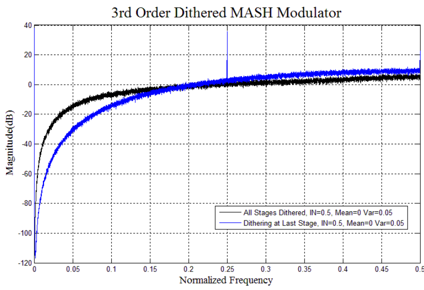


**Figure 11. Output Power Spectrum of MASH 1-1-1 Dithered Modulators**

In Figure 12 and 13, the output power spectrums of modulators "chaotic, dithered (applied in the last stage, at the input of the one bit quantizer) and loaded with irrational initial condition (applied in the first stage, at the unit delay element)" are given for MASH 1-1-1 and MASH 1-1-1-1 structures, respectively. Please note that in these figures, by considering the input of the modulator as IN=0.5, the values a=0.01 for chaotic, random sequences with Mean = 0, Var =0.05 for dithered, and initial condition=pi/33 for initial condition loaded modulators were taken.

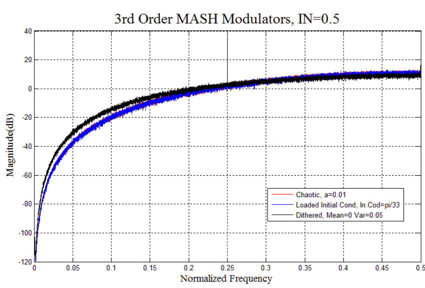


**Figure 12. Output Power Spectrum of MASH 1-1-1 Modulators**

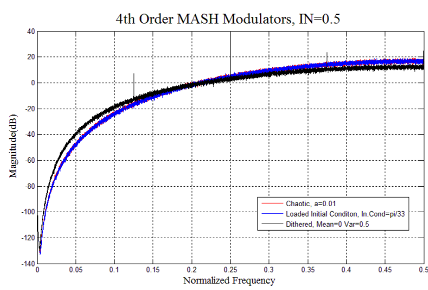


**Figure 13. Output Spectrum of MASH 1-1-1-1 Modulators**

According to Figures 12 and 13, the chaotic and the initial condition loaded modulators have no tones at their output power spectrums but dithered one has one (at normalized frequency 0.25) for $3^{rd}$ order and four (at normalized frequencies 0.125, 0.25, 0.375 and 0.5) for $4^{th}$ order cases. It is also clear that the output power spectrums of the chaotic and the initial condition loaded modulators are almost the same.

## 3. Conclusion

DSMs can be thought as a nonlinear chaotic system that may exhibit tonal behaviour at their output spectrums. To provide for the mitigation of structured tones, application of dithering, using chaotic modulators, loading irrational initial conditions and maintaining controllable maximum sequence lengths are commonly used and advised techniques primarily in MASH-DSMs. Higher order MASH-DSMs are less problematic and are commonly used in many high speed and low noise frequency synthesiser circuits for generating fractional part of the division ratio due to their noise shaping characteristics.

In this paper, other than maintaining very long controllable sequence lengths, the tone mitigation techniques, namely application of dithering, using chaotic modulators and loading irrational initial conditions, for MASH 1-1-1 and MASH 1-1-1-1 modulators were implemented and their noise shaping performances presented and compared.

According to the numerous simulations, it is observed that the output power spectrums of dithered and chaotic modulators may have tones and are not always tone free. However the spectrums of loaded irrational initial condition modulators are free of tones for all input values. Moreover, due to less number of components to implement, loaded irrational initial condition method is advised. However, the drawback of this technique is, for fixed point digital implementation, it is not possible to realize irrational initial condition. It is advised and proved [6] that if the internal digital word length of a fixed point DDSM is greater than 15, for $3^{rd}$ or higher order MASH, irrational initial conditions can be approximated by odd numbers.

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