Pricing and Hedging American Fixed-Income Derivatives with Implied Volatility Structures in the Two-Factor Heath–Jarrow–Morton Model

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Most previous empirical studies using the Heath–Jarrow–Morton model (hereafter referred to as the HJM model) have focused on the one-factor model. In contrast, this study implements the Das (1999) two-factor Poisson–Gaussian version of the HJM model that incorporates a jump component as the second-state variable. This study aims at examining the performance of the two-factor model through comparing it with the one-factor model in pricing and hedging the Eurodollar futures option. The degree of impact arising from the jump factor also is examined. In addition, three new volatility specifications are constructed to enhance further the pricing performance of the model. Their performances are compared according to three performance yardsticks—in-sample fitting, out-of-sample pricing, and the hedging test. The result indicates that the two-factor model outperforms the one-factor model in both the

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in-sample and out-sample price fitting, but the one-factor model performs better in the hedging test. In addition, the HJM model, coupled with the proposed volatility specification, leads to good fitting results that will be of considerable use to practitioners and academics in guiding model choice for interest-rate derivatives.
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INTRODUCTION

This study aims at contributing to the literature in three respects. First, most studies done previously have focused on the implementation of one-factor HJM models, but a two-factor model generally is believed to describe better the term structure. Bliss and Ritchken (1996) showed that a two-factor model is more appropriate to explain movements in interest rates. This study extends empirical tests to the two-factor HJM model, which can capture further the subtleties of the forward rate process. Das (1999) developed a Poisson–Gaussian version of the HJM model by adding a jump component to the pure-Gaussian model. This enabled the two-factor model to capture better the observed skewness and kurtosis of the interest rates (Das, 1999). Das briefly illustrated the model using U.K. bond-market data. The present study conducts a detailed empirical examination of this two-factor HJM model by applying a set of distinct volatility structures and implementing it on actively traded Eurodollar futures and futures options. Moreover, a detailed comparison is made between the one- and two-factor models. From this comparison, the degree of pricing impact of the jump component is highlighted. The ability of the models in forecasting the future option price by testing its out-of-sample fit of daily option prices also is examined. For this predictive test, the previous day's option and asset prices are used to infer the required parameters and volatility values, which then are used to forecast the current day's option prices.

Second, as the option valuation under the HJM framework is determined mainly by the volatility function of the forward rates (Heath, Jarrow, & Morton, 1992), the specifications of volatility structure have a strong bearing on the pricing accuracy of the claims. Hence, it is by all means beneficial to identify more-sophisticated volatility functions that can improve further the pricing performance.

Amin and Morton (1994) empirically tested a large set of volatility functions using a one-factor HJM model. This study tests the fitting performance of volatility functions using both the one- and two-factor HJM models. Moreover, a new set of volatility specifications is introduced for the analysis. One of the volatility functions being tested is that proposed by Das (1999), which allows for a humped structure of the term structure of volatility. The present study also constructs three new volatility specifications that allow for the humped term structure of volatilities and compares these three volatility functions with those designed by Das (1999) and Amin and Morton (1994) on the goodness of fit to the market option values. From the comparison, critical factors that affect the pricing performance are identified.

Third, the performance of the five volatility functions in hedging the options is studied. The delta hedges are estimated for all of the volatility structures being tested. The deltas then are used to hedge the options using their underlying assets. Two types of hedging strategies are applied to each volatility function. Errors for hedges that are rebalanced either every day or every five days are calculated to measure the hedging effectiveness of each volatility function.

The lattice approach is used for implementing the two HJM models because it can price American options in the same manner as Eurodollar futures options, while the Monte Carlo Simulation generally only values European options. The Poisson–Gaussian version of the two-factor model cannot be represented by a binomial tree as can the pure-Gaussian one-factor model. However, when the volatility specification that leads to the recombining process is used, a hexanomial tree can be used for the two-factor model (Das, 1999).

American options are priced by a backward induction procedure that takes at each node the maximum of the intrinsic value and the option value should it not be exercised (Bühler, Uhrig-Homburg, Walter, & Weber, 1999).

MODEL

The model originally was developed and modified by Heath, Jarrow, and Morton (1990, 1991, 1992). However, this study considers the multi-factor HJM models as illustrated by Inui and Kijima (1998). The one-factor HJM model is discussed first. The evolution of the forward rate in the one-factor model is generated by incorporating the drift-adjustment terms (DATs) developed by Grant and Vora (1999) in the forward rates to avoid the occurrence of arbitrage opportunities.

This study then proceeds to the two-factor HJM model under the Das (1999) framework. A jump factor is taken as the second state variable in the two-factor model for developing the lattice of forward rates.

At trading time t, the HJM term-structure model drives the dynamics of the forward rate curves for all maturities $T(T > t)$ by the following stochastic differential equation (Inui & Kijima, 1998):

$$f(t + h, T) = f(t, T) + \delta(t, T,)h + \sum_{i=1}^{N} \xi_i(t, T, f(t, T))\Delta Z_i(t), \quad T \in [0, \tau] \qquad (1)$$

where $f(t, T)$ represents the one-period forward rate at maturity T, as observed at time t; h is the discrete time interval; $\delta(t, T,)$ is the drift coefficients of the forward rate $f(t, T)$, while $\xi_i(t, T, f(t, T))$ denotes the ith random shock; and $Z = [Z_1(t), \ldots, Z_N(t)]$ denotes the independent N-dimensional Brownian motions.

The one- and two-factor models are developed in discrete time to facilitate the actual implementation.

One-Factor HJM Model

For $N = 1$, Eq. (1) is expressed as a one-factor model.

$$f(t + h, T) = f(t, T) + A(t, T,.)h + \sigma(t, T, f(t, T))\Delta Z_1(t) \quad \text{for } t \le T, \quad T \in [0, \tau] \qquad (2)$$

where $A(t, T,.)$ is the drift term and $\sigma(t, T, f(t, T))$ is the volatility coefficient of the forward rate.

Grant and Vora (1999) derived the drift term $A(t, T,.)$ by means of the local-expectation hypothesis. The expected rate of return of the underlying asset over a single period should equal the spot rate in that period under the condition of local expectations (Ritchken, 1996). Hence,

$$\ln\left[\frac{E(P(t + h, T))}{P(t, T)}\right] = f(t, t) \qquad (3)$$

$$\ln\left[\frac{E(\exp(-\sum_{k=t/h+1}^{T/h-1} f(t + h, kh)h))}{\exp(-\sum_{k=t/h}^{T/h-1} f(t, kh)h)}\right] = f(t, t) \qquad (4)$$

where E stands for the expectation operator under the martingale probability measure. Equation (4) can be rewritten as

$$\ln\left[E\left(\exp\left(-\sum_{k=t/h+1}^{T/h-1} f(t+h, kh)h\right)\right)\right]$$

$$= f(t, t)\ln\left[\exp\left(-\sum_{k=t/h}^{T/h-1} f(t, kh)h\right)\right] \quad (5)$$

The forward rate $f(t, T)$ is assumed to be distributed normally with mean μ and variance σ^2, where

$$\mu = f(t, T) + A(t, T,.)$$

$$\sigma^2 = \sigma^2(t, T)$$

as

$$E(\exp(-Y)) = \exp\left(-\mu + \frac{1}{2}\sigma^2\right), \quad \text{where } Y \sim N(\mu, \sigma^2) \quad (6)$$

Hence,

$$\ln\left[E\left(\exp\left(-\sum_{k=t/h+1}^{T/h-1} f(t+h, kh)\right)\right)\right]$$

$$= \ln\left[\exp\left(-\sum_{k=t/h+1}^{T/h-1} f(t, kh)\right) - \sum_{k=t/h+1}^{T/h-1} A(t, kh,.)\right.$$

$$\left. + \frac{1}{2}\sum_{i=t/h+1}^{T/h-1}\sum_{j=t/h+1}^{T/h-1} \sigma(t, ih)\sigma(t, jh)\right] \quad (7)$$

The general form of the drift is denoted as

$$A(t, T,.) = \frac{1}{2}\sum_{i=t/h+1}^{T/h}\sum_{j=t/h+1}^{T/h} \sigma(t, ih)\sigma(t, jh) - \sum_{k=t/h+1}^{T/h-1} A(t, kh,.) \quad (8)$$

The evolution of the forward rate then can be derived by the HJM tree

$$f(t+h, T) = \begin{cases} f(t, T) + A(t, T,.)h + \sigma(t, T)\sqrt{h} & \text{with probability } \frac{1}{2} \\ f(t, T) + A(t, T,.)h - \sigma(t, T)\sqrt{h} & \text{with probability } \frac{1}{2} \end{cases} \quad (9)$$

Two-Factor HJM Model

For $N = 2$, Eq. (1) becomes a two-factor HJM equation. Das (1999) derived the model in a discrete time format as follows:

$$f(t + h, T) = f(t, T) + \alpha(t, T,.)h + \sigma(t, T, f(t, T))X_1(t + h)\sqrt{h} + X_2(t + h)N_{\bar{\lambda}h}, \quad \forall T > t \tag{10}$$

where $\alpha(t, T,.)$ is the drift term of the forward rate, $\sigma(t, T, f(t, T))$ is the Gaussian coefficient, and $X_1(.)$ and $X_2(.)$ are the random shocks for the jump-diffusion process where

$$X_1 \sim N(0, 1)$$

$$X_2 = \begin{cases} \mu + \gamma & \text{with probability } \frac{1}{2} \\ \mu - \gamma & \text{with probability } \frac{1}{2} \end{cases}$$

X_1 denotes the diffusion component while X_2 is the jump component with mean μ and variance γ^2. Its distribution is governed by point-process N, which takes on a value of 1 with a probability of $1 - e^{-\lambda}$, where λ is the probability of a jump at any time interval h, and is expressed as $\lambda = \bar{\lambda}h$

$$N = \begin{cases} 0 & \text{with probability } 1 - \bar{\lambda}h + o(h) \\ 1 & \text{with probability } \bar{\lambda}h + o(h) \\ >1 & \text{with probability } o(h) \end{cases}$$

Under the martingale condition, the risk-neutral drift $\alpha(t, T)$ is expressed as

$$\left[\sum_{j=t/h+1}^{T/h} \alpha(t, jh)h\right]h = \ln\left[E\left\{\exp\left(\left[-\sum_{j=t/h+1}^{T/h} \sigma(t, jh)X_1(t + h)\sqrt{h} + \frac{1}{h}(T - t)X_2(t + h)N(t + h)\right]h\right)\right\}\right] \tag{11}$$

where the right-hand side could be denoted as $\ln(A)$. The expectation term A is expanded as follows:

$$A = \exp\left(\left[-\sum_{j=t/h+1}^{T/h} \sigma(t, jh)\sqrt{h} + \frac{1}{h}(T - t)(-\mu - \gamma)\right]h\right)x\,\frac{\lambda}{4} + \exp\left(\left[-\sum_{j=t/h+1}^{T/h} \sigma(t, jh)\sqrt{h}\right]h\right)x\,\frac{1 - \lambda}{2}$$

$$+ \exp\left(\left[-\sum_{j=t/h+1}^{T/h} \sigma(t, jh)\sqrt{h} + \frac{1}{h}(T-t)(-\mu+\gamma)\right]h\right)x\frac{\lambda}{4}$$

$$+ \exp\left(\left[\sum_{j=t/h+1}^{T/h} \sigma(t, jh)\sqrt{h} + \frac{1}{h}(T-t)(-\mu-\gamma)\right]h\right)x\frac{\lambda}{4}$$

$$+ \exp\left(\left[\sum_{j=t/h+1}^{T/h} \sigma(t, jh)\sqrt{h}\right]h\right)x\frac{1-\lambda}{2}$$

$$+ \exp\left(\left[\sum_{j=t/h+1}^{T/h} \sigma(t, jh)\sqrt{h} + \frac{1}{h}(T-t)(-\mu+\gamma)\right]h\right)x\frac{\lambda}{4}$$

If the volatility specification $\sigma(\cdot)$ does not include the state variable, $f(t, T)$, then the process is path recombining. Thus, the evolution of the forward rate can be generated by a hexanomial tree.

The Eurodollar futures call option is valued by backward induction using the following equation proposed by Jarrow (1996):

$$C(t, s_t) = \max\left[\frac{E_t(C(t+1, s_{t+1})}{r(t, s_t)}, P(t, s_f) - K_t\right] \tag{12}$$

where $C(t, s_t)$ is the call option price at time t; E_t denotes the risk-neutral expectation conditional on the information set at time t; $r(t, s_t)$ and $P(t, s_t)$ represent the spot rate and futures price, respectively, at time t and state s; and K_t is the strike price at time t.

Volatility Functions

There is a rich class of volatility structures available for HJM models, and five distinct forms of volatility specification are examined here. These volatility functions can be nested into a general form as follows:

$$\sigma[t, T, f(t, T)] = \{\sigma_0 + [\sigma_1 + \sigma_2(T - \kappa t)]e^{-\eta(T-t)}\} f(t, T)$$

This general form captures the five volatility functions as special cases and synthesizes with the HJM model into an unified framework. These five volatility functions are as follows.

Humped Model: $\sigma[t, T, f(t, T)] = \sigma_0 + [\sigma_1 + \sigma_2(T - t)]e^{-\eta(T-t)}$

Heath, Jarrow, Morton, and Spindel (1992) indicated the humped shapes of the historical volatility function of Treasury rates. Other studies of fixed-income derivatives also exhibited humped shapes in the volatility structure of interest rates (Moraleda & Vost, 1997). Das (1999) proposed a generalized volatility function (also labeled the humped model) that generates a humped term structure of volatilites expressed as:

$$\sigma(T) = A + B \exp\left(-\frac{T}{D}\right) + C\frac{T}{D}\exp\left(-\frac{T}{D}\right)$$

where A governs the level of the term structure of volatilities, B controls the slope, C captures the curvature, and D is a damping parameter (Das, 1999).

The above function is modified slightly into the general form expressed by Ritchken and Chuang (1999). The function also is modified to contain a *time invariance* property (Amin & Morton, 1994) that depends on $(T - t)$ instead of only T.

$$\sigma[t, T, f(t, T)] = \sigma_0 + [\sigma_1 + \sigma_2(T - t)]e^{-\eta(T-t)}$$

where

$$\sigma_0 = A$$
$$\sigma_1 = B$$
$$\sigma_2 = \frac{C}{D}$$
$$\eta = \frac{1}{D}$$

$\sigma[t, T, f(t, T)]$ and η are non-negative.

Humped-and-Curvature-Adjusted Model:
$\sigma[t, T, f(t, T)] = \sigma_0 + [\sigma_1 + \sigma_2(T - \kappa t)]e^{-\eta(T-t)}$

A new volatility function is constructed as an extension of the Das (1999) humped volatility function by multiplying the time t with an adjustment factor $\kappa(T > \kappa t)$. This gives the curvature parameter σ_2 more flexibility in controlling the curvature of the term structure of volatilities.

As in the Das (1999) humped volatility function, the humped-and-curvature-adjusted model leads to a recombining path. Hence, it will reduce greatly computational time and cost.

Humped-and-Proportional Model:
$\sigma[t, T, f(t, T)] = \{\sigma_0 + [\sigma_1 + \sigma_2(T - t)]e^{-\eta(T-t)}\} f(t, T)$

A new volatility function is designed to be proportional to the forward rate level by multiplying a time t forward rate with the humped volatility structure.

The function is constructed according to the requirements of the HJM class volatility structure and can be expressed as $\varphi(t, T)$ $\min[f(t, T), M]$, where $\varphi(t, T)$ is a non-negative deterministic function as derived by Das (1999), and M is a very large positive constant. Hence, the function is proportional to the value of the forward rate at time t. The proportionality implies that forward rates are non-negative (Jarrow, 1996), but it also makes the dynamics of the forward-rate process non-Markovian. This is to test whether the proportionality will improve the goodness of fit in option pricing.

Linear-Exponential Model: $\sigma[t, T, f(t, T)] = [\sigma_1 + \sigma_2(T - t)]e^{-\eta(T-t)}$

The specification of the damping volatility structure provides the mean-reversion property that can reduce the change of occurrence of negative rates on the tree. Moreover, the function allows tree recombining (Das, 1999) and greatly reduces computational effort.

Linear-Proportional Model: $\sigma[t, T, f(t, T)] = [\sigma_1 + \sigma_2(T - t)]\, f(t, T)$

The function was shown by Amin and Morton (1994) to provide better price-fitting performance. This two-parameter model is used to compare with the other newly created forms to analyze their relative pricing performance.

The five volatility functions are compared to examine their goodness of fit to the market data.

Parameter Estimation

The parameters of the volatility functions and the jump component must be determined to construct the evolution of the forward rates. First, the initial term structure of the forward rate is generated using the current futures prices of different maturities. The term structure of the forward rate then is estimated based on the method proposed by Amin and Morton (1994), using the implied volatility figures of the previous day. Second, M futures options of the same maturity are taken at the same point of time. M should be greater than or equal to the number of the parameters to be estimated. The parameters Φ then are varied to minimize the sum of the squared errors of the following equation.

$$\text{Sum of Squared Error} = \sum_{i=1}^{M} [\Omega_{mod,i}(\Phi) - \Omega_i] \qquad (13)$$

where $\Omega_{mod,i}(\Phi)$ and Ω_i are the model price and market price of the ith option of the same maturity date. This results in an estimate of the implied option value and the corresponding parameter values at that day. With the same procedure executed for each day, a time series of the volatility parameters in the sample period is generated. The estimation procedure is repeated for each term of maturity and each volatility specification.

Empirical Analysis

Data

Three-month Eurodollar futures and futures option prices are used in this study for several reasons. First, they are traded actively on the Chicago Merchant Exchange. This study will be most beneficial to practitioners should it identify any better alternatives in pricing the futures options. Second, as the term of maturity of the Eurodollar futures contract is normally less than one year, it is possible to construct the whole initial-term structure. All of these factors facilitate the implementation of the HJM model (Amin & Morton, 1994). The sample period ranges from December 13, 1999 to March 7, 2000. The daily ask quote futures prices of each maturity and the corresponding futures option prices are obtained from the Bloomberg database. There are 836 observations in the sample. Any incomplete data are discarded.

Empirical Tests

Three sets of empirical tests are conducted for the HJM models. These tests measure the performance of the models in respect of in-sample fitting, out-of-sample pricing, and hedging. The out-of-sample pricing indicates the model mis-specification and how well the models forecast the future option prices. On the other hand, the hedging test examines the ability of the models to capture the dynamic relationship between the option and underlying asset values (Bakshi, Cao, & Chen, 1997).

In-sample performance. The two HJM models undergo tests of their fitting performances to the true market-option values in the in-sample data set. To implement this procedure, the call options available on each day are used to imply the parameter estimates of the model.

The parameters then are used to infer the corresponding model option values of that day. The mean of the sum of the square of the pricing errors for the in-sample then is computed for the five volatility functions daily.

The in-sample fitting performances of the HJM models are assessed based on their mean price deviations against the mean strike prices. In addition, the degree of price impact of the jump-state variable is examined in the two-factor model by comparing mean-square errors across the moneyness in the two HJM models. The five volatility functions then are ranked according to their mean-square errors versus moneyness.

Out-of-sample performance. In addition to the test of pricing performance of the HJM models with the in-sample data, the predictability of the model for future option values is evaluated by extending the test of fitness to the out-of-sample data set. This test also will help to examine whether the enhancement of the pricing performance of a particular volatility function is due to the increase in the number of parameters or the improvement of the structural form of the model (Bakshi et al., 1997).

To implement the predictive test, the parameter estimates and the volatility function of the previous day are derived by using the call options available that day. They then are used to forecast the current day's model option values. The computed model prices are subtracted from the corresponding market option values of the current day to estimate the mean-square error. This procedure is repeated daily for each volatility function.

The results in three aspects are analyzed further. First, the overall predictability of the two HJM models is examined by comparing the pricing errors generated in the in-sample and out-of-sample data sets. Second, the two HJM models are ranked according to their pricing performances of option values in the out-of-sample data set. Finally, the mean-square errors generated by the five volatility structures across the entire moneyness are examined to identify the possible factors that influence the ranking of fitting performance of the volatility functions.

Hedging performance. The models and volatility functions are examined according to their hedging performances using the single-instrument hedging method illustrated by Bakshi et al. (1997).

According to this approach, the futures call option is hedged by its underlying futures. The instrument delta is defined as the ratio of the change of the model futures option prices triggered by an instantaneous shift in the term structure to the corresponding change of the futures prices (Amin & Morton, 1994).

With this hedging method, the parameter estimates of previous day are derived and used to model the futures option price of the current day. The short position of the futures call option is hedged by going long with its underlying futures contract. Then the hedged position is liquidated either in the following day or five days later (Bakshi et al., 1997). The mechanism is derived from the Bakshi et al. (1997) proposal, which is summarized as follows:

At time t, let $D_X(t)$ be the delta under the volatility function X. Hence, the value of the replicating portfolio at time t is expressed as

$$V_X(t) = C(t) - D_X(t)F(t) \tag{14}$$

where $V_X(t)$ is the portfolio value at time t, while $C(t)$ and $F(t)$ represent the futures call option value and the underlying futures price at time t, respectively.

The portfolio is rebalanced at every time interval Δt. The portfolio value is invested at a risk-free rate, $r(t)$. Consequently, the change of the portfolio value equivalent to the hedging error is expressed as

$$\Delta V_X(t) = V(t)e^{r(t)\Delta t} - V(t + \Delta t) \quad \text{or}$$

$$\Delta V_X(t) = D_X(t)F(t + \Delta t) - C(t + \Delta t) + \{C(t) - D_X(t)F(t)\}e^{r(t)\Delta t} \tag{15}$$

The portfolio is reconstructed, and the hedging error is calculated at every time interval Δt. Then the average hedging error is

$$E[V_X(t)] = \frac{1}{N}\sum_{w=1}^{N} \Delta V_X(t + W\Delta t) \tag{16}$$

The hedging errors are squared to penalize the extreme values (Skinner, 1998).

The mean square of hedging errors is computed according to the following formula.

$$E[\Delta V_X^2(t)] = \frac{1}{N}\sum_{w=1}^{N} \Delta V_X^2(t + W\Delta t) \tag{17}$$

The absolute hedging error is formulated as

$$\text{ABS}[\Delta V_X(t)] = \frac{1}{N}\sum_{w=1}^{N} |\Delta V_X(t + W\Delta t)| \tag{18}$$

The hedging errors and the above three measures are computed for each volatility structure under the two HJM models. There are three steps in the hedging procedure. The parameters first are determined using the call option values of the previous day. These estimates then are used to calculate the current day's option value and to construct the delta hedge. Finally, the portfolio is rebalanced either daily or after five days. The process is repeated daily for each option in the sample.

There are other possible ways for hedging, such as the Vega hedge. The Vega measures the sensitivity of an option to the changes in the implied volatility. Hence, one could reduce the sensitivity of the option to the volatility by using the Vega hedge. This helps eliminate some model risk. However, it is not easy to implement the Vega hedge in practice. Moreover, the volatility of the underlying asset is not known with certainty. To forecast what it will become in the future is difficult. Therefore, the delta hedge is adopted for empirical study in this study, but it will be insightful and valuable to examine hedging performance using the Vega hedge in future research.

EMPIRICAL RESULTS

In-Sample Performance

One-Factor Model vs Two-Factor Model (Jump)

Figures 1 and 2 plot the market and model option prices derived by the one- and two-factor models across the mean strike prices. The model option prices estimated by the two-factor HJM framework fit the market values better.

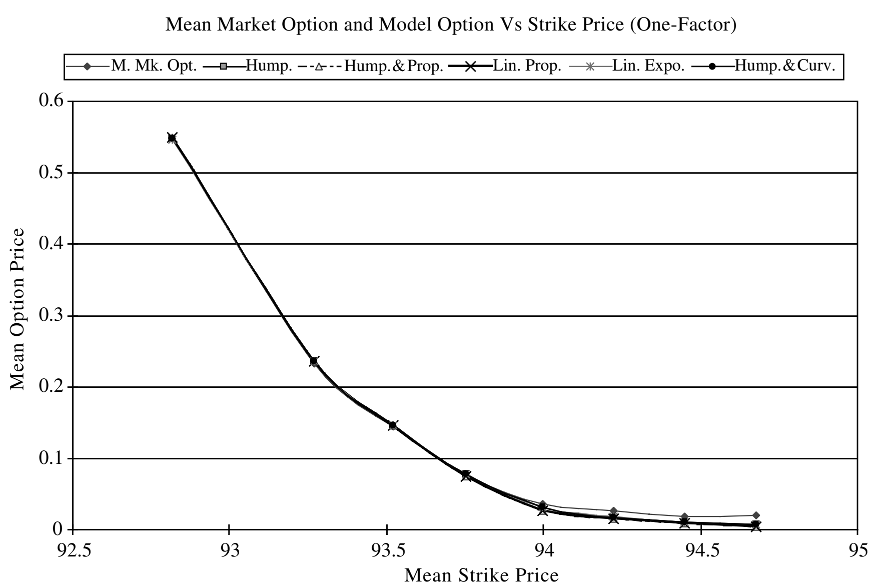

FIGURE 1
Mean market and model option prices derived by the one-factor HJM model across the mean strike prices. This figure plots the mean market and model option prices across different strike prices. The model option prices are derived by the one-factor HJM model coupled with various volatility functions. All the options are sorted by the strike prices and grouped into eight categories. The average strike price for each of the eight groups is plotted on the horizontal axis. The sample period is from December 13, 1999 to March 7, 2000. There are 836 observations in the sample.

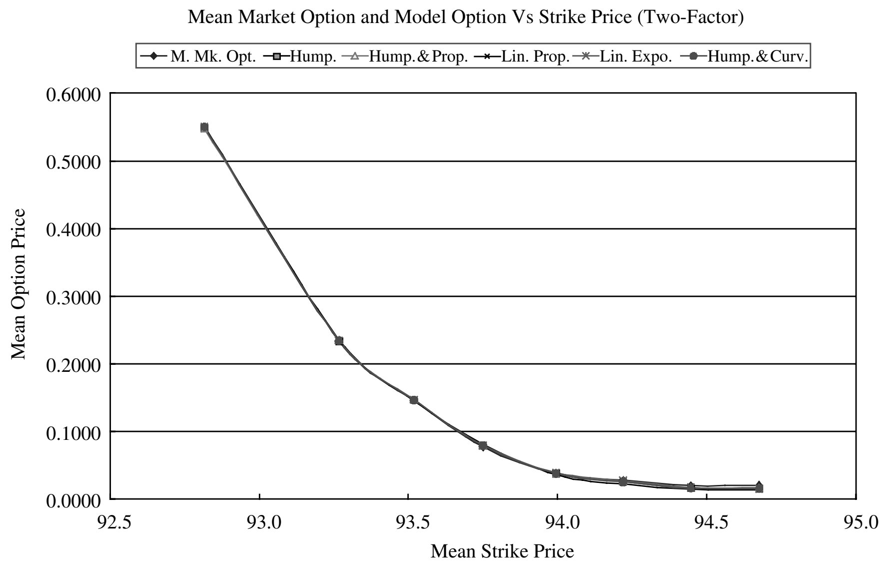

FIGURE 2

Mean market and model option prices derived by the two-factor HJM model across the mean strike prices. This figure plots the mean market and model option prices across different strike prices. The model option prices are derived by the two-factor HJM model coupled with various volatility functions. All the options are sorted by the strike prices and grouped into eight categories. The average strike price for each of the eight groups is plotted on the horizontal axis. The sample period is from December 13, 1999 to March 7, 2000 for almost all call options with maturities in June, September, and December. There are 836 observations in the sample.

The deviation is illustrated in Figure 3. For all the volatility functions, the values derived from the two-factor HJM model have much smaller deviations from the actual data.

Table I indicates that the maximum price deviation generated by the two-factor model is only 43% of that generated by the one-factor model.

Moreover, the mean price deviations for the one- and two-factor models increase with the strike price, while the pricing error is more pronounced at higher strike price for the one-factor model. This indicates that the jump factor in the two-factor model helps to capture the skewness and kurtosis in the interest-rate derivative (Das, 1999).

The pricing performance of the two models is illustrated in Figure 4, which displays the mean-square errors of the humped-and-curvature-adjusted model in pricing the option according to both the one- and two-factor HJM framework. Similar patterns are found in other volatility functions.

For the one-factor model, the mean-square errors of the option prices increase from the in-the-money (ITM) region to the out-of-the-money (OTM) region. The errors jump significantly at the OTM region. However, the two-factor model consistently generates much smaller

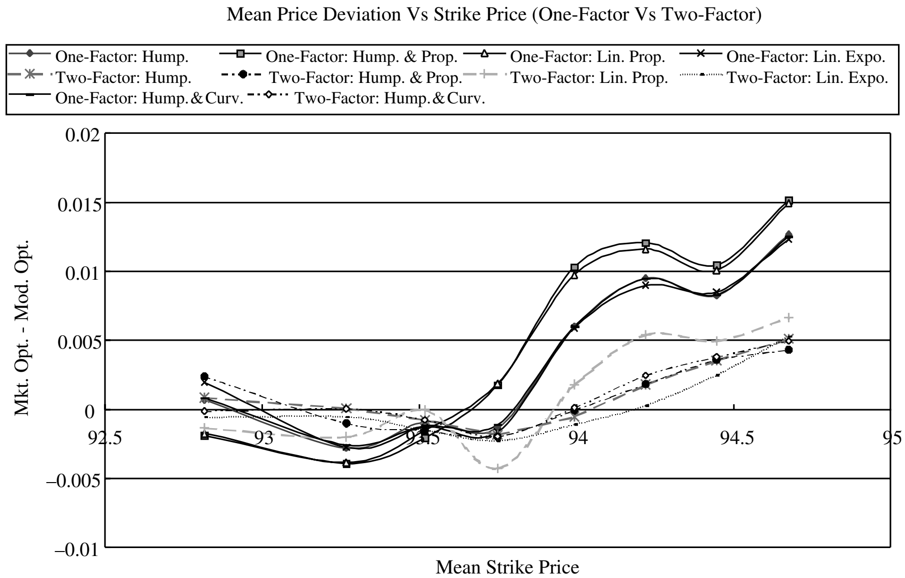

FIGURE 3

Mean option-price deviation across the mean strike prices. This figure plots the mean price deviation as a function of the strike prices. The solid lines represent the price deviations generated by the one-factor HJM model coupled with various volatility functions. The dotted lines represent the price deviations generated by the two-factor HJM model coupled with various volatility functions. All the options are sorted by the strike prices and grouped into eight categories. The average strike price for each of the eight groups is plotted on the horizontal axis. The sample period is from December 13, 1999 to March 7, 2000. There are 836 observations in the sample. The mean price deviation is defined as the market price minus the model price.

mean-square errors. Moreover, the errors are distributed more evenly over the entire range of moneyness.

As the data-sampling interval is on a daily basis, the kurtosis is much higher than that of monthly observations. The kurtosis normally leads to fat-tailed distribution, hence generating the *smile* effect in the option pricing. Consequently, the OTM options are traded at much higher volatilities than the at-the-money (ATM) options. Furthermore, the skewness will result in an asymmetric distribution of the interest rate (Das, 1999).

The two-factor model under discussion was developed by Das (1999) according to the Poisson–Gaussian process in the HJM framework. A jump component is added as the second state variable. This helps to capture better the skewness and kurtosis effects in the pricing of Eurodollar futures options. This test supports the claims and shows that the two-factor model does fit the data much better than the normal one-factor HJM model.

Figure 5 shows the mean pricing deviation of the model option prices derived by the five volatility functions against the moneyness in the one-factor and two-factor models.

TABLE I
Comparison of Option-Pricing Performance of Volatility Functions for the One- and Two-Factor HJM Models

	Mean Strike Price							
	92.8	93.3	93.5	93.8	93.9	94.2	94.4	94.7
Panel A: One-Factor Model								
Mean Mkt. Opt. Value	0.54923	0.23361	0.14543	0.07707	0.03755	0.02774	0.01976	0.02046
Mean Model Opt. Value								
Humped	0.54859	0.23643	0.14637	0.07843	0.03157	0.01826	0.01152	0.00783
Hump.&Curv.	0.54842	0.23620	0.14663	0.07820	0.03155	0.01830	0.01145	0.00795
Hump.&Prop.	0.55114	0.23754	0.14751	0.07532	0.02727	0.01569	0.00932	0.00532
Lin. Expo.	0.54799	0.23635	0.14642	0.07868	0.03166	0.01862	0.01146	0.00807
Lin. Prop.	0.55099	0.23748	0.14699	0.07522	0.02781	0.01615	0.00972	0.00556
Mean Price Deviation								
Humped	0.00064	−0.00283	−0.00094	−0.00136	0.00598	0.00948	0.00824	0.01263
Hump.&Curv.	0.00081	−0.00260	−0.00120	−0.00114	0.00599	0.00944	0.00831	0.01251
Hump.&Prop.	−0.00191	−0.00394	−0.00207	0.00175	0.01028	0.01205	0.01044	0.01514
Lin. Expo.	0.00124	−0.00275	−0.00099	−0.00161	0.00589	0.00912	0.00830	0.01240
Lin. Prop.	−0.00176	−0.00388	−0.00155	0.00184	0.00974	0.01159	0.01004	0.01490
Panel B: Two-Factor Model								
Mean Mkt. Opt. Value	0.54923	0.23361	0.14543	0.07707	0.03755	0.02774	0.01976	0.02046
Mean Model Opt. Value								
Humped	0.54841	0.23356	0.14622	0.07895	0.03807	0.02595	0.01625	0.01537
Hump.&Curv.	0.54938	0.23357	0.14622	0.07903	0.03747	0.02531	0.01597	0.01555
Hump.&Prop.	0.54686	0.23464	0.14697	0.07909	0.03765	0.02593	0.01618	0.01616
Lin. Expo.	0.54980	0.23414	0.14693	0.07935	0.03863	0.02748	0.01727	0.01519
Lin. Prop.	0.55062	0.23562	0.14552	0.08137	0.03579	0.02237	0.01482	0.01385
Mean Price Deviation								
Humped	0.00082	0.00004	−0.00079	−0.00188	−0.00052	0.00179	0.00351	0.00510
Hump.&Curv.	−0.00015	0.00004	−0.00079	−0.00197	0.00008	0.00243	0.00379	0.00491
Hump.&Prop.	0.00237	−0.00104	−0.00154	−0.00202	−0.00010	0.00181	0.00358	0.00430
Lin. Expo.	−0.00057	−0.00053	−0.00150	−0.00228	−0.00109	0.00026	0.00249	0.00527
Lin. Prop.	−0.00138	−0.00202	−0.00009	−0.00430	0.00176	0.00537	0.00494	0.00661

Note. This table summarizes the pricing deviation from the market value for the five volatility functions. Panel A records the data for the one-factor model and panel B is for the two-factor model. The sample period is from December 13, 1999 to March 7, 2000 for almost all call options with maturities in June, September, and December. There are 836 observations in the sample. The mean price deviation is defined as the market price minus the model price.

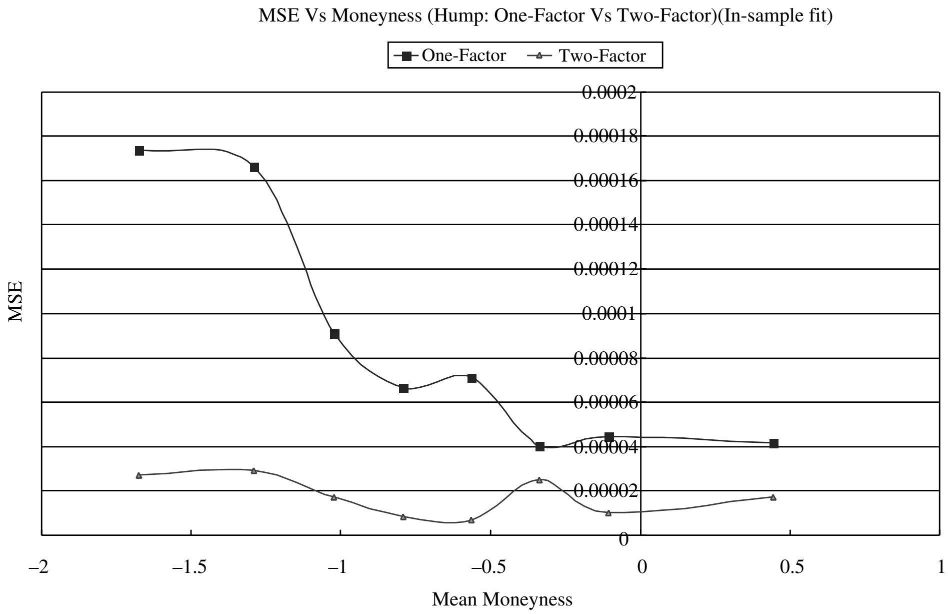

FIGURE 4
Mean-square error across the mean moneyness. This figure plots the mean-square error for pricing the call options as a function of the moneyness. The model option prices are estimated using in-sample dataset. The line with *square* marks indicates the option prices derived by the one-factor HJM model while the line with *triangle* marks shows the option values derived by two-factor HJM model. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

The pricing deviation under the two-factor model generally is much smaller than that of the one-factor model across the moneyness.

Moreover, the pricing deviation of the volatility functions generally exhibits a *valley* shape for the two-factor model. Along the line of moneyness, the volatility functions tend to underprice the OTM futures call options. However, the underpricing becomes smaller for the less OTM options. The volatility functions then overprice the call options again at the region between less OTM and ATM.

Subsequently, the volatility specifications underprice the options at the region between ATM and less ITM, Finally, the degree of underpricing decreases as the options become more ITM.

For the one-factor model, the mean pricing deviation that arises with the five volatility functions shows a similar pattern to the two-factor model across the moneyness, although the deviations for the one-factor model generally are higher. The functions generally underprice the OTM call options and overprice the less OTM calls and the ITM futures call options. However, the degree of overpricing is much smaller than that of underpricing.

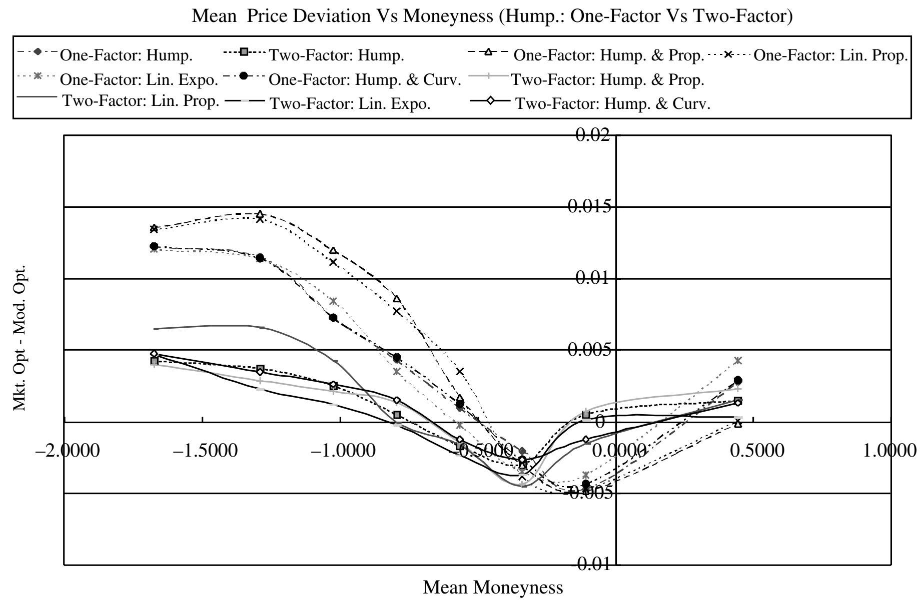

FIGURE 5
Mean price deviation across the mean moneyness. This figure plots the mean price deviation of the call options as a function of the moneyness. The dotted lines represent the price deviations generated by the one-factor HJM model coupled with various volatility functions. The solid lines represent the price deviations generated by the two-factor HJM model coupled with various volatility functions. The mean price deviation is defined as the market price minus the model price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

Volatility Functions

As shown in Table II, the five volatility functions, the humped and proportional model outperforms the other four in fitting the market option values. It has the smallest mean-square errors across the entire range of moneyness. A similar result also is found in the study of Amin and Morton (1994). The authors derived a linear-proportional model with volatility function dependent on the level of interest rates. On pricing the Eurodollar futures options, the model generated lower average absolute error and better fitting performance.

Out-of-Sample Performance

One-Factor Model vs Two-Factor Model (Jump)

Panel A of Table III summarizes the mean-square errors of the two HJM models in the in-sample and out-of-sample data sets. The errors in the out-of-sample data set for both models are only slightly larger than those in the in-sample data set. The out-of-sample error also represents the

TABLE II

Comparison of Mean-Square Error of Futures Option Against Moneyness for Five Volatility Functions (In-Sample)
One-Factor Model vs Two-Factor Model

	Mean-Square Error Mean Moneyness—All							
Volatility Function	*−1.6739*	*−1.2889*	*−1.0240*	*−0.7932*	*−0.5644*	*−0.3367*	*−0.1073*	*0.4438*
Panel A: One-Factor Model								
Humped	1.738E–04	1.663E–04	9.126E–05	6.652E–05	7.090E–05	4.052E–05	4.465E–05	4.143E–05
Humped & Curvature	1.744E–04	1.627E–04	9.177E–05	6.909E–05	6.965E–05	3.578E–05	3.749E–05	4.472E–05
Humped & Proportional	2.144E–04	2.424E–04	1.741E–04	9.583E–05	8.096E–05	3.871E–05	3.864E–05	1.477E–05
Linear Exponential	1.708E–04	1.623E–04	9.324E–05	6.583E–05	7.615E–05	4.177E–05	4.269E–05	4.677E–05
Linear Proportional	2.098E–04	2.299E–04	1.581E–04	8.586E–05	6.266E–05	4.629E–05	3.920E–05	1.576E–05
	−1.6739	*−1.2889*	*−1.0240*	*−0.7932*	*−0.5644*	*−0.3367*	*−0.1073*	*0.4438*
Panel B: Two-Factor Model								
Humped	2.687E–05	2.929E–05	1.695E–05	8.464E–06	6.727E–06	2.493E–05	1.002E–05	1.709E–05
Humped & Curvature	2.760E–05	2.358E–05	1.977E–05	1.190E–05	9.265E–06	2.118E–05	1.328E–05	2.181E–05
Humped & Proportional	2.137E–05	1.843E–05	1.594E–05	8.791E–06	9.400E–06	3.015E–05	1.645E–05	1.956E–05
Linear Exponential	3.531E–05	2.191E–05	2.009E–05	2.293E–05	2.610E–05	3.834E–05	2.256E–05	1.367E–05
Linear Proportional	5.453E–05	6.997E–05	6.132E–05	4.521E–05	3.851E–05	7.822E–05	5.010E–05	3.009E–05

Note. This table reports the mean-square error vs moneyness for the five distinct volatility functions. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness and divided into eight portions with similar numbers of observations. The average moneyness is calculated for each portion. The sample period is from December 13, 1999 through March 7, 2000. The comparison is made for futures option of each term of maturity. Panel A records the results from the one-factor model and panel B displays the results from the two-factor model.

TABLE III
Out-of-Sample and In-Sample Pricing Performances for the Volatility Functions*

	Out-of-Sample		In-Sample	
Panel A: Overall Mean-Square Error				
Volatility Functions	One-Factor	Two-Factor	One-Factor	Two-Factor
Humped	1.0895E–04	5.7338E–05	8.6696E–05	1.7540E–05
Humped & Proportional	4.0511E–04	5.0549E–05	1.1200E–04	1.7521E–05
Humped & Curvature	1.0677E–04	7.5193E–05	8.5508E–05	1.8563E–05
Linear Proportional	3.9902E–04	3.0073E–04	1.0552E–04	5.3382E–05
Linear Exponential	1.1203E–04	1.6285E–04	9.0278E–05	2.5059E–05

Panel B: Akaike Information Criterion (AIC)†

Volatility Functions	Parameter	One Factor	Two Factor	One Factor	Two Factor
Humped	4	— 1,995	— 2,532	— 2,186	— 3,522
Humped & Proportional	4	— 897	— 2,637	— 1,972	— 3,523
Humped & Curvature	5	— 2,010	— 2,303	— 2,196	— 3,473
Linear Proportional	3	— 912	— 1,148	— 2,024	— 2,593
Linear Exponential	2	— 1,976	— 1,663	— 2,156	— 3,228

*Panel A illustrates the mean-square errors of the two HJM models in fitting the out-of-sample and in-sample data. The reported mean-square error is the sum of square of the difference between the model option prices and true market values in the sample. The table also measures forecast errors for the five volatility functions under the out-of-sample category. The out-of-sample forecast option values are derived based on the previous day's parameter estimates. The in-sample model option values are based on the parameter estimates of that day. Panel B illustrates the comparison of alternative model using Akaike information criterion (AIC). The smaller the AIC, the better the model in price fitting is. The sample period is from December 13, 1999 to March 7, 2000 for almost all call options with maturities in June, September, and December. There are 836 observations in the sample.

†AIC = $N \times \ln$(residual sum of squares) + $2n$, where N is the number of usable observation, n is the number of parameter estimated.

forecast error and is close to zero. The HJM model is deemed to be good generally at forecasting the options. When the pricing errors under the out-of-sample category for the one- and two-factor model are compared, the two-factor model is found consistently to outperform the one-factor model in pricing the option. The former generally has lower mean-square errors than the latter.

Table III shows the mean-square errors for the five volatility functions for both the one- and two-factor models. These errors generally range from 4×10^{-4} to 1.7×10^{-5}. Hence, the pricing performances of the volatility functions under test are quite close.

In examining the empirical quality of the one- and two-factor models, the methodology of Bühler et al. (1999) is used to compare the deviation between the market and model price. The two-factor model outperforms the one-factor model.

The results in Table III are validated further by the regression results of the market and model option values. Panels A and B in

TABLE IV
Regression Results of Market Option Prices and Model Option Prices
(Market Price = $\beta_0 + \beta_1$ Model Price + ε)*

Volatility Functions	β_0	β_1	R^2	*F Stat.*†
Panel A: One-Factor Model				
Humped	0.00532 (13.79)	0.98751 (9.61)	0.99796	58.8
Hump.&Curv.	0.00669 (9.22)	0.98503 (9.60)	0.99276	56.5
Hump.&Prop.	−0.00124 (−1.68)	0.96546 (14.35)	0.99283	158.5
Lin. Expo.	0.00472 (11.76)	0.98771 (9.03)	0.99779	49.2
Lin. Prop.	−0.00116 (−1.59)	0.96569 (14.23)	0.99293	152.6
Panel B: Two-Factor Model				
Humped	−0.0003 (−0.97)	0.99440 (4.57)	0.99874	47.8
Hump.&Curv.	−0.0006 (−1.74)	0.99144 (6.26)	0.99842	9.29
Hump.&Prop.	−0.00015 (−0.50)	0.99729 (2.31)	0.99885	12.1
Lin. Expo.	−0.00081 (−1.58)	0.98924 (5.29)	0.99649	67.3
Lin. Prop.	0.00424 (6.10)	0.98459 (5.55)	0.99344	34.0

Volatility Function	*Humped*	*Hump. &Curv.*	*Hump. &Prop.*	*Lin. Expo.*	*Lin. Prop.*
Panel C: F Test: One- vs Two-Factor Model					
F Test	514.6	2,986.9	4,368.9	–	64.9

*This table reports the regression results of the market prices and model prices generated by the five distinct volatility functions. The *t* statistics for $\beta_0 = 0$ and $\beta_1 = 1$ are reported in parentheses. The sample period is from December 13, 1999 to March 7, 2000 for almost all call options with maturities in June, September, and December. There are 836 observations in the sample.
†*F* statistic for joint test of $\beta_0 = 0$ and $\beta_1 = 1$.

Table IV report the systematic bias between the market and model prices. The *F*-statistics of the joint test that $\beta_0 = 0$ and $\beta_1 = 1$ significantly are rejected for all models. A similar observation was made by Amin and Morton (1994).

The β_1 coefficients of the volatility functions in the two-factor HJM model are consistently higher than those in the one-factor model. This again indicates that the two-factor model gives a better estimation of the market data. Moreover, $\beta_1 < 1$ for all of the volatility functions. This result reconciles with that of Amin and Morton (1994).

Panel C of Table IV reveals the F-test result between the one- and the two-factor models. The F test is used to compare the fit of the two models, that is, the one-factor model is assumed to be *nested* within the two-factor model. The F test generally is far greater than one, which means that the two-factor model is better than the one-factor model in pricing the options. However, for the linear exponential model, the R^2 in the one-factor model is higher than that in the two-factor model. Therefore, under the linear exponential volatility function, the one-factor model has higher goodness of fit in the regression analysis than the two-factor model. The model that incorporates the linear-exponential function with the jump-diffusion process is internally less stable and generates more pricing errors in the out-of-sample fit. This result also is reflected in Table III. Future investigation should apply the same test to a new data set.

Volatility Functions

Overall performance. As with the result of the in-sample fit, the humped and proportional model overwhelms the other four volatility functions in forecasting the market option values. The model consistently has the lowest mean-square errors across the entire range of moneyness in the out-of-sample data set as illustrated in Table V.

The Akaike information criterion (AIC) also is used for the model selection. According to the calculation shown in Panel B of Table III, the ranking of pricing performance of the volatility functions further is supported. The humped and proportional model attains the smallest AIC figure and hence gives the best price fitting.

Four sets of comparisons are made of the volatility functions in their relative pricing performances to identify the possible factors that affect pricing accuracy.

Path dependence. The first comparison is illustrated in Figure 6, which shows the pricing performances of the humped and proportional model and the humped model. Both models have the same number of parameters. However, as the humped and proportional model has the forward rate attached in the function, the forward-rate process is nonrecombining.

Figure 6 indicates the mean-square errors derived by the humped and proportional model and humped model across the moneyness. The humped and proportional model generally prices the option better and

TABLE V

Comparison of Mean-Square Error of Futures Option Against Moneyness for Five Volatility Functions (Out-of-Sample)— One-Factor Model vs Two-Factor Model

	Mean-Square Error							
	Mean Moneyness—All							
Volatility Function	*−1.6739*	*−1.2889*	*−1.0240*	*−0.7932*	*−0.5644*	*−0.3367*	*−0.1073*	*0.4438*
Panel A: One-Factor Model								
Humped	1.774E–04	1.717E–04	9.575E–05	7.064E–05	9.515E–05	7.163E–05	1.181E–04	7.274E–05
Humped & Curvature	1.776E–04	1.712E–04	9.867E–05	7.079E–05	9.559E–05	6.883E–05	9.804E–05	7.473E–05
Humped & Proportional	1.698E–04	1.525E–04	1.157E–04	1.567E–04	4.806E–04	8.086E–04	9.920E–04	3.665E–04
Linear Exponential	1.603E–04	1.591E–04	9.747E–05	7.760E–05	1.231E–04	9.189E–05	1.152E–04	7.305E–05
Linear Proportional	1.663E–04	1.547E–04	1.138E–04	1.801E–04	4.784E–04	8.205E–04	9.056E–04	3.737E–04
	−1.6739	*−1.2889*	*−1.0240*	*−0.7932*	*−0.5644*	*−0.3367*	*−0.1073*	*0.4438*
Panel B: Two-Factor Model								
Humped	2.541E–05	2.957E–05	2.727E–05	2.613E–05	6.218E–05	1.410E–04	8.815E–05	5.889E–05
Humped & Curvature	2.332E–05	2.342E–05	3.075E–05	2.574E–05	7.417E–05	1.848E–04	1.472E–04	9.152E–05
Humped & Proportional	2.174E–05	2.273E–05	2.617E–05	2.155E–05	4.230E–05	1.169E–04	1.138E–04	3.960E–05
Linear Exponential	3.595E–05	2.608E–05	3.370E–05	4.069E–05	7.959E–05	1.709E–04	1.676E–04	7.266E–04
Linear Proportional	4.478E–05	7.053E–05	1.097E–04	1.418E–04	2.722E–04	6.101E–04	5.953E–04	5.517E–04

Note. This table reports the mean-square error vs moneyness for the five distinct volatility functions. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for futures call option. All of the options are sorted by moneyness and divided into eight portions with similar number of observations. The average moneyness is calculated for each portion. The sample period is from December 13, 1999 through March 7, 2000. The comparison is made for futures option of each term of maturity. Panel A records the results from the one-factor model and panel B displays the results from the two-factor model.

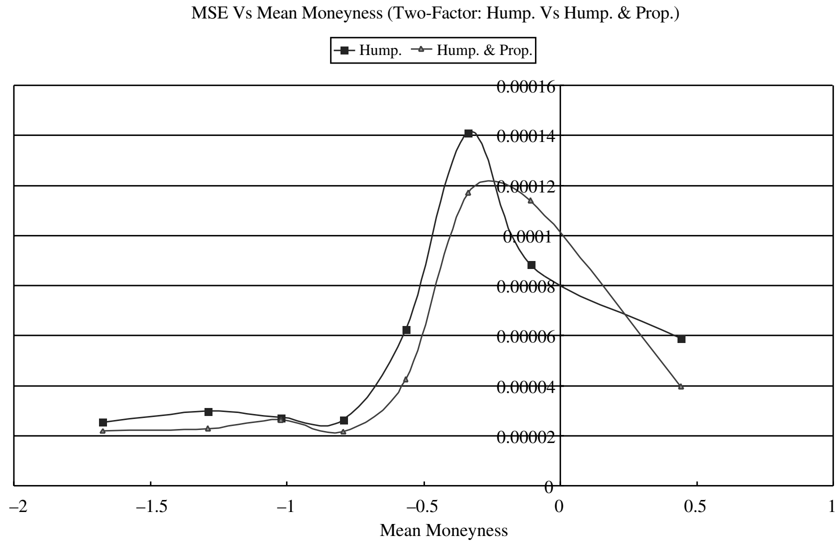

FIGURE 6

Comparison of humped model and humped and proportional model—mean-square error across the mean moneyness. This figure compares the mean-square error derived by two-factor models coupled with the humped model with that coupled with the humped and proportional model. The error is plotted as a function of the mean moneyness. The line with *square* marks is derived using the humped model while the line with *triangle* marks is by the humped and proportional model. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

has lower mean-square errors at both the OTM and ITM regions. Therefore, the forward rate in the humped and proportional model helps to improve the pricing performance.

However, although the humped and proportional model is better in pricing performance, the degree of price improvement is limited. For the path-independent humped model, the volatility structure allows mapping onto a recombining tree. This greatly reduces the computational time. Hence, it is a wiser choice to use the path-independent volatility function to price the simple American derivatives.

Number of parameters. The humped model and the linear exponential model are compared. Both functions could lead to a Markovian term structure and exhibit the humped feature in the volatility structure. However, the former is given an additional level s0 coefficient to ensure the non-negativity of interest rates. This makes the humped model fit the market values better.

Figure 7 shows that the mean square error of the humped model is smaller than that of the linear exponential model. Moreover, the humped model has better goodness of fit to both the OTM and ITM option

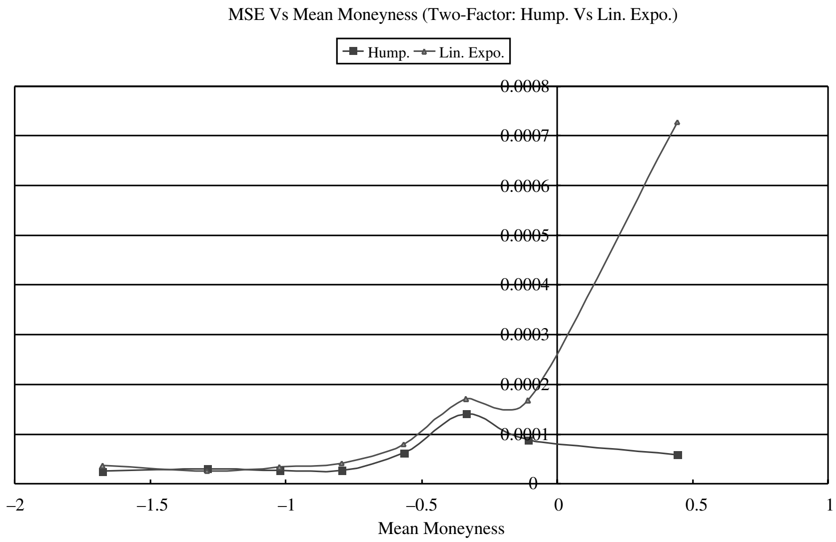

FIGURE 7

Comparison of the humped model and linear-exponential model—mean-square error across the mean moneyness. This figure compares the mean-square error derived by two-factor models coupled with the humped model with that coupled with the linear-exponential model. The error is plotted as a function of the mean moneyness. The line with *square* marks is derived using the humped model while the line with *triangle* marks is derived by the linear-exponential model. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

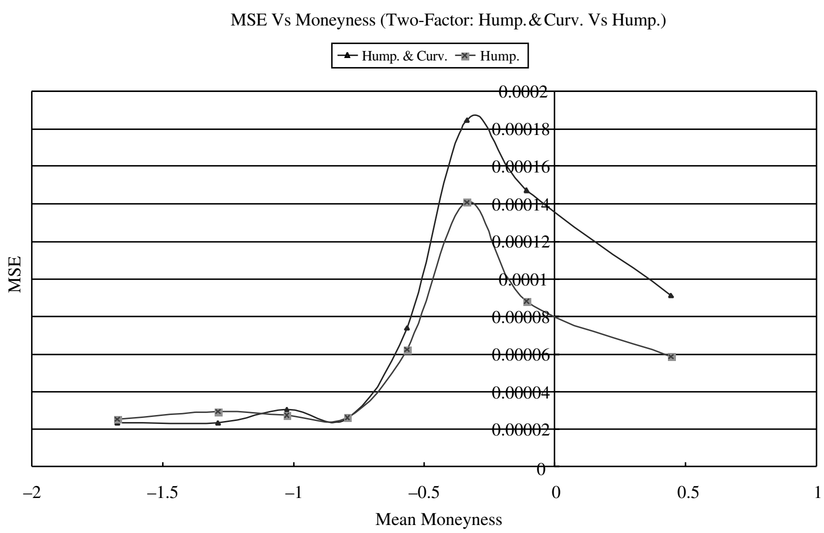

FIGURE 8

Comparison of the humped model and the humped-and-curvature-adjusted model—mean-square error across the mean moneyness. This figure compares the mean-square error derived by two-factor models coupled with the humped model with that coupled with the humped-and-curvature-adjusted model. The error is plotted as a function of the mean moneyness. The line with *square* marks is derived using the humped model while the line with *triangle* marks is by the humped-and-curvature-adjusted model. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

values, while the mean-square error for the linear exponential model increases significantly at the OTM region.

The results indicate that the humped model with an additional level parameter helps to price the option better. This reconciles with the finding of Amin and Morton (1994) that the pricing performance of the volatility specification improves as the number of parameters increase.

The humped and curvature-adjusted model and the humped model are examined according to their relative pricing performance. As shown in Figure 8, although the humped and curvature-adjusted model has one more parameter than the humped model, the humped model has a smaller mean-square error than the humped model at the ATM and ITM regions. One of the possible reasons is that the curvature-adjustment factor that attaches to the time invariance does not help to improve the structural fitting, and it makes the form of the model unstable and overfitting to noise during the in-sample fit (Bakshi et al., 1997). This eventu-

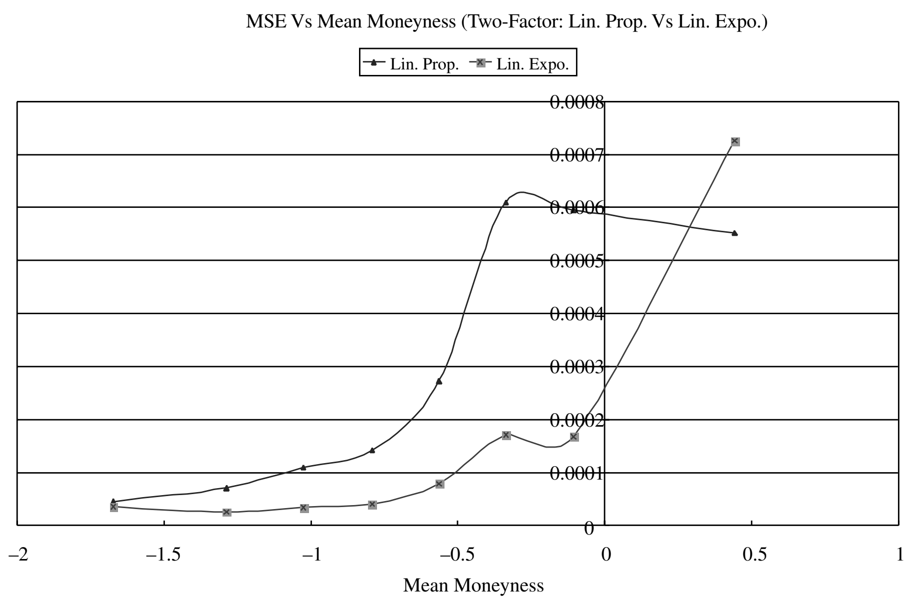

FIGURE 9

Comparison of the linear-exponential model and the linear-proportional model—mean-square error across the mean moneyness. This figure compares the mean-square error derived by two-factor models coupled with linear-exponential model with that coupled with the linear-proportional model. The error is plotted as a function of the mean moneyness. The line with *square* marks is derived using the linear exponential model while the line with *triangle* marks is derived by the linear-proportional model. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

ally leads to lower predictability of next-day option prices when using the previous day's parameter estimates.

The linear-exponential and linear-proportional models are compared in Figure 9. The linear-exponential model outperforms the linear-proportional model although the mean-square error increases at the ITM region. This again reconciles with the findings of Amin and Morton (1994), who indicated that the additional parameter in the linear-exponential model leads to better pricing performance of the traded options.

The relative performances of the five volatility functions are shown in Figure 10. The relative-pricing performances measured by the mean-square errors for the five volatility functions are ranked in the following order:

1. the humped and proportional model (the best);
2. the humped model;
3. the humped- and curvature-adjusted model;
4. the linear-exponential model; and
5. the linear-proportional model.

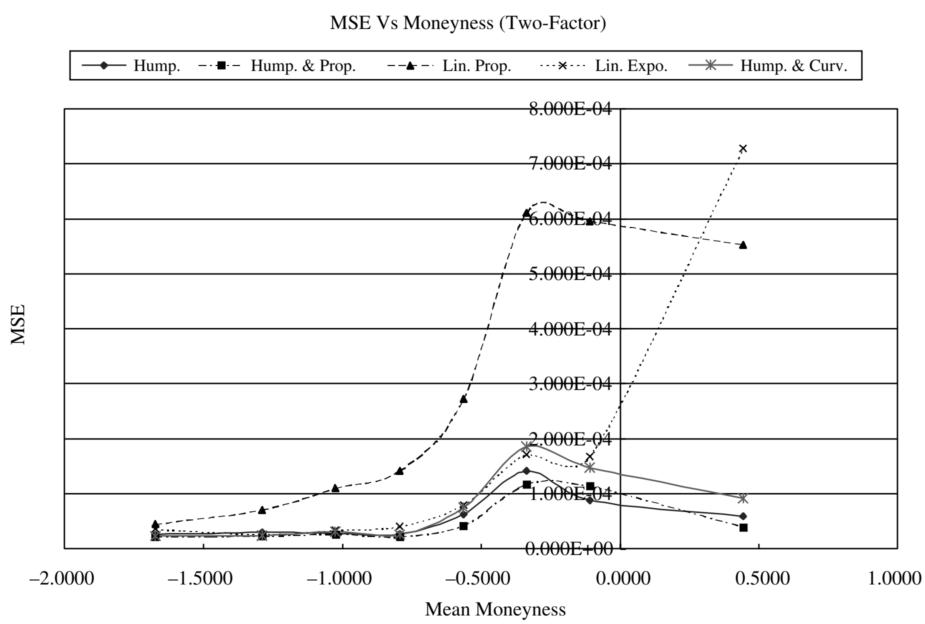

FIGURE 10

Relative pricing performance of the five volatility functions—mean-square error across the mean moneyness. This figure compares the mean-square errors derived by two-factor HJM models coupled with the five volatility functions being tested. The error is plotted as a function of the mean moneyness. The mean-square error is the mean of the square of the difference between market option price and model option price. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness.

Hedging Performance

One-Factor Model vs Two-Factor Model (Jump)

Table VI shows the overall performance of the two HJM models in hedging the futures call options. The two models are compared in terms of average hedging errors, absolute hedging errors, and mean-squared hedging errors. For both models, the errors of the five-day hedging are higher than those of one-day hedging. This is probably due to the nonlinear relationship between the futures option values and the underlying futures prices. Therefore, the hedge ratio changes over time. The deviation of the hedge ratio increases when the time interval of rebalancing is widened.

Based on the average hedging error, the two-factor model generally performs better in hedging the option for both the one- and five-day hedges.

However, according to the mean-squared hedging errors, the hedging performance of one-factor HJM models is better. A similar result is found in the work of Bakshi et al. (1997). A particular reason for this is illustrated in the following two figures.

Figures 11 and 12 illustrate the hedging errors generated under the one- and two-factor models across the moneyness. The hedging errors

TABLE VI

Overall Hedging Performance of the One-and Two-Factor HJM Models

	Average Hedging Error		*Absolute Hedging Error*		*Mean-Squared Hedging Error*	
	One-Factor	*Two-Factor*	*One-Factor*	*Two-Factor*	*One-Factor*	*Two-Factor*
Panel A: One-Day Hedging						
Humped	−2.056E–03	1.098E–03	3.847E–03	7.818E–03	3.772E–05	4.813E–04
Humped & Curvature	−2.069E–03	2.144E–05	3.839E–03	5.175E–03	3.735E–05	8.784E–05
Humped & Proportional	−2.097E–03	−2.637E–03	3.838E–03	5.061E–03	3.643E–05	7.015E–05
Linear Exponential	−1.728E–03	−1.264E–03	3.790E–03	5.047E–03	3.633E–05	7.147E–05
Linear Proportional	−2.071E–03	−1.778E–03	3.839E–03	6.242E–03	3.660E–05	1.760E–04
Panel B: Five-Day Hedging						
Humped	−8.981E–03	6.972E–05	1.133E–02	1.780E–02	4.063E–04	1.859E–03
Humped & Curvature	−9.004E–03	−8.280E–04	1.136E–02	1.205E–02	4.058E–04	4.753E–04
Humped & Proportional	−9.238E–03	−1.245E–02	1.147E–02	1.467E–02	4.247E–04	5.980E–04
Linear Exponential	−7.678E–03	−6.047E–03	1.028E–02	1.317E–02	3.709E–04	4.861E–04
Linear Proportional	−9.141E–03	−4.538E–03	1.141E–02	1.618E–02	4.216E–04	1.195E–03

Note. This table summarizes the overall hedging performance of the two HJM models. The delta hedge of current day is derived using the parameter estimates of the previous day. The replicating portfolio is determined and rebalanced in the following day. The absolute hedging error is defined as the mean of the sum of the absolute value of the hedging error generated. The average hedging error is estimated for each call option in the sample. The mean-squared hedging error is defined as the sum of the square of the hedging error generated. The sample period is from December 13, 1999 through March 7, 2000. Panel A records the results for one-day hedging and panel B displays the results for five-day hedging.

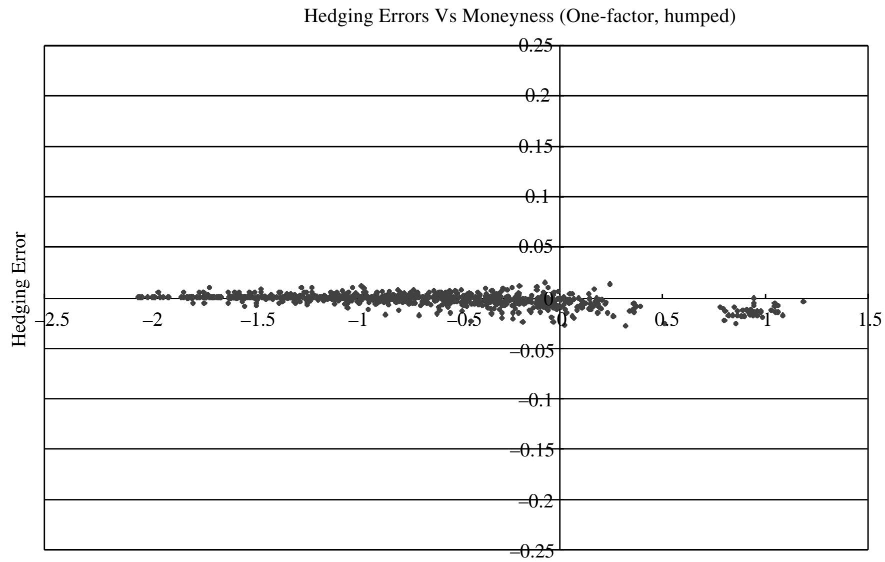

FIGURE 11
One-day hedging error across the moneyness for the one-factor HJM model coupled with the humped model. This figure compares the one-day hedging error derived by one-factor HJM models coupled with the humped volatility function across the moneyness. The hedging error is defined as the change of value of the replicating portfolio over the time interval of one day. The moneyness is defined as the market futures price minus the strike price for the futures call option.

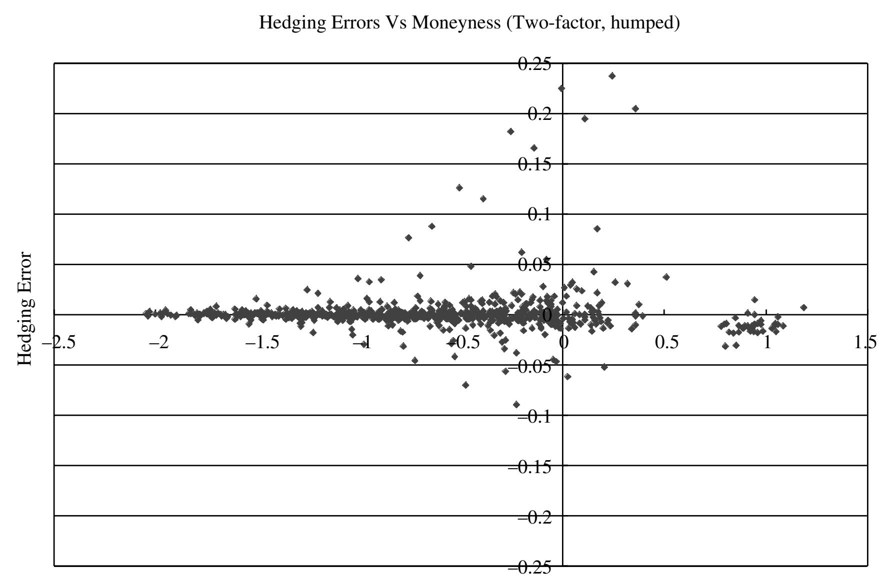

FIGURE 12
One-day hedging error across the moneyness for the two-factor HJM model coupled with the humped model. This figure compares the one-day hedging error derived by two-factor HJM models coupled with the humped volatility function across the moneyness. The hedging error is defined as the change of value of the replicating portfolio over the time interval of one day. The moneyness is defined as the market futures price minus the strike price for the futures call option.

generated fluctuate symmetrically along the line of moneyness. However, the scale of fluctuation under the two-factor model is larger. Although the average hedging errors are smaller for the two-factor model due to the inter-canceling effect, the absolute values of the errors are larger.

A possible explanation for the unsatisfactory hedging performance of the two-factor model stems from the short hedging-rebalancing period. The average jump-frequency parameter is estimated to be around 0.2 times per year. The possibility of the occurrence of a significant jump is small during a one- or five-day interval. Thus, the hedging performance may not be improved by incorporating jumps into the option-pricing framework (Bakshi et al., 1997). Rather, a one-factor HJM model may be adequate in hedging the option.

Volatility Functions

The hedging performances of the five volatility specifications are ranked according to the absolute value of their hedging errors. Table VII presents the absolute hedging errors of the volatility functions against the moneyness. The absolute hedging errors generally increase from the OTM region to the ITM region. One of the possible reasons for this is that the options at the OTM region are relatively less liquid and infrequently traded. Their option prices vary little after one or five days. Consequently, the hedging errors in this region are relatively smaller, but the trading volumes of the options increase when their strike prices approach the market values. This may lead to the possibility of large price movements of the option even at the one-day interval, which will make the hedging errors increase when the moneyness moves toward the ITM region.

The ranking of the hedging performances of the volatility functions is evaluated further for both one- and five-day hedges under each HJM model. The results are illustrated in the following four figures.

Figure 13 shows the one-day absolute hedging errors of the five volatility structures under the two-factor model. The humped-and-curvature-adjusted model performs the best among the five volatility functions in hedging the OTM calls, while the humped-and-proportional model is the best performer in hedging ATM and ITM call options. The order of ranking of the five volatility functions in hedging the options in these regions is:

1. the humped and proportional model (the best);
2. the linear exponential model;

TABLE VII

Comparison of the Absolute Single-Instrument Hedging Errors of Futures Call Options for Five Volatility Functions: One-Factor Model vs Two-Factor Model

	Absolute Hedging Error					
Volatility Functions	*−1.6110*	*−1.1590*	*−0.8436*	*−0.5384*	*−0.2320*	*0.3081*
Panel A: One-Factor Model						
			Moneyness—One-Day Hedging			
Humped	1.224E–03	1.537E–03	2.468E–03	3.548E–03	5.061E–03	9.087E–03
Humped & Curvature	1.222E–03	1.537E–03	2.496E–03	3.593E–03	4.973E–03	9.055E–03
Humped & Proportional	1.212E–03	1.519E–03	2.477E–03	3.489E–03	4.993E–03	9.178E–03
Linear Exponential	1.323E–03	1.595E–03	2.593E–03	3.416E–03	4.867E–03	8.799E–03
Linear Proportional	1.221E–03	1.554E–03	2.516E–03	3.501E–03	4.949E–03	9.132E–03
			Moneyness—Five-Day Hedging			
Humped	2.843E–03	2.755E–03	4.996E–03	8.070E–03	1.386E–02	3.475E–02
Humped & Curvature	2.841E–03	2.765E–03	5.011E–03	8.005E–03	1.396E–02	3.486E–02
Humped & Proportional	2.794E–03	2.642E–03	4.776E–03	7.712E–03	1.373E–02	3.640E–02
Linear Exponential	3.126E–03	2.898E–03	4.405E–03	6.489E–03	1.103E–02	3.307E–02
Linear Proportional	2.760E–03	2.662E–03	4.769E–03	7.602E–03	1.360E–02	3.630E–02
Panel B: Two-Factor Model						
			Moneyness—One-Day Hedging			
Humped	1.747E–03	2.637E–03	5.192E–03	7.951E–03	1.141E–02	1.768E–02
Humped & Curvature	1.447E–03	1.816E–03	3.081E–03	4.517E–03	7.444E–03	1.253E–02
Humped & Proportional	2.320E–03	2.880E–03	4.065E–03	4.889E–03	5.802E–03	1.025E–02
Linear Exponential	1.875E–03	2.337E–03	3.529E–03	4.674E–03	6.152E–03	1.152E–02
Linear Proportional	1.744E–03	1.972E–03	3.633E–03	5.588E–03	8.747E–03	1.549E–02
			Moneyness—Five-Day Hedging			
Humped	3.718E–03	6.051E–03	9.597E–03	1.671E–02	2.453E–02	4.536E–02
Humped & Curvature	2.607E–03	2.894E–03	4.714E–03	8.318E–03	1.635E–02	3.671E–02
Humped & Proportional	5.775E–03	7.505E–03	1.095E–02	1.405E–02	1.713E–02	3.211E–02
Linear Exponential	3.596E–03	4.559E–03	7.364E–03	1.079E–02	1.635E–02	3.568E–02
Linear Proportional	3.955E–03	4.175E–03	7.573E–03	1.335E–02	1.956E–02	4.752E–02

Note. This table illustrates the hedging performance of the five volatility functions under the two HJM models. The delta hedge of the current day is derived using the previous day's parameter estimates. The replicating portfolio is determined and rebalanced the following day or five days later. The absolute hedging error is defined as the absolute value of the hedging error estimated for each call option in the sample. The moneyness is defined as the market futures price minus the strike price for futures call option. All of the options are sorted by moneyness. The average moneyness is calculated for each portion.

3. the humped and curvature adjusted model;
4. the linear proportional model; and
5. the humped model.

The five-day hedging performance of the two-factor model is illustrated in Figure 14, which displays the absolute hedging errors of the five

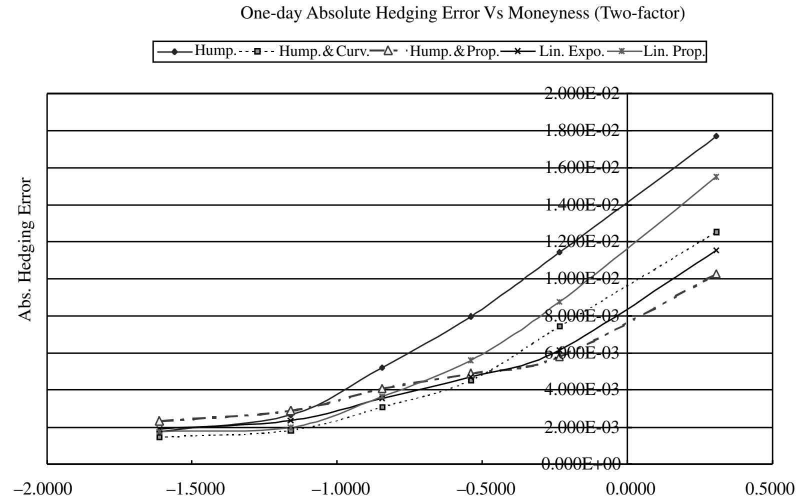

FIGURE 13

One-day absolute hedging error across the mean moneyness for the two-factor HJM model coupled with various volatility functions. This figure compares the one-day absolute hedging error derived by two-factor HJM models coupled with various volatility functions across the mean moneyness. The absolute hedging error is defined as the absolute value of the change of value of the replicating portfolio over the time interval of one day. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness and divided into six portions with similar numbers of observations. The average moneyness is calculated for each portion.

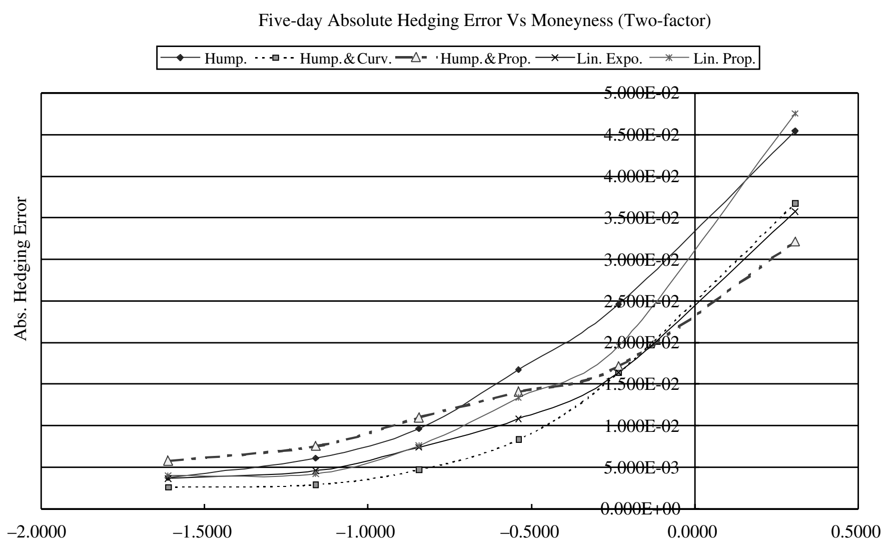

FIGURE 14

Five-day absolute hedging error across the mean moneyness for the two-factor HJM model coupled with various volatility functions. This figure compares the five-day absolute hedging error derived by two-factor HJM models coupled with various volatility functions across the mean moneyness. The absolute hedging error is defined as the absolute value of the change of value of the replicating portfolio over the time interval of five days. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness and divided into six portions with similar numbers of observations. The average moneyness is calculated for each portion.

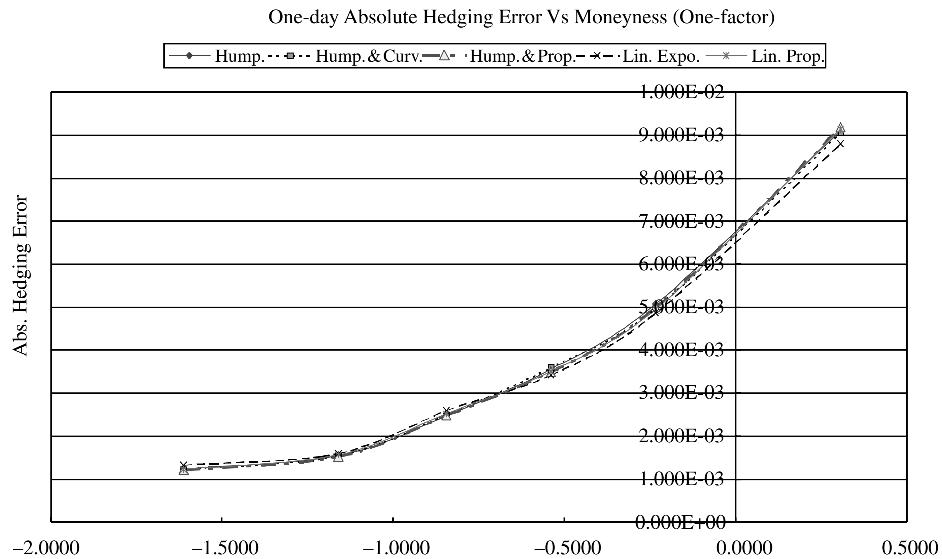

FIGURE 15

One-day absolute hedging error across the mean moneyness for the one-factor HJM model coupled with various volatility functions. This figure compares the one-day absolute hedging error derived by one-factor HJM models coupled with various volatility functions across the mean moneyness. The absolute hedging error is defined as the absolute value of the change of value of the replicating portfolio over the time interval of one day. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness and divided into six portions with similar numbers of observations. The average moneyness is calculated for each portion.

volatility specifications across the moneyness. The five-day hedge exhibits a pattern similar to that of the one-day hedge, although it generally has larger hedging errors compared with the latter. The humped-and-curvature-adjusted model has the lowest absolute hedging errors in the OTM region. Moreover, similar to the ranking in the one-day hedge in the ATM and ITM regions, the humped-and-proportional model performs the best, followed by the linear-exponential model, and then by humped-and- curvature-adjusted model, the linear-proportional model, and the humped model.

Figures 15 and 16 show the absolute hedging errors of the volatility function under the one-factor model in one- and five-day hedges, respectively. For both types of hedge, the hedging performances of the five volatility specifications are indistinguishable. The pattern is similar to that in the two-factor model. In general, the humped-and-proportional model is good at hedging options in the OTM region, while the linear-exponential model performs better in the ATM and ITM regions. The same ranking is shown in both the one- and five-day hedges.

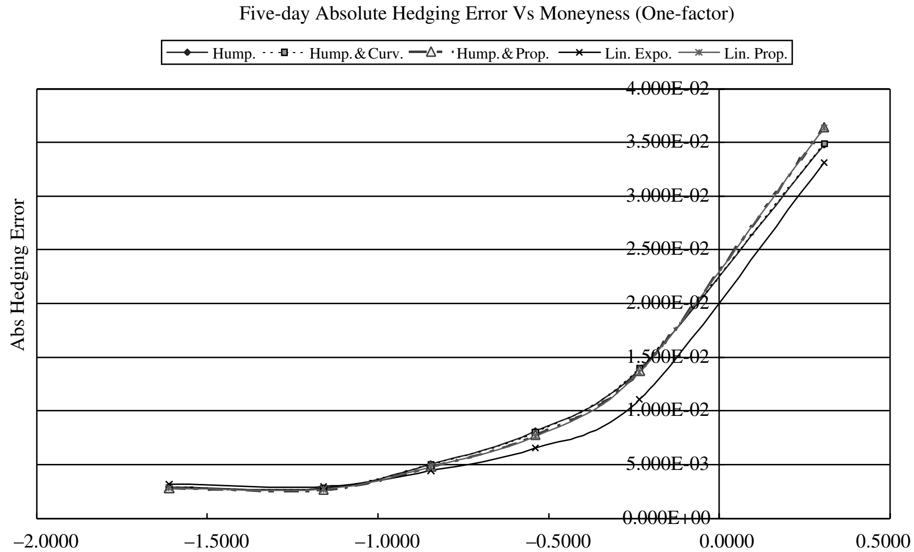

FIGURE 16

Five-day absolute hedging error across the mean moneyness for the one-factor HJM model coupled with various volatility functions. This figure compares the five-day absolute hedging error derived by one-factor HJM models coupled with various volatility functions across the mean moneyness. The absolute hedging error is defined as the absolute value of the change of value of the replicating portfolio over the time interval of five days. The moneyness is defined as the market futures price minus the strike price for the futures call option. All of the options are sorted by moneyness and divided into six portions with similar numbers of observations. The average moneyness is calculated for each portion.

CONCLUSION

This article studies the issue of option pricing by employing the two-factor Heath–Jarrow–Morton (HJM) model according to the Das (1999) framework. A jump factor is incorporated in the model to develop a lattice of forward rates.

The study employs the methodology used by Amin and Morton (1994) in the parameter estimation to facilitate the comparison. One- and two-factor HJM models are compared under the same conditions to examine the degree of pricing impact arising from the jump component in the two-factor model. Moreover, the study introduces three new volatility structures to examine their relative performances on option pricing.

The performances of the HJM models and the volatility functions are examined in relation to in-sample fitting, out-of-sample predictability, and single-instrument hedging. For the in-sample fit, the empirical evidence indicates that the model option prices estimated by the two-factor HJM model better fit the market values. The test indicates that the jump component in the two-factor model helps to capture better the kurtosis effects in the pricing of the Eurodollar futures option.

Regarding the volatility functions, all three new models give better estimations of the option values. Moreover, among the five volatility functions under test, the humped-and-proportional model gives the lowest overall mean-square errors. The order of ranking of fitting performance of the five volatility specifications is:

1. the humped-and-proportional model (the best);
2. the humped model;
3. the humped-and-curvature-adjusted model;
4. the linear-exponential model; and
5. the linear-proportional model.

For the out-of-sample test, the two-factor model once again outperforms the one-factor model in forecasting the future call option prices. Moreover, the relative forecasting performances of the volatility functions are ranked in the same order as in the in-sample test.

Moreover, the humped-and-proportional model and the humped model are compared to examine the degree of price impact from the path-dependence factor. Both models have the same number of parameters. The path-dependent humped-and-proportional model is found to have lower overall mean-square errors in pricing the option. The path-dependence factor does help to improve the pricing performance. However, the degree of price improvement is limited. Moreover, the use of path-independent volatility function generally can shorten the computation time because the function allows mapping onto a recombining tree in valuing the option. Hence, it is preferable to use the path-independent volatility function for valuing the plain vanilla options.

Moreover, the pricing performances of the humped model and the linear-exponential model are compared. The additional level parameter in the volatility structure helps the humped model to outperform the linear-exponential model. A similar result is found in the comparison of the linear-exponential model and the linear-proportional model, in which the former has one more parameter than the latter. Therefore, the number of parameters in the model does affect the performance of the volatility function. This reconciles with the findings of Amin and Morton (1994).

Regarding the hedging performances of the one- and two-factor models in general, the hedging errors for the five-day hedge generally are larger than the one-day hedge. This is probably due to the nonlinear relationship between the option values and their underlying futures prices. The hedge ratio changes over time and is more likely to vary when the time interval of rebalancing is widened. Moreover, based on the average hedging-error measures, the two-factor model performs better. However, the one-factor model generally has smaller mean-squared hedging errors and absolute hedging errors. This is due to the fact that while the hedging errors generated by both models fluctuate symmetrically along the line of moneyness, the scale of fluctuation under the two-factor model is larger. Therefore, although the average hedging errors are smaller for the two-factor model because of the intercanceling effect, the absolute value of hedging errors under that model are larger.

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