The connectedness and hedging between gold and Islamic securities in the short, medium and long term

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Abstract

This paper investigates the dynamic connectedness between gold, sukuk and Islamic equities at multiple investment horizons, it also computes optimal hedge ratios and portfolio weights for these assets. Our findings suggest that gold hedges the risk of sukuk in the short and medium terms. We find also that gold plays an average but stable role in hedging and diversifying Islamic equities across all investment horizons. Moreover, we find that gold–Islamic assets portfolio provided a better risk diversification in the short term. These empirical findings are important as they highlight the role of gold in diversifying and managing the risks of portfolios that invest in Islamic assets.

Keywords: Sharia-compliant stocks, Sukuk, Gold, Optimal hedge ratio, Connectedness, Frequency, Wavelet analysis.

JEL Classification: C32, C18, G32

1. Introduction

Many studies in the literature find weak connectedness between gold, equities and bonds. These studies point to the advantages of gold in hedging and diversifying investment portfolios (see for example, Sherman, 1982; Smith, 2002; Michaud et al. 2011; Ratner and Klein, 2008; and Wozniak 2008; Baur and Lucey, 2010; Hood and Malik, 2013; Ghazali et al. 2015; Bredin et al. 2015, Bekiros et al. 2017; Śmiech and Papież, 2017; Junttila et al. 2018). In all of these studies only one investment horizon is considered and the connectedness between gold and Sharia-compliant securities is largely ignored.[1] Therefore, in this paper, we fill this gap and examine the benefit of gold in hedging Islamic portfolios at varying investment horizons.

The recent years have witnessed a big interest in the Islamic finance industry. For instance, in 2017 it is estimated that there are around 2.05 trillion dollars invested in the Islamic finance industry. While most of these are invested in banking assets, still more than 20% or 0.4 trillion of this figure is invested in the Sukuk market (IFSI, 2018).[2] Surprisingly, the astonishing success of the market of Islamic funding took place during the global financial crisis period.[3] Hence, it is important to identify the patterns of information transmission between these growing markets and other assets such as gold.

In the related literature on information transmission, usually one-time horizon is considered.[4] This is restrictive as investors have different time horizons with varying nature of connectedness and information transmission. Therefore, showing the whole spectrum of information transmission at varying horizons is crucial which is the aim of our study. In particular, we investigate the

[1] The only study that consider varying time horizons is Bredin et al. 2015. But they did not investigate Islamic equities and Sukuk

[2] This point has been brought to our attention by one of the referees.

[3] See for example Chapra (2011); Hasan and Dridi (2011); Beck et al. (2013).

[4] Note that the literature distinguishes between short term and long term association or co-integration, but the association within a time horizon is not widely considered, i.e. at various frequencies.

dynamic interdependence between gold, sukuk and Islamic equities in frequency domain to reveal short, medium and long term association. Our study makes a significant contribution to the literature because the link between gold and Islamic products at varying time horizons has not hitherto been investigated.

We compute a spectral representation of the forecast error variance decompositions in frequency domain. Then, the portion of forecast error variance that could be attributed to a shock in another market is computed at varying time horizons. This procedure has been recently proposed by Barunik and Křehlík (2018).

The role of gold in diversifying and hedging Islamic securities at various time horizons is inferred by applying the multivariate generalized autoregressive conditional heteroscedastic dynamic conditional correlation model (MGARCH-DCC) in a frequency domain. The model is used to get dynamic conditional correlations between gold, sukuk and equities at varying time scales. Using these correlations, we compute the dynamic hedge ratios and the weights of optimal portfolio which contains gold and Islamic assets at varying investment horizons.

Our results indicate that sukuk and Islamic equities have less correlations with gold in the short term than they do in the longer term. This indicates that gold is a good diversifier, particularly in short term horizons. We also find that the role of gold in hedging against sukuk fluctuations is limited; especially for longer investment horizons. This is confirmed by the negligible dynamic hedge ratios, which implies small and very short-lived hedging opportunities. Overall, the results show that gold is effective in hedging against the short-term fluctuations of investing in sukuk but not Islamic stocks.

The rest of the paper is organized as follows: The next section provides background information on the development of gold usage in Islam. Section 3 outlines the directional connectedness measures proposed by Barunik and Křehlík (2018) and the Wavelet-based DCC-GARCH approach. Section 4 provides a description of the data set and some preliminary statistics. In Section 5, we present the empirical results. Last section concludes.

2. Islamic products and gold

The growth of Islamic finance market during the global financial crisis has attracted attention to the performance of the Islamic finance industry. This success is attributed to the Sharia-compliant products which follow a different model than that of conventional securities. The Sharia-compliant instruments prohibit interest payments (riba), making these financial products less sensitive to interest rates or other related macroeconomic factors that normally drive expected returns on conventional assets. Another distinguishing feature is the restriction imposed on contracts that involve risk or speculation (Gharar). Other restrictions include gambling, derivatives, short selling, and arbitrage transactions.[5] These restrictions make Islamic assets as a good source for risk management strategy and may reduce investors' need for hedging, (Maghyereh and Awartani, 2016).

Despite these restrictions, sharia-complaint securities are not immune to financial system risks. In fact, a number of studies have shown that the performance of these securities exhibits a significant dependence with global conventional markets. For instance, Hammoudeh, et al. (2014) and Ajmi, et al. (2014) examine the nonlinear causal relationship between Islamic and global conventional stock markets during several global economic and financial shocks. The hypothesis of independence from conventional counterparts and global shocks is rejected indicating that Islamic products are not really immune. However, these studies did not investigate the association

[5] Apart from its diversifying role, gold can be viewed as an asset free from riba and gharar which are strictly prohibited in Islam.

between Islamic products, such as sukuk, with gold at multi investment horizon which we try to accomplish in this study.

The bulk of literature finds that gold performs a good hedging role against wealth losses and against extreme financial market fluctuations. For example, McCown and Zimmerman (2006) show that gold has approximately the same mean return as a Treasury bill and bears no market risk. Baur and Lucey (2010) show that gold is a safe haven for stocks in the United States, the United Kingdom and Germany. More recently, Beckmann et al. (2015) adopt a novel regime-dependent framework to perform a broad study that includes data from 18 individual economies and five regional indices. They show that gold serves as a hedge and as a safe haven depending on the specific economic environment under consideration. Finally, Bredin et al., (2015) use wavelet analysis and find that gold acts as a good hedge for a variety of international equity and debt markets over multiple time horizons that extends up to one year.

There is a limited number of studies that examined the hedging and safe haven properties of gold versus Islamic securities and sukuk. To the best of our knowledge, the only study that investigated these three assets together is Maghyereh, et al. (2018). However, In this study, we examine the role gold plays by using a new methodology that maps the dynamic association into frequency domain, as proposed by Barunik and Křehlík (2018).[6] This is important as the Islamic finance market has experienced a spectacular growth and it provided investors with a wide range of investment opportunities. Thus, it is crucial to investigate the dynamics between this market and

[6] In a recent study Mensi et al. (2017) studied risk spillovers between the two major commodity markets, crude oil, gold, the aggregate Dow Jones Islamic index and ten stock sectors. However, they did not examine sukuk.

other financial markets such as the gold market.[7] The next section discusses the methodology of our study. [8]

3. Methodology

In this paper, the interest lies in measuring the extent of the connectedness between markets at varying horizons. The Diebold Yilmaz (2012) spill over indices are very suitable, but as we consider longer time horizons, these indices might not be optimal as the VAR model will be estimated over a smaller number of observations.[9] Instead, Barunik and Křehlík (2018) has proposed similar measures in the context of VAR that are based on the spectral representations of the variance compositions of forecast errors and hence, the VAR model can still be estimated with the same number of observations. However, as opposed to Diebold and Yilmaz, the variance decompositions are computed in the frequency domain which enables getting the linkages at varying horizons.

To see how the results may change using another methodology, we estimate MGARCH-DCC over the wavelet transform of the returns series. The same as the spectral of the variance decompositions allows for obtaining directional association measures over various horizons, the wavelets of the return series, can be used to obtain dynamic conditional correlations over varying horizon by simply estimating a dynamic conditional correlation model such as the MGARCH-DCC model over the wavelets. [10]

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[7] See Mensi et al. (2015) for more on the growth of Islamic financial market

[8] In comparison to the bulk of related literature, our methodology is unique. It has been recently adopted by Lau et al. (2017) who examined the return and volatility transmission across cryptocurrencies, gold, bond, equities and the global volatility index (VIX); and Batten, et al. (2017) who examined the interconnectedness of the global steam coal market.

[9] The Diebold and Yilmaz measures are based on the generalized variance decompositions of the forecast errors of vector autoregression model of the variables. To get the association over longer horizons, the VAR has to be estimated over weekly, biweekly or monthly independent data and this hence, it will be estimated over a smaller number of observations.

[10] Note that Barunik and Křehlík (2018) have derived the spectral representation of variance decompositions in order to compute directional connectedness over varying horizons. They did not compute wavelets of the actual return

Next, we briefly describe how the wavelet-based DCC-GARCH method will be employed to obtain dynamic correlations over various horizons.

3.1 Connectedness measure

The Barunik and Křehlík (2018) procedure (BK hereafter) to measure connectedness relies on obtaining the spectral representation of variance decompositions.[11] In particular, the spectral density of the return series X_t at frequency ω is defined as the Fourier transform of the $MA(\infty)$ filtered series as[12]

$$S_X(\omega) = \sum_{h=-\infty}^{\infty} E(X_t X'_{t-h}) e^{-i\omega h} = \Psi\left(e^{-i\omega}\right) \sum \Psi' \left(e^{+i\omega}\right) \tag{1}$$

where the spectral representation for the covariance is $E(X_t X'_{t-h}) = \int_{-\pi}^{\pi} S_X(\omega) e^{i\omega h} d\omega$, $\Psi\left(e^{-i\omega}\right) = \sum_h e^{-i\omega h} \Psi_h$ is the Fourier transform of the impulse response Ψ_h, and ∞ implies that infinite horizon relation is included in the setting. The variance decomposition at a given frequency ω can be obtained by

$$\left(\Theta(\omega)\right)_{i,j} = \frac{\sigma_{jj}^{-1} \sum_{h=0}^{\infty} \left(\Psi\left(e^{-ih\omega}\right)\Sigma\right)_{i,j}^2}{\sum_{h=0}^{\infty} \left(\Psi(e^{-ih\omega})\Sigma(e^{ih\omega})\right)_{i,i}} \tag{2}$$

Following BK, Eq. 2 is standardised as

$$\left(\widetilde{\Theta}(\omega)\right)_{i,j} = \frac{\left(\Theta(\omega)\right)_{i,j}}{\sum_{j=1}^{k} \left(\Theta(\omega)\right)_{i,j}} \tag{3}$$

The generalized variance decompositions at frequency band $D = (\alpha, b): a, b \in (-\pi, \pi)$ is defined as

series as we do here in this study. Therefore, the MGARCH-DCC over the wavelets is completely independent from Barunik and Křehlík (2018).

[11] This procedure has been recently adopted by Lau et al. (2017).

[12] In time domain, connectedness is obtained using impulse response function Ψ. The Fourier transform of Ψ is used as the response function in frequency domain and it is defined as $\Psi(e^{-i\omega}) = \sum_h e^{-i\omega h} \Psi_h$, with $i = \sqrt{-1}$.

$$\left(\widetilde{\Theta}_d\right)_{i,j} = \int_a^b \left(\widetilde{\Theta}(\omega)\right)_{i,j} d\omega \tag{4}$$

The decomposition above can be used to obtain the overall connectedness within the frequency band D. This is computed as

$$C^D = \frac{\sum_{i=1,i\neq j}^{k}\left(\widetilde{\Theta}_D\right)_{i,j}}{\sum_{i,j}\left(\widetilde{\Theta}_D\right)_{i,j}} = 1 - \frac{\sum_{i=1}^{k}\left(\widetilde{\Theta}_D\right)_{i,j}}{\sum_{i,j}\left(\widetilde{\Theta}_D\right)_{i,j}} \tag{5}$$

Note that when C^D is close to unity it implies strong connections within band $D = (\alpha, b)$. This may occur even when the aggregate connectedness amongst the variables is low.

The above variance decomposition can be used to compute the directional connectedness among markets as within system connectedness on the frequency band D as follows:

A. The contribution of markets j to market $i, where\ (i \neq j)$, can be defined as *within from* connectedness as :

$$C_{i\leftarrow\cdot}^{D} = \sum_{i=1,i\neq j}^{k} \left(\widetilde{\Theta}_D\right)_{i,j} \tag{6}$$

B. The variance transmission from market i to other market j is defined as the contribution to the connectedness :

$$C_{i\rightarrow\cdot}^{D} = \sum_{i=1,i\neq j}^{k} \left(\widetilde{\Theta}_D\right)_{j,i} \tag{7}$$

C. Then, we compute the *within net connectedness* as the difference between variance given and variance received from a particular market by offsetting (6) and (7) as

$$C_{i,net}^{D} = C_{i\leftarrow\cdot}^{D} - C_{i\rightarrow\cdot}^{D} \tag{8}$$

A positive /negative net connectedness indicates that i market is a net giver / receiver of information to / from all other markets.

D. The pairwise connectedness between markets i and j is computed as

$$C_{i,j}^{D} = \left(\widetilde{\Theta}_{D}\right)_{j,i} - \left(\widetilde{\Theta}_{D}\right)_{i,j} \tag{9}$$

E. Finally, the aggregate measure of connectedness in the system is calculated as:

$$\tilde{C}^{D} = C^{D}\Gamma(D) \tag{10}$$

where C^D the total connectedness measure obtained from the connectedness tables $\left(\widetilde{\Theta}_{D}\right)_{i,j}$ that corresponds to frequency band D; $\Gamma(D)$ is the spectral weight of the frequency band D that measures its relative contribution in the overall connectedness in the VAR system. It is computed as $\Gamma(D) = \frac{\sum_{i,j=1}^{k}(\widetilde{\Theta}_{D})_{i,j}}{\sum_{i,j}(\widetilde{\Theta}_{D})_{i,j}}$. If we sum across aggregate measures of connectedness in (10) across all frequencies, we obtain an overall measure of connections in the system $\sum_{D}\tilde{C}^{D}$.[13]

In this paper, we investigate dynamics at three frequency bands; $D_1 \in [2,8]$ days, $D_2 \in [8,32]$ days, and $D_3 \in [32, 256]$ days. These bands correspond to the short, the medium, and the long-term. To estimate variance decompositions and impulses in time domain we use a lag length of two in the VAR system and we forecast 100 days ahead to compute the variance decomposition of forecast errors. [14]

3.2 Wavelet-based DCC-GARCH approach

The hybrid wavelet-based DCC-GARCH model is a normal DCC-GARCH model that is estimated over the wavelets. It provides a framework to examine co-movement in time and frequency domains [15] and hence it allows for measuring dependence at different time scales. Moreover, unlike the Fourier analysis which is not robust, the MGARCH-DCC accommodates

[13] This measure is synonymous to the spill over index reported by (Diebold and Yilmaz, 2012).
[14] We follow the standard literature (i.e., Diebold and Yilmaz, 2012, 2014, 2015; Batten et al. 2017).
[15] Recently, this approach is used by Baruník et al. (2016), Boubaker and Raza (2017) and Maghyereh et al. (2019) to study the dynamic correlations between different gold, oil, and stocks.

potential nonlinearities, non-stationary data, asymmetry, long-term trend and jumps in the time series of returns (Crowley, 2007).

To estimate the dependence over varying time horizons we first obtain the wavelet transform of return series of gold, Sukuk and Islamic stocks using the maximal overlap discrete wavelet transformation (MODWT hereafter). Then, we use the time series of the wavelet transformation at each frequency to estimate the DCC-GARCH model and the dynamic correlations. These are subsequently used to compute dynamic hedge ratios and optimal portfolio weights for various horizons.

In the following two subsections we briefly introduce the Wavelet-based DCC-GARCH methodology. We start with the domain frequency method of wavelet analysis and then we introduce the DCC GARCH model.

3.2.1 Wavelet-based approach

The first stage of the analysis, we obtain the wavelets of the time series of returns in frequency domain by using the maximal overlap discrete wavelet transform (MODWT).[16] Comparing with the discrete wavelet transform method (DWT), the MODWT can handle input data of any length and does not require the length of the series to be power of two.[17] Furthermore, the MODWT does not introduce phase shifts in the wavelet coefficients (Gençay et al. 2002), therefore any peaks or troughs in the original time series will be correctly aligned with similar events in the multiresolution analysis (Masset, 2008). Furthermore, the MODWT produces more asymptotically efficient wavelet variance than that based on the DWT (Lien and Shrestha, 2007).

[16] Note that most notations in this section are quoted from Baruník et al. (2016) and Khalfaoui and Boutahar (2012).

[17] For more details about the properties of MODWT see Cornish et al. (2013) and Crowley (2007). Note that we could have transformed using the discrete wavelet transform method, however this method is sensitive to the choice of the starting point (Percival and Walden, 2000).

The jth level wavelet coefficients $(W_{j,t})$ and scaling coefficients $(V_{j,t})$ for a return series $r(t)$ are obtained via the following formulas:

$$\widetilde{W}_{i,j} = \sum_{I=0}^{L_j-1} \tilde{h}_{j,l} r_{t-j \ mod\, T} \quad t = 0, \dots, T-1, \tag{11}$$

$$\tilde{V}_{i,j} = \sum_{I=0}^{L_j-1} \tilde{g}_{j,l} r_{t-j \ mod\, T} \quad t = 0, \dots, T-1, \tag{12}$$

where L is the length of the filter, $\tilde{h}_{j,l} = \frac{h_{j,l}}{2^{j/2}}$ is the wavelet filter, and $\tilde{g}_{j,l} = \frac{g_{j,l}}{2^{j/2}}$ is the scale filter.[18]

To better capture economic and financial characteristics of the time series, we use the least asymmetric wavelet method of Daubechies (1988, 1992) with a filter length of L = 8 to obtain multiscale decomposition of the return series.[19] This is also called Symlet and it is used over other wavelet filters such as the Haar wavelet, the classic Daubechies, the Mexican Hat, and Coiflets because it provides a good approximation to an ideal band-pass filter (Gencay et al. 2002; Lien and Shrestha, 2007) and a more smooth wavelet coefficients that is more appropriate in financial time series. As a result, the decomposed signals for multi resolution analysis in the MODWT is defined as

$$r(t) = S_J(t) + \sum_{j=1}^{J} D_j(t) \tag{13}$$

[18] The MODWT filters satisfy the following properties: (1) $\sum_{l=0}^{L-1} \tilde{h}_l = 0$, $\sum_{l=0}^{L-1} \tilde{g}_l = 1$; (2) $\sum_{l=0}^{L-1} \tilde{h}_l^2 = \sum_{l=0}^{L-1} \tilde{g}^2{}_l = \frac{1}{2^l}$; (3) $\sum_{-\infty}^{+\infty} \tilde{h}_l \tilde{h}_{l+2n} = \sum_{-\infty}^{+\infty} \tilde{g}_l \tilde{g}_{l+2n}$ (see Khalfaoui et al. 2015).

[19] The construction of the Daubechies' least asymmetric wavelet is very similar to the classic Daubechies orthogonal wavelets but its symmetry is stronger. To obtain more asymmetry and smooth wavelet coefficients, Daubechies (1988, 1992) modified the procedure of the classic Daubechies orthogonal wavelets to be nearly linear-phase filters.

where $S_J(t) = \sum_{l=-\infty}^{+\infty} h(l) S_{J-1}(t + 2^{J-1} \times l)$ represents the smoothed version of series $r(t)$ at scale J; and $D_j(t) = \sum_{l=-\infty}^{+\infty} g(l) S_{j-1}\left(t + 2^{j-1} \times l\right)$ represents the wavelet details which captures local fluctuations over the whole period of time series at each scale j $\{j = 1, \dots, J\}$.[20]

In this study, we choose to decompose our return series into eight details $(d_1, \dots, d_8)$ and one smooth component (S_8). The wavelet scales are: $d_1([2-4]\, days)$, $d_2([4-8]\, days)$, $d_3([8-16]\, days)$, $d_4([16-32]\, days)$, $d_5([32-64]\, days)$, $d_6([64-128]\, days)$, $d_7([128-256]\, days)$, and $d_8([256-512]\, days)$. The short term horizon is defined as $\{D_1 = (d_1 + d_2)\}$ and it represents the gold and sharia-compliant security price variations due to shocks occurring from 2 to 8 days. The medium term horizon is defined as $\{D_2 = (d_3 + d_4)\}$ which represents fluctuations due to shocks occurring from 8 to 32 days. Finally, the long term horizon is defined as $\{D_3 = (d_5 + d_6 + d_7)\}$ which represents fluctuations occurring from 32 to 256 days.[21]

3.2.2 DCC-GARCH model

The second stage of the analysis consists of applying the DCC-GARCH developed by Engle (2002) on the results of the wavelets multiscale decomposition to analyse dynamics conditional correlations of the markets on a scale-by-scale basis.

Let $D_t(j)$ be a $k \times 1$ vector of wavelet return series at time t and j scale of gold and sharia-compliant securities. Then, the conditional mean equations at the different scales can be written as:

$$A(L) D_t(j) = \varepsilon_t, \text{ where } \varepsilon_t \mid \Omega_{t-1} \sim N(0, H_t), \text{and } t = 1, \dots, T \tag{14}$$

[20] After applying MODWT to decompose all our return series on a scale-by scale basis, we can also study wavelet correlation $\rho_{r_x,r_y}(j)$ between two return series r_x and r_y at different scales j $\{j = 1, \dots, J\}$ using the form: $\rho_{r_x,r_y}(j) = \frac{\gamma_{r_x,r_y(j)}}{\mathcal{V}_{r_x}(j)\, \mathcal{V}_{r_y}(j)}$, where γ_{r_x,r_y} is the estimator of wavelet covariance at scale j and $\mathcal{V}_{r_x}(j)^2$ $\mathcal{V}_{r_y}(j)^2$ are estimators of wavelet variance and covariance, respectively (see Whitcher et al. 2005).

[21] These are consistent with time-frequency connectedness analysis (see Khalfaoui and Boutahar, 2012).

where A is a matrix of parameters to be estimated, L is the lag operator and ε_t is the vector of innovations based on the information filter Ω_{t-1}. The conditional variance–covariance matrix of the vector of errors ε_t can be written as:

$$H_t(j) = \Lambda_t(j) D_t \Lambda_t(j) \tag{15}$$

where $\Lambda_t(j) = diag\left(h_t^{1/2}(j), \dots, h_{N,t}^{1/2}(j)\right)$ is a diagonal matrix that contains the conditional volatilities. The $h_t(j)$ can be estimated by using a univariate GARCH (1,1) model. The D matrix is written as $D_t(j) = diag\big(Q_t(j)\big)^{-\frac{1}{2}} Q_t diag\big(Q_t(j)\big)^{-\frac{1}{2}}$.

The time varying conditional correlations between gold and Islamic asset are elements in the time varying D matrix. These are going to be the focus of our analysis and are computed as:

$$\rho_{ik,t} = q_{ik,t}(j)/\sqrt{q_{ii,t}(j) q_{kk,t}(j)} \;\; for\ i \neq k \tag{16}$$

The $Q_t(j) \equiv q_{ik,t}$ matrix is a square symmetric positive-definite matrix that is obtained from the estimated univariate GARCH models of the variables and it is given by $Q_t(j) = (1 - \theta_1 - \theta_2)\bar{Q} + \theta_1 u_t(j) \acute{u}_{t-1}(j) + \theta_2 Q_{t-1}(j)$, where $u_t = (u_{1t}, \dots, u_{kt})'$ is an $k \times 1$ vector of the standardized residuals obtained from the first-step estimation. Q is a $k \times k$ unconditional variance matrix of u_t. Finally θ_1 and θ_2 are non-negative scalar parameters that capture the effects of previous shocks and dynamic conditional correlations on current correlations. The parameters of the multi-variate GARCH models are estimated by QMLE and the likelihood is maximized using BFGS criteria. [22]

The estimated results derived from the DCC model can be used for rebalancing and managing risk by maintaining an optimal portfolio weights. In this study, we use estimated correlations to

[22] The MGARCH specification is in line with most of the previous studies (Sadorsky, 2012, 2014; Lin et al. 2014; Lin and Li, 2015; Basher and Sadorsky; 2016; Maghyereh et al. 2017; Maghyereh et al. 2018; Maghyereh et al. 2019; among many others).

compute optimal hedge ratios and optimal portfolio weights at each wavelet scale (short, medium and long term). The risk minimizing hedge ratio $\left(\beta^*_{ik,t}(j)\right)$ for two assets (i,k) at time t and at wavelet scale j is computed as:

$$\beta^*_{ik,t}(j) = \frac{h_{ik,t}(j)}{h_{kk,t}(j)} \tag{17}$$

where $h_{ik,t}(j)$ is the conditional covariance between i and k and $h_{kk,t}(j)$ is the conditional variance of k.[23]

In the same way, we use conditional volatilities from the DCC model to construct an optimal portfolio that minimizes the portfolio risk. Following Kroner and Ng (1998), among others, the optimal weight $(w^*_{ik,t}(j))$ for a two-asset portfolio (i,k) at time t and at wavelet scale j is obtained by:

$$w^*_{ik,t}(j) = \frac{h_{kk,t}(j) - h_{ik,t}(j)}{h_{ii,t}(j) - 2h_{ik,t}(j) + h_{kk,t}(j)}, \tag{18}$$

$$\text{with } w^*_{ik,t}(j) = \begin{cases} 0, & if \quad w^*_{ik,t}(j) < 0 \\ w^*_{ik,t}(j), & if \quad 0 \le w^*_{ik,t}(j) \le 1 \\ 1, & if \quad w^*_{ik,t}(j) > 1 \end{cases} \tag{19}$$

where $w^*_{ik,t}(j)$ is the weight of asset i in a one-dollar portfolio; $h_{ik,t}(j)$ is the conditional covariance between i and j; $h_{ii,t}(j)$ and $h_{kk,t}(j)$ are the conditional variances of assets i and k respectively. The weight of asset k in the considered portfolio is computed as $(1 - w^*_{ik,t}(j))$.

4. Data set and descriptive statistics

The data set includes the daily prices of the Dow Jones Citigroup Sukuk Index, the Dow Jones Islamic stock market index and the cash price of an ounce of gold in dollars from the 30th of

[23] Hedge ratios are computed similar to Kroner and Sultan (1993).

September 2005 to the 5th of February 2018, corresponding to the data availability of sukuk prices. The use of daily data allows increasing the number of observations as so to adequately capture the rapidity and intensity of the dynamic interactions among assets' prices. The data is retrieved from Thomson Reuters DataStream and consists of 3,223 daily observations. For each series, we compute the continuously compounded returns as $r_t = ln(p_t/p_{t-1})$, where p_t is the daily closing price at time t. Figure 1 exhibits the evolution of the prices and returns of gold, sukuk and Islamic stock index during the chosen sample period.

The Dow Jones Citigroup Sukuk Index is composed of dollar denominated sukuk that are issued in the global market.[24] This index is launched on April 2, 2006 with a history that dates back to September 30, 2005. [COMMENT 4, reviewer 1]: At the start of the Index, it included only seven Sukuk issues, but afterwards the number of issues has increased significantly to reach seventy-one in 2018. Around 62% of the Sukuk in the index are issued by either sovereigns or quasi-sovereign entities. The rest are issued by corporates and Islamic banks. The origin of these issues is concentrated in the Gulf Cooperation Council countries where 65% of the issues originate. The largest issuer included in the index is Saudi Arabia which controls around 34% of the index; it is followed by United Arab Emirates at 20%; Malaysia at 10% and finally Indonesia at 17%.[25] To be included in this index, bonds must have at least a minimum maturity of one year and a minimum issue size of US$250 million. All bonds included in the index are of investment grade which are rated by leading rating agencies.[26]

[24] The bond must pass screens and meet the standards issued by the Accounting and Auditing Organization for Islamic Financial Institutions (AAOIFI).
[25] Source: Bloomberg and CIMB-Principal Islamic Asset Management Sdn Bhd as at end-December 2018.
[26] To be eligible for inclusion in the index, bonds must have minimum quality BBB-/Baa3 by S&P and Moody's respectively.

The Dow Jones Islamic stock market index provides a global investable universe of stocks that comply with Islamic finance principles. The start of the index is May 1999 and its constituents are reviewed quarterly.[27] The index excludes companies with high debt in their capital structure, and also excludes companies that are deemed unethical according to Islam, such as corporations engaged in gambling, alcohol, tobacco, weapons production, pork food products, and conventional interest-based financial services, such as banks and insurance companies. The index also excludes companies with intangible assets. In September 2017, it included 2,876 companies from 51 countries.[28]

[INSERT FIGURE 1 HERE]

Table 1 presents the descriptive statistics for the returns of gold, sukuk and Islamic equities. As can be seen in panel A of the table, the three assets' returns during the study period are positive but tiny, with gold offering the highest daily average returns of 1.4%, followed by Islamic equity at 0.95% and finally by sukuk at 0.02%. Gold also exhibits the highest volatility compared to Islamic assets as measured by the standard deviation. However, its volatility is still close to the volatility of Islamic equities, but it is 2.5 folds the volatility of sukuk. On a risk adjusted basis, gold offers the highest return per unit of risk. It is followed by Islamic equities, and then by Sukuk which offers relatively a very tiny return per unit of risk. The skewness coefficients are negative for all asset return series with sukuk having a relatively high coefficient compared to gold and equities. It is around -13 in sukuk compared to -0.47 and -0.49 in gold and Islamic equities respectively. This shows that the distributional characteristics of sukuk is similar to conventional bonds and debt

[27] For more information, see http://us.spindices.com/indices/equity/dow-jones-islamic-market-world-index.

[28] The Dow Jones Global stock market index, for instance, contains 12,000 companies from 77 countries. See Al-Zubi and Maghyereh (2007); Hammoudeh et al. (2014); Charles et al. (2015) for information about the constituents of Islamic indices.

instruments with distributions that are negatively skewed.[29] There is excess kurtosis in all return series and it is more pronounced in the sukuk return series. The null hypothesis of normality is rejected by the Jarque-Bera statistic (JB) for all assets at conventional levels. The results of the unit root tests are also reported in Panel A of Table 1. The panel reports the Augmented Dicky Fuller test statistics, the KPSS test statistics and the Phillips Perron (PP) test statistics. The three tests show that all return series are covariance stationary over the sample period.

In Panel B of Table 1, we report the pairwise unconditional correlation coefficients between asset returns. The correlation between gold and Islamic assets is low. It is a round 0.057 and 0.135 with sukuk and Islamic equity respectively. The correlation between Islamic equities and sukuk is also low and it is around -0.009. These low correlations indicate that substantial diversification benefits may be achieved in a portfolio that contains gold, Islamic equities and sukuk.

[INSERT TABLE 1 HERE]

As we are interested in assessing the evolution of correlations in varying time horizons, we decompose our raw return series into 8 scales using the maximal overlap discrete wavelet transform (MODWT).[30] Figure 2 plots the wavelet smooth part of gold, sukuk and Islamic stock returns over the full sample period. The figure shows that the three wavelet smooth series exhibit nonlinear deterministic trend. However, the smooth parts of Islamic stocks and sukuk are moving in tandem more than with gold. Hence, we may say that the smooth long term returns variation of Islamic stocks and sukuk are positively correlated with each other more than their correlation with Gold.

Figure 3 plots the wavelet multiple correlation at detailed wavelet scales with the 95 percent confidence intervals. As can be seen in the figure, wavelet correlations are higher and tend to

[29] The limited upside potential which is bounded by the nominal rate on the fixed income instrument and the big downside drop that may occur in the case of default leads to negative skewed distributions in fixed income instruments.
[30] The MODWT is used with Daubechies least asymmetric wavelet filter.

increase as time scale increases. For instance, while the correlation between gold and Islamic stock/Sukuk is around 0.32/0.5 in the long -time scale of 128-256 days, it is less than 0.05/0.1 at 2 to 4-day scale. It is worth to mention here that Sukuk is only positively correlated with Islamic stock in the long- term while it exhibits negative association with Islamic stocks in the short term. These preliminary results show that portfolios in the long run are not as diversified as indicated by short-term associations. It also highlights the importance of using multi scale approach in managing portfolio risks and allocations.

[INSERT FIGURES 2&3 HERE]

Table 2 presents the statistical properties of the wavelet series of gold, sukuk and Islamic stock in frequency domain computed at three scales: the short, medium and long term. The results are similar to the statistical properties in time domain and they indicate that sukuk exhibits the lowest standard deviation of all series at all wavelet scales. There is skewness and kurtosis at every scale and the data is not normal as indicated by the JB statistics. However, as we move from low scale (low frequency) to high scale (high frequency), the data is closer to normal with less skewness and kurtosis. Surprisingly, the gold negative skewness has changed to positive at the highest scale.[31] However, the negative skewness of Islamic stock and sukuk is maintained across all scales. Sukuk has the lowest volatility and, generally, volatilities are lower as we move to a higher scale. The table also presents the tests for stationarity in the wavelet time-series at the three scales using the Augmented Dickey–Fuller test (ADF), the Kwiatkowski-Phillips-Schmidt-Shin test (KPSS), and the Phillips-Perron test (PP). All test statistics indicated that assets returns are stationary at the 1% level of significance at all scales.

[31] Probably, this is more consistent with commodity markets that is more likely to exhibit positive skewness due to supply shocks.

[INSERT TABLE 2 HERE]

5. Empirical results

5.1 Connectedness evidence

As mentioned previously, we measure connectedness using the Barunik and Křehlík measures in frequency domain. The aim is to obtain the spectrum of the information transmission among markets at varying scales, i.e. in the short, medium and long term. Figure 4 shows the dynamics of aggregate connectedness of a system with three markets at the three scales considered. The connectedness at varying scales follows similar patterns except across the time domain. However, the intensity with which connectedness changes across time is different for various scales. For instance, during the global financial crisis, the connectedness has gone down across the three scales, but the drop is more severe in the short term compared to the medium and long term.[32] This partly shows that the short spells of diversification may disappear in the longer term as variables retain stronger association.

The main observation from the figure is that connectedness is higher for longer time horizons (when $d3 \in [32, 256]$). The result here supports the higher long-term correlation between gold and Islamic assets recorded in table 3. The observation of higher connectedness at lower frequencies can be translated into less diversification and more uncertainty regarding the portfolio, which contains gold and Islamic securities, performance in the long term. So far, the finding suggests that gold may not be effective in diversifying sharia-compliant portfolios in the long term but there is diversification potential in the short term.

[INSERT FIGURE 4 HERE]

[32] Probably, the main reason lies in the rally of gold following the global financial crisis among other distressed assets.

The matrix presented in table 3 reports the full sample cross market connectedness between gold and sharia-compliant securities in long, medium and short-term. The diagonal elements of the matrix represent the own market connectedness which is not the focus of this study. The off-diagonal elements of the matrix measure the directional connectedness among the markets.

[INSERT TABLE 3 HERE]

The off diagonal elements are part of the forecast error variance of the i^{th}market that is attributed to the j^{th} market. In particular, the (i, j) value is the estimated contribution to the variance of the 10 step ahead volatility forecast error of market i coming from innovations to returns of market j. The elements in the bottom row denoted "To Abs" give the amount of variance going from the j^{th} market to all other markets and they are the sum of elements in each column excluding the own share. If we include own share, then we refer to it as "To Wth" in the following row. Likewise, in the last two columns, the elements give the amount of the variance contributed by i^{th} market to all other j^{th} markets excluding its own share as denoted "From Abs" or including its own share as denoted "From Wth". Finally, the number denoted C in the bottom right corner gives the total connectedness. The three panels of the table correspond to the matrix of information transmission at the three scales considered: short, medium and long term.

Panel A shows that gold is a net transmitter to sukuk and Islamic stock markets in the short-term scale. Its contribution to the forecast error variance of other markets "To Abs" is 4.27 which is higher than the contribution of other markets to its forecast error variance "From Abs" 0.55. However, this is not the case in Panels B and C that show the dynamics in the medium and longer terms. In these Panels, gold is net recipient of volatility shocks. The contribution of other markets to gold forecast error variance is almost four folds what gold contributes to the other markets. Gold gives 1.89, 0.88 and receives 9.56, 9.11 in the medium- and long-term scales respectively.

The pairwise connectedness between gold and sukuk is skewed in favour of gold in the short term. Gold transmits 9.9 and receives only 1.45 from sukuk, making gold as a net transmitter to sukuk in the short term. The same applies to the pairwise directional connectedness between gold and Islamic stock. The association is established more by the information transmission from the gold market to the Islamic stock market. These relationships do not carry on in the medium and longer term where the Islamic markets become a net provider to the gold market.

It should be noted that the association in both directions between gold and Islamic products is higher in the long term than in the short horizon. For instance, the total connectedness between gold and Islamic stock in the short-term is 3.21 while it is 16.08 in the long term.[33] The higher information transmission in longer horizons compared to shorter horizons indicates that gold greatest diversification benefit lies in the short-term.

The cross-over from sukuk to Islamic stocks in the short-term is around 11.78 which is much higher than the transmission in the medium/long term, 0.13/0.12. On the contrary, Islamic stocks spill more on sukuk in the medium/ long term by an amount of 20.97/15.41 compared to the only 5.75 transmission in the short term. Therefore, the portfolio that consists of sukuk and Islamic stocks is diversified in terms of the amount of information transmissions across the various scales.

5.2 Wavelets DCC-GARCH

[33] These numbers are obtained by summing up the pairwise information cross over in both directions, i.e. 2.91 plus 0.21 in the short term and 0.01 plus 16.07 in the long term.

Table 4 presents the results of the wavelet DCC-GARCH model for the three scales. Panel A of the table presents the parameter estimates of the conditional mean and variance of the wavelet returns series of gold, Islamic equities and sukuk in the short, the medium and the long terms. As shown in the table, the sign of auto-regressive parameter of the conditional mean changes from negative to positive when we move from the short to the medium and/or the long term. This indicates that while the short-term process is mean reverting, the long-term data is not.

The table also shows that the volatility process over all horizons is stationary and persistent.[34] Short, medium- and long-term volatility exhibit stickiness and clustering. However, the persistence parameter of the volatility process weakens as we move to longer horizons. For instance, the parameter associated with previous volatility of gold is 0.892 in the short term, but it drops to 0.542 / 0.576 in the medium / long term. A similar pattern is observed in sukuk and Islamic stock. On the contrary, the influence of the short-term shocks on volatility strengthens as we move across horizons. In the short term, the influence of shocks to the conditional mean on volatility is tiny and negligible and this is reflected in the small value of the associated parameter. For instance, the parameter associated with short term shocks is only 0.09 and 0.02 in gold and sukuk respectively. This increases to 0.312 and 0.256 in the long term. The same applies to Islamic stocks.

[INSERT TABLE 4 HERE]

To recap, there is some evidence of volatility clustering over all horizons. However, in the longer terms, the shocks to the conditional mean play more role in the determination of future volatility.

Panel B of Table 4 presents the parameter estimates of the multivariate dynamic conditional correlation equation. The values of θ_2 show that correlations among the assets are more persistent

[34] In all cases $\theta_1 + \theta_2$ is close to 1. Note that persistence decreases over the longer term.

in the short and long term. The medium-term correlations are more influenced by the shocks to the conditional mean and this is indicated by the relatively higher value of $\theta_1 = 0.379$ in the table. Similar to volatilities, correlations are more persistent in the short term. Finally, all correlations are mean reverting.

The panel also presents the dynamic correlation parameter between the three assets. The correlations are bigger in the longer term. For instance, the short/ medium- and long-term correlation of gold with Islamic stock (0.194/0.248 and 0.384) are higher than the gold correlations with sukuk (0.127/0.185 and 0.257). This also applies to the correlation between Islamic stock and sukuk. In the following, we discuss the hedging and diversification potential of gold in Islamic portfolios.

5.3 Hedging and diversification performance

In this section, we assess whether gold is useful in hedging Islamic assets. In order to do that, we compute the optimal hedge ratios of gold required to hedge sukuk (or Islamic stock) exposure and to minimize the variance of the position. The computation is done at the three wavelet scales using equation (17). Panel A of Table 5 reports only averages of hedge ratios during the sample period.

[INSERT TABLE 5 HERE]

The optimal hedge ratio values reported in the table 5 indicate the amount of gold that should be bought / sold in order to hedge a $1 of sukuk/Islamic equities at various time scales. As shown, gold constitutes as a good hedge against sukuk in the short and medium term. On average, a $1 portfolio of sukuk can be hedged with a long position of 82 / 92 cents of gold in the short/medium term.[35] For longer term, it seems gold is not useful as the hedge ratio is tiny and negligible. The hedge ratio for Islamic stocks indicates that a position in gold with one third of the value of the exposure should be established to minimize variation. The ratio is stable and does not change across horizons. The table also shows that sukuk plays an important role in hedging Islamic equities but only in the short and medium term. On average, a $1 portfolio of Islamic stock can be hedged with a long position of 58 cents of gold in the short term and .2 cents in the long term.

Following the Kroner and Ng (1998) approach, we construct the optimal weights of gold required to minimize the variance of portfolios of sukuk and/or Islamic stock without reducing the expected return. Results are reported in panel B of Table 6. The table shows on average, the optimal weight of gold to minimize the volatility of a sukuk portfolio is 90% in the long term but only 3.2 % and 3.6% in the short and medium term respectively. The table also shows that sukuk is a good diversifier of Islamic equity except in the long term. The optimal weights of sukuk in the Islamic stock are 95%, 96% and 0.1% in the short, medium and long term respectively.

To see if gold improves on the efficient frontier of an Islamic stocks and Sukuk portfolio, we construct minimum variance portfolios as in Markowitz's (1952). [36]The Sharpe Ratios of portfolio I which only includes Islamic assets and portfolio II which adds gold are displayed in Table 6 for all investment horizons.[37] The table shows that an Islamic portfolio with gold has a higher Sharpe ratio in the short term but not in the medium and long term.[38]

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[35] Note that this hedge is not cheap as it needs an investment in gold that is almost equivalent to the established position in sukuk.
[36] Specifically, let w be $n \times 1$ vector portfolio weights, H be conditional variance-covariance matrix from DCC model, and n be the number of assets. The optimal weights are then calculated by solving the following optimization problem: $Minimiz_w w'H_t w \quad s.t. \sum_{i=1}^{n} w_i = 1, 0 \leq w_i \leq 1 \; for \; i = 1, \dots, n.$
[37] This has been brought to our attention thankfully by one of the reviewers.
[38] In a theory all risky assets have positive expected returns. However, as we constructed on the basis of wavelets we got negative Sharpe ratios.

Moreover, Table 6 presents the risk reduction effectiveness (RRE) indicator that computes the amount of volatility that can be reduced by adding gold to the portfolio. The risk reduction is computed as

$$REE(j) = \frac{\sigma^2_{Portfolio\,II}(j)}{\sigma^2_{Portfolio\,I}(j)},$$

Where $\sigma^2_{Portfolio\,II}(j)$ and $\sigma^2_{Portfolio\,I}(j)$ are the conditional variances of portfolio I and Portfolio II at scale $j = 1, 2, 3$, respectively.

Higher values of $REE(j)$ indicates higher reduction in portfolio volatility when we allocate to gold. As can be seen in Table 6, adding gold reduces volatility but only in the short term. However, in the long term, gold is a source of risk and increase the overall risk of an Islamic portfolio.

In Table 6, we also display the Value-at-Risk (VaR) of the portfolio that includes gold, and as can be seen in the table gold only reduces downside risk of Islamic assets in the short- term.

The results obtained from the Sharpe Ratio, the Risk Reduction Effectiveness and Value at risk are consistent and they show that the role of gold in the short-term risk reduction is important while in the long term it may increase the volatility of an Islamic portfolio.

[INSERT TABLE 6 HERE]

6. Conclusion

Theoretically commodities improve the risk and return profile of traditional portfolios that contain equities and bonds. A particular commodity that is thought to have an important role to play during market falls and crashes is gold. Under extreme uncertainties regarding the performance of capital markets and the economies, investors tend to allocate to gold in order to shelter their investments. Therefore, the expected negative association between the performance of gold and equities is interesting from a risk management perspective, portfolio diversification and hedging.

Furthermore, gold is believed to increase in value over the long term. Hence, it is expected to be able to maintain the purchasing power of fixed income investments and to provide hedge against inflation. A particular asset class that is very sensitive to inflation is bonds and thus gold has an important role to play in the management and risk of bond portfolios.

In this study, we measure the association of gold with Islamic equities and Sukuk over varying time horizon in order to shed lights on the role that gold may play in managing Islamic assets. Moreover, we provide guidance on hedge ratios and on how much gold is needed in order to minimize the volatility of portfolios that contain Islamic assets. The evidence we provide extends up to one year.

In terms of association, our measures show that while the short-term association with gold is weak, the long-term linkages are relatively strong. The implication for investors is that gold is important diversifier when investing over short periods of time that are less than one month. Portfolio managers whose investment horizon is longer should think of other ways to diversify their portfolios. A potential reason for this finding is the real nature of assets that underline Islamic investments which are more likely to correlated with gold, particularly in the long term. In the short-term gold is found to be a net transmitter of information to Islamic securities while in the longer terms we find the other way around.

We proceed by estimating the Wavelet DCC-GARCH in order to obtain the dynamic conditional correlations at various investment horizons. The estimation shows that the linkages between gold and Islamic securities is higher in the longer investment horizons. The hedge ratios computed from the dynamic conditional correlations imply that gold is a good hedger of sukuk in the short and medium term. The results also show that gold is a good diversifier of Islamic portfolio in the short term.

The hedge ratios that we have computed based on the GARCH-DCC model of the Wavelets indicate that gold is more suitable for hedging, but it is only good in reducing the volatility of portfolios that are invested for periods of less than one month. The numbers also show that more gold is needed to hedge Sukuk than to hedge equities. The practical implication is little gold is needed in Islamic equity portfolios and relatively more gold is required in order to hedge short- or medium-term Islamic security exposures.

An allocation of 30% of gold is needed to minimise the volatility of an Islamic equity portfolio regardless of the horizon. However, as Sukuk volatility is low compared to gold volatility, the minimum variance portfolio is achieved with only 3% of gold. The implication is that only a slight allocation to gold is needed if there is substantial allocation to Sukuk in the Islamic portfolio, while more allocations and up to one third is needed if the portfolio is concentrated with Islamic equity as compared to Sukuk.

Our work contributes to the previous studies by considering the association across various time scales, rather than using one-time scale. This is important for portfolio managers who invest in Islamic securities and gold. The results show that gold is a good hedger in the short and medium term and also a good diversifier in the short term only.

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Table 1: Descriptive statistics of the raw data

	Gold	Sukuk	Islamic Stock
Panel A: Descriptive statistics			
Mean	0.0140	0.0002	0.0095
Maximum	2.9815	3.2744	4.2454
Minimum	-4.4131	-6.8005	-3.5549
Std.Dev.	0.5180	0.18279	0.4370
Skewness	-0.4760	-13.570	-0.4964
Kurtosis	5.2585	681.210	10.8880
JB	1178.2***	15116.0***	3270.6***
ADF	-32.7343***	-31.7194***	-32.1778***
KPSS	0.3636	0.0645	0.0713
PP	-2977.15***	-3047.34***	-1469.42***
Panel B: Unconditional correlation			
Gold	1.000		
Sukuk	0.057***	1.000	
	(0.001)		
Islamic Stock	0.135***	-0.009	1.000
	(0.000)	(0.592)	

Notes: The data for returns is daily and covers the period that extends from September 30, 2005 to February 5, 2018. JB is the value of the Jarque–Bera statistic, testing for normality. The ADF stands for Augmented Dickey–Fuller test, KPSS for Kwiatkowski-Phillips-Schmidt-Shin test and PP for Phillips-Perron test. In ADF, PP, the null hypothesis is defined as ‘the series has a unit root against the alternative of stationarity’, while for KPSS the null hypothesis states ‘the series is stationary. All unit root tests are carried out with a constant and a time trend where the optimal lag length has been chosen using the Akaike information criterion. The p-values are in brackets. ***, ** and * refer to significance levels at 1%, 5% and 10%, respectively.

Table 2: Descriptive statistics of the wavelet components

	Gold	Sukuk	Islamic Stock
Panel A: *Frequency of short-term,* $D_1 \in [2,8]$ *days*			
Mean	7.78e-20	1.59e-20	3.93e-20
Maximum	0.0245	0.0230	0.0398
Minimum	-0.0266	-0.0393	-0.0327
Std.Dev.	0.0044	0.0015	0.0037
Skewness	-0.1434	-3.6056	0.4020
Kurtosis	2.5074	181.09	15.334
JB	464.33***	48515.0***	5192.0***
ADF	-27.61***	-28.37***	-27.79***
KPSS	0.007	0.007	0.012
PP	-302.82***	-273.49***	-246.50***
Panel B: *Frequency of medium-term,* $D_2 \in [8,32]$ *days*			
Mean	8.58e-20	-2.34e-20	3.68e-20
Maximum	0.0125	0.0072	0.0182
Minimum	-0.0118	-0.0131	-0.0123
Std.Dev.	0.0022	0.0008	0.0021
Skewness	-0.0811	-2.4355	0.4450
Kurtosis	2.3260	55.477	7.1078
JB	420.15***	11506.0***	1872.6***
ADF	-18.59***	-18.82***	-19.38***
KPSS	0.070	0.016	0.150
PP	-9.00***	-9.49***	-3.41**
Panel C: *Frequency of long-term,* $D_3 \in [32, 256]$ *days*			
Mean	-4.67e-20	-1.33e-20	-2.23e-20
Maximum	0.0038	0.0020	0.0031
Minimum	-0.0050	-0.0027	-0.0055
Std.Dev.	0.0012	0.0003	0.0010
Skewness	0.0053	-0.6336	-0.6184
Kurtosis	0.3430	10.300	1.9498
JB	15.106***	2821.9***	231.5***
ADF	-7.29***	-5.65***	-6.53***
KPSS	0.010	0.009	0.006
PP	-6.97***	-7.14***	-6.74***
Panel D*: Wavelet correlation*			
Scale	Gold-Sukuk	Gold-Islamic Stock	Sukuk-Islamic Stock
Frequency of short-term,$D_1 \in [2,8]$ *days*	0.049	0.147	-0.066
Frequency of medium-term,$D_2 \in [8,32]$ *days*	0.175	0.329	-0.057
Frequency of long-term,$D_3 \in [32, 256]$ *days*	0.224	0.464	0.288

Notes: The data for returns is daily and covers the period September 30, 2005 to February 5, 2018. Short term=d1+d2 represents the short-term variations due to shocks occurring at a time scale of 2 to 4 days (d1) and 4 to 8 days (d2). Medium term=d3+d4 represents fluctuations due to shocks occurring from 8 to 16 days (d3) and 16 to 32 days (d4). Long term horizon=d5+d6+d7 represents fluctuations due to shocks occurring in the periods of 32 to 64 days (d5), 64 to 128 days (d6) and of 128 to 256 days (d7). JB is the value of the Jarque–Bera statistic, testing for normality. The ADF stands for Augmented Dickey–Fuller test, KPSS for Kwiatkowski-Phillips-Schmidt-Shin test and PP for Phillips-Perron test. In ADF, PP, the null hypothesis is defined as 'the series has a unit root against the alternative of stationarity', while for KPSS the null hypothesis states 'the series is stationary'. All unit root tests are carried out with a constant and a time trend where the optimal lag length has been chosen using the Akaike information criterion. ***, ** and * refer to significance levels at 1%, 5% and 10%, respectively.

Table 3: Dynamic connectedness with frequency bands, Barunik and Křehlík approach

	From market j				
To market i	Gold	Sukuk	Islamic Stock	*From Abs*	*From Wth*
Panel A: Frequency of short-term, C_D $with$ $D_1 \in [2,8]$ *days*					
Gold	3.15	1.45	0.21	0.55	1.27
Sukuk	9.90	31.29	5.75	5.22	11.95
Islamic Stock	2.91	11.78	64.52	4.90	11.22
To Abs	4.27	4.41	1.99	10.67	
To Wth	9.79	10.10	4.55		24.44
Panel B: Frequency of medium-term, C_D $with$ $D_2 \in [8,32]$ *days*					
Gold	15.72	14.18	14.52	9.56	20.70
Sukuk	5.58	24.99	20.97	8.85	19.15
Islamic Stock	0.09	0.13	42.46	0.07	0.16
To Abs	1.89	4.77	11.83	18.49	
To Wth	4.09	10.32	25.60		40.01
Panel C: Frequency of long-term, C_D $with$ $D_3 \in [32, 256]$ *days*					
Gold	14.93	11.27	16.07	9.11	29.91
Sukuk	2.64	12.04	15.41	6.02	19.74
Islamic Stock	0.01	0.12	18.93	0.04	0.14
To Abs	0.88	3.80	10.49	15.17	
To Wth	2.89	12.47	34.43		49.79

Notes: The (i, j) value is the estimated contribution to the variance of the 100 step ahead return forecast error of market i coming from innovations to return of market j. To Abs and To Wth refer to absolute and within the estimated system. The table is estimated using the Barunik and Křehlík (2018) methodology.

Table 4: Estimated coefficients of the wavelet-based DCC-GARCH model

	Frequency of short-term, $D_1 \in [2,8]$ days			Frequency of medium-term, $D_2 \in [8,32]$ days			Frequency of long-term, $D_3 \in [32,256]$ days		
	Gold	Sukuk	Islamic Stock	Gold	Sukuk	Islamic Stock	Gold	Sukuk	Islamic Stock
Panel A: Univariate GARCH estimates and univariate diagnostic tests									
Conditional mean equation									
φ_0	--0.0234	0.012	0.012	0.005	0.012	-0.082	-0.091***	-0.048***	-0.041***
	(0.615)	(0.354)	(0.977)	(0.650)	(0.354)	(0.380)	(0.000)	(0.000)	(0.000)
φ_1	-0.296***	-0.221***	-0.229***	0.842***	0. 666***	0.847***	0.635 ***	0.476	0.608
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)***
Conditional variance equation									
θ_0	0.311*	0.115	0.138***	0.098***	0.046	0.036***	0.050***	0.014***	1.050***
	(0.054)	(0.890)	(0.000)	(0.000)	(0.251)	(0.000)	(0.000)	(0.000)	(0.000)
θ_1	0.090***	0.023***	0.157***	0.417***	0.070	0.448***	0.312***	0.256***	0.288***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.120)	(0.000)	(0.000)	(0.000)	(0.000)
θ_2	0.892***	0.897***	0.836***	0.542***	0.720***	0.550***	0.576***	0.569***	0.615***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$\theta_1 + \theta_2$	0.982	0.920	0.993	0.959	0.790	0.998	0.888	0.825	0.903
Univariate diagnostic									
$Q(20)$	9.947	3.275	11.568* (0.071)	15.90***	18.66*	10.43***	12.571*	9.934	9.758
	(0.133)	(0.401)		(0.050)	(0.052)	(0.110)	(0.092)	(0.120)	(0.118)
$Q^2(20)$	2.639	4.148	4.584	6.927	7.593	3.986	3.694	2.347	3.155
	(0.918)	(0.748)	(0.751)	(0.515)	(0.542)	(0.731)	(0.748)	(0.792)	(0.790)
Panel B: conditional correlation estimates and multivariate diagnostic tests									
Multivariate DCC equation									
θ_1	0.060***			0.379***			0.082***		
	(0.000)			(0.000)			(0.000)		
θ_2	0.874 ***			0.482			0.763***		
	(0.000)			(0.000)			(0.000)		
$\theta_1 + \theta_2$	0.934			0.861			0.845		
Dynamic conditional correlations									
$\rho_{\text{Gold-Sukuk}}$	0.127***			0.185***			0.257		
	(0.000)			(0.000)			(0.145)		
$\rho_{\text{Gold-Islamic Stock}}$	0.194***			0.248***			0.384***		
	(0.000)			(0.000)			(0.000)		
$\rho_{\text{Sukuk-Islamic Stock}}$	0.071**			0.186***			0.425***		
	(0.030)			(0.000)			(0.000)		
Multivariate diagnostic									
$Li - McL\ Q(20)$	353.531			933.58			785.378		
	(0.721)			(0.361)			(0.581)		
$Li - McL\ Q^2(20)$	165.226			173.35			91.67		
	(0.521)			(0.501)			3 (0.705)		
LL	48823.4			61962.7			6370.018		

Notes: This table presents the results of the wavelet-based DCC-GARCH model. $Q(20)$and $Q^2(20)$ are Box-Pierce statistics for autocorrelations of the standardized residuals and the squared standardized residuals. $Li - McL\ Q(20)$ *and* $Li - McL\ Q^2(20)$ are the multivariate Li and McLeod's (1981) test statistics for serial correlation in standardized and squared residuals, respectively. LL is the log-likelihood function value. The values in parentheses are the actual probability values. ***, ** and * refer to significance levels at 1%, 5% and 10%, respectively.

Table 5: Optimal portfolio weights and average hedge ratio (wavelet-based DCC-GARCH model)

	Short-term, $D_1 \in [2,8]$ days	Medium-term, $D_2 \in [8,32]$ days	Long-term, $D_3 \in [32,256]$ days
Panel A: Average hedge ratio			
Gold/Sukuk	0.816	0.920	0.001
Gold/Islamic Stock	0.285	0.307	0.399
Sukuk/ Islamic Stock	0.588	0.446	0.002
Panel B: Optimal portfolio weights			
Gold/Sukuk	0.032	0.036	0.901
Gold/Islamic Stock	0.317	0.371	0.327
Sukuk/ Islamic Stock	0.949	0.961	0.001

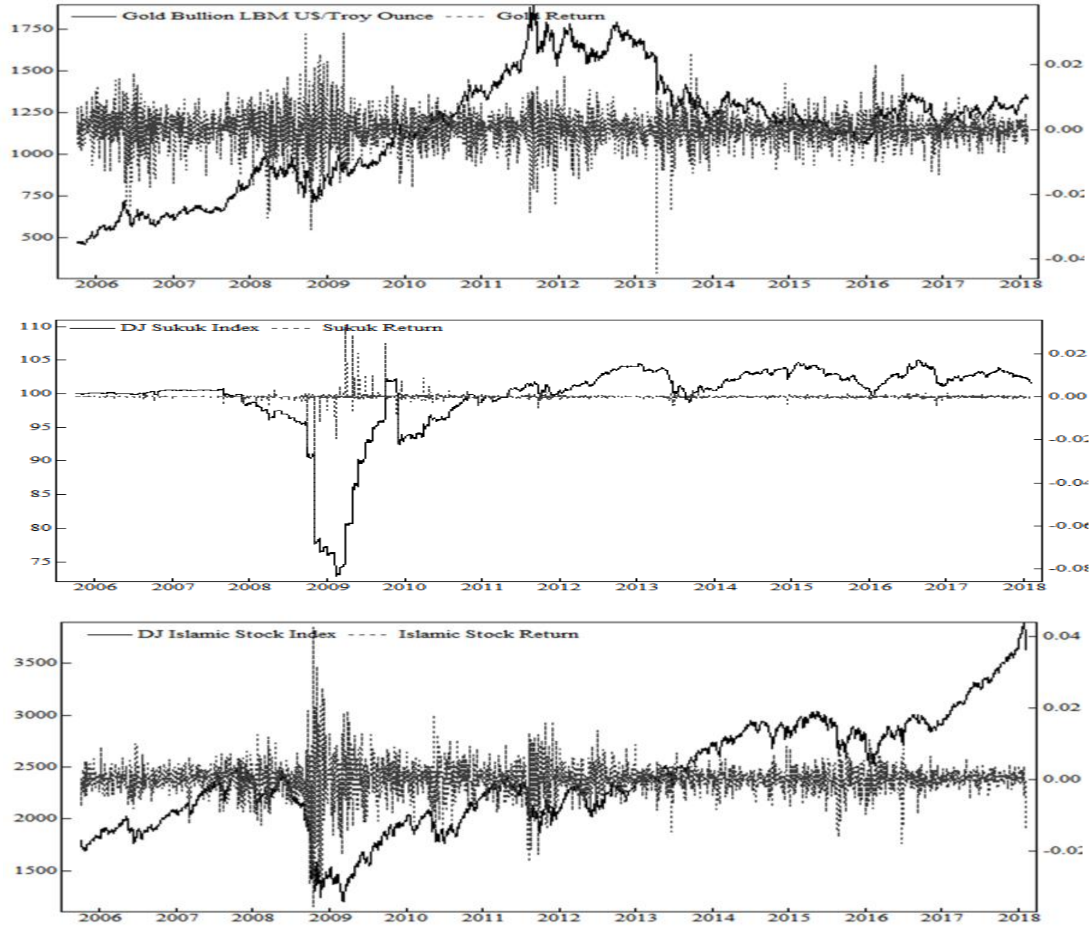
Note: The table reports average optimal weights and hedge ratios between pairs of assets in 1$ portfolio.

Table 6: Performance of minimum variance portfolios

	Optimal portfolio excluding gold (Portfolio I)			Optimal portfolio including gold (Portfolio II)		
	Short-term $D_1 \in [2,8]$ days	Medium-term $D_2 \in [8,32]$ days	Long-term $D_3 \in [32,256]$ days	Short-term $D_1 \in [2,8]$ days	Medium-term $D_2 \in [8,32]$ days	Long-term $D_3 \in [32,256]$ days
Return	1.979E-18	-1.514E-18	-2.008E-18	2.571E-18	-6.984E-19	-1.981E-18
Standard Deviation	2.173E-04	4.917E-05	1.294E-05	2.076E-04	4.715E-05	1.258E-05
Sharpe Ratio	9.105E-15	-3.080E-14	-1.550E-13	1.238E-14	-1.481E-14	-1.475E-13
Risk reduction effectiveness				4.46E-02	4.11E-02	2.78E-02
VaR (95%)	-3.575E-02	-8.088E-03	-2.129E-03	-3.216E-02	-7.756E-03	-3.069E-03
Optimal portfolio weights						
Sukuk	0.8633	0.8303	0.0489	0.8097	0.7913	0.0393
Islamic Stock	0.1367	0.1697	0.9511	0.1213	0.1451	0.9121
Gold				0.0690	0.0636	0.0486

Notes: The actual weights of minimum variance portfolios are computed using a standard Markowitz (1952) mean variance procedure. Sharpe ratio measures the risk-adjusted returns, which reflects the mean-variance efficiency of the portfolio under consideration. Risk reduction effectiveness VaR represents maximum potential loss in the value of a portfolio at a given probability and time horizon.

Figure 1: Time Series plot of prices and returns for Gold, Sukuk and Islamic stock



Note: Left axis represents s price levels. Right axis represents returns levels

Figure 2: Wavelet smooth component for Gold, Sukuk and Islamic stock

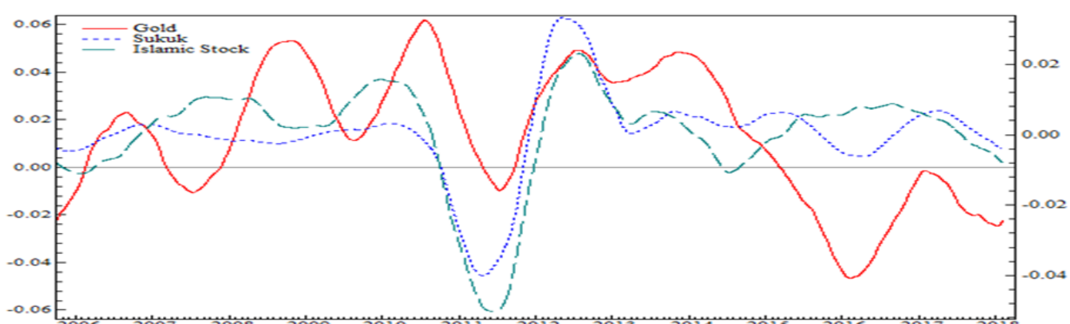
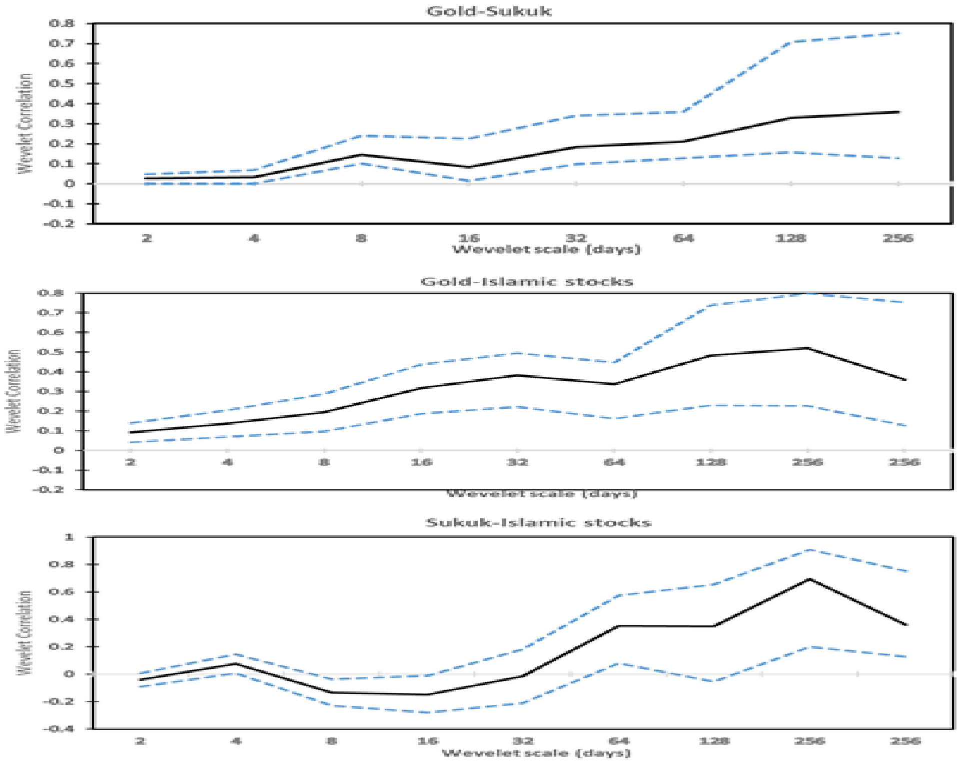


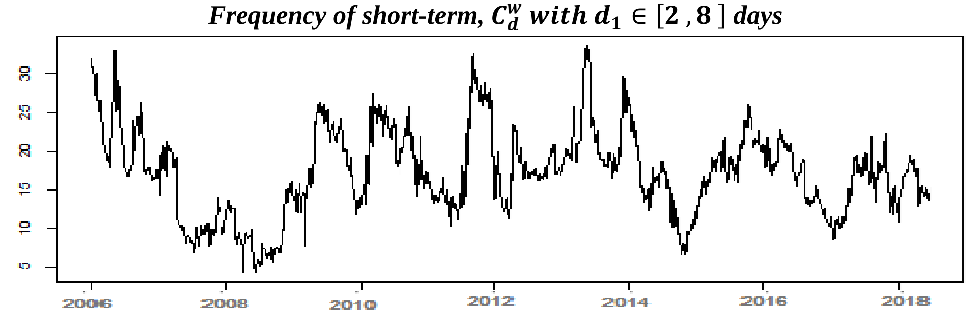
Figure 3: Wavelet multiple correlations at different time scales



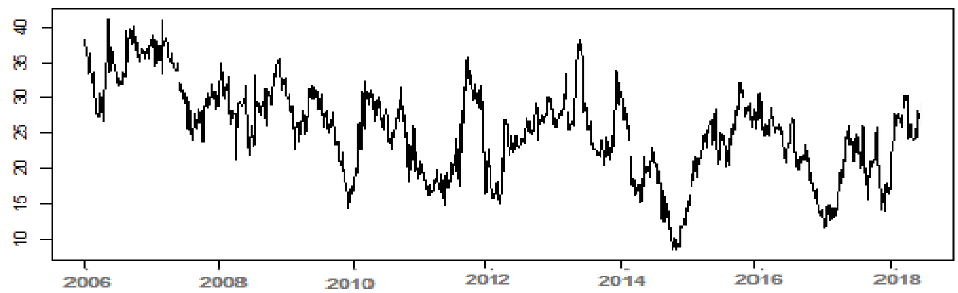
Note: The figure presents the plot of wavelet multiple correlations between gold, Sukuk and Islamic stock. The wavelet correlations are estimated based on the daubechies least asymmetric wavelet filter of length 8. The black line stand for correlation stock returns and the dash lines indicate the upper and lower bounds for 95% confidence interval.

Figure 4: Overall dynamic frequency connectedness, Barunik and Krehlik approach

Frequency of short-term, C_d^w ***with*** $d_1 \in [2\,,8\,]$ ***days***



Frequency of medium term, C_d^w ***with*** $d_2 \in [8, 32]$ ***days***



Frequency of long-term, C_d^w with $d_3 \in [32, 256]$ days