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# The Use of Almost Linear Phase IIR filters in DFT Modulated Filter Banks for Communication Systems

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**Abstract**—This paper is on the use and performance of  $M$ -path polyphase Infinite Impulse Response (IIR) filters for channelisation, conventionally where Finite Impulse Response (FIR) filters are preferred. This paper specifically focuses on the Discrete Fourier Transform (DFT) modulated filter banks, which are known to be an efficient choice for channelisation in communication systems. In this paper, the low-pass prototype filter for the DFT filter bank has been implemented using an  $M$ -path polyphase IIR filter and we show that the spikes present at the stopband can be avoided by making use of the guardbands between narrowband channels. It will be shown that the channelisation performance will not be affected when polyphase IIR filters are employed instead of their counterparts derived from FIR prototype filters. Detailed complexity and performance analysis of the proposed use will be given in this article.

**Index Terms**—Digital Communication, DFT, filter banks, Linear-phase polyphase IIR filter

## I. INTRODUCTION

Discrete Fourier Transform (DFT) modulated filter banks are in use in a wide range of electronic applications such as audio and image processing, biomedical electronics and telecommunications. In the field of telecommunications, the DFT filter bank provides a cost effective solution for channelisation [1] [2]. The common practice is to use a Finite Impulse Response (FIR) to realise the low-pass prototype filter of the DFT modulated filter bank due to many benefits that FIR designs offer. Rather than using FIR filters, it is also possible to employ Infinite Impulse Response (IIR) filters for low-pass filtering. IIR filters are known to meet identical specifications with lower orders, and therefore reduce the overall complexity of the filtering operation. They also require lower coefficient precision with respect to FIR filters. The disadvantages of IIR filtering are the instability and limit cycles, which can be avoided by following careful design steps. On the other hand, IIR filters have non-linear phase response, which is contrary to the distortion free filtering requirements of communication systems. In the literature there are a wide range of approaches to linearise the phase response [3], [4] which are well established and being widely used. However, it should be noted that none of these phase linearisation techniques provide a perfect linear phase response, and therefore the IIR filters designed based on these techniques are referred as "Almost linear phase" [5].

The efficient implementation of DFT modulated filter banks necessitates the polyphase decomposition of the low-pass

prototype filter [1]. Even though all of its handicaps listed in the previous paragraph can be avoided, the use of IIR filtering for DFT modulated filter banks are not preferred due to its polyphase decomposition causing problems in the stopband region of the filter response. The polyphase decomposition of IIR filters creates spikes in the stopband at odd multiples of the filter's cutoff frequency [6]. These spikes reduce the stopband attenuation as well as causing aliasing effects. To overcome this problem for polyphase IIR filter based the DFT modulated filter banks, the same denominator is assigned to the transfer functions of the all of the sub-filters within the polyphase structure [7]-[10]. Alternatively an extra set of filters can be designed to filter out the unwanted stopband response [11]. Obviously all of these approaches contribute towards increasing the hardware complexity of the filter bank.

In this paper we will make use of an almost linear phase IIR filter and its direct  $M$ -path polyphase decomposition. We will show that the spikes at the stopband would not affect the overall channelisation performance if they are aligned with the guard bands inserted in between user channels. In fact, this is a well known approach for interpolation filters [12] but to the best of the authors' knowledge it hasn't been presented or analysed for channelisation purposes.

## II. DFT MODULATED FILTER BANKS FOR CHANNELISATION

A channelisation scenario is depicted in Figure 1. The spectrum of  $K$  users are first applied to an Analysis Filter Bank (AFB) which decomposes the frequency multiplexed

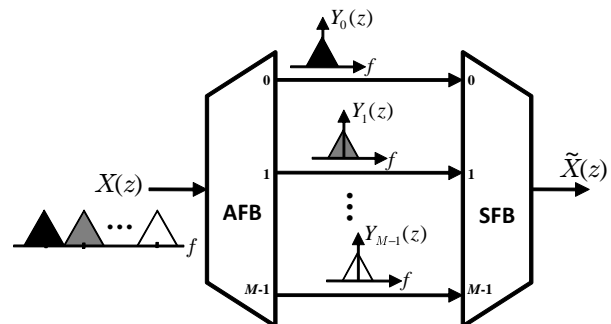


Fig. 1. The analysis and synthesis filter banks and their use to decompose and reconstruct the frequency spectrum of a frequency modulated signal  $X(z)$ .

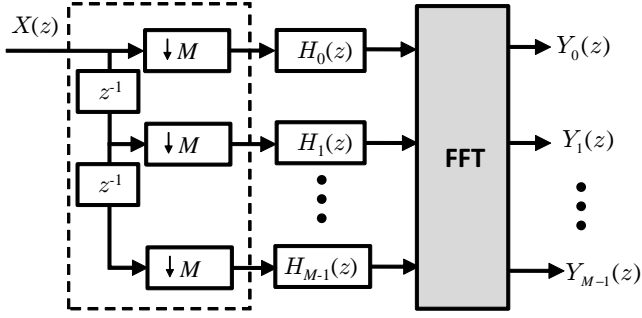


Fig. 2. Analysis Polyphase Filter bank being composed of the polyphase components of  $H(z)$ , a DFT block (usually Fast Fourier Transform (FFT) is used being a cost effective approach) and a series of unit delays and downsamplers. The structure highlighted in dashed lines can be implemented using a commutator.

signal  $X(z)$  into  $M$  baseband signals, i.e.  $Y_m(z)$  for  $m = 0, \dots, M - 1$ , each corresponding to a signal transmitted over one narrowband channel, where  $M=2 \times K$ . The purpose of the Synthesis Filter Bank (SFB) on the other hand is to generate the input spectrum back again with the aim for a perfect or near perfect reconstruction. For a perfect reconstruction  $\tilde{X}(z)$  is expected to be the delayed version of the input  $X(z)$ .

The general structure for an AFB is shown in Figure 2, where DFT modulation is utilised for channelisation. As can be seen in Figure 2, the DFT filter bank is formed of  $M$  sub-filters, i.e.  $H_m(z)$  for  $m = 0, \dots, M - 1$ , which are the  $M$ -path polyphase components of the low-pass prototype filter  $H(z)$ . If  $H(z)$  is an IIR filter, then

$$H(z) = \frac{1}{M} \sum_{m=0}^{M-1} z^{-m} H_m(z^M) = \frac{1}{M} \sum_{m=0}^{M-1} z^{-m} \prod_{n=1}^{N_a} A_{m,n}(z^M) \quad (1)$$

and

$$A_{m,n}(z) = \frac{\sum_{i=0}^{N_b} \beta_{m,n}[i] z^{-i}}{\sum_{j=0}^{N_b} \beta_{m,n}[N_b - j] z^{-j}} \quad (2)$$

where  $N_a$  is the number of all-pass filters per branch and  $\beta_{m,n}[b]$  for  $b = 0, \dots, N_b$  are the all-pass coefficients of the  $n$ th all-pass filter on the  $m$ th branch.  $N_b$  represents the number of all-pass coefficients per branch and  $H_m(z)$  is the down-sampled polyphase component.  $\beta_{m,n}[0] = 1$  for  $\forall m, n$  and  $N_b$  distinct coefficients are required to implement the  $n$ th all-pass filter of  $H_m(z)$ . Hence, the total number of coefficients used to implement a polyphase components of (1) is  $N_c = N_a \times N_b$ .

#### A. Guardband Allocations for Individual Channels

The frequency response of IIR polyphase filter  $H(z)$ , formulated in (1), is depicted in Figure 3.(a) where the number of users is  $K=4$ . As can be seen in this figure, the use of the polyphase IIR filter has introduced undesired aliasing at odd multiples of  $f_s/2M$  (i.e. at  $3f_s/16$ ,  $5f_s/16$  and  $7f_s/16$  for the chosen 4-user scenario), which is inevitable when the IIR prototype filter is decomposed into its  $M$ -path polyphase

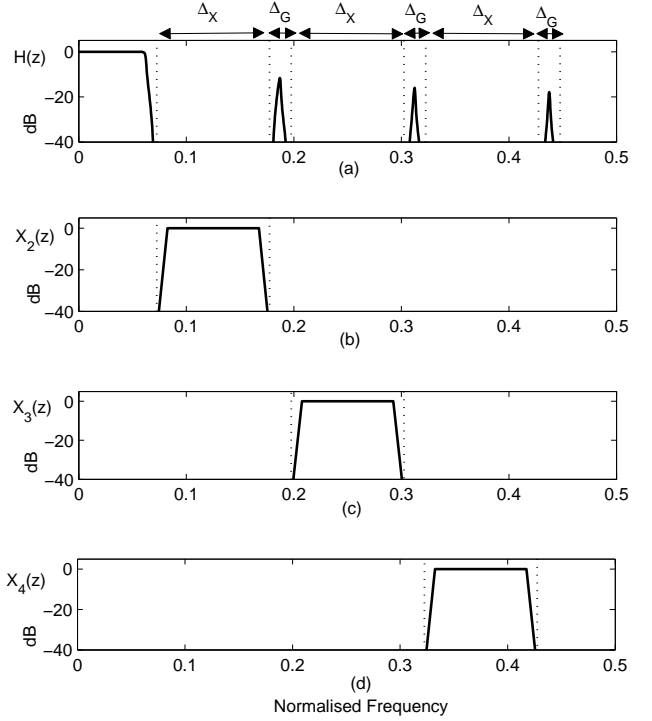


Fig. 3. (a) The frequency response of IIR polyphase filter  $H(z)$  for  $K=4$ . Spectrums for the (b) second (c) third and (d) fourth users

components implementing Figure 2. The use of guardbands avoids interchannel interference between neighboring users located side by side in the frequency spectrum. Although the use of guardbands reduces the spectral efficiency (as no signal is transmitted in this region and the bandwidth assigned to guardbands will stay unused), they are widely adopted in most of the communication systems. In this article we aim to demonstrate a novel use of guardband allocations to mitigate the spikes present in the response of IIR polyphase filter  $H(z)$ .

The bandwidth for  $X_k(z)$ , which corresponds to the signal for user  $k$ , will be

$$\Delta_{X_k} = f_s/2K - \Delta_{G_k}, \quad \text{for } k = 1, \dots, K \quad (3)$$

where  $\Delta_{G_k}$  is the guardband between the  $k$ th and the  $k+1$ st channel. In this article we will assume that all user bandwidths and guardbands are equal and a single parameter will be used to represent one guardband i.e.  $\Delta_X = \Delta_{X_k}$  and  $\Delta_G = \Delta_{G_k}$  as can be observed in Figure 3.(b) to (d), which depicts the communication signals of the 2nd, 3rd and 4th users, which have equal bandwidths. Please note that the user signals do not overlap with the spikes in Figure 3.(a) due to the use of guardbands.

It is a well known phenomenon that the number of coefficients used to implement a digital filter in turn affects the performance of a digital filter. Same for  $M$ -path polyphase IIR filters, a higher order filter at each polyphase branch (i.e. when  $N_c$  is increased) it would be possible to reduce both the magnitude and width of the spikes present in the stopband region. Figure 4 shows the response of 3 almost linear-phase polyphase IIR filters with different transition bands and spike

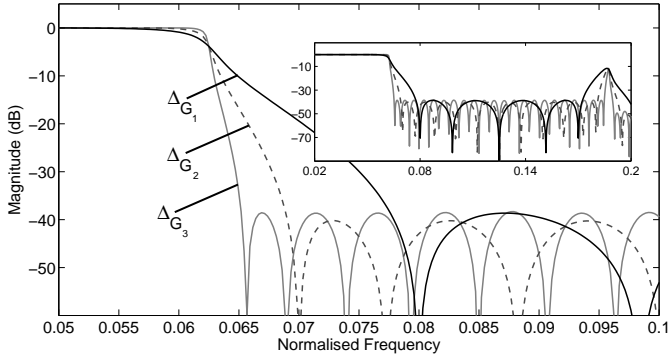


Fig. 4. Comparison of three 8-path polyphase IIR filters. Each filter has been designed to achieve a stopband attenuation of 40 dB. This guarantees a 40 dB attenuation for user channels occupying a bandwidth of  $\Delta_X$ .

widths all designed for the same 4-user scenario observed in Figure 3. All three filters depicted in Figure 4 achieve  $-40$  dB stopband attenuation. For repeatability of our results, we have made use of the MATLAB tool *lineardesign2.m* available in [12]. This tool calculates the all-pass coefficients almost linear-phase polyphase IIR filters based on a set of parameters given by the users. The three filters shown in Figure 4 have different guardband requirements i.e. the filter with wider transition band will have wider stopband spikes and vice versa for the filters with narrow transition band. Therefore the guardband widths will be  $\Delta_{G_1} > \Delta_{G_2} > \Delta_{G_3}$ . Note  $N_c$  was chosen to be 2, 5 and 11 for  $\Delta_3$ ,  $\Delta_2$  and  $\Delta_1$  respectively.

In Figure 5 it is possible to observe the relationship between  $N_c$ , the stopband attenuation level and the spectral efficiency  $S$  all in one plot. For this study stopband attenuation means the lowest attenuation level that is observed within  $\Delta_X$ . Figure 5 also shows that a spectral efficiency up to 95% is achievable using polyphase IIR filter banks, which is high enough to meet the guardband requirements of almost all communication systems.

### III. SIMULATION RESULTS

In the previous section we have shown that it is possible to use polyphase IIR filters for channelisation despite the disadvantageous stopband response. In this section we will compare the performance of IIR and FIR polyphase filters for DFT modulated filter banks. The aim is to show that if guardband allocations are made in line with the stopband spikes of the IIR polyphase filters, it is possible to reduce the complexity of channelisers by replacing the FIR filters with their IIR counterparts.

**Computational Complexity:** To compare their computational complexity, six FIR filters were designed with different numbers of coefficients and almost linear-phase polyphase IIR filters are computed to match the specifications of these six FIR filters in terms of the bandwidth of the transition region and stopband attenuation. The results can be seen in Figure 6. Complex operations are represented in real additions and real

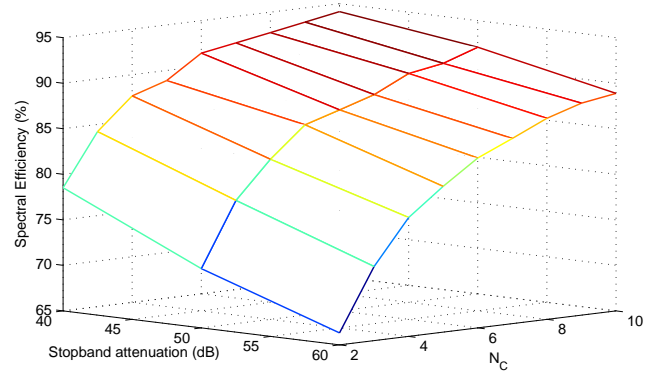


Fig. 5. The relationship between  $N_c$ , the stopband attenuation level and  $S$  all in one plot. This result also shows that a spectral efficiency up to 95% is achievable using polyphase IIR filter banks.

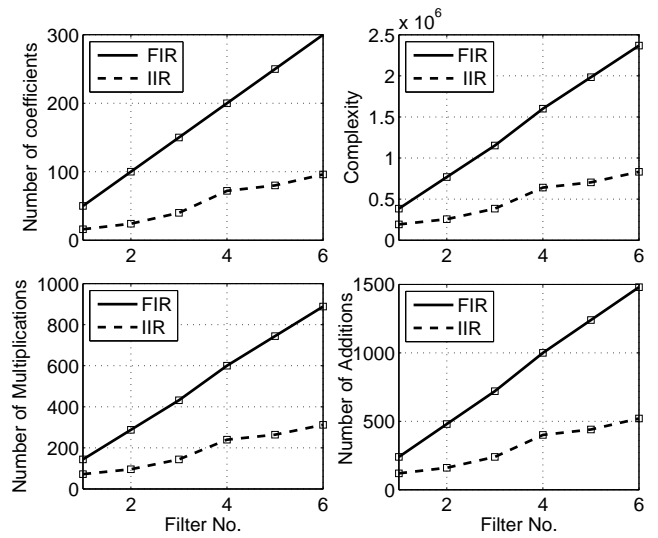


Fig. 6. Complexity comparisons of polyphase FIR and IIR filters used in the Analysis Filter Banks

multiplications in Figure 6. As the number of filter coefficients increase, the computational savings (in terms of number of additions and multiplications) will be higher if IIR filters are preferred due to the linear increase in the number of processing components for both filter types.

**BER Performance:** In this subsection we compare the Bit Error Rate (BER) performance of DFT modulated channelisers using IIR and FIR polyphase filters. Quadrature Pulse Shift Keying (QPSK) symbols are created in a communication scenario for  $K=4$ . Square root raised cosine filtering is implemented where the roll off factor is 0.5 and  $f_s/f_d$  represents the ratio of the sampling frequency to the digital input signal frequency. To analyse the change in BER with respect to  $f_s/f_d$  and Signal to Noise Ratio (SNR) Figure 7 was generated. It can be observed in this figure that the BER performance of the FIR and IIR approaches are almost identical.

A communication scenario is modelled for 4 users in Figure

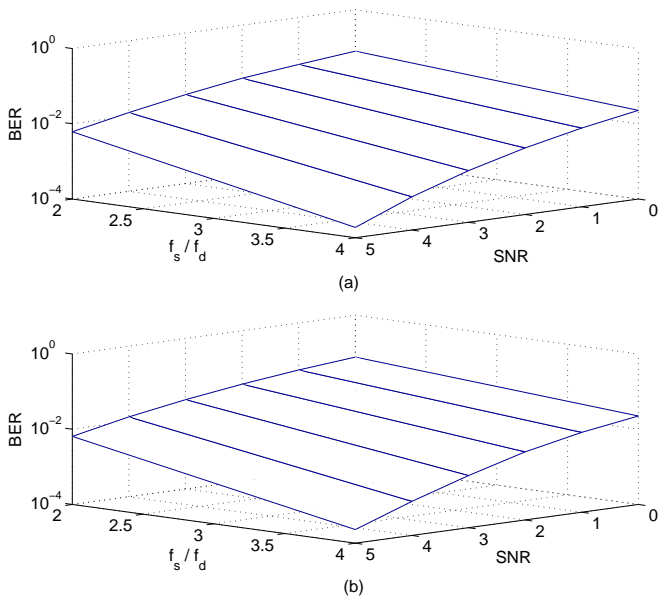


Fig. 7. The change of the BER with respect to changes in SNR and  $f_s/f_d$  ratio for polyphase (a) FIR and (b) IIR filters. For these simulations square root raised cosine filtering is utilized with roll-off factor of 0.5.

8. Figure 8.(a) is received signal to be channelised. Figure 8.(b) is the output of the AFB and SFB pair when polyphase FIR filters are used and Figure 8.(c) is when IIR is in use. Although it is apparent that both filters, FIR and IIR, have successfully recovered the signals back, the spectrum of the signal obtained using the polyphase IIR filter is a bit more noisy (still being below -50 dB in the stopband region) due to the aliasing introduced by the spikes in the stopband. However, it has been observed in Figure 7 that this will not affect the BER performance.

#### IV. CONCLUSION

In this article we presented an alternative way to make use of almost linear-phase  $M$ -path polyphase IIR filters for DFT modulated filter banks. The strategy was set to align the spikes in stopband region with the guardbands placed between neighbouring communication channels. It has shown that the complexity of the filtering process can be reduced using IIR filters while keeping the BER performance almost the same. Although this provides a practical solution to reduce the implementation costs and power dissipation of digital filters, in environments with high levels of noise and interference the performance of this approach could be lower and needs to be further investigated. Linked to this paper the MATLAB code for the experiments performed are available on the ADVRG web-site <http://advrg.wmin.ac.uk/polyphaseIIRFilterBanks.html>

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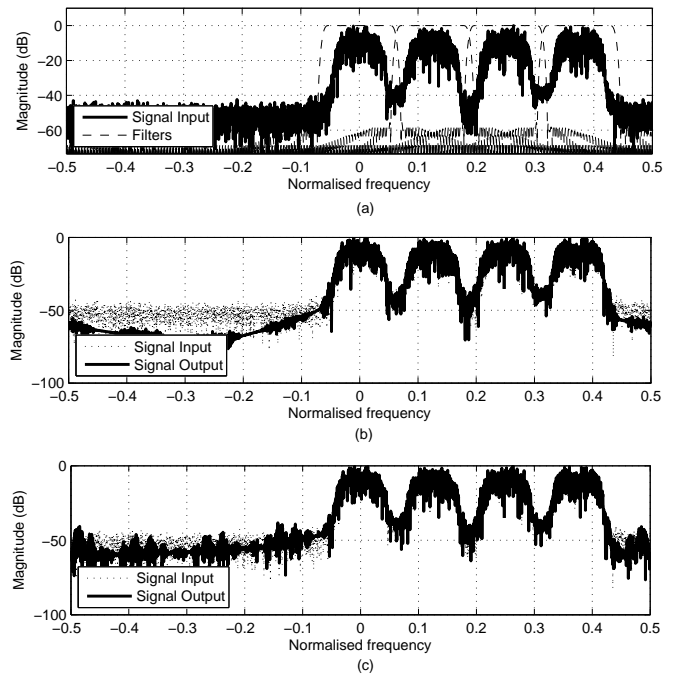


Fig. 8. (a) Spectrum of 4 users and the response of the filters corresponding to each user (b) The output of the AFB and SFB pair when polyphase FIR filters are used (c) The output of the AFB and SFB pair when polyphase IIR filters are used

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